Neutrino Oscillations from Quantum Field Theory

Partly based on: EA, J. Kopp & M. Lindner, JHEP 0805:005,2008 [arXiv:0802.2513] and EA & J. Kopp, work in progress

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Why QFT ?

Two “standard” approaches to ν oscillations

Evolution of the lavor eigenstate

$$|\nu_a^{\rm fl}\rangle \;=\; \sum_i U_{ai}^*\,|\nu_i^{\rm mass}\rangle \quad\Rightarrow\quad \sum_i U_{ai}^*\,e^{-i\phi_i}\,|\nu_i^{\rm mass}\rangle\,,$$

$$\phi_i = E_i t - p_i x$$

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I. Same momentum prescription: Assume the emitted neutrino state has a well defined momentum (plane wave) $\;\Rightarrow\; \Delta p \;=\; 0$.

For ultra-relativistic neutrinos $\;E_i \;=\; \sqrt{p^2 + m_i^2} \;\simeq p + \frac{m_i^2}{2p} \quad \Rightarrow$

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$$\Delta E \;\simeq\; \frac{m_2^2 - m_1^2}{2p} \;\equiv\; \frac{\Delta m^2}{2p} \,; \qquad\qquad \Delta\phi \;=\; \frac{\Delta m^2}{2p}\, t$$

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Also, assume $L \;\approx\; t$ (“time-to-space conversion”)

$\Rightarrow$ The standard formula is obtained

II. Same energy prescription: Assume the emitted neutrino state has a well defined energy (stationary state) $\Rightarrow$ $\Delta E = 0$.

$$\Delta\phi = \Delta E \cdot t - \Delta p \cdot L \quad \Rightarrow \quad - \Delta p \cdot L$$

For ultra-relativistic neutrinos $p_i = \sqrt{E^2 - m_i^2} \simeq E - \frac{m_i^2}{2E} \quad \Rightarrow$

$$-\Delta p \equiv p_1 - p_2 \approx \frac{\Delta m^2}{2E};$$

$\Rightarrow$ The standard formula is obtained

Stand. phase $\Rightarrow$ $\boxed{(l_{\rm osc}) = \frac{4\pi p}{\Delta m^2} \simeq 2.5\ m\, \frac{p\,(\rm MeV)}{\Delta m^2\ \rm eV^2}}$

No “time-to-space conversion” necessary

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For on-shell free particles: $E_i^2 = p^2 + m_i^2 \quad \Rightarrow \quad \boxed{p\sigma_p = E\sigma_E}$

$\Rightarrow$ fixed momentum $(\sigma_p = 0)$ means $\sigma_E = 0$ for each ν_i

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II. Stationary states: no time evolution

"Time-to-space conversion" not necessary

Cannot describe decoherence by wave packet separation

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The wave packet approach

In quantum theory free propagating particles are describes by wave packets.

The evolved produced state:

$$\langle\vec{x}|\nu_a(t)\rangle \;=\; \sum_i U_{ai}^* \int \frac{d^3p}{(2\pi)^{3/2}}\, f_{iS}(\vec{p}-\vec{p}_i)\, e^{i\vec{p}\vec{x}-iE(p)t}\, |\nu_i\rangle$$

The detected state:

$$\langle\vec{x}|\nu_b\rangle \;=\; \sum_i U_{bi}^* \int \frac{d^3p}{(2\pi)^{3/2}}\, f_{iD}(\vec{p}-\vec{p}_i)\, e^{i\vec{p}(\vec{x}-\vec{L})}\, |\nu_i\rangle$$

The oscillation amplitude:

$$\mathcal{A}_{ab}(L,T) \;=\; \sum_i U_{ai}^* U_{bi} \int d^3x \langle\nu_b|\vec{x}\rangle\langle\vec{x}|\nu_a(T)\rangle$$

$$=\; \sum_i U_{ai}^* U_{bi} \int d^3p f_{iD}(\vec{p}-\vec{p}_i)^* f_{iS}(\vec{p}-\vec{p}_i) e^{-iE_i(\vec{p})T+i\vec{p}\vec{L}}$$

The wave packet approach – contd.

Neutrino production and detection times usually not measured

$\Rightarrow$ the oscillation probability obtained upon integration over T:

$$P_{ab}(L) = \int dT |\mathcal{A}_{ab}(L,T)|^2$$

Must satisfy the unitarity conditions

$$\sum_a P_{ab}(L) = \sum_b P_{ab}(L) = 1$$

Not automatically satisfied for standard normalization of the wave packets $\Rightarrow$ the proper normalization has to be imposed "by hand".

The wave packet approach – contd.

