

Why the GSI anomaly cannot be explained by Quantum Beats

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Heidelberg

- AM: *Why a splitting in the final state cannot explain the GSI-Oscillations*, to appear soon
- H. Kienert, J. Kopp, M. Lindner, AM: *The GSI anomaly*, J. Phys. Conf. Ser. **136**, 022049, 2008, arXiv:0808.2389

SFB Tr 27 Meeting, Project C1, Heidelberg, 2009

Outline

- 1 Introduction
- 2 Quantum Beats
- 3 One atom of type I
- 4 One atom of type II
- 5 Two atoms of type II
- 6 Conclusions

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2 Quantum Beats

3 One atom of type I

4 One atom of type II

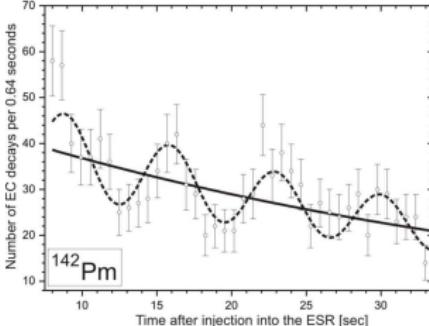
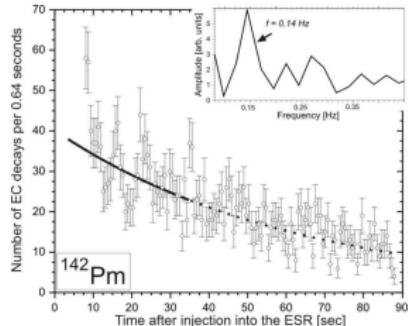
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The measurement at GSI

