Neutrino Oscillations from Quantum Field Theory

[arXiv:0802.2513] and EA & J. Kopp, work in progress

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Why QFT?
Two “standard” approaches to $\nu$ oscillations

Evolution of the labor eigenstate

$$|\nu^a_{fl}\rangle = \sum_i U^*_{ai} |\nu^\text{mass}_i\rangle \implies \sum_i U^*_{ai} e^{-i\phi_i} |\nu^\text{mass}_i\rangle,$$

$$\phi_i = E_i t - p_i x$$
Two “standard” approaches to $\nu$ oscillations

Evolution of the lavor eigenstate

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$$\phi_i = E_i t - p_i x$$

Oscillation phase:

$$\Delta \phi = \phi_i - \phi_k = \Delta E t - \Delta px$$
Two “standard” approaches to $\nu$ oscillations

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Oscillation phase:

$$\Delta \phi = \phi_{i} - \phi_{k} = \Delta E t - \Delta p x$$

I. Same momentum prescription: Assume the emitted neutrino state has a well defined momentum (plane wave) $\Rightarrow \Delta p = 0$.

For ultra-relativistic neutrinos $E_{i} = \sqrt{p^{2} + m_{i}^{2}} \simeq p + \frac{m_{i}^{2}}{2p} \Rightarrow$
Two “standard” approaches to $\nu$ oscillations

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$$\Delta E \simeq \frac{m_2^2 - m_1^2}{2p} \equiv \frac{\Delta m^2}{2p}; \quad \Delta \phi = \frac{\Delta m^2}{2p} t$$
Two “standard” approaches to $\nu$ oscillations

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For ultra-relativistic neutrinos $E_i = \sqrt{p^2 + m_i^2} \approx p + \frac{m_i^2}{2p}$ $\Rightarrow$

$$\Delta E \approx \frac{m_2^2 - m_1^2}{2p} \equiv \frac{\Delta m^2}{2p} \quad ; \quad \Delta \phi = \frac{\Delta m^2}{2p} t$$

Also, assume $L \approx t$ (“time-to-space conversion”)

$\Rightarrow$ The standard formula is obtained
II. Same energy prescription: Assume the emitted neutrino state has a well defined energy (stationary state) \( \Rightarrow \Delta E = 0 \).

\[
\Delta \phi = \Delta E \cdot t - \Delta p \cdot L \quad \Rightarrow \quad -\Delta p \cdot L
\]

For ultra-relativistic neutrinos

\[
p_i = \sqrt{E^2 - m_i^2} \approx E - \frac{m_i^2}{2E} \quad \Rightarrow
\]

\[
-\Delta p \equiv p_1 - p_2 \approx \frac{\Delta m^2}{2E};
\]

\( \Rightarrow \) The standard formula is obtained

Stand. phase \( \Rightarrow \quad (l_{osc}) = \frac{4\pi p}{\Delta m^2} \approx 2.5 \; m \; \frac{p\text{(MeV)}}{\Delta m^2\text{eV}^2} \)

No “time-to-space conversion” necessary
I. Plane waves: have the same probability throughout the whole space
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$L$ - dependence: only through “time-to-space conversion”
(dubious at least!) – valid only for pointlike particles
Problems with plane waves and stat. states

1. Plane waves: have the same probability throughout the whole space

$L$-dependence: only through “time-to-space conversion” (dubious at least!) – valid only for pointlike particles

For on-shell free particles:

\[ E^2_i = p^2 + m_i^2 \quad \Rightarrow \quad p\sigma_p = E\sigma_E \]

\[ \Rightarrow \quad \text{fixed momentum} \quad (\sigma_p = 0) \quad \text{means} \quad \sigma_E = 0 \quad \text{for each} \quad \nu_i \]

– no production/detection coherence
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$L$-dependence: only through “time-to-space conversion” (dubious at least!) – valid only for pointlike particles

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$\Rightarrow$ fixed momentum ($\sigma_p = 0$) means $\sigma_E = 0$ for each $\nu_i$

– no production/detection coherence

II. Stationary states: no time evolution

“Time-to-space conversion” not necessary

Cannot describe decoherence by wave packet separation

For on-shell free particles $\sigma_E = 0$ means $\sigma_p = 0$ for each $\nu_i$
The wave packet approach

