Description of Neutrinoless double beta decay in deformed nuclei with realistic forces

---calculation method for nuclear matrix element

Fang Dong-Liang(ITP, Uni Tuebigen)
Under guidance of Prof. A. Faessler and Dr V. Rodin

TR-27
C5

Introduction

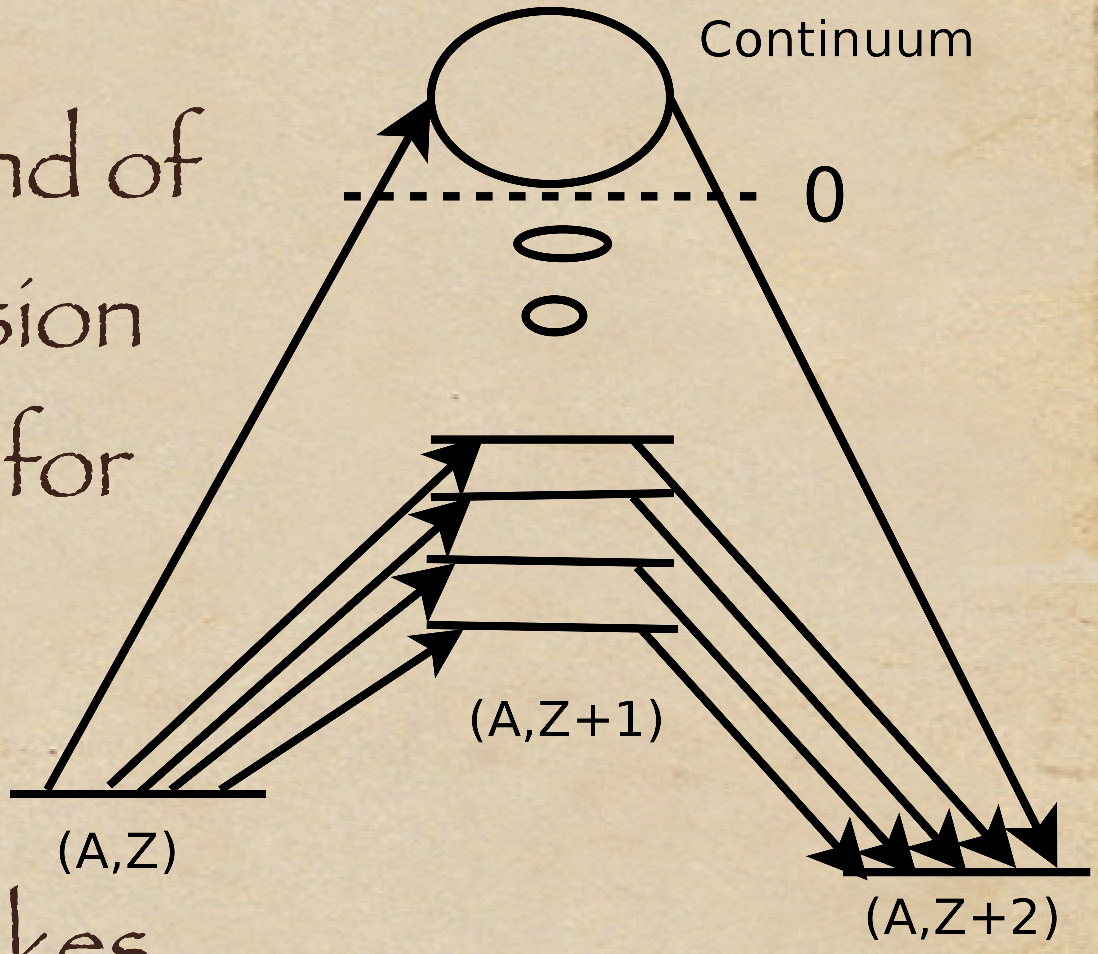
- What is double-beta-decay and neutrinoless-double-beta-decay?
- Why we are interested in neutrinoless-double-beta-decay?
- How we simulate this process?

Introduction

- Double beta decay is kind of nuclear decay with emission of two e^- instead of one for normal beta decay.
- The existence is due to nuclear pairing which makes the separation energy larger for even-even nuclei

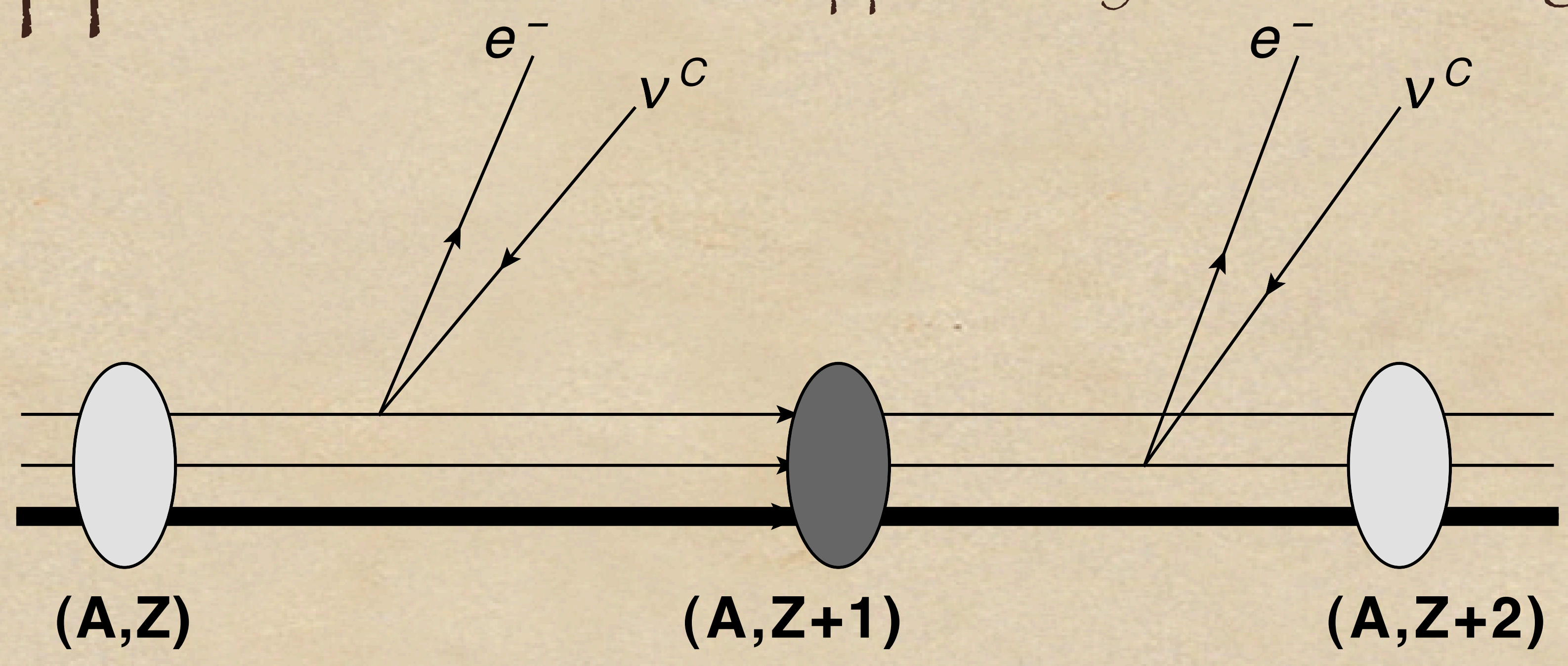
Introduction

- Double beta decay is kind of nuclear decay with emission of two e⁻ instead of one for normal beta decay.
- The existence is due to nuclear pairing which makes the separation energy larger for even-even nuclei