- measurement: lifetime of several H-like ions with respect to EC (electron capture)
- observation: cos-modulation superimposed on the exponential decay law
- oscillation frequency ~ 7 sec $\Rightarrow \sim 10^{-15}$ eV!!!
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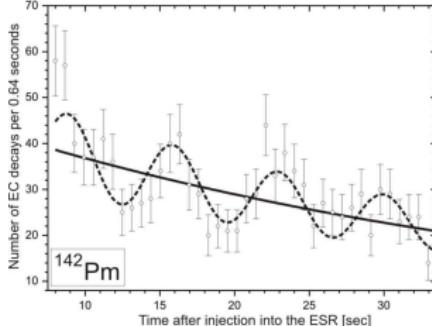
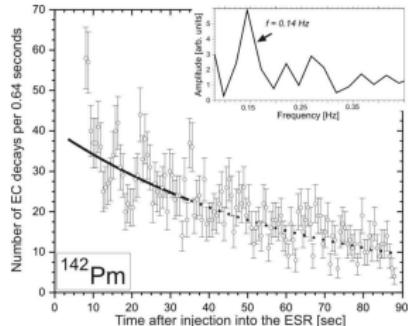
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Y. A. Litvinov et al.,
Phys. Lett. **B664**,
162 (2008),
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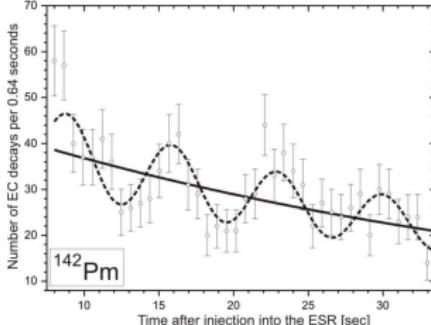
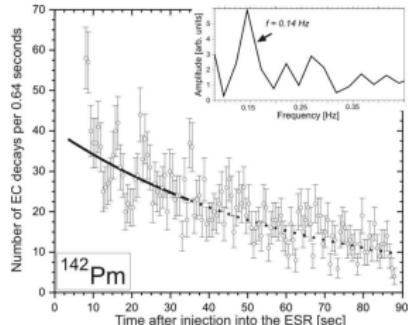
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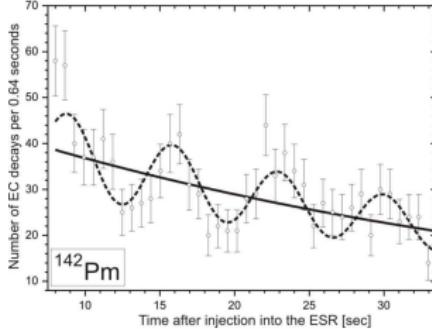