In quantum theory free propagating particles are described by wave packets. The evolved produced state:

\[
\langle \vec{x} | \nu_a(t) \rangle = \sum_i U_{ai}^* \int \frac{d^3 p}{(2\pi)^{3/2}} f_{iS}(\vec{p} - \vec{p}_i) e^{i\vec{p}\vec{x} - iE(p)t} |\nu_i\rangle
\]

The detected state:

\[
\langle \vec{x} | \nu_b \rangle = \sum_i U_{bi}^* \int \frac{d^3 p}{(2\pi)^{3/2}} f_{iD}(\vec{p} - \vec{p}_i) e^{i\vec{p}(\vec{x} - \vec{L})} |\nu_i\rangle
\]

The oscillation amplitude:

\[
A_{ab}(L, T) = \sum_i U_{ai}^* U_{bi} \int d^3 x \langle \nu_b | \vec{x} \rangle \langle \vec{x} | \nu_a(T) \rangle
\]

\[
= \sum_i U_{ai}^* U_{bi} \int d^3 p f_{iD}(\vec{p} - \vec{p}_i)^* f_{iS}(\vec{p} - \vec{p}_i) e^{-iE_i(\vec{p})T + i\vec{p}\vec{L}}
\]
The wave packet approach – contd.

Neutrino production and detection times usually not measured
⇒ the oscillation probability obtained upon integration over $T$:

$$P_{ab}(L) = \int dT |A_{ab}(L,T)|^2$$

Must satisfy the unitarity conditions

$$\sum_a P_{ab}(L) = \sum_b P_{ab}(L) = 1$$

Not automatically satisfied for standard normalization of the wave packets ⇒ the proper normalization has to be imposed “by hand”.

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Result for Gaussian wave packets:

\[ P(\nu_a \rightarrow \nu_b ; L) = \sum_{i,k} U_{ai}^* U_{ak} U_{bi} U_{bk}^* e^{-i \frac{\Delta m_{ik}^2}{2E}} L \]

\[
\times \exp \left[ - \left( \frac{L}{(l_{\text{coh}})_{ik}} \right)^2 - 2\pi^2 \xi^2 \left( \frac{\sigma_x}{(l_{\text{osc}})_{ik}} \right)^2 \right]
\]

First exponential: loss of coherence due to wave packet separation for \( L \gtrsim l_{\text{coh}} \):

\[
l_{\text{coh}} = 2\sqrt{2} \frac{\sigma_x}{\Delta v_g} = \frac{4\sqrt{2}E^2}{\Delta m_{ik}^2} \sigma_x
\]

Second exponential: suppression of oscillations due to averaging when \( \xi \sigma_x \) is large compared to the oscillation length \( l_{\text{osc}} \).

\[ \Delta E \simeq \xi \left( \frac{m_i^2}{2E} \right) \]

Accounts for possible coherence violation at neutrino production and detection.
The wave packet approach – contd.

Avoids problems of plane-wave and stationary state ("same momentum" and "same energy") approaches.
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Accounts for possible decoherence effects due to the wave packet separation and/or lack of production and detection coherence.
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- The shape and width of neutrino wave packets have to be postulated; the parameter $\xi$ cannot be calculated
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- The shape and width of neutrino wave packets have to be postulated; the parameter $\xi$ cannot be calculated.

- Production and detection processes are not properly taken into account; neutrinos are assumed to be always on-shell.
The wave packet approach – contd.

Avoids problems of plane-wave and stationary state ("same momentum" and "same energy") approaches.

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Problems:

- The shape and width of neutrino wave packets have to be postulated; the parameter $\xi$ cannot be calculated
- Production and detection processes are not properly taken into account; neutrinos are assumed to be always on-shell
- Normalization "by hand" is invoked
QFT approach

- Most rigorous and consistent approach to neutrino oscillations
- Fully takes into account neutrino production and detection processes
- Treats neutrino production, propagation and detection as a single process
- Avoids unjustified assumptions about the neutrino wave function ("same energy", "same momentum"), shape and width of the wave packets, etc.; automatic normalization
- Requires knowledge of the wave functions of particles accompanying neutrino production and detection
Neutrino production, propagation and detection described by a single Feynman diagram:

Coordinate-space Feynman rules have to be used

Propagation over macroscopic distances — neutrinos are essentially on the mass shell.