Introduction

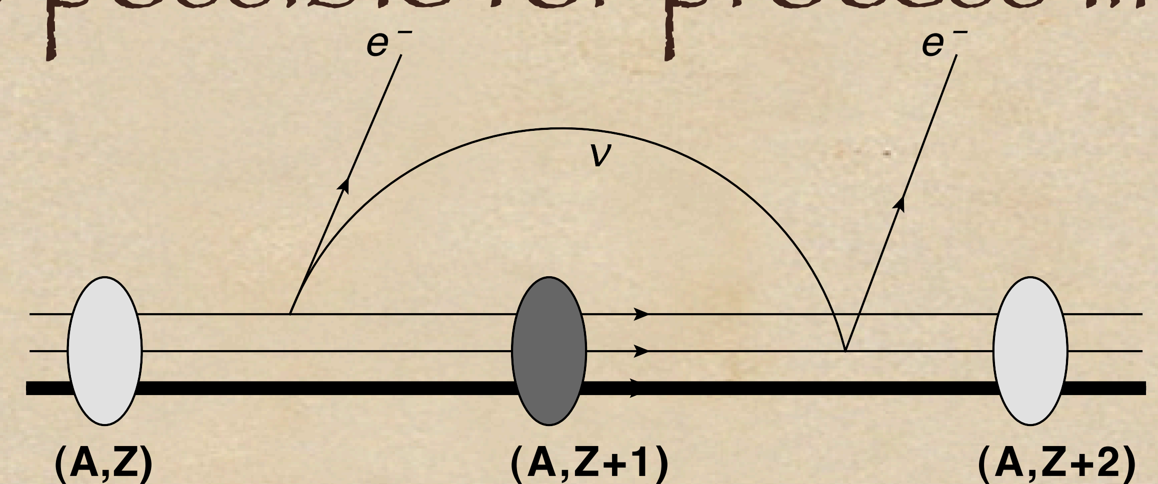
- From SM we know that this process can happen (Thesis: Maria Goeppert-Mayer 1935 Goettingen)



- Instead of two successive β decay

Introduction

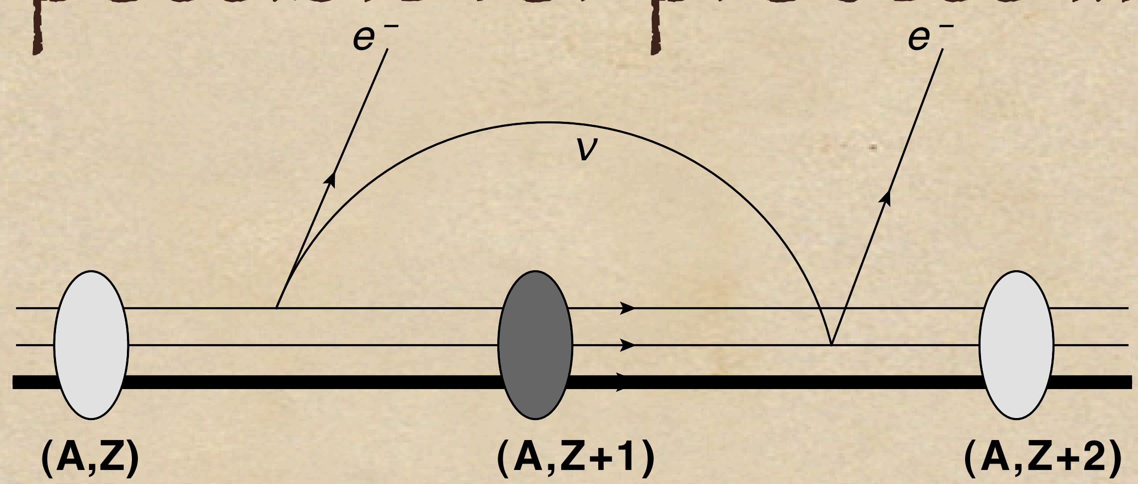
- But there are also some hypothesis that neutrino is Majorana particle, so it is also possible for process like



- Lepton Flavor violation;massive neutrino

Introduction

- But there are also some hypothesis that neutrino is Majorana particle, so it is also possible for process like