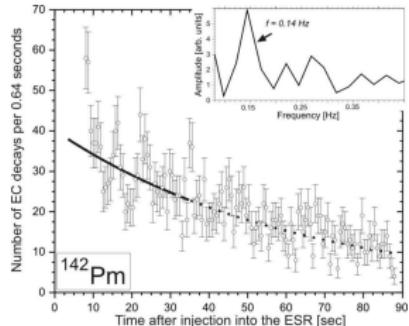
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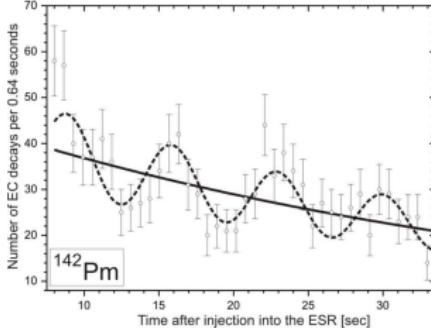
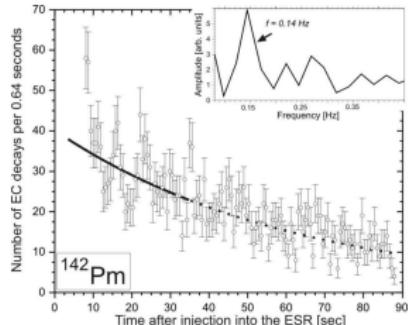
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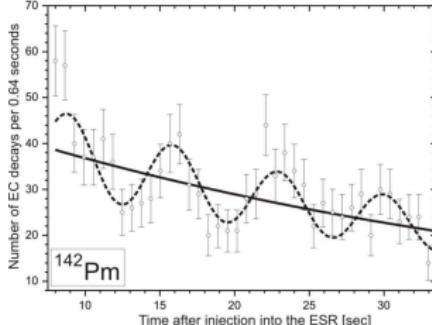
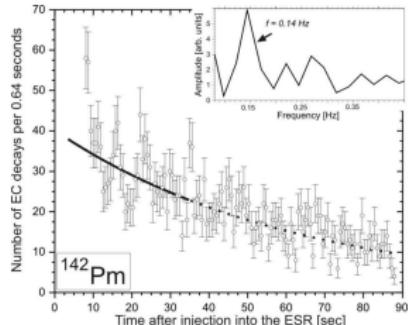
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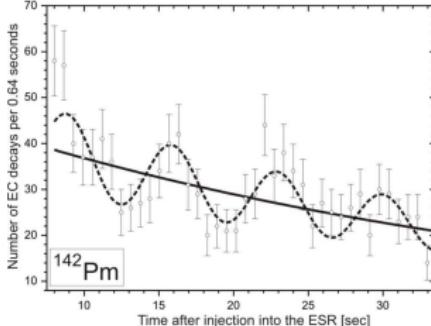
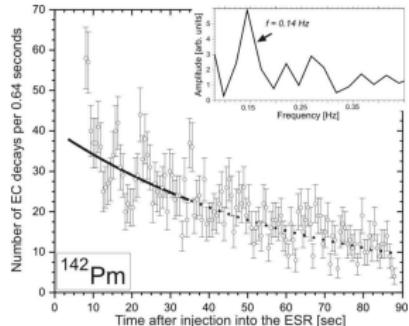
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Type I



Type II

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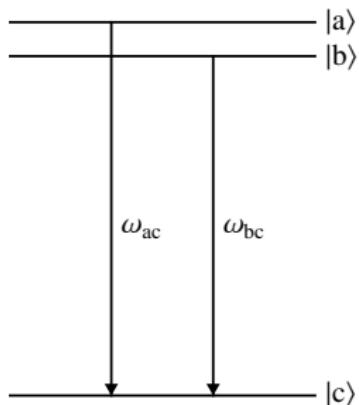