Result for Gaussian wave packets:

$$P(\nu_a \to \nu_b\,;\,L) \;=\; \sum_{i,k} U_{ai}^* U_{ak} U_{bi} U_{bk}^*\, e^{-i\frac{\Delta m_{ik}^2}{2E}L}$$

$$\times\;\exp\left[-\left[\frac{L}{(l_{\rm coh})_{ik}}\right]^2 - 2\pi^2\xi^2\left[\frac{\sigma_x}{(l_{\rm osc})_{ik}}\right]^2\right]$$

First exponential: loss of coherence due to wave packet separation for $L \gtrsim l_{\rm coh}$:

$$l_{\rm coh} \;=\; 2\sqrt{2}\,\frac{\sigma_x}{\Delta v_g} \;=\; \frac{4\sqrt{2}E^2}{\Delta m_{ik}^2}\sigma_x$$

Second exponential: suppression of oscillations due to averaging when $\xi\sigma_x$ is large compared to the oscillation length $l_{\rm osc}$. [ξ : $\Delta E \simeq \xi(m_i^2/2E)$]

Accounts for possible coherence violation at neutrino production and detection.

The wave packet approach – contd.

Avoids problems of plane-wave and stationary state (“same momentum” and “same energy”) approaches.

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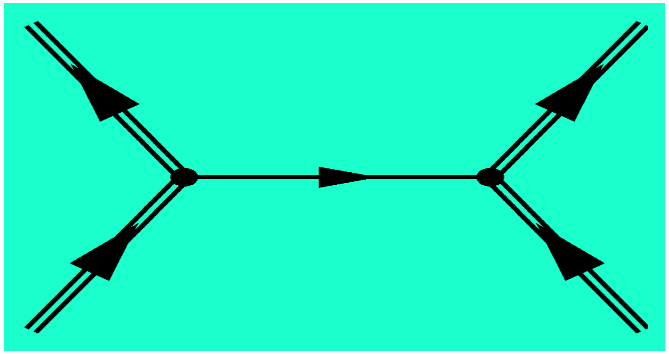
- The shape and width of neutrino wave packets have to be postulated; the parameter ξ cannot be calculated
- Production and detection processes are not properly taken into account; neutrinos are assumed to be always on-shell
- Normalization “by hand” is invoked

QFT approach

- Most rigorous and consistent approach to neutrino oscillations
- Fully takes into account neutrino production and detection processes
- Treats neutrino production, propagation and detection as a single process
- Avoids unjustified assumptions about the neutrino wave function (“same energy”, “same momentum”), shape and width of the wave packets, etc.; automatic normalization
- Requires knowledge of the wave functions of particles accompanying neutrino production and detection

QFT approach – contd.

Neutrino production, propagation and detection described by a single Feynman diagram:



Coordinate-space Feynman rules have to be used

Propagation over macroscopic distances – neutrinos are essentially on the mass shell.

Described by propagators rather than by wave functions – no questions about the properties of neutrino w. functions

The rate of the overall prod. - propag. - detect. process calculated. The oscillation probability is obtained by dividing by the rates of the production and detection processes

QFT approach – contd.

The amplitude of the overall process:

$$i\mathcal{A}_{ab} \;=\; \sum_i U_{ai}^* U_{bi} \int d^4x_1 \int d^4x_2\, \tilde{\mathcal{M}}_{iP}(x_1)\, S_{Fi}(x_1 - x_2)\, \tilde{\mathcal{M}}_{iD}(x_2)$$

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$$= \sum_i U_{ai}^* U_{bi} \int d^4x_1 \int d^4x_2\, \tilde{\mathcal{M}}_{iP}(x_1)\, \tilde{\mathcal{M}}_{iD}(x_2) \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x_2-x_1)}\,(\not{p} + m_i)}{p^2 - m_i^2 + i\epsilon}$$

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Using $\not{p} + m_i = \sum_s \bar{u}_i(p,s) u_i(p,s)$, $x_1 = x_S + x_1'$, $x_2 = x_D + x_2'$

with $\vec{L} = \vec{x}_D - \vec{x}_S\,,\quad T = t_D - t_S\,,\quad \Psi(p^0,\vec{p}) = \Psi(p^0,\vec{p})_S\, \Psi(p^0,\vec{p})_D \quad \Rightarrow$

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$$\Psi(p^0, \vec{p})_S = \int d^4x_1'\, \mathcal{M}_P(x_1')\, e^{ipx_1'}\,, \quad \Psi(p^0, \vec{p})_D = \int d^4x_2'\, \mathcal{M}_D(x_2')\, e^{-ipx_2'}$$

QFT calculation – Grimus-Stockinger f-la.