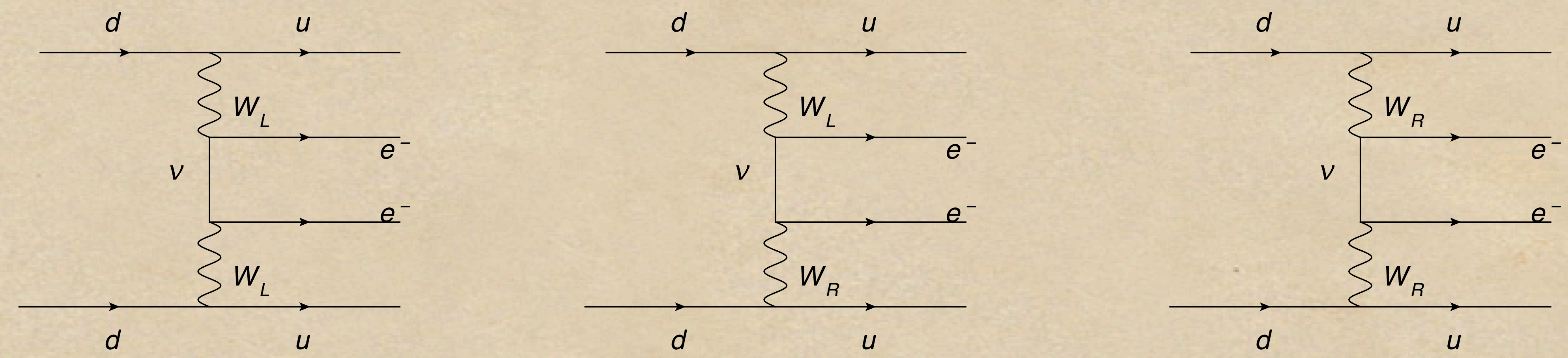


$\nu = \nu^c$

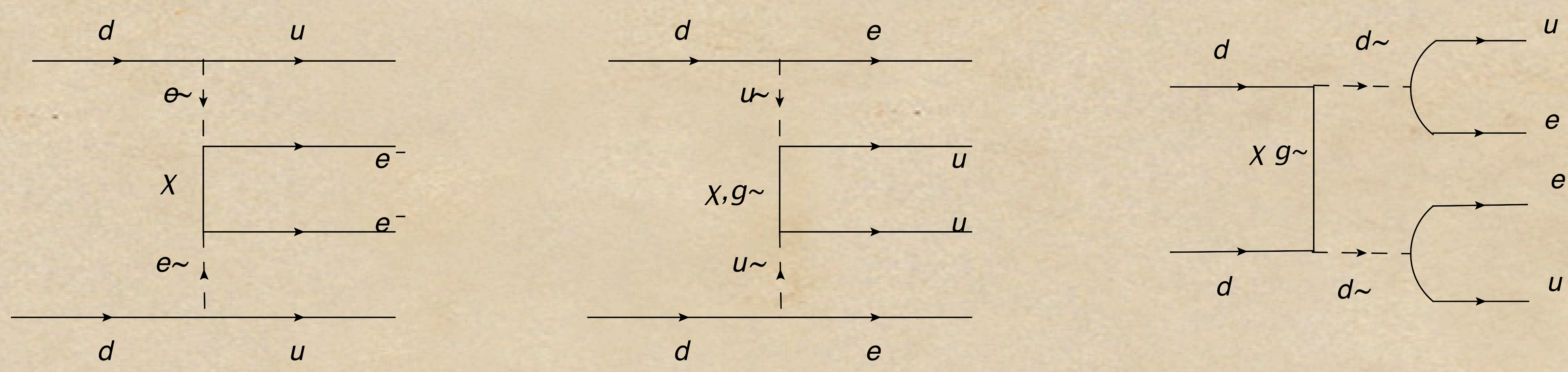
$m_\nu \neq 0$

- Lepton Flavor violation;massive neutrino

Introduction



- Examples for non-supersymmetric models; J. Phys. G. 24;2139



- Example for R-parity violated supersymmetric models; Phys. Lett. B352,1

Introduction

- The neutrinoless double beta decay probability from Fermi Golden-rule

$$T = \int dE_k(\nu) \sum_k \frac{\langle f|\hat{H}|k\rangle\langle k|\hat{H}|i\rangle}{E_{i0^+} - (E_k(e_1^-) + E_k(\nu) + E_{Kern}(k))}$$

- Incorporating above emission Hamiltonians

$$T = M_m \langle m_\nu \rangle + M_\theta \langle tg\vartheta \rangle + M_{WR} < (\frac{M_1}{M_2})^2 > + M_{SUSY} \lambda'^2_{111} + M_{VR} < \frac{m_p}{m_{VR}} > + \ldots$$

Introduction

- The neutrinoless double beta decay probability from Fermi Golden-rule

$$T = \int dE_k(\nu) \sum_k \frac{\langle f | \hat{H} | k \rangle \langle k | \hat{H} | i \rangle}{E_{i0^+} - (E_k(e_1^-) + E_k(\nu) + E_{Kern}(k))}$$

- Incorporating above emission Hamiltonians

$$T = M_m \langle m_\nu \rangle + M_\theta \langle tg\vartheta \rangle + M_{WR} < (\frac{M_1}{M_2})^2 > + M_{SUSY} \lambda'^2_{111} + M_{VR} < \frac{m_p}{m_{VR}} > + \ldots$$

Leading term, I will talk about later

The transition matrix element is as important as the data to determine the absolute scale of neutrino mass

Introduction

- The neutrinoless double beta decay probability from Fermi Golden-rule

$$T = \int dE_k(\nu) \sum_k \frac{\langle f|\hat{H}|k\rangle\langle k|\hat{H}|i\rangle}{E_{i0^+} - (E_k(e_1^-) + E_k(\nu) + E_{Kern}(k))}$$

- Incorporating above emission Hamiltonians

$$T = M_m \langle m_\nu \rangle + M_\theta \langle tg\vartheta \rangle + M_{WR} < (\frac{M_1}{M_2})^2 > + M_{SUSY} \lambda'^2_{111} + M_{VR} < \frac{m_p}{m_{VR}} > + \ldots$$

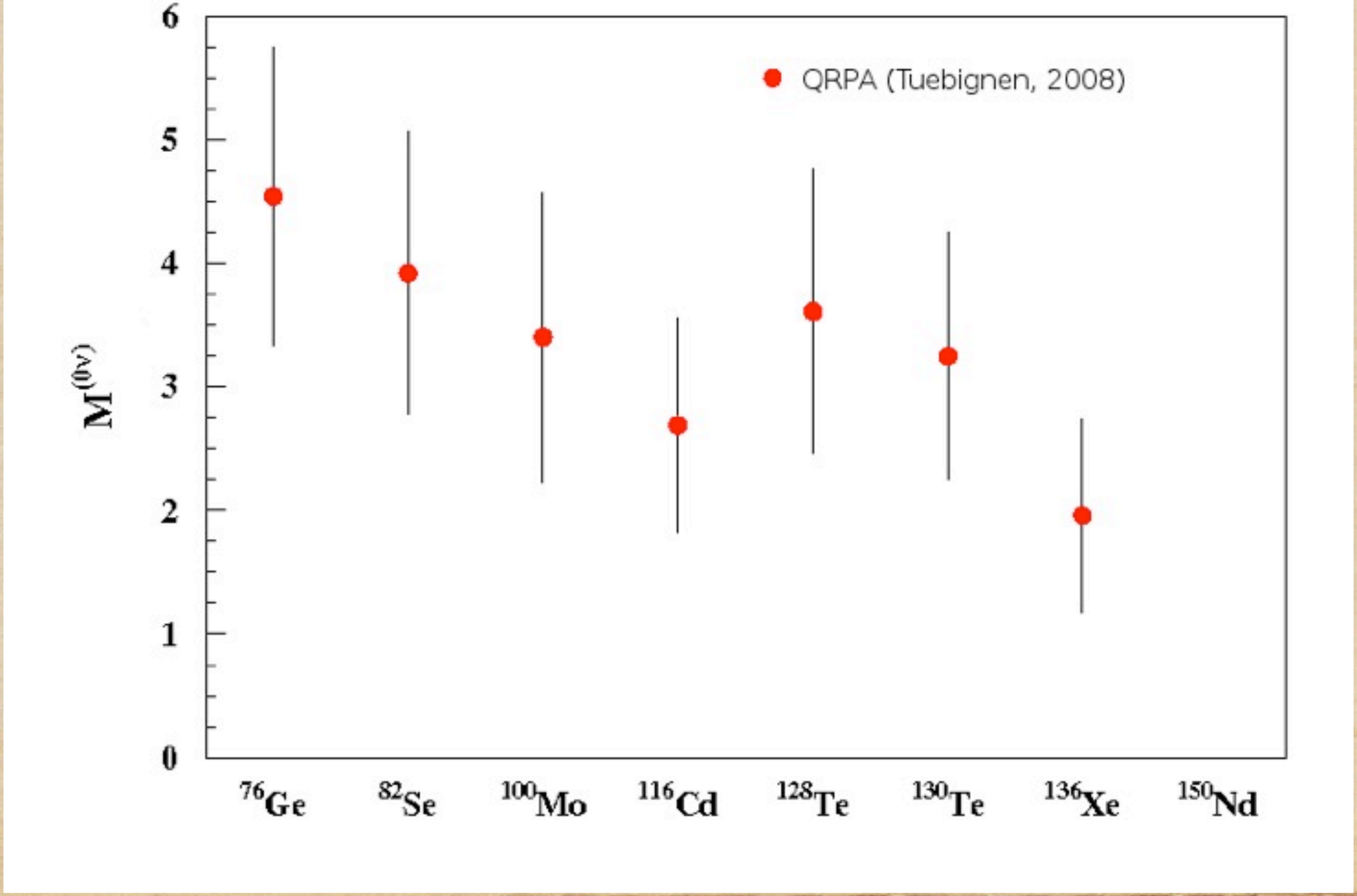
Leading term, I will talk about later

$$\omega = \frac{2\pi}{\hbar} |T|^2 \rho_f \leq 4.4 \times 10^{-33} [\sec^{-1}]\ {}^{128}\mathrm{Te}$$

