Three-Flavour Effects in Neutrino Oscillations

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3f effects in neutrino oscillations

- General properties of 3f oscill. probabilities
- Analytic solutions
- 3f effects in oscillations of solar, atmospheric, reactor and accelerator neutrinos
- $\not{C}\not{P}$ and $\not{T}$ in ν oscillations in vacuum
- $\not{C}\not{P}$ and $\not{T}$ in ν oscillations in matter
- The problem of U_{e3}

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These days: 3f analyses a must!

Two types of 3f effects

I. “Trivial” effects

- Existence of new physical channels – in addition to $\nu_e \leftrightarrow \nu_\mu$ there are $\nu_e \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_\tau$; mutual influence of channels through unitarity (conservation of probability).

- New “parameter channels” for the same physical channel. E.g.: $\nu_e \leftrightarrow \nu_\mu$ oscill. can be governed by $(\Delta m^2_{21},\, \theta_{12})$ and $(\Delta m^2_{31},\, \theta_{13})$

Two types of 3f effects – contd.

II. Nontrivial effects

- Fundamental $\not{C}\not{P}$ and $\not{T}$
- Matter-induced $\not{T}$
- Interference of different "parameter channels" – specific contributions to oscillation probabilities
- Matter effects on $\nu_\mu \leftrightarrow \nu_\tau$ oscillations

Nontrivial 3f effects (except the last one): disappear if at least one mixing angle is 0 or 90°, or at least one $\Delta m^2_{ij} = 0$

Leptonic mixing

$$\nu_a = U_{ai}\,\nu_i$$

Oscillation probability in vacuum:

$$P(\nu_a \to \nu_b; t) = \left| \sum_i U_{bi}\; e^{-iE_i t}\; U_{ai}^* \right|^2$$

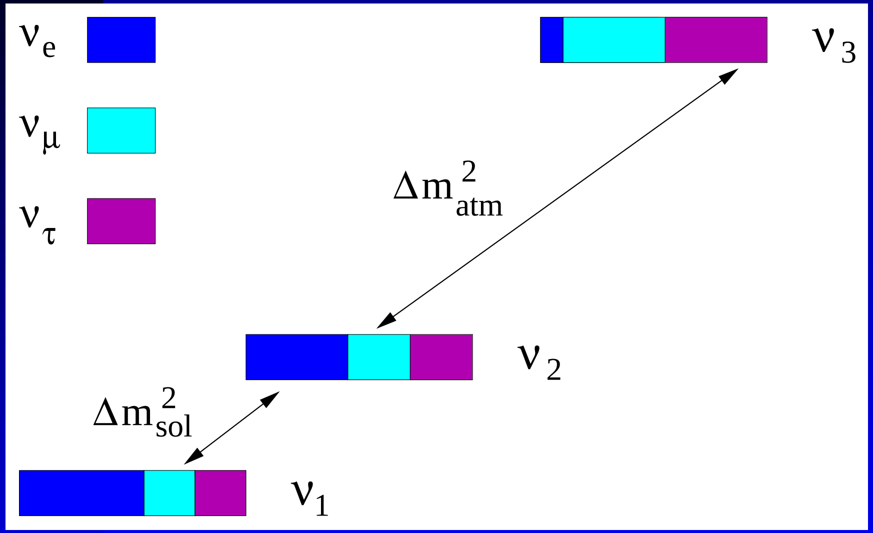
3f mixing matrix ($c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$):

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\rm CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\rm CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

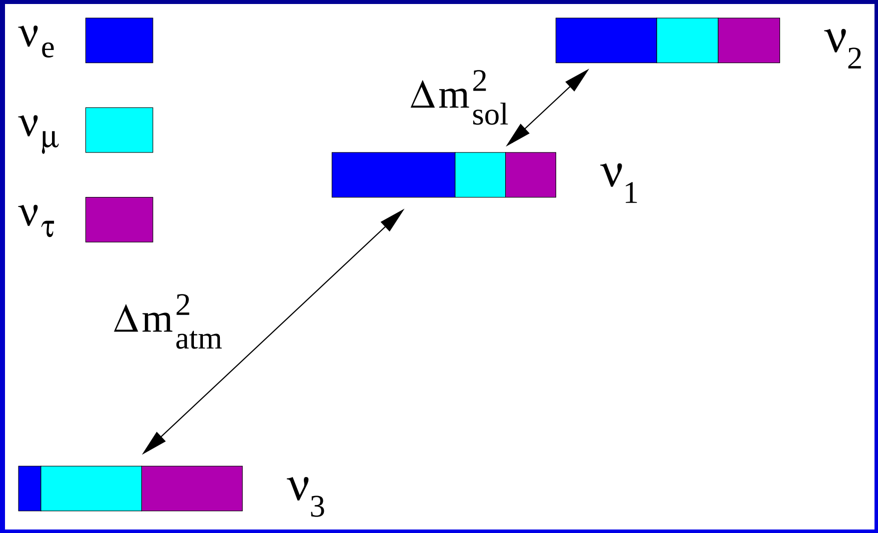
Leptonic mixing – contd.

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & c_{13}c_{23} \end{pmatrix}$$

Normal hierarchy:



Inverted hierarchy:



Neutrino oscillations in matter

Evolution equation:

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \left[U\begin{pmatrix}E_1 & 0 & 0\\ 0 & E_2 & 0\\ 0 & 0 & E_3\end{pmatrix}U^\dagger + \begin{pmatrix}V(t) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p}\,; \qquad t \simeq r$$

$$V(t) = [V(\nu_e)]_{CC} = \sqrt{2}G_F N_e(t)$$

$[V(\nu_e)]_{NC} = [V(\nu_\mu)]_{NC} = [V(\nu_\tau)]_{NC}$ – do not contribute

(Modulo tiny radiative corrections)

General properties of P_{ab}

3 flavours $\Rightarrow$ $3 \times 3 = 9$ probabilities

$$P_{ab} = P(\nu_a \to \nu_b),$$

plus 9 probabilities for antineutrinos $P_{\bar{a}\bar{b}}$.

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5 indep. conditions $\Rightarrow$ 9-5=4 indep. probabilities left.

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Additional symmetry: the matrix of matter-induced potentials $diag(V(t), 0, 0)$ commutes with O_{23} $\Rightarrow$ additional relations between probabilities.