Grimus-Stockinger theorem for $\vec{p}$ - integration of the neutrino propagator (limit $L \to \infty$):

$$\int d^3p \, \frac{\psi(\vec{p})\, e^{i\vec{p}\vec{L}}}{A - \vec{p}^2 + i\epsilon} \xrightarrow{|\vec{L}|\to\infty} -\frac{2\pi^2}{L}\psi(\sqrt{A}\tfrac{\vec{L}}{L})e^{i\sqrt{A}L} + \mathcal{O}(L^{-\frac{3}{2}})$$

$p \;\Rightarrow\; (p_0, (p_0^2 - m_i^2)^{1/2}\, \vec{L}/L)$. When is it actually valid?

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$$L \gg \frac{p}{\sigma_p^2} \, .$$

In the opposite limit – no factor $1/L$. Reason: no transverse spreading of the wave packets in this regime; "perfectly collimated beam".

$t_{transv} \sim E/\sigma_p^2, \;\; t_{long.} \sim E^3/\sigma_p^2 m^2$.

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$\Rightarrow$ explains the strange “plane wave behaviour” found in this limit by Ioannissian & Pilaftsis (1998) – not clear if can be realized in any experimental setting.

QFT approach – contd.

First consistent derivation in a (simplified) QFT framework – Kobzarev et al., 1980. At production: plane-wave charged leptons collide with infinitely heavy nuclei. At detection: neutrinos collide with infinitely heavy nuclei and charged leptons (and other nuclei) are produced.

$(\sigma_p)_{lept} = 0$, $(\sigma_p)_{nucl} \to \infty$ (nuclei are localized) $\Rightarrow \quad \Psi_{nucl}(p) = const.$

$$\Psi(p^0, \vec{p})_S \sim \delta(p^0 - E_{in})\delta(p^0 - E_{out})$$

The amplitude for process with propagation of ν_i:

$$\mathcal{A}_i \sim \delta(E_{in} - E_{out}) \int d^3p \, \frac{e^{i\vec{p}\vec{L}}}{E_{in}^2 - \vec{p}^2 - m_i^2 + i\epsilon} \sim \frac{1}{L}\delta(E_{in} - E_{out}) e^{i\vec{p}_i\vec{L}}$$

Leads to the standard formula for the oscillation probability of relativistic neutrinos in vacuum:

$\diamondsuit$ $$\boxed{P(\nu_a \to \nu_b; L) = \left| \sum_i U_{bi} \, e^{-i\frac{\Delta m_{i1}^2}{2E}L} \, U_{ai}^* \right|^2}$$

QFT approach

- Calculations in a realistic model with Gaussian wave packets: Giunti et al. 1993, Dolgov et al. 2004, Beuthe, 2003...
- Calculation with localized stationary external states: Grimus & Stockinger, 1996; Ioannissian &Pilaftsis (1998); ...
- A good review (up to 2003) – M. Beuthe, Phys. Rep. 375 (2003) 105.
- QFT calculation for Mössbauer neutrinos): EA, J. Kopp & M. Lindner, 2008.

QFT calculation for Mössbauer neutrinos

The amplitude for zero linewidths:

$$i\mathcal{A} \;=\; \int d^3x_1\,dt_1 \int d^3x_2\,dt_2\,\Psi^*_{He,S}(\vec{x}_1)e^{+iE_{He,S}\,t_1}\;\Psi_{H,S}(\vec{x}_1)e^{-iE_{H,S}\,t_1}$$

$$\cdot\;\Psi^*_{H,D}(\vec{x}_2)e^{+iE_{H,D}\,t_2}\;\Psi_{He,S}(\vec{x}_2)e^{-iE_{He,D}\,t_2}$$

$$\cdot\sum_j \mathcal{M}^{\mu}_S\mathcal{M}^{\nu *}_D|U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(t_2-t_1)+i\vec{p}(\vec{x}_2-\vec{x}_1)}$$

$$\cdot\;\bar{u}_{e,S}\gamma_\mu(1-\gamma_5)\,\frac{i(\not{p}+m_j)}{p_0^2-\vec{p}^2-m_j^2+i\epsilon}\,\gamma_\nu(1-\gamma_5)u_{e,D}$$

Here

$$\mathcal{M}^{\mu}_{S,D} = \frac{G_F\cos\theta_c}{\sqrt{2}}\,\psi_e(R)\,\bar{u}_{He}\,(M_V\,\delta^\mu_0 - g_A M_A\sigma_i\,\delta^\mu_i/\sqrt{3})\,u_H\,\kappa^{1/2}_{S,D}$$

QFT calculation – contd.