Introduction

Introduction

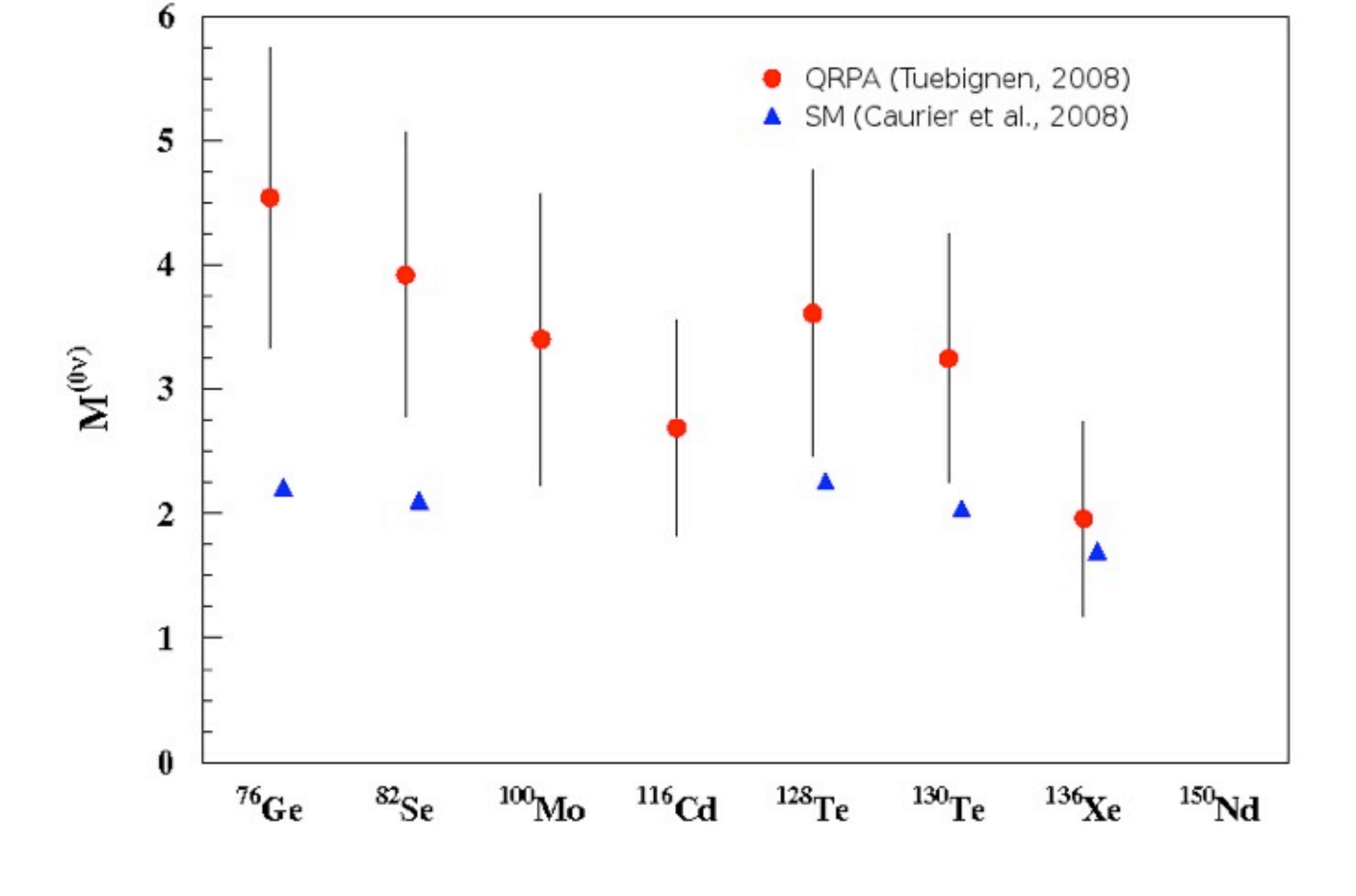
QRPA and RQRPA(Tue) results with errors from basis size,exp. errors of two-neutrino decay, axial coupling constant and short-range correlations



Graphs by S. Schoenert

Introduction

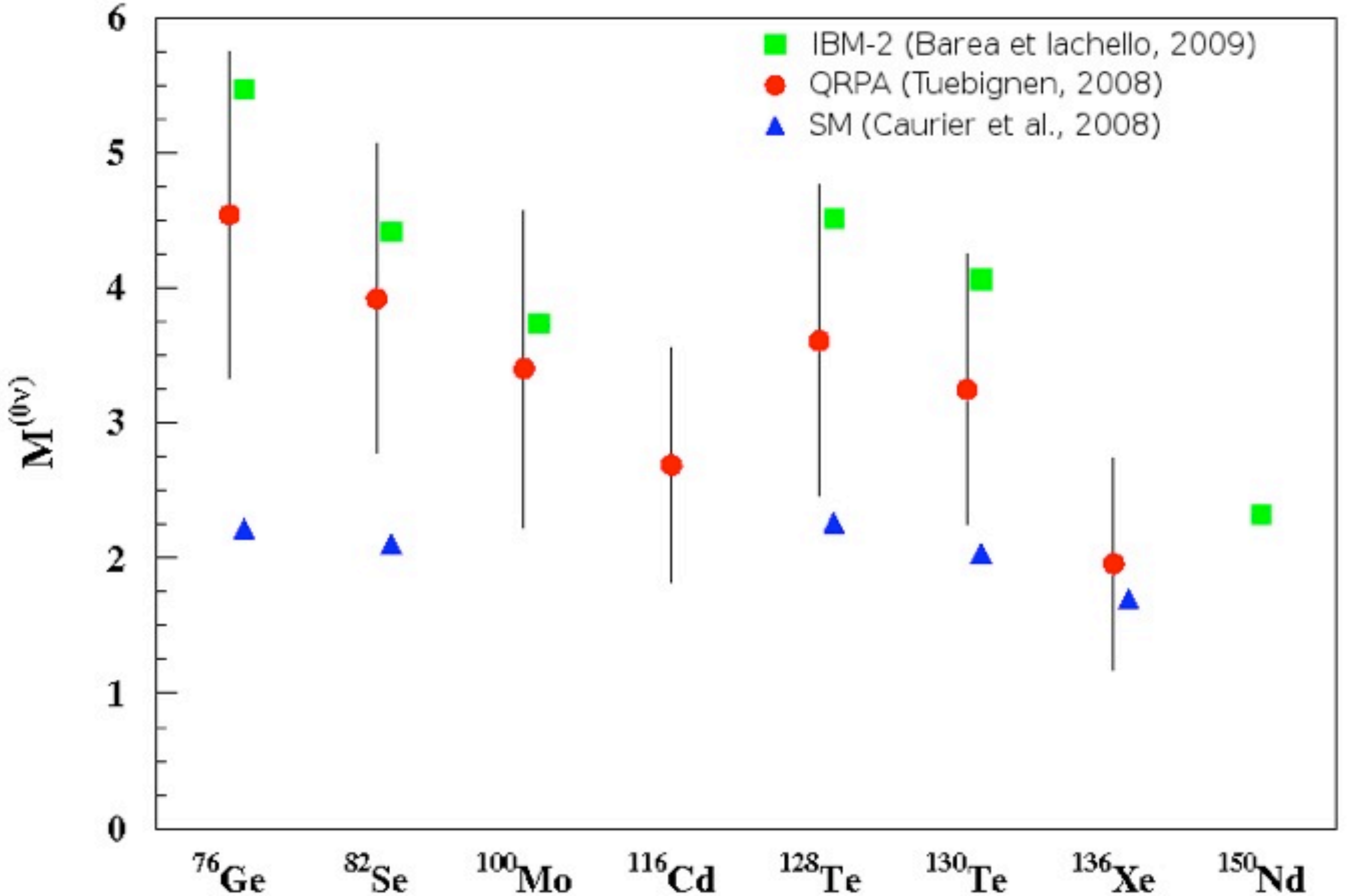
QRPA and RQRPA(Tue) results with errors from basis size,exp. errors of two-neutrino decay, axial coupling constant and short-range correlations