Define

$$\tilde{P}_{ab} = P_{ab}(s_{23}^2 \leftrightarrow c_{23}^2,\ \sin 2\theta_{23} \to -\sin 2\theta_{23})$$

(e.g., $\theta_{23} \to \theta_{23} + \pi/2$).

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$$P_{\bar{a}\bar{b}} = P_{ab}(\delta_{\rm CP} \to -\delta_{\rm CP}, V \to -V)$$

$\Rightarrow$*All 18 ν and $\bar{\nu}$ probab. can be expressed through just two*

General dependence on $\delta_{\rm CP}$

Another use of essentially the same symmetry

$$O_{23} \to O'_{23} = O_{23} \times diag(1, 1, e^{i\delta_{\rm CP}})$$

The matrix of matter-induced potentials $diag(V(t), 0, 0)$ commutes with O'_{23} $\Rightarrow$

General dependence of probabilities on $\delta_{\rm CP}$:

$$P_{e\mu} = A_{e\mu}\cos\delta_{\rm CP} + B_{e\mu}\sin\delta_{\rm CP} + C_{e\mu}$$

$$P_{\mu\tau} = A_{\mu\tau}\cos\delta_{\rm CP} + B_{\mu\tau}\sin\delta_{\rm CP} + C_{\mu\tau}$$
$$+ D_{\mu\tau}\cos 2\delta_{\rm CP} + E_{\mu\tau}\sin 2\delta_{\rm CP}$$

(Yokomakura, Kimura & Takamura, 2002)

Analytic solutions

For constant-density matter closed form solutions exist

(Barger, Whisnant, Pakvasa & Phillips, 1980; Zaglauer & Schwartzer, 1988; Ohlsson & Snellman, 1999; Xing, 2000; Kimura, Takamura & Yokomakura, 2002)

But: Expressions rather complicated and not easily tractable

For a general $N_e \neq const$ no closed form solutions exist

Approximate analytic solutions desirable!

Approximations based on two small parameters:

$$(1) \quad \frac{\Delta m^2_\odot}{\Delta m^2_{\rm atm}} = \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \sim 1/30$$

$$(2) \quad |U_{e3}| = |\sin\theta_{13}| \lesssim 0.2 \ \ \text{(CHOOZ)}$$

$\Delta m^2_{21} = 0$ or $U_{13} = 0$ – effective 2f limits

Analytic solutions – contd.

Matter of constant density – a good first approximation for LBL experiments (neutrinos traverse Earth's mantle). Not very useful for solar, atmospheric and supernova neutrinos

Different approach: matter with arbitrary density profile, reduce the problem to an effective 2f one + easily calculable 3f corrections. Both approaches pursued using

(1) Expansion to 1st order in $\alpha = \Delta m^2_{21}/\Delta m^2_{31}$,
exact dependence on $s_{13} = \sin\theta_{13}$

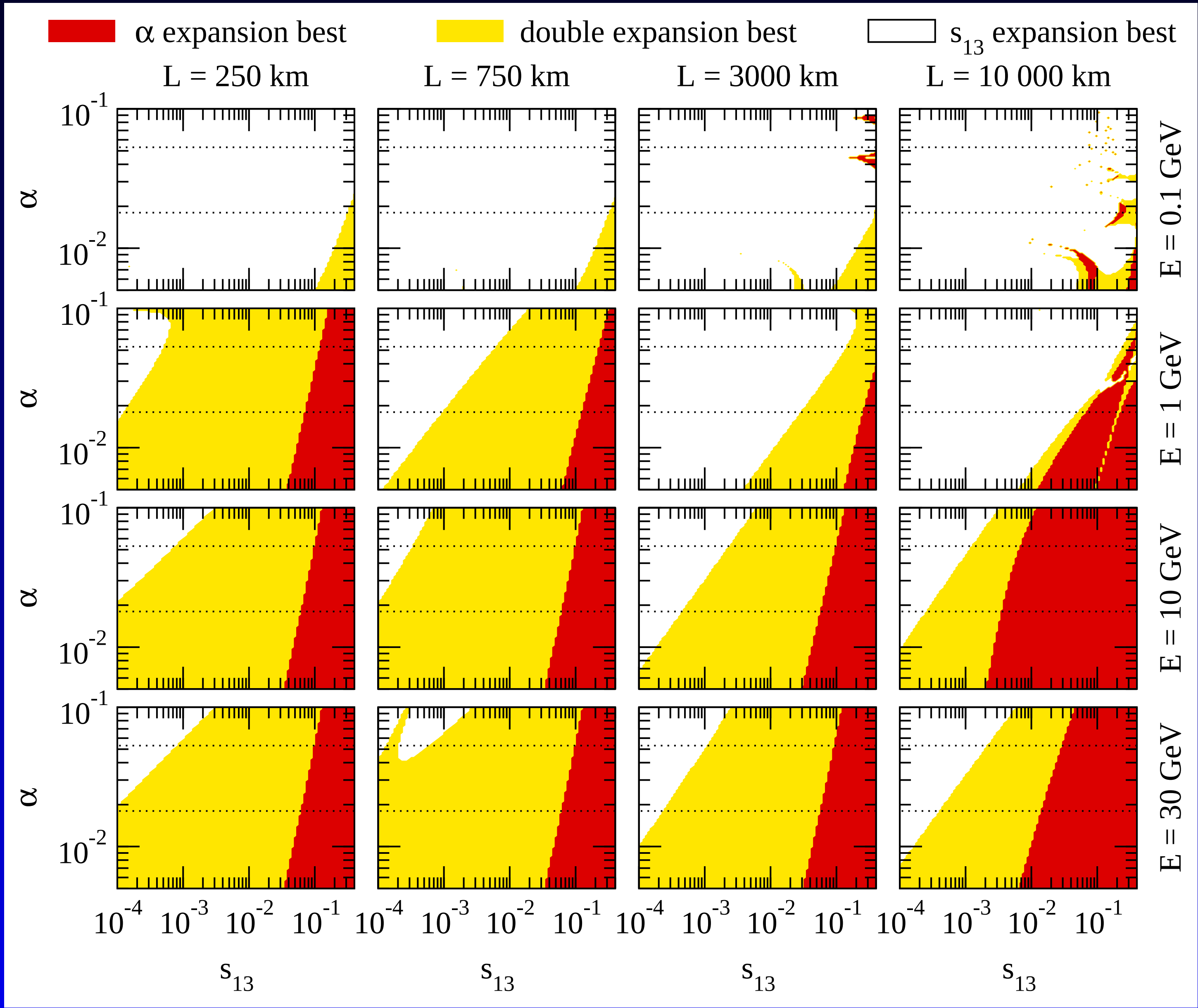
(2) Expansion to 1st order in s_{13}, exact dependence on α

(3) Expansion to 2nd order in both s_{13} and α

Recent discusion and summary:

E.A., Johansson, Ohlsson, Lindner & Schwetz, 2004

Comparison of different approximations



3f effects in solar ν oscillations

What do the solar ν_e oscillate to?