Described by propagators rather than by wave functions — no questions about the properties of neutrino wave functions

The rate of the overall production - propagation - detection process calculated. The oscillation probability is obtained by dividing by the rates of the production and detection processes
The amplitude of the overall process:

\[ iA_{ab} = \sum_i U_{ai}^* U_{bi} \int d^4x_1 \int d^4x_2 \tilde{M}_{iP}(x_1) S_{Fi}(x_1 - x_2) \tilde{M}_{iD}(x_2) \]
The amplitude of the overall process:

\[ iA_{ab} = \sum_i U_{ai}^* U_{bi} \int d^4 x_1 \int d^4 x_2 \tilde{\mathcal{M}}_i P(x_1) S_{F_i}(x_1 - x_2) \tilde{\mathcal{M}}_i D(x_2) \]

\[= \sum_i U_{ai}^* U_{bi} \int d^4 x_1 \int d^4 x_2 \tilde{\mathcal{M}}_i P(x_1) \tilde{\mathcal{M}}_i D(x_2) \int \frac{d^4 p}{(2\pi)^4} \frac{e^{-ip(x_2-x_1)}(p + m_i)}{p^2 - m_i^2 + i\epsilon} \]
QFT approach – contd.

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Using \( \not{p} + m_i = \sum_s \bar{u}_i(p, s) u_i(p, s) \), \( x_1 = x_S + x'_1 \), \( x_2 = x_D + x'_2 \)

with \( \vec{L} = \vec{x}_D - \vec{x}_S \), \( T = t_D - t_S \), \( \Psi(p^0, \vec{p}) = \Psi(p^0, \vec{p})_S \Psi(p^0, \vec{p})_D \Rightarrow \)
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\[ \diamond \quad iA_{ab} = \sum_i U_{ai}^* U_{bi} \int \frac{d^4 p}{(2\pi)^4} \Psi(p^0, \vec{p}) e^{-ip^0T + i\vec{p}\vec{L}} \frac{e^{-ip^0T + i\vec{p}\vec{L}}}{p^2 - m_i^2 + i\epsilon} \]
QFT approach – contd.

The amplitude of the overall process:

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\[ \Psi(p^0, \not{p})_S = \int d^4x_1' M_P(x'_1) e^{ipx'_1}, \quad \Psi(p^0, \not{p})_D = \int d^4x_2' M_D(x'_2) e^{-ipx'_2} \]
Grimus-Stockinger theorem for $\vec{p}$ - integration of the neutrino propagator (limit $L \to \infty$):

$$\int d^3p \frac{\psi(\vec{p}) e^{i\vec{p}\vec{L}}}{A - \vec{p}^2 + i\epsilon} \xrightarrow{|\vec{L}| \to \infty} - \frac{2\pi^2}{L} \psi(\sqrt{A} \frac{\vec{L}}{L}) e^{i\sqrt{A}L} + O(L^{-\frac{3}{2}})$$

$p \Rightarrow (p_0, (p_0^2 - m_i^2)^{1/2} \frac{\vec{L}}{L})$. When is it actually valid?
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$$L \gg \frac{p}{\sigma_p^2}.$$

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SFB day, Heidelberg  
July 9, 2009  
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$$

$p \Rightarrow (p_0, (p_0^2 - m_i^2)^{1/2} \bar{L}/L)$. When is it actually valid?

$L \gg \frac{p}{\sigma_p^2}$.

In the opposite limit – no factor $1/L$. Reason: no transverse spreading of the wave packets in this regime; “perfectly collimated beam”.

$t_{\text{transv}} \sim E/\sigma_p^2$, $t_{\text{long.}} \sim E^3/\sigma_p^2 m^2$. 
Grimus-Stockinger theorem for $\vec{p}$ - integration of the neutrino propagator (limit $L \to \infty$):

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\int d^3 p \ \frac{\psi(\vec{p}) e^{i\vec{p}\vec{L}}}{A - \vec{p}^2 + i\epsilon} \quad \stackrel{|\vec{L}| \to \infty}{\longrightarrow} \quad - \frac{2\pi^2}{L} \psi(\sqrt{A\frac{L}{L}}) e^{i\sqrt{AL}} + O(L^{-\frac{3}{2}})
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$$

⇒ explains the strange “plane wave behaviour” found in this limit by Ioannissian & Pilaftsis (1998) – not clear if can be realized in any experimental setting.
First consistent derivation in a (simplified) QFT framework – Kobzarev et al., 1980. At production: plane-wave charged leptons collide with infinitely heavy nuclei. At detection: neutrinos collide with infinitely heavy nuclei and charged leptons (and other nuclei) are produced.