Graphs by S. Schoenert

Introduction

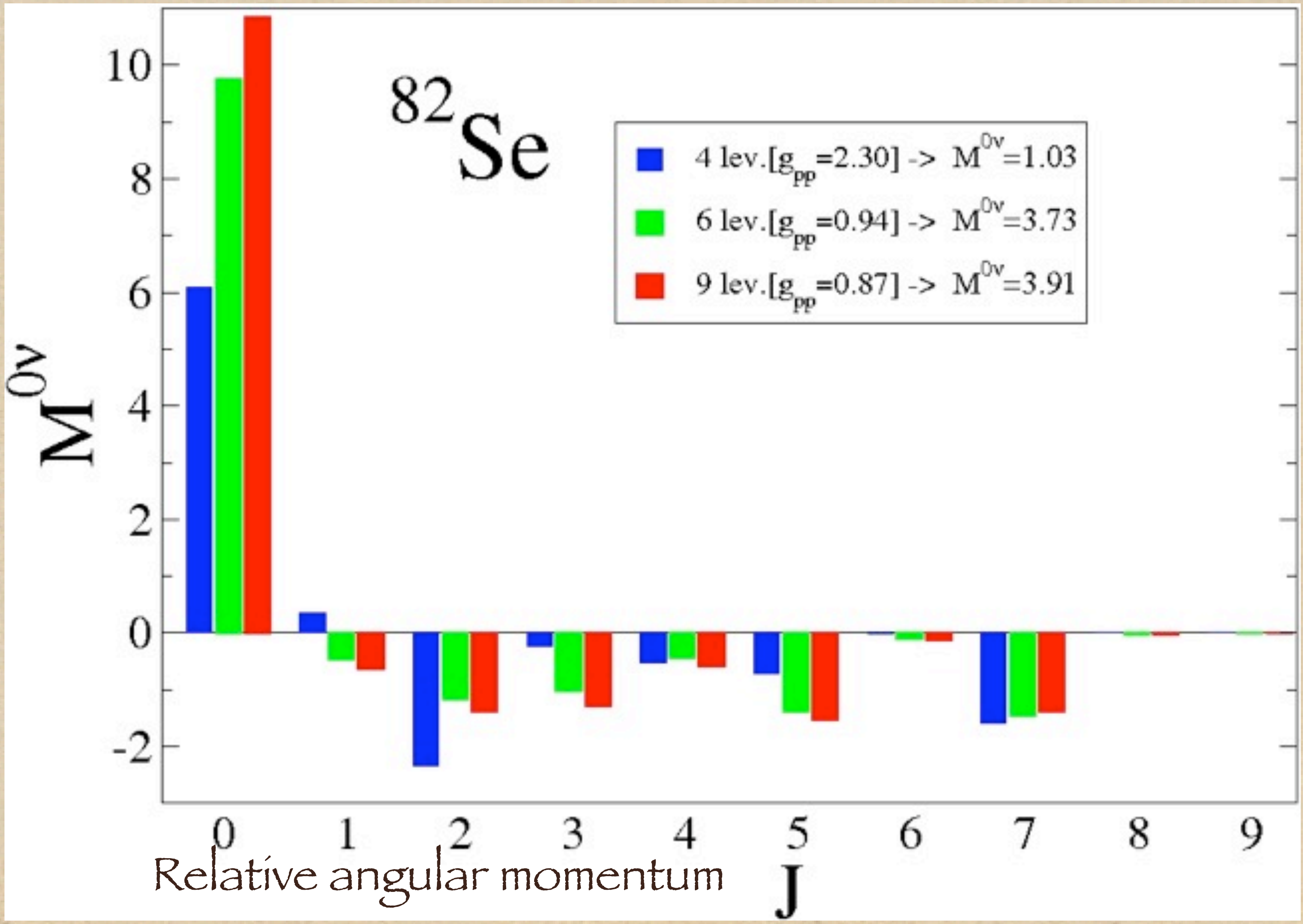
QRPA and RQRPA(Tue) results with errors from basis size,exp. errors of two-neutrino decay, axial coupling constant and short-range correlations



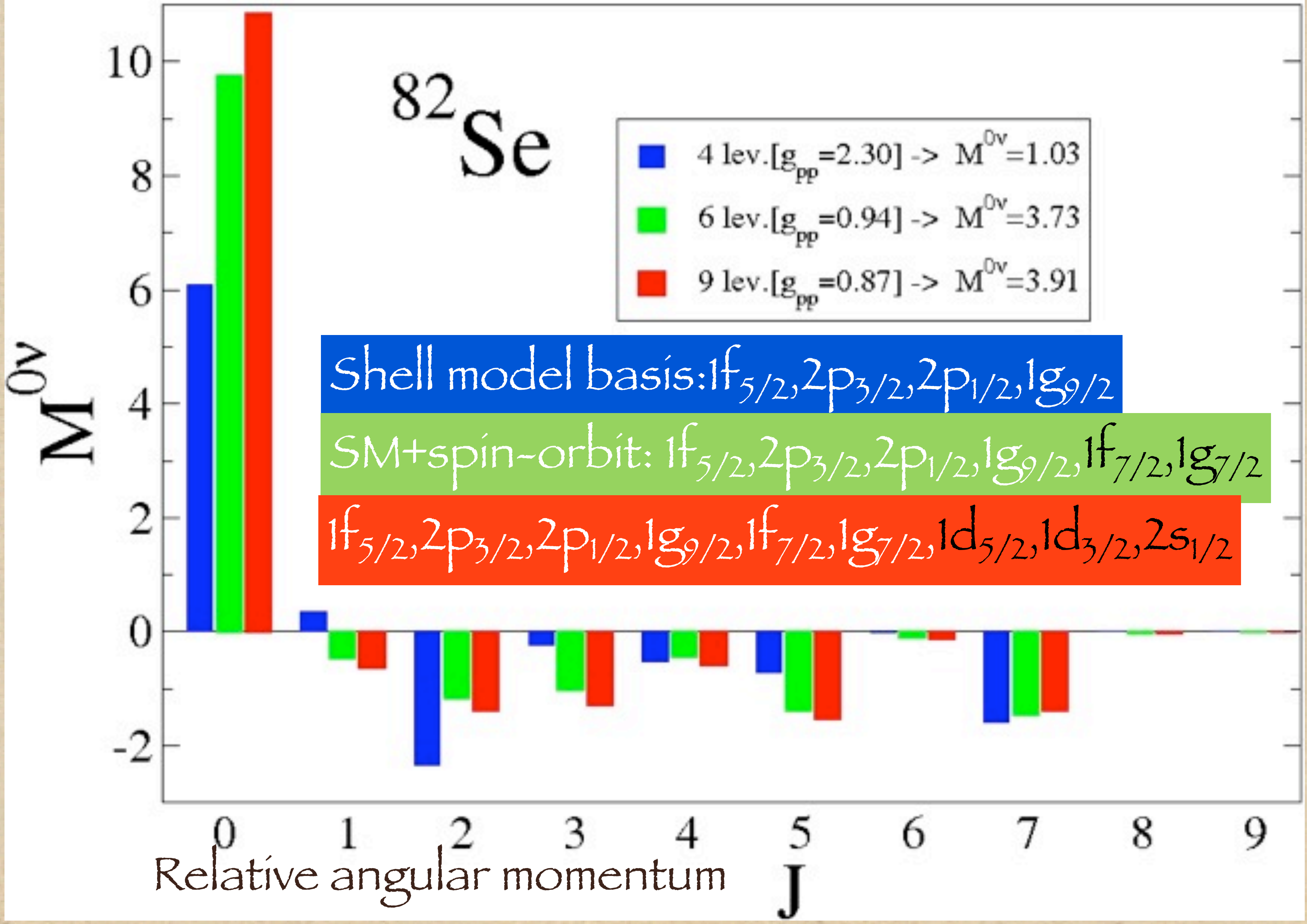
IBM-2: S and D nucleon p and n pairs with SDI mapped on the s and d boson