From $|U_{e3}| \ll 1$:

$$\nu_3 \simeq s_{23}\,\nu_\mu + c_{23}\,\nu_\tau$$

$\Rightarrow$ ν_3 practically does not participate in $\nu_\odot$ oscillations.

From unitarity of U: solar ν oscillations between

$$\nu_e \quad \text{and} \quad \nu' = c_{23}\,\nu_\mu - s_{23}\,\nu_\tau \qquad \Rightarrow$$

$$P(\nu_e \to \nu_\mu)/P(\nu_e \to \nu_\tau) \simeq c_{23}^2/s_{23}^2$$

$\theta_{23} \approx 45^\circ$ from $\nu_{\rm atm}$ oscillations; $\Rightarrow$

◇ *Solar ν_e oscillate into a superposition of ν_μ and ν_τ with almost equal weights*

(N.B.: The same argument applies to oscillation of reactor $\bar\nu_e$'s – KamLAND)

Solar ν_e survival probability

At low E ν_μ and ν_τ experimentally indistinguishable $\Rightarrow$ all observables depend just on $P(\nu_e \to \nu_e)$

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Averaging over fast oscillations due to large $\Delta m^2_{\rm atm} = \Delta m^2_{31}$:

$$P(\nu_e \to \nu_e) \simeq c^4_{13} P_{2ee}(\Delta m^2_{21}, \theta_{12}, V_{\rm eff}) + s^4_{13} \;,$$

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NC expts. free of <u>both</u> astrophysics and θ_{13} uncertainties!

Day-Night effect

Earth matter (regeneration) effect on solar ν_e:

(Nigt-time signal) $\neq$ (Day-time signal)

How does the Day-Night difference of the solar ν_e flux at the detectors depend on θ_{13}? While $P_D(\nu_e) \propto c_{13}^4$,

$$P_N(\nu_e) - P_D(\nu_e) \propto c_{13}^6$$

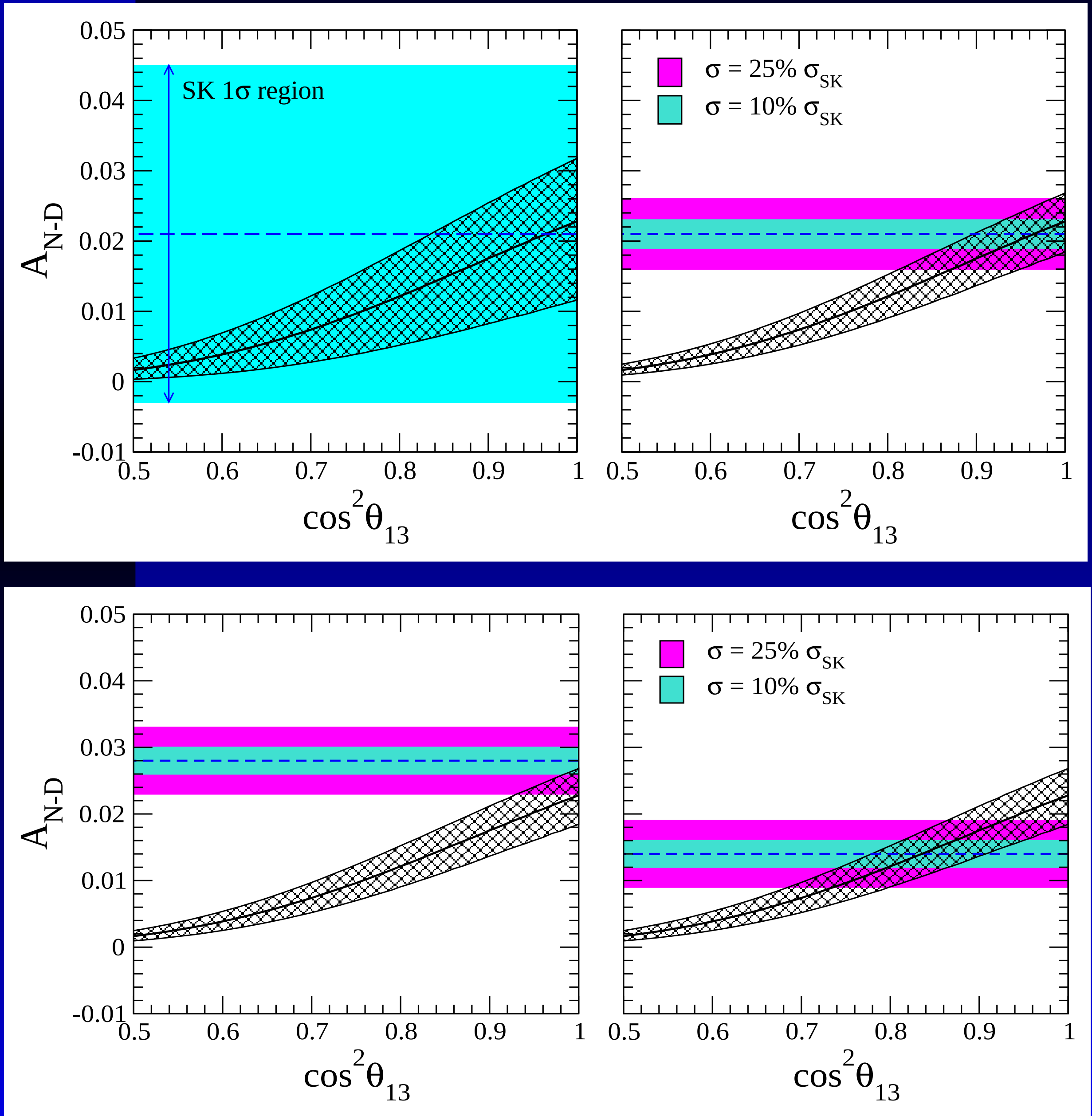
(Const. density approximation: Blennow, Ohlsson & Snellman, 2004);
arbitrary density profile: E.A., Tortola & Valle, 2004)

Night-Day asymmetry of the signal:

$$A_{ND} = 2\frac{N-D}{N+D} \propto c_{13}^2$$

Can one learn anything about θ_{13} by measuring A_{ND}?