\[
(\sigma_p)_{\text{lept}} = 0, \quad (\sigma_p)_{\text{nucl}} \to \infty \quad \text{(nuclei are localized)} \quad \Rightarrow \quad \Psi_{\text{nucl}}(p) = \text{const}.
\]

\[
\Psi(p^0, \vec{p})_{S} \sim \delta(p^0 - E_{\text{in}})\delta(p^0 - E_{\text{out}})
\]

The amplitude for process with propagation of \( \nu_i \):

\[
A_i \sim \delta(E_{\text{in}} - E_{\text{out}}) \int d^3p \frac{e^{i\vec{p}\vec{L}}}{E_{\text{in}}^2 - \vec{p}^2 - m_i^2 + i\epsilon} \sim \frac{1}{L} \delta(E_{\text{in}} - E_{\text{out}}) e^{i\vec{p}_i\vec{L}}
\]

Leads to the standard formula for the oscillation probability of relativistic neutrinos in vacuum:

\[
\begin{align*}
\diamond & \quad P(\nu_a \to \nu_b; L) = \left| \sum_i U_{bi} e^{-i\frac{\Delta m^2}{2E_{\text{L}}}} L U^*_{ai} \right|^2
\end{align*}
\]
QFT approach

- Calculations in a realistic model with Gaussian wave packets: Giunti et al. 1993, Dolgov et al. 2004, Beuthe, 2003...

- Calculation with localized stationary external states: Grimus & Stockinger, 1996; Ioannissian & Pilaftsis (1998); ...


The amplitude for zero linewidths:

\[
iA = \int d^3 x_1 \, dt_1 \int d^3 x_2 \, dt_2 \, \Psi^*_{H_e,S}(\vec{x}_1)e^{+iE_{H,e,S}t_1} \, \Psi_{H,S}(\vec{x}_1)e^{-iE_{H,S}t_1} \\
\quad \cdot \Psi^*_{H,D}(\vec{x}_2)e^{+iE_{H,D}t_2} \, \Psi_{H,e,S}(\vec{x}_2)e^{-iE_{H,e,D}t_2} \\
\quad \cdot \sum_j M^\mu_S M^\nu_D |U_{e,j}|^2 \, \sqrt{\frac{2}{2\pi}} e^{-ip_0(t_2-t_1)+i\vec{p}(\vec{x}_2-\vec{x}_1)} \\
\quad \cdot \bar{u}_{e,S}(1 - \gamma_5) \, \frac{i(p + m_j)}{p_0^2 - \vec{p}^2 - m_j^2 + i\epsilon} \, \gamma_\nu(1 - \gamma_5) u_{e,D}
\]

Here

\[
M^\mu_{S,D} = \frac{G_F \cos \theta_c}{\sqrt{2}} \psi_e(R) \, \bar{u}_{H,e} \left( M_V \delta^\mu_0 - g_A M_A \sigma_i \delta^\mu_i / \sqrt{3} \right) u_H \, \kappa_{S,D}^{1/2}
\]
For Lorentzian energy distributions of external particles:

\[ \rho_{A,B}(E_{A,B}) = \frac{\gamma_{A,B}/2\pi}{(E_{A,B} - E_{A,B,0})^2 + \gamma_{A,B}^2/4} \]

\( A = \{H, He\}, \ B = \{S, D\}, \ E_{A,B,0} = m_A + \frac{1}{2}\omega_{A,B} \) \Rightarrow

\[ \Gamma \approx \frac{\Gamma_0 B_0}{4\pi L^2} Y_S Y_D \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \exp \left[-\frac{(p_{jk}^{\text{min}})^2}{\sigma_p^2}\right] \exp \left[-\frac{|\Delta m_{jk}^2|}{2\sigma_p^2}\right] \]

\[ \cdot \frac{1}{2} \left( e^{-L/L_{jk,B}} + e^{-L/L_{jk,B}} \right) \exp \left[-i \frac{\Delta m_{jk}^2}{2E} L \right] \frac{(\gamma_S + \gamma_D)/2\pi}{(E_{S,0} - E_{D,0})^2 + \frac{(\gamma_S + \gamma_D)^2}{4}} \]

\[ L_{jk,B}^{\text{coh}} \quad \text{– coherence lengths:} \]

\[ L_{jk,B}^{\text{coh}} = \frac{4\bar{E}^2}{\gamma_B|\Delta m_{jk}^2|} = \frac{\sigma_x}{\Delta v_g}, \quad \sigma_x = \frac{2}{\gamma_B} \quad (B = S, D) \]
Generalized Lamb – Mössbauer (Debye – Waller) factor

$$\exp \left[ - \frac{p_j^2 + p_k^2}{2\sigma_p^2} \right] = \exp \left[ - \frac{(p_{jk}^{\text{min}})^2}{\sigma_p^2} \right] \exp \left[ - \frac{\Delta m_{jk}^2}{2\sigma_p^2} \right]$$