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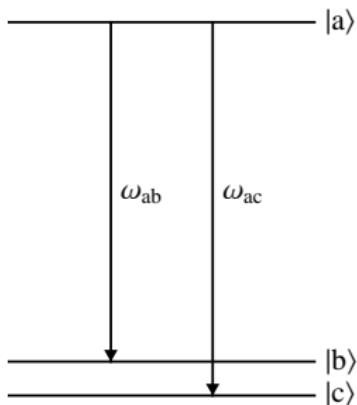
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- important: the states $|a\rangle$, $|b\rangle$, and $|c\rangle$ correspond to different energy eigenvalues
⇒ They are orthogonal!
- this orthogonality is not touched by the uncertainty relation
- BUT: an uncertainty allows for a coherent superposition

Quantum Beats (Chow et al., Phys. Rev. A, 11, 1380)



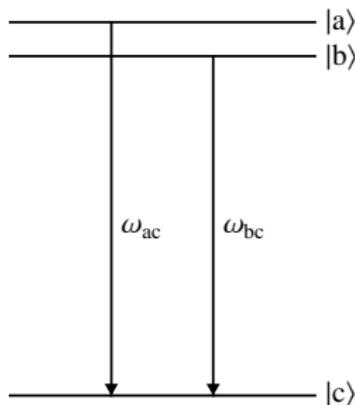
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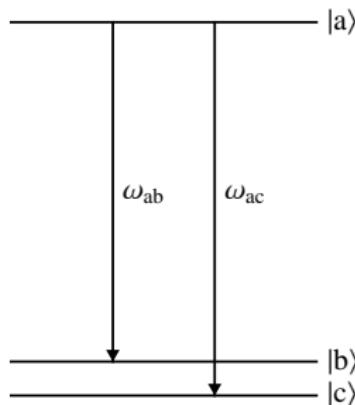
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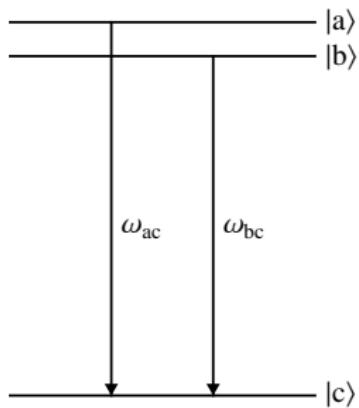
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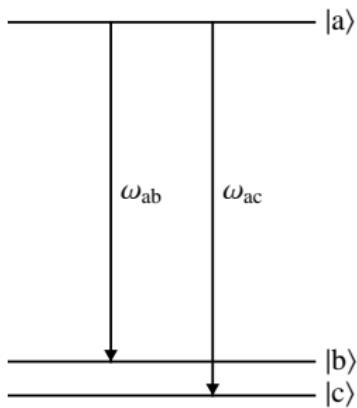
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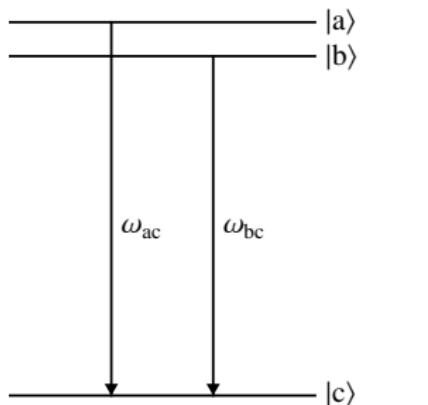
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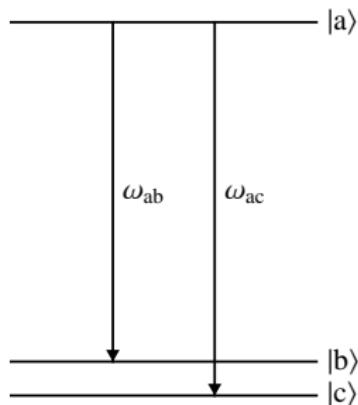
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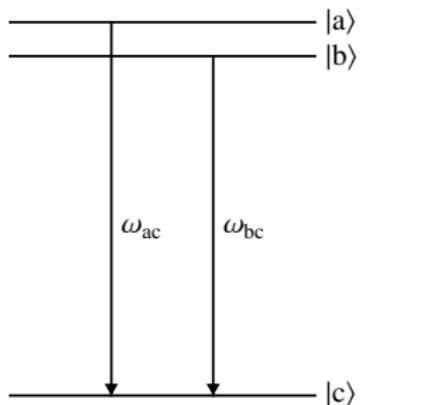