At present, exp. errors too large; future experiments (UNO, Hyper-K) can give useful information



Depending on the error and central value of A_{ND}, the upper bound on θ_{13} may be improved or even a lower bound may appear!

CHOOZ:

$$c_{13}^2 > 0.95$$

3f effects in atm. ν oscillations

- Dominant channel $\nu_\mu \leftrightarrow \nu_\tau$
  In 2f case – no matter effects (neglecting tiny $V_{\mu\tau}$ caused by rad. corrections). Independent from the sign of Δm^2_{31} (normal vs inverted hierarchy). In 3f case – sensitivity to matter effects, sign of Δm^2_{31} (may be strong!)

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  In 2f limits ($\Delta m^2_{21} \to 0$ or $\theta_{13} \to 0$) – suppression of oscillation effects on e-like events

Matter effects on $\nu_\mu \leftrightarrow \nu_\tau$ osc.

In 2f approximation: no matter effects on $\nu_\mu \leftrightarrow \nu_\tau$ oscillations [$V(\nu_\mu) = V(\nu_\tau)$ modulo tiny rad. corrections].

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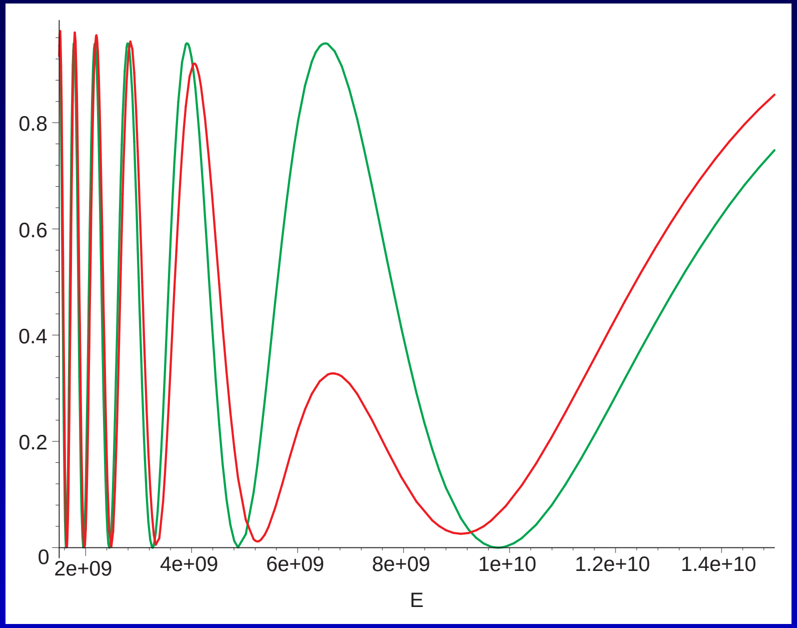
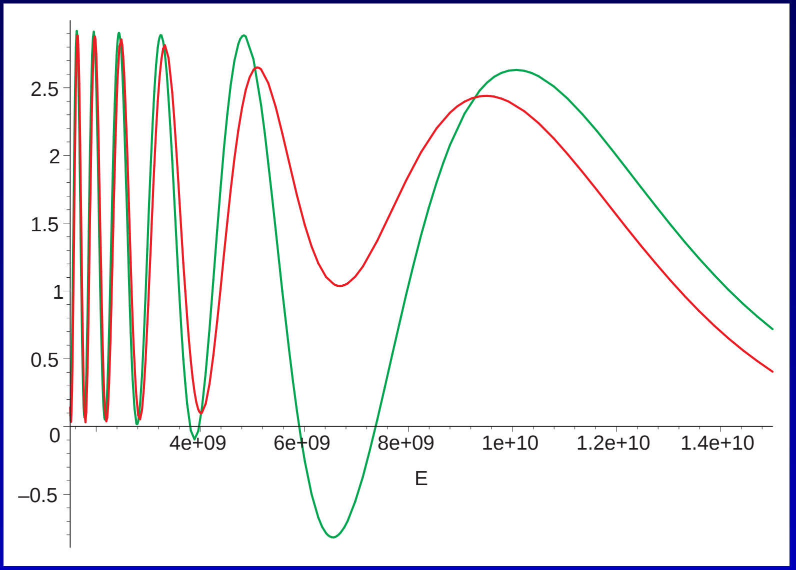


$P_{\mu\tau}$



Oscillated flux of atm. ν_μ

$\Delta m^2_{31} = 2.5 \times 10^{-3}$ eV^2, $\sin^2\theta_{13} = 0.026$, $\theta_{23} = \pi/4$,
$\Delta m^2_{21} = 0$, $L = 9400$ km

Red curves – w/ matter effects, green curves – w/o matter effects on $P_{\mu\tau}$

3f effects in atm. ν osc. – contd.

- $\Delta m^2_{21} \to 0$ (E.A., Dighe, Lipari & Smirnov, 1998) :

$$\frac{F_e - F_e^0}{F_e^0} = P_2(\Delta m^2_{31},\, \theta_{13}, V_{\rm CC}) \cdot (r\, s^2_{23} - 1)$$

- $s_{13} \to 0$ (Peres & Smirnov, 1999):

$$\frac{F_e - F_e^0}{F_e^0} = P_2(\Delta m^2_{21},\, \theta_{12}, V_{\rm CC}) \cdot (r\, c^2_{23} - 1)$$

At low energies $r \equiv F_\mu^0/F_e^0 \simeq 2$; also $s^2_{23} \simeq c^2_{23} \simeq 1/2$ – a conspiracy to hide oscillation effects on e-like events! Results from a peculiar flavour composition of the atmospheric ν flux. (Because of $\theta_{23} \simeq 45^\circ$, $P_{e\mu} = P_{\mu e}$ is $\simeq P_{e\tau}$; but the original ν_μ flux is ~ 2 times larger than ν_e flux $\Rightarrow$ compensation of transitions from and to ν_e state).