Graphs by S. Schoenert

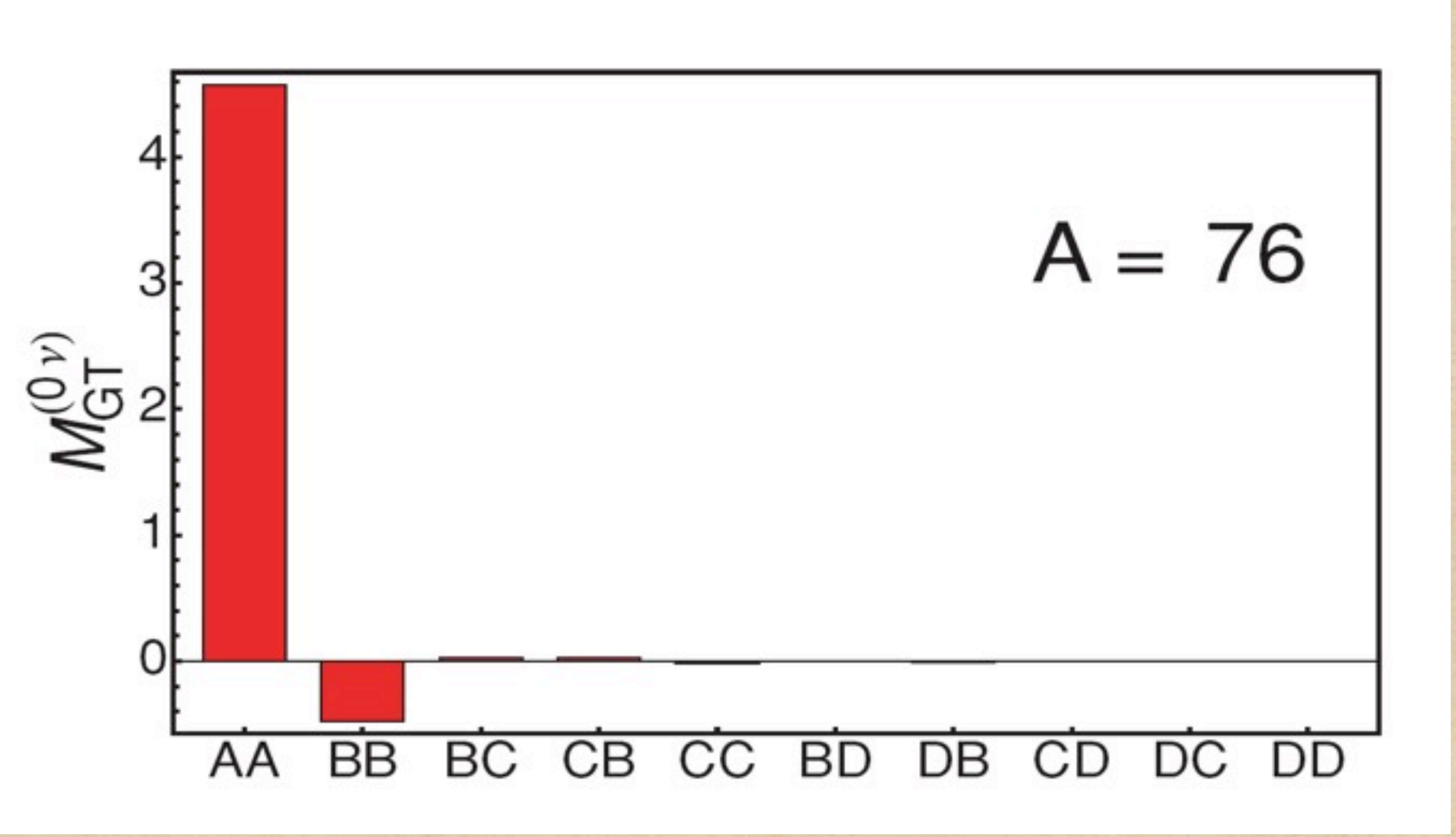
Introduction



Introduction



Introduction



Introduction

- Renormalized QRPA $0\nu\beta\beta$ Half Lives calculated in TUE with the Bonn CD force and Jastrow correlations for quenched $g_A = 1.00$; errors from $2\nu\beta\beta$ experiments for $<m_\nu> = 50$ meV

Nuclei	half life (year)
$^{76}Ge \to ^{76}Se$	$(1.10 \pm 0.13) \times 10^{27}$
$^{82}Se \to ^{82}Kr$	$(3.50 \pm 0.42) \times 10^{26}$
$^{100}Mo \to ^{100}Ru$	$(3.33 \pm 0.45) \times 10^{26}$
$^{128}Te \to ^{128}Xe$	$(7.35 \pm 1.00) \times 10^{27}$
$^{150}Nd \to ^{150}Sm$	$(3.55 \pm 0.50) \times 10^{25}$

Introduction

- Renormalized QRPA $0\nu\beta\beta$ Half Lives calculated in TUE with the Bonn CD force and Jastrow correlations for quenched $g_A = 1.00$; errors from $2\nu\beta\beta$ experiments for $<m_\nu> = 50$ meV

Nuclei	half life (year)
$^{76}Ge \to ^{76}Se$	$(1.10\pm0.13)\times10^{27}$
$^{82}Se \to ^{82}Kr$	$(3.50\pm0.42)\times10^{26}$
$^{100}Mo \to ^{100}Ru$	$(3.33\pm0.45)\times10^{26}$
$^{128}Te \to ^{128}Xe$	$(7.35\pm1.00)\times10^{27}$
$^{150}Nd \to ^{150}Sm$	$(3.55\pm0.50)\times10^{25}$