First factor $\Rightarrow$ suppression of emission and absorption, i.e. a generalized Lamb-Mössbauer factor, second factor $\Rightarrow$ suppression of oscillations.
Generalized Lamb – Mössbauer (Debye – Waller) factor

\[
\exp \left[ - \frac{p_j^2 + p_k^2}{2\sigma_p^2} \right] = \exp \left[ - \frac{(p_{jk}^{\text{min}})^2}{\sigma_p^2} \right] \exp \left[ - \frac{\Delta m_{jk}^2}{2\sigma_p^2} \right]
\]

First factor \( \Rightarrow \) suppression of emission and absorption, i.e. a generalized Lamb-Mössbauer factor, second factor \( \Rightarrow \) suppression of oscillations.

\[|\Delta m_{jk}^2| \lesssim 2\sigma_p^2 \Rightarrow \text{localization condition: Spatial localization } \sigma_x \sim 1/\sigma_p.\]

Oscillations would be suppressed only if \[|\Delta m_{jk}^2| \gtrsim 2\sigma_p^2.\]
Generalized Lamb – Mössbauer (Debye – Waller) factor

\[
\exp \left[ - \frac{p_j^2 + p_k^2}{2\sigma_p^2} \right] = \exp \left[ - \frac{(p_{jk}^{\min})^2}{\sigma_p^2} \right] \exp \left[ - \frac{\Delta m_{jk}^2}{2\sigma_p^2} \right]
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First factor \(\Rightarrow\) suppression of emission and absorption, i.e. a generalized Lamb-Mössbauer factor, second factor \(\Rightarrow\) suppression of oscillations.

\(|\Delta m_{jk}^2| \lesssim 2\sigma_p^2 \Rightarrow\) localization condition: Spatial localization \(\sigma_x \sim 1/\sigma_p\).

Oscillations would be suppressed only if \(|\Delta m_{jk}^2| \gtrsim 2\sigma_p^2\).

In reality: \(|\Delta m_{jk}^2|_{\text{max}} \approx 2.5 \cdot 10^{-3} \text{ eV}^2;\quad \sigma_p^2 \sim (10 \text{ keV})^2 \Rightarrow\) oscillations will not be suppressed.
Conclusions

- For a consistent derivation of the probability of neutrino oscillations one needs either wave packet (QM) or QFT approach.
- QFT has an advantage of fully taking into account the neutrino production and detection processes.
- Does not treat neutrinos to be always on the mass shell.
  \[ \Rightarrow \text{allows } \sigma_P \neq \sigma_E \]
- Does not require postulating the shapes and widths of the wave packets.
- Automatically leads to the correct normalization of \( P_{ab}(L) \).
- Indispensable for accurate description of transition from coherence to decoherence regime.
- Clarifies many subtle issues of the theory of neutrino oscillations.
- Once used to derive the osc. probability, can be forgotten in most situations of practical interest.
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$\Rightarrow$ Localization condition violated $\Rightarrow$ oscillations washed out (Kayser, 1981)
Longitudinal vs. transversal w.p. dispersion

Spreading of the wave packets: consequence of the fact that there is a spread of momenta inside of the wave packets and of the $p$-dependence of the group velocity.

$$v_{spr}^i \approx \frac{\partial v_i}{\partial p^j} \sigma_p^j = \frac{1}{E} (\delta_{ij} - v_i v_j) = \frac{1}{E} [\sigma_p^i - v_i (\vec{v} \sigma_p^i)]$$

This gives

$$v_{spr.}^\perp = \frac{\sigma_p}{E}, \quad v_{spr.}^\parallel = \frac{\sigma_p}{E} (1 - v^2) = \frac{\sigma_p}{E} \frac{m^2}{E^2}$$

$$t_{transv} \sim \frac{E}{\sigma_p^2}, \quad t_{long.} \sim \frac{E^3}{\sigma_p^2 m^2}.$$