Breaking the conspiracy – 3f effects in atmospheric neutrino oscillations (Peres & Smirnov, 2002)

$$\frac{F_e - F_e^0}{F_e^0} \simeq P_2(\Delta m^2_{31},\, \theta_{13}) \cdot (r\, s^2_{23} - 1) + P_2(\Delta m^2_{21},\, \theta_{12}) \cdot (r\, c^2_{23} - 1) - 2 s_{13}\, s_{23}\, c_{23}\, r\, \mathrm{Re}(\tilde{A}^*_{ee}\, \tilde{A}_{\mu e})$$

Interference term not suppressed by the flavour composition of the $\nu_{\rm atm}$ flux; may be (partly) responsible for observed excess of upward-going sub-GeV e-like events

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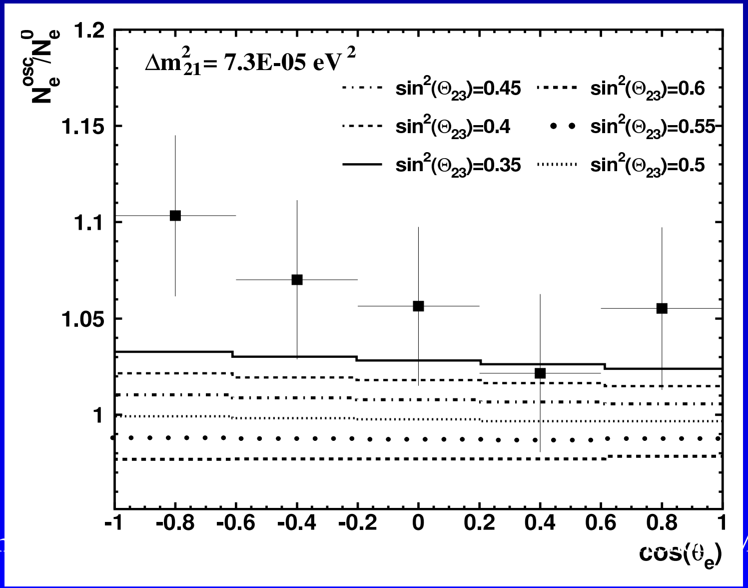

Interf. term may not be sufficient to fully explain the excess of low-E e-like events – a hint of $\theta_{23} \neq 45°$? (Peres & Smirnov, 2004)

Reactor $\bar{\nu}_e$ oscillations

$\bar{\nu}_e$ survival probability:

♢ $$P_{\bar{e}\bar{e}} \simeq 1 - \sin^2 2\theta_{13} \cdot \sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right) - c_{13}^4 \sin^2 2\theta_{12} \cdot \sin^2\left(\frac{\Delta m_{21}^2}{4E}L\right)$$

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- CHOOZ, Palo Verde, ... $\quad (L \lesssim 1$ km)

$$\overline{E} \sim 4\ \text{MeV}\,; \qquad \frac{\Delta m^2_{31}}{4E}\,L \sim 1\,; \qquad \frac{\Delta m^2_{21}}{4E}\,L \ll 1$$

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$$\diamondsuit \quad P_{\bar{e}\bar{e}} \simeq 1 - \sin^2 2\theta_{13} \cdot \sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right) - c_{13}^4 \sin^2 2\theta_{12} \cdot \sin^2\left(\frac{\Delta m_{21}^2}{4E}L\right)$$

- CHOOZ, Palo Verde, ... $(L \lesssim 1$ km$)$

$$\overline{E} \sim 4\ \mathrm{MeV}\,; \qquad \frac{\Delta m_{31}^2}{4E}\,L \sim 1\,; \qquad \frac{\Delta m_{21}^2}{4E}\,L \ll 1$$

One mass scale dominance (2f) approximation:

$$\diamond \quad P(\bar{\nu}_e \to \bar{\nu}_e; L) = 1 - \sin^2 2\theta_{13} \cdot \sin^2\left(\frac{\Delta m_{31}^2}{4E}\,L\right)$$

(Note: Term $\sim \sin^2 2\theta_{12}$ cannot be neglected if $\theta_{13} \lesssim 0.03$, which is about the reach of currently discussed future reactor experiments.)

Reactor $\bar{\nu}_e$ oscillations – contd.

- KamLAND $(\bar{L} \simeq 170$ km)

$$\frac{\Delta m_{21}^2}{4E}\,L \gtrsim 1\,; \qquad \frac{\Delta m_{31}^2}{4E}\,L \gg 1$$

Averaging over fast oscillations due to $\Delta m_{\rm atm}^2 = \Delta m_{31}^2$:

$$\diamond \quad P(\bar{\nu}_e \to \bar{\nu}_e) = c_{13}^4 P_{2\,\bar{e}\,\bar{e}}(\Delta m_{21}^2, \theta_{12}) + s_{13}^4$$