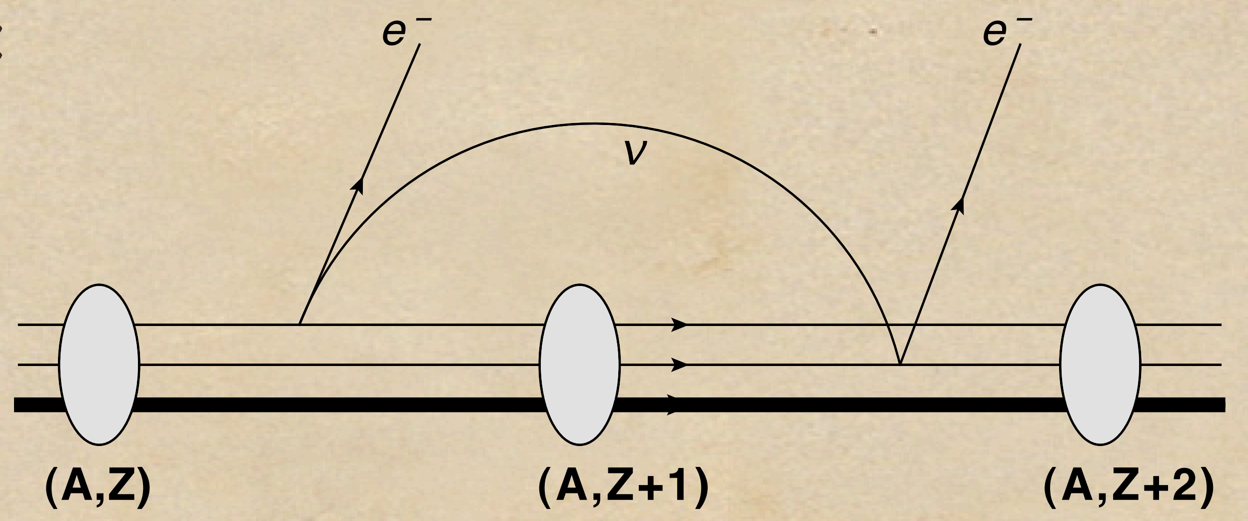
Seems the best experiment candidate , But strongly deformed

Simulation of NME

- Detailed decay width one intermediate Virtual light Majorana neutrino

$$\Gamma_{0\nu} = 2\pi \int\int \sum_{spin} \left| R_{0\nu} \right|^2 \delta(\varepsilon_1 + \varepsilon_2 + E_F - E_I) \frac{d\vec{p}_1}{(2\pi)^3} \frac{d\vec{p}_2}{(2\pi)^3}$$

- With:

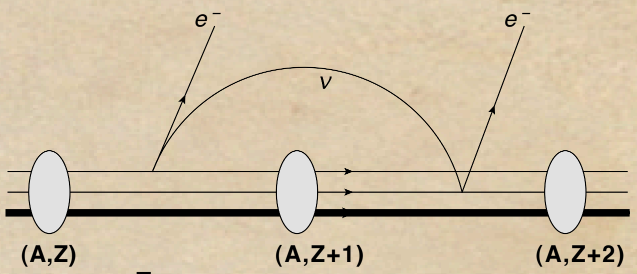


Simulation of NME

- Detailed decay width one intermediate Virtual light Majorana neutrino

$$\Gamma_{0\nu} = 2\pi \int\int \sum_{spin} \left| R_{0\nu} \right|^2 \delta(\varepsilon_1 + \varepsilon_2 + E_F - E_I) \frac{d\vec{p}_1}{(2\pi)^3} \frac{d\vec{p}_2}{(2\pi)^3}$$

- With: $R_{0\nu} = \sqrt{\frac{1}{2}} (\frac{G\cos\theta_C}{\sqrt{2}})^2 \sum_{i=1}^{2n} \sum_M \sum_s \int d\vec{x} d\vec{y}$



$$\times \int \frac{d\vec{k}}{(2\pi)^3} \left\langle f \left| J_i^{\nu+}(y) \right| M \right\rangle \left\langle M \left| J_i^{\mu+}(x) \right| i \right\rangle [1 - P(e_1, e_2)]$$

$$\times \frac{\bar{e}_{p_2 s'_2}(\vec{y}) \gamma_\nu (1-\gamma_5) N_{iks}(\vec{y}) \bar{e}_{p_1 s'_1}(\vec{x}) \gamma_\mu (1-\gamma_5) N_{iks}(\vec{x})}{\varepsilon_1 + \omega + E_M - E_I}$$

Calculation of NME

- And we can get the expression as
  $1/T_{1/2}^{0\nu} = G_{0\nu}\left|M_{0\nu}\right|^2 \left\langle m_\nu \right\rangle_{ee}^2$

Calculation of NME

- And we can get the expression as

$$1/T_{1/2}^{0\nu} = G_{0\nu}\left|M_{0\nu}\right|^2 \left\langle m_\nu \right\rangle_{ee}^2 \longleftarrow \left\langle m_\nu \right\rangle_{ee} = \left|\sum_j U_{ej}^2 m_j\right|$$

Calculation of NME

- And we can get the expression as

$$1/T_{1/2}^{0\nu} = G_{0\nu}\left|M_{0\nu}\right|^2 \left\langle m_\nu \right\rangle_{ee}^2 \longleftarrow \left\langle m_\nu \right\rangle_{ee} = \left|\sum_j U_{ej}^2 m_j\right|$$

phase space integration

Calculation of NME

- And we can get the expression as

$$1/T_{1/2}^{0\nu} = G_{0\nu}\left|M_{0\nu}\right|^2 \langle m_\nu \rangle_{ee}^2 \longleftarrow \langle m_\nu \rangle_{ee} = \left|\sum_j U_{ej}^2 m_j\right|$$

phase space integration

NME

Calculation of NME

- And we can get the expression as

$$1/T_{1/2}^{0\nu} = G_{0\nu}\left|M_{0\nu}\right|^2 \langle m_\nu \rangle_{ee}^2 \longleftarrow \langle m_\nu \rangle_{ee} = \left|\sum_j U_{ej}^2 m_j\right|$$

phase space integration

NME

$$M_{0\nu} = \sum_{M_i,i,J} C_i \langle f | O_J | M_i \rangle \langle H(r_{12}) \rangle_{i,J} \langle M_i | O_J | i \rangle$$

Calculation of NME

- And we can get the expression as

$$1/T_{1/2}^{0\nu} = G_{0\nu}\left|M_{0\nu}\right|^2 \langle m_\nu \rangle_{ee}^2 \longleftarrow \langle m_\nu \rangle_{ee} = \left|\sum_j U_{ej}^2 m_j\right|$$

phase space integration

NME

$$M_{0\nu} = \sum_{M_i,i,J} C_i \langle f | O_J | M_i \rangle \langle H(r_{12}) \rangle_{i,J} \langle M_i | O_J | i \rangle$$

$H(r_{12})$--loop integration over neutrino momentum as a function of distance between two nucleon

Why Deformation

- ^{150}Nd has 60 protons and 90 neutrons, for heavy nuclei,experiments have suggested that it is heavily deformed

	Nuclear reorientation method		BE2 transition probability		Relativistic mean field theory β_2
	deformation β_2	Quad. mom. Q(b)	deformation β_2	Quad. mom. Q(b)	
^{150}Nd	+0.367(86)	-2.00(51)	0.2848(21)	5.258(38)	0.221
^{150}Sm	+0.230(30)	-1.32(19)	0.1931(22)	3.684(41)	0.176