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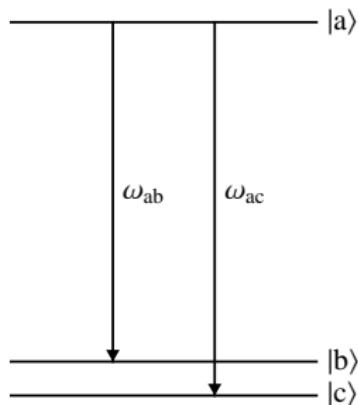


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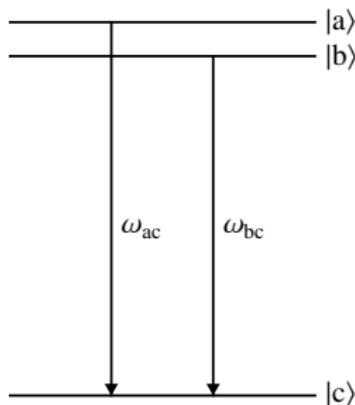


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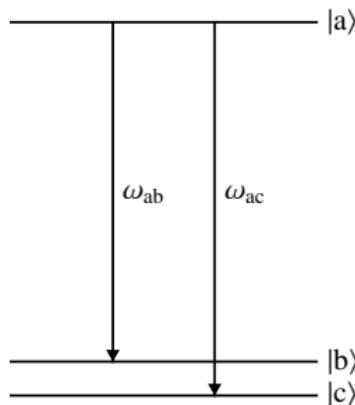


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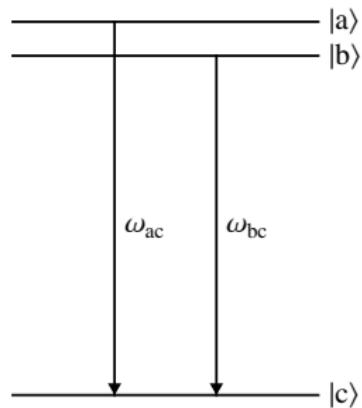
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 - initially:
$$|\Psi(0)\rangle = \mathcal{A}_0|a\rangle|0\rangle_\gamma + \mathcal{B}_0|b\rangle|0\rangle_\gamma + \mathcal{C}_0|c\rangle|0\rangle_\gamma$$
$$\rightarrow \text{with: } |\mathcal{A}_0|^2 + |\mathcal{B}_0|^2 + |\mathcal{C}_0|^2 = 1$$
 - no photons emitted yet \rightarrow vacuum $|0\rangle_\gamma$
 - time-evolution \Rightarrow lower state gets populated by photon emission
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- corresponding state $|\Psi(t)\rangle$:
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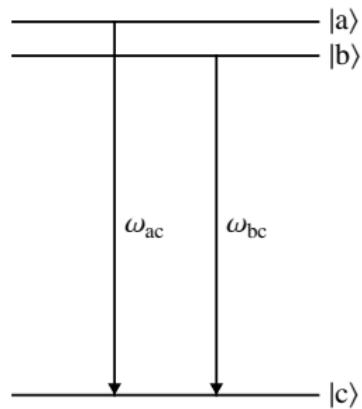
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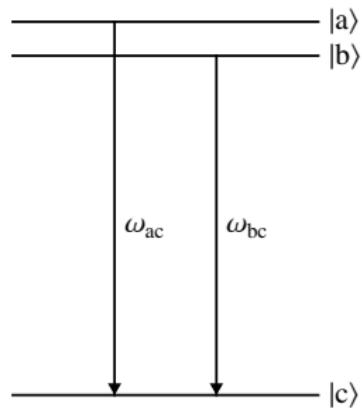
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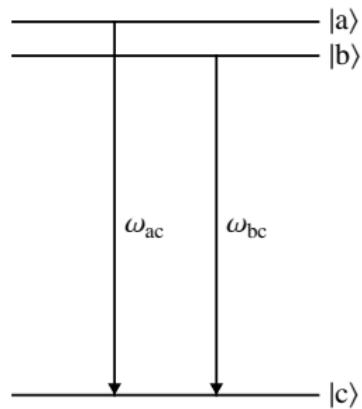
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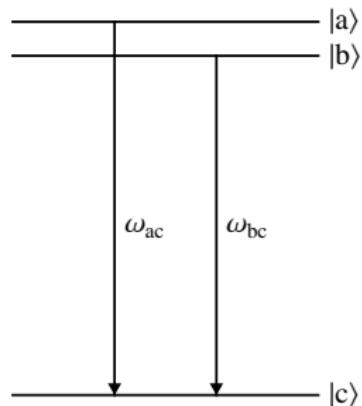
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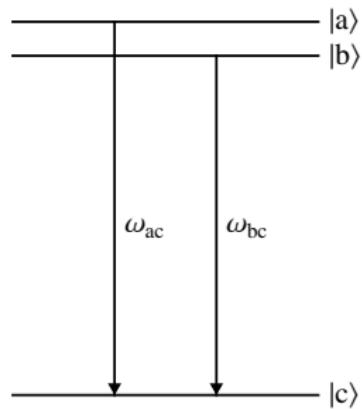
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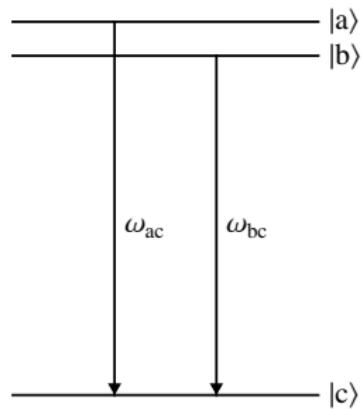
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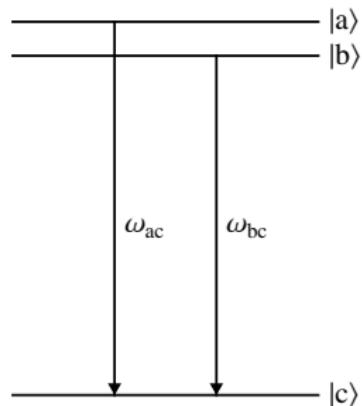
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↪ \vec{k} : momentum, λ : polarization