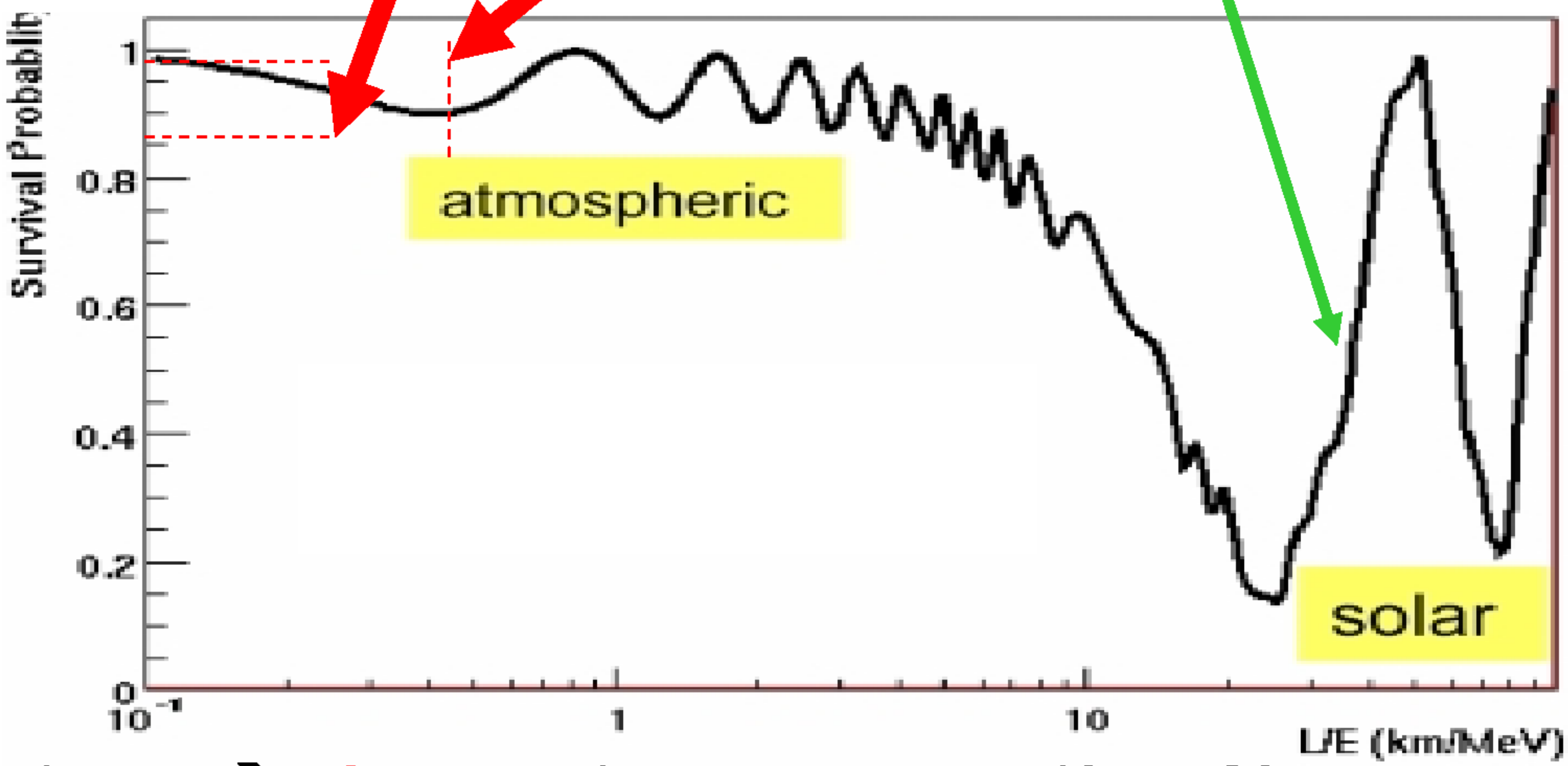
Can differ from 2f probability by as much as $\sim 10\%$

(energy-independent suppression)

N.B.: Matter effects a few % – can be comparable with effects of $\theta_{13} \neq 0$!

Reactor $\bar{\nu}_e$ oscillations

$$P_{ee} \approx 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{31} L}{4E_\nu} + \left(\frac{\Delta m^2_{21} L}{4E_\nu}\right)^2 \cos^4\theta_{13} \sin^2 2\theta_{12}$$



E=4MeV → 2km 4km 40km 80km

Manfred Lindner

3f effects in LBL experiments

- ν_μ disappearance:

3f corrections to probability up to $\sim 10\%$, mainly due to

$$\sin^2(2\theta_{\mu\tau})_{\text{eff}} = c_{13}^4 \sin^2 2\theta_{23}$$

Matter effects in $\nu_\mu \leftrightarrow \nu_\tau$ oscillations (typically small); contribution of subdominant $\nu_\mu \to \nu_e$ oscillations

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- $\nu_\mu(\nu_\tau)$ appearance at ν factories; ν_e appearance in experiments with superbeams

Driven by $\nu_e \leftrightarrow \nu_{\mu,\tau}$ oscillations. For $E \sim 10$ GeV and

$$3 \cdot 10^{-3} \lesssim \theta_{13} \lesssim 3 \cdot 10^{-2}$$

– competition between two parameter channels [$(\Delta m_{31}^2, \theta_{13})$ and $(\Delta m_{21}^2, \theta_{12})$] in $P(\nu_e \leftrightarrow \nu_\mu)$

LBL experiments – contd.

- Unlike in $\nu_{\rm atm}$, no suppression of $\nu_e \leftrightarrow \nu_{\mu,\tau}$ oscill. effects due to flavour composition of the original flux
- Matter effects in $\nu_\mu \leftrightarrow \nu_\tau$ oscillations vanish only when both Δm^2_{21} and U_{e3} vanish
- Dependence of all P_{ab} on CP-violating phase $\delta_{\rm CP}$ (both $\sim \sin\delta_{\rm CP}$ and $\sim \cos\delta_{\rm CP}$) comes from interf. terms – pure 3f effect
- 3f effects especially important for precision measurements (ν factories!)

$\not{CP}$ and $\not{T}$ in ν osc. in vacuum

$\nu_a \to \nu_b$ oscillation probability:

$$\diamond \quad P(\nu_a, t_0 \to \nu_b; t) = \left| \sum_i U_{bi}\, e^{-iE_i(t-t_0)}\, U_{ai}^* \right|^2$$

- CP: $\nu_{a,b} \leftrightarrow \bar{\nu}_{a,b} \quad \Rightarrow \quad U_{ai} \to U_{ai}^* \quad (\{\delta_{\rm CP}\} \to -\{\delta_{\rm CP}\})$
- T: $t \rightleftarrows t_0 \quad \Leftrightarrow \quad \nu_a \leftrightarrow \nu_b$

  $\Rightarrow \quad U_{ai} \to U_{ai}^* \quad (\{\delta_{\rm CP}\} \to -\{\delta_{\rm CP}\})$

T-reversed oscillations (“backwards in time”) $\Leftrightarrow$ oscillations between interchanged initial and final flavours

$\diamond$ *$\not{CP}$ and $\not{T}$ – absent in 2f case, pure $N \geq 3$f effects!*

- CPT: $\nu_{a,b} \leftrightarrow \bar{\nu}_{a,b}$ & $t \rightleftarrows t_0$ $(\nu_a \leftrightarrow \nu_b)$

  ◇ $P(\nu_a \to \nu_b) \to P(\bar{\nu}_b \to \bar{\nu}_a)$

The standard formula for P_{ab} in vacuum is CPT invariant!