For Lorentzian energy distributions of external particles:

$$\rho_{A,B}(E_{A,B}) = \frac{\gamma_{A,B}/2\pi}{(E_{A,B} - E_{A,B,0})^2 + \gamma_{A,B}^2/4}$$

$$\left(A = \{H, He\},\ B = \{S, D\},\ E_{A,B,0} = m_A + \tfrac{1}{2}\omega_{A,B}\right) \quad \Rightarrow$$

$$\Gamma \simeq \frac{\Gamma_0\, B_0}{4\pi L^2}\, Y_S Y_D \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \exp\left[-\frac{(p_{jk}^{\min})^2}{\sigma_p^2}\right] \exp\left[-\frac{|\Delta m_{jk}^2|}{2\sigma_p^2}\right]$$

$$\cdot\ \frac{1}{2}\left(e^{-L/L^{\rm coh}_{jk,S}} + e^{-L/L^{\rm coh}_{jk,D}}\right) \exp\left[-i\frac{\Delta m_{jk}^2}{2\bar{E}}L\right] \frac{(\gamma_S + \gamma_D)/2\pi}{(E_{S,0} - E_{D,0})^2 + \frac{(\gamma_S+\gamma_D)^2}{4}}$$

$L^{\rm coh}_{jk,B}$ – coherence lengths:

$$L^{\rm coh}_{jk,B} = \frac{4\bar{E}^2}{\gamma_B |\Delta m_{jk}^2|} = \frac{\sigma_x}{\Delta v_g}, \qquad \sigma_x = \frac{2}{\gamma_B} \qquad (B = S, D)$$

QFT calculation – contd.

Generalized Lamb – Mössbauer (Debye – Waller) factor

$$\exp\left[-\frac{p_j^2+p_k^2}{2\sigma_p^2}\right] = \exp\left[-\frac{(p_{jk}^{\min})^2}{\sigma_p^2}\right]\exp\left[-\frac{|\Delta m_{jk}^2|}{2\sigma_p^2}\right]$$

First factor $\Rightarrow$ suppression of emission and absorption, i.e. a generalized Lamb-Mössbauer factor, second factor $\Rightarrow$ suppression of oscillations.

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$|\Delta m_{jk}^2| \lesssim 2\sigma_p^2 \quad \Rightarrow$ localization condition: Spatial localization $\sigma_x \sim 1/\sigma_p$. Oscillations would be suppressed only if $|\Delta m_{jk}^2| \gtrsim 2\sigma_p^2$.

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Oscillations would be suppressed only if $|\Delta m_{jk}^2| \gtrsim 2\sigma_p^2$.

In reality: $|\Delta m_{jk}^2|_{\max} \simeq 2.5\cdot 10^{-3}\ \mathrm{eV}^2$; $\sigma_p^2 \sim (10\ \mathrm{keV})^2 \quad\Rightarrow$
oscillations will <u>not</u> be suppressed.

Conclusions

- For a consistent derivation of the probability of neutrino oscillations one needs either wave packet (QM) or QFT approach
- QFT has an avantage of fully taking into account the neutrino production and detection processes
- Does not treat neutrinos to be always on the mass shell
  $\Rightarrow$ allows $\sigma_P \neq \sigma_E$
- Does not require postulating the shapes and widths of the wave packets
- Automatically leads to the correct normalization of $P_{ab}(L)$
- Indispensible for accurate description of transition from coherence to decoherence regime
- Clarifies many subtle issues of the theory of neutrino oscillations
- Once used to derive the osc. probability, can be forgotten in most situations of practical interest

Backup slides

Coherence at neutrino production

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$\Rightarrow$ Localization condition violated $\Rightarrow$ oscillations washed out (Kayser, 1981)

Longitudinal *vs.* transversal w.p. dispersion

Spreading of the wave packets: consequence of the fact that the there is a spread of momenta inside of the wave packets and of the p-dependence of the group velocity.

$$v^i_{spr} \simeq \frac{\partial v_i}{\partial p^j}\sigma^j_p = \frac{1}{E}(\delta_{ij} - v_i v_j) = \frac{1}{E}[\sigma^i_p - v_i(\vec{v}\vec{\sigma_p})]$$

This gives

$$v^{\perp}_{spr.} = \frac{\sigma_p}{E}\,, \qquad v^{||}_{spr.} = \frac{\sigma_p}{E}(1-v^2) = \frac{\sigma_p}{E}\frac{m^2}{E^2}$$

$t_{transv} \sim E/\sigma_p^2,\; t_{long.} \sim E^3/\sigma_p^2 m^2$.