- In the second column, deformation is formulated by $\beta_2 = \sqrt{\frac{\pi}{5}} \frac{Q_p}{Zr_c^2}$
- But we tried another way by calculating $\langle Q_{20} \rangle$ with deformed wave function

Why Deformation

- ^{150}Nd has 60 protons and 90 neutrons, for heavy nuclei,experiments have suggested that it is heavily deformed

	Nuclear reorientation method		BE2 transition probability		Relativistic mean field theory β_2
	deformation β_2	Quad. mom. Q(b)	deformation β_2	Quad. mom. Q(b)	
^{150}Nd	+0.367(86)	-2.00(51)	0.2848(21)	5.258(38)	0.221
^{150}Sm	+0.230(30)	-1.32(19)	0.1931(22)	3.684(41)	0.176

- In the second column, deformation is formulated by $\beta_2 = \sqrt{\frac{\pi}{5}}\frac{Q_p}{Zr_c^2}$
- But we tried another way by calculating $\langle Q_{20} \rangle$ with deformed wave function

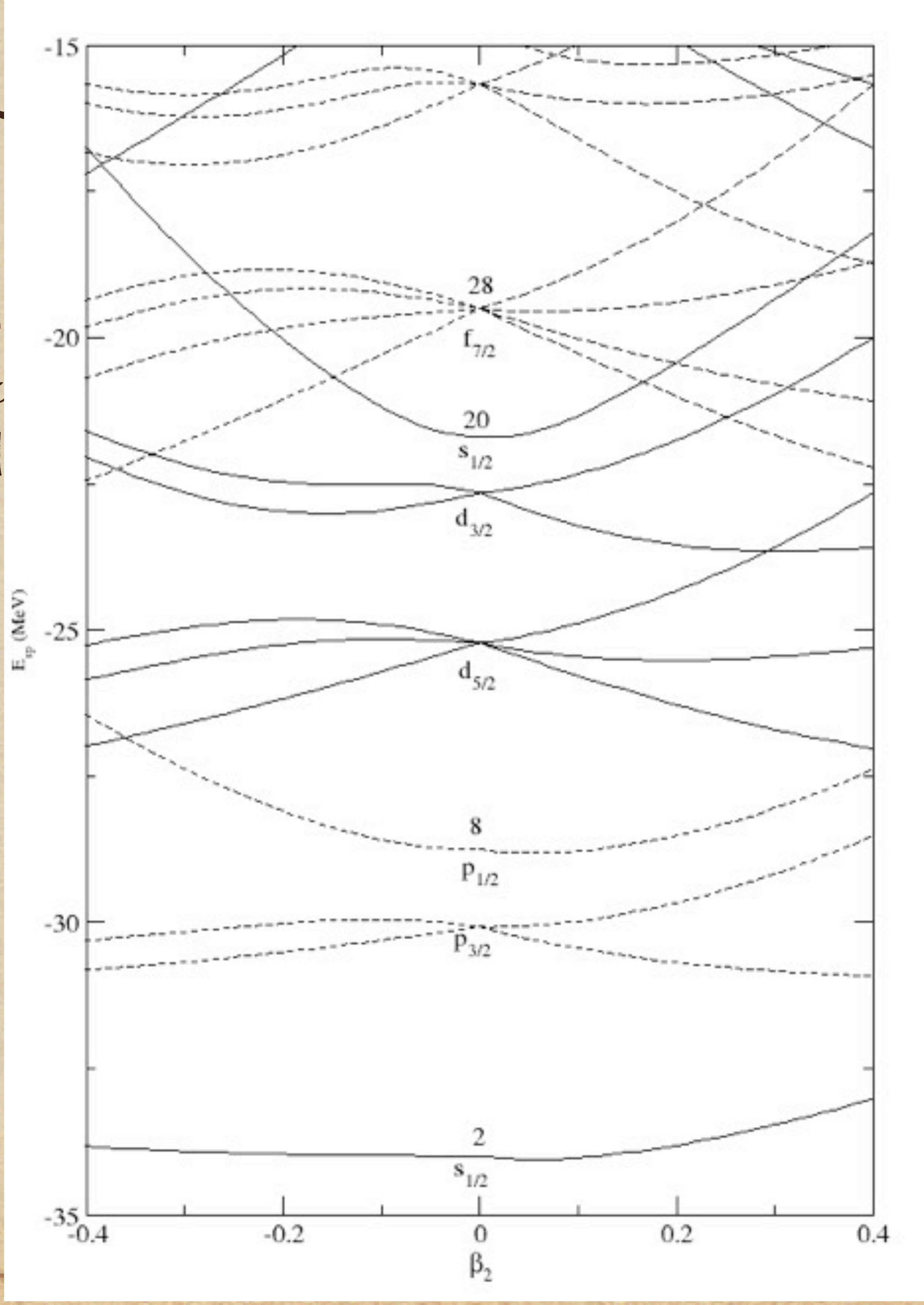
^{150}Nd	0.183
^{150}Sm	0.114

Ground states

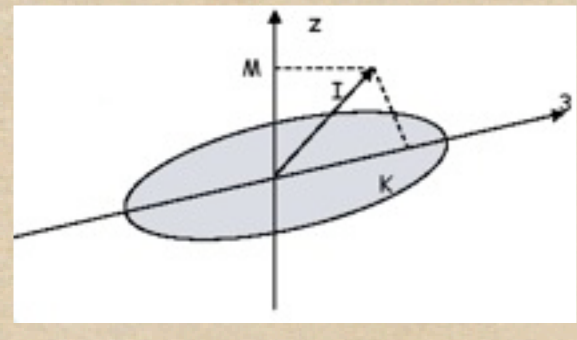
- So from now on we will work in deformed basis and using deformed wave function obtained by solution of Schroedinger Equation with deformed Woods-Saxon potential, the single particle wave can be written as

Grou

- So from n ng deformed wave func equation with deformed wave can be written as



Ground states

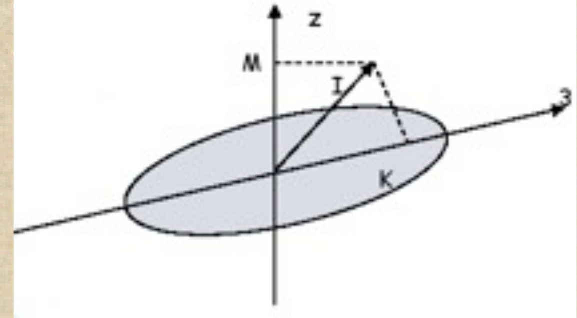


- So from now on we will work in deformed basis and using deformed wave function obtained by solution of Schroedinger Equation with deformed Woods-Saxon potential, the single particle wave can be written as

$$\left|\tau\rho_\tau\right\rangle = \sum_{N_d n_z} [b^{(+)}_{Nn_z\Omega_\tau} \left|N_d n_z \Lambda_\tau, \Omega_\tau = \Lambda_\tau + 1/2\right\rangle \left|\Sigma = 1/2\right\rangle$$

$$+ b^{(-)}_{Nn_z\Omega_\tau} \left|N_d n_z \Lambda_\tau + 1, \Omega_\tau = \Lambda_\tau + 1 - 1/2\right\rangle \left|\Sigma = -1/2\right\rangle]$$

Ground states