$\not{C}\not{P} \Leftrightarrow \not{T}$ – *consequence of CPT*

Measures of $\not{C}\not{P}$ and $\not{T}$ – probability differences:

$$\Delta P_{ab}^{\rm CP} \equiv P(\nu_a \to \nu_b) - P(\bar{\nu}_a \to \bar{\nu}_b)$$

$$\Delta P_{ab}^{\rm T} \equiv P(\nu_a \to \nu_b) - P(\nu_b \to \nu_a)$$

From CPT:

◇ $\Delta P_{ab}^{\rm CP} = \Delta P_{ab}^{\rm T}$; $\Delta P_{aa}^{\rm CP} = 0$

3f case

One $\not{C}\not{P}$ Dirac-type phase $\delta_{\rm CP}$ *(NB: Majorana phases do not affect ν oscillations!)* $\Rightarrow$ one $\not{C}\not{P}$ and $\not{T}$ observable:

- $\Delta P^{\rm CP}_{e\mu} = \Delta P^{\rm CP}_{\mu\tau} = \Delta P^{\rm CP}_{\tau e} \equiv \Delta P$

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- $\Delta P^{\rm CP}_{e\mu} = \Delta P^{\rm CP}_{\mu\tau} = \Delta P^{\rm CP}_{\tau e} \equiv \Delta P$

$$\Delta P = -4 s_{12}\, c_{12}\, s_{13}\, c_{13}^2\, s_{23}\, c_{23}\, \sin\delta_{\rm CP}$$
$$\times \left[\sin\left(\frac{\Delta m^2_{12}}{2E}\,L\right) + \sin\left(\frac{\Delta m^2_{23}}{2E}\,L\right) + \sin\left(\frac{\Delta m^2_{31}}{2E}\,L\right)\right]$$

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Vanishes when

- At least one $\Delta m^2_{ij} = 0$

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Vanishes when

- At least one $\Delta m^2_{ij} = 0$
- At least one $\theta_{ij} = 0$ or 90°

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Vanishes when

- At least one $\Delta m^2_{ij} = 0$
- At least one $\theta_{ij} = 0$ or 90°
- $\delta_{\rm CP} = 0$ or 180°

3f case

One $\not{CP}$ Dirac-type phase $\delta_{\rm CP}$ *(NB: Majorana phases do not affect ν oscillations!)* $\Rightarrow$ one $\not{CP}$ and $\not{T}$ observable:

$$\diamond \quad \Delta P^{\rm CP}_{e\mu} = \Delta P^{\rm CP}_{\mu\tau} = \Delta P^{\rm CP}_{\tau e} \equiv \Delta P$$

$$\Delta P = -4 s_{12}\, c_{12}\, s_{13}\, c_{13}^2\, s_{23}\, c_{23}\, \sin\delta_{\rm CP}$$
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Vanishes when

- At least one $\Delta m^2_{ij} = 0$
- At least one $\theta_{ij} = 0$ or 90°
- $\delta_{\rm CP} = 0$ or 180°
- In the averaging regime

3f case

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- In the averaging regime
- In the limit $L \to 0$ (as L^3)

3f case

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- At least one $\theta_{ij} = 0$ or $90°$
- $\delta_{\rm CP} = 0$ or $180°$
- In the averaging regime
- In the limit $L \to 0$ (as L^3)

Very difficult to observe!

$\not{C}\not{P}$ and $\not{T}$ in ν osc. in matter

- CP: $\nu_{a,b} \leftrightarrow \bar{\nu}_{a,b} \quad \Rightarrow \quad U_{ai} \to U_{ai}^* \quad (\{\delta_{\rm CP}\} \to -\{\delta_{\rm CP}\})$

  $V(r) \to -V(r)$

- T: $t \rightleftarrows t_0 \quad \Leftrightarrow \quad \nu_a \leftrightarrow \nu_b$

  $\Rightarrow \quad U_{ai} \to U_{ai}^* \quad (\{\delta_{\rm CP}\} \to -\{\delta_{\rm CP}\})$

  $V(r) \to \tilde{V}(r)$

$$\tilde{V}(r) = \sqrt{2} G_F \tilde{N}(r)$$

$\tilde{N}(r)$: corresponds to interchanged positions of ν source and detector. Symmetric density profiles: $\tilde{N}(r) = N(r)$

◇ *The very presence of matter violates C, CP and CPT!*

$\Rightarrow$ Fake (extrinsic) $\not{C}\not{P}$ which may complicate the study of fundamental (intrinsic) $\not{C}\not{P}$

$\not{C}\not{P}$ in matter

- Exists even in 2f case (in $\geq$ 3f case exists even when all $\{\delta_{\rm CP}\} = 0$) due to matter effects:

$$P(\nu_a \to \nu_b) \neq P(\bar{\nu}_a \to \bar{\nu}_b)$$

E.g., MSW effect can enhance $\nu_e \leftrightarrow \nu_\mu$ and suppress $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ or vice versa.

- Survival probabilities are not CP-invariant:

$$P(\nu_a \to \nu_a) \neq P(\bar{\nu}_a \to \bar{\nu}_a)$$

To disentangle fundamental $\not{C}\not{P}$ from the matter induced one in LBL experiments – need to measure energy dependence of oscillated signal or signal at two baselines – a difficult task

(Difficult) alternatives:

- Low-E LBL experiments ($E \sim 0.1 - 1$ GeV, $L \sim 100 - 1000$ km) (Koike & Sato, 1999; Minakata & Nunokawa, 2000, 2001);
- Indirect measurements:

  (A) CP-even terms $\sim \cos\delta_{CP}$ (Lipari, 2001)

  (B) Area of leptonic unitarity triangle (Farzan & Smirnov, 2002; Aguilar-Saavedra & Branco, 2000; Sato, 2000)

$\not{C}\not{P}$ cannot be studied in SN ν experiments because of experimental indistinguishability of low-energy ν_μ and ν_τ

$\not{T}$ in matter

CPT not conserved in matter $\Rightarrow$ $\not{C}\not{P}$ and $\not{T}$ are not directly related!