Single atom of type I

- electric field operator: $\vec{E}(\vec{x}, t) = \sum_{\vec{k}, \lambda} \epsilon_{\vec{k}, \lambda} (a_{\vec{k}, \lambda} e^{-ikx} + a_{\vec{k}, \lambda}^\dagger e^{+ikx})$
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Relation to the GSI-experiment: atom \rightarrow ion, photon \rightarrow neutrino

- if there is a splitting in the initial state, this can cause oscillations in the decay rate
- HOWEVER: splitting $\sim 10^{-15}$ eV \Rightarrow tiny, not at all explained...
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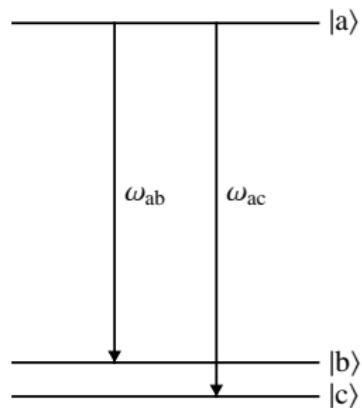
Outline

- 1 Introduction
- 2 Quantum Beats
- 3 One atom of type I
- 4 One atom of type II
- 5 Two atoms of type II
- 6 Conclusions

Single atom of type II

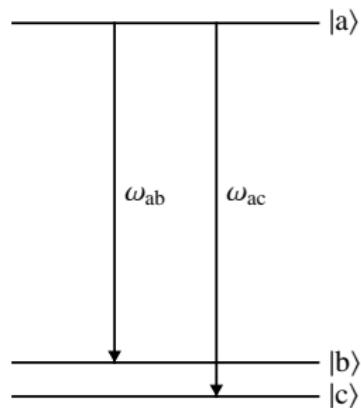
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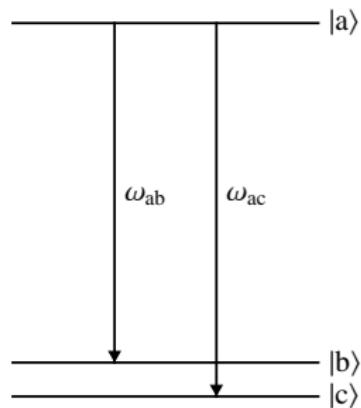
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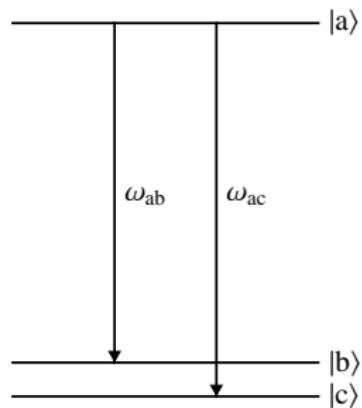


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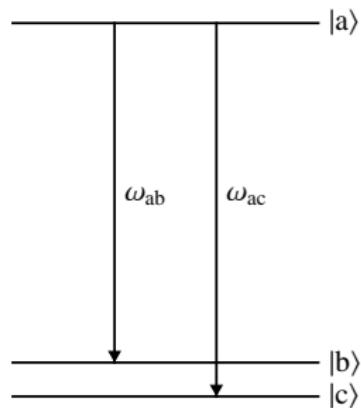


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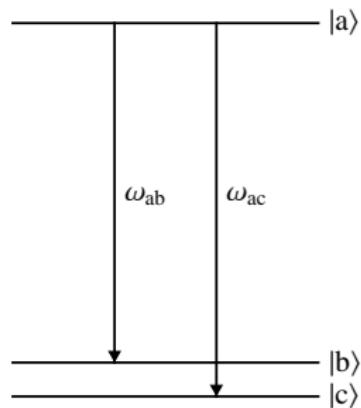


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Single atom of type II

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By waiting long enough, one could determine the photon's energy by measuring the atomic final state. \Rightarrow No interferences expected!

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- the neutrino is not expected to interact before losing its coherence (estimates: $L_{\text{coh}} \lesssim 10^{19}$ m, mean free path $\sim 10^{40}$ m in the Galaxy)
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Outline

- 1 Introduction
- 2 Quantum Beats
- 3 One atom of type I
- 4 One atom of type II
- 5 Two atoms of type II
- 6 Conclusions

Two atoms of type II

- if the spatial separation of the two atoms is smaller than the wavelength of the photon, one can write down a combined state:

$$\begin{aligned} |\Psi(0)\rangle = & \mathcal{A}_0 |a\rangle_1 |a\rangle_2 |0\rangle_\gamma + \mathcal{B}_0 |b\rangle_1 |b\rangle_2 |0\rangle_\gamma + \mathcal{C}_0 |c\rangle_1 |c\rangle_2 |0\rangle_\gamma + \\ & + \mathcal{D}_{1,0} |a\rangle_1 |b\rangle_2 |0\rangle_\gamma + \mathcal{D}_{2,0} |b\rangle_1 |a\rangle_2 |0\rangle_\gamma + \mathcal{E}_{1,0} |a\rangle_1 |c\rangle_2 |0\rangle_\gamma + \\ & + \mathcal{E}_{2,0} |c\rangle_1 |a\rangle_2 |0\rangle_\gamma + \mathcal{F}_{1,0} |b\rangle_1 |c\rangle_2 |0\rangle_\gamma + \mathcal{F}_{2,0} |c\rangle_1 |b\rangle_2 |0\rangle_\gamma \end{aligned}$$

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- there can indeed be oscillatory terms, e.g. $\mathcal{J}_1^* \mathcal{K}_1 e^{-i\Delta t}$, which is proportional to:

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Relation to the GSI-experiment: atom \rightarrow ion, photon \rightarrow neutrino

- even in runs where there was only one EC-decay, there might have been more ions in the ring \rightarrow this possibility has to be considered!
 - the wavelength has to be replaced by the de Broglie wavelength of the neutrino
 - Y. A. Litvinov et al., Phys. Lett. **B664**, 162 (2008), arXiv:0801.2079
 $\Rightarrow Q$ -value ~ 1 MeV
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THANK YOU!!!