- Matter does not necessarily induce $\not{T}$ (only asymmetric matter with $\tilde{N}(r) \neq N(r)$ does)
- There is no $\not{T}$ (either fundamental or matter induced) in 2f case – a consequence of unitarity:

$$P_{ee} + P_{e\mu} = 1$$

$$P_{ee} + P_{\mu e} = 1$$

$$\Downarrow$$

$$P_{e\mu} = P_{\mu e}$$

- In 3f case – only one T-odd probability difference for ν's (and one for $\bar{\nu}$'s) irrespective of matter density profile – a consequence of unitarity in 3f case (Krastev & Petcov, 1988)

$$\Delta P^T_{e\mu} = \Delta P^T_{\mu\tau} = \Delta P^T_{\tau e}$$

Matter-induced $\not{T}$:

◇ An interesting, pure 3f matter effect; absent in symmetric matter (e.g., $N(r) = const$)

◇ Does not vanish in the regime of complete averaging

◇ May fake fundamental $\not{T}$ and complicate its study (extraction of $\delta_{\rm CP}$ from experiment)

◇ Vanishes when either $U_{e3} = 0$ or $\Delta m^2_{21} = 0$ (2f limits)

$\Rightarrow$ doubly suppressed by both these small parameters

$\Rightarrow$ *Perturbation theory can be used to get analytic expressions*

General structure of T-odd probability differences:

$$\Delta P^T_{e\mu} \;=\; \underbrace{\sin\delta_{\rm CP}\cdot Y}_{\text{fundam. } \not{T}} \;+\; \underbrace{\cos\delta_{\rm CP}\cdot X}_{\text{matter-ind. } \not{T}}$$

In adiabatic approximation: $X = J_{\rm eff}\cdot$ (oscillating terms) ,

$\diamondsuit$ $$J_{\rm eff} \;=\; s_{12}\, c_{12}\, s_{13}\, c_{13}^2\, s_{23}\, c_{23}\, \frac{\sin(2\theta_1 - 2\theta_2)}{\sin 2\theta_{12}}$$

(E.A., Huber, Lindner & Ohlsson, 2001)

Compare with the vacuum Jarlskog invariant:

$$J \;=\; s_{12}\, c_{12}\, s_{13}\, c_{13}^2\, s_{23}\, c_{23} \sin\delta_{\rm CP}$$

$$\Rightarrow \qquad \sin\delta_{\rm CP} \;\Leftrightarrow\; \frac{\sin(2\theta_1 - 2\theta_2)}{\sin 2\theta_{12}}$$

To extract fundamental $\not{T}$ need to measure:

$$\Delta P_{ab} \equiv P_{\rm dir}(\nu_a \to \nu_b) - P_{\rm rev}(\nu_b \to \nu_a) \propto \sin\delta_{\rm CP}$$

Even survival probabilities P_{aa} $(a = \mu, \tau)$ can be used!
(Fishbane & Kaus, 2000)

$$P_{\rm dir}(\nu_a \to \nu_a) - P_{\rm rev}(\nu_a \to \nu_a) \sim \sin\delta_{\rm CP} \quad (a \neq e)$$

In 3f case P_{ee} does not depend on $\delta_{\rm CP}$ (Kuo & Pantaleone, 1987; Minakata & Watanabe, 1999) – not true if $\nu_{\rm sterile}$ is present!

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Matter-induced $\not{T}$ in LBL experiments due to imperfect sphericity of the Earth density distribution cannot spoil the determination of $\delta_{\rm CP}$ if the error in $\delta_{\rm CP}$ is $> 1\%$ at 99% C.L.
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$\Rightarrow$ *No need to interchange positions of ν source and detector!*

Experimental study of $\not{T}$ difficult because of problems with detection of $e^{\pm}$

Matter-induced $\not{T}$:

- Negligible effects in terrestrial experiments
- Cannot be observed in supernova ν oscillations due to experimental indistinguishability of low-E ν_μ and ν_τ
- Can affect the signal from $\sim$GeV neutrinos produced in annihilations of WIMPs inside the Sun (de Gouvêa, 2000)

“CPT in matter”

Is there a relation between $\not{C}\not{P}$ and $\not{T}$ in matter?

For symmetric density profiles (i.e. $\tilde{V}(r) = V(r)$)

$$P(\nu_a \to \nu_b;\, \delta_{\rm CP}, V(r)) \;=\; P(\bar{\nu}_b \to \bar{\nu}_a;\, \delta_{\rm CP}, -V(r))$$

(Minakata, Nunokawa & Parke, 2002)

Easy to generalize to the case of an arbitrary density profile:

$$P(\nu_a \to \nu_b;\, \delta_{\rm CP}, V(r)) \;=\; P(\bar{\nu}_b \to \bar{\nu}_a;\, \delta_{\rm CP}, -\tilde{V}(r))$$

Unlike CPT in vacuum, does not directly relate observables

Can be useful for cross-checking theoreticl calculations

Why study U_e3 (*A hymn to* U_{e3})

- The least known of leptonic mixing parameters
- Discriminates between various neutrino mass models (Barr & Dorsner, 2000; Tanimoto, 2001)
- Unexplained smallness (rel. to $\Delta m^2_\odot / \Delta m^2_{\rm atm}$?)
- The (likely) bottleneck for studying fundamental $\not{C}\not{P}$ and $\not{T}$ effects and matter-induced $\not{T}$ in neutrino oscillations
- Important for measuring the sign of Δm^2_{31} in future LBL experiments – normal vs inverted ν mass hierarchy

A hymn to U_e3 – contd.

- Governs subdominant oscillations of atmospheric neutrinos in multi-GeV region and interf. term in sub-GeV region

- Governs the Earth matter effects on supernova neutrino oscillations

- The only opportunity to see the “canonical” MSW effect (strong matter enhancement of small mixing)?

- Drives the parametric enhancement of oscillations of core-crossing neutrinos inside the Earth

Conclusions

- Two types of 3f effects – “trivial” (existence of new channels, their inter-dependence through unitarity) and nontrivial (interference of different parameter channels, qualitatively new effects – fundamental CP and T-violation, and matter – induced T violation
- 3f corrections to probabilities of oscillations of solar, atmospheric, reactor and acceler. neutrinos depend on $|U_{e3}| = |\sin\theta_{13}|$; can reach $\sim$10%
- Possible interesting 3f effects for SN neutrinos – depend significantly on the value U_{e3}

Conclusions – contd.

♢ Manifestations of ≥ 3 flavours in neutrino oscillations:

- Fundamental $\not{CP}$ and $\not{T}$
- Matter-induced $\not{T}$
- Matter effects in $\nu_\mu \leftrightarrow \nu_\tau$ oscillations
- Specific CP and T conserving interference terms in oscillation probabilities

♢ U_{e3} plays a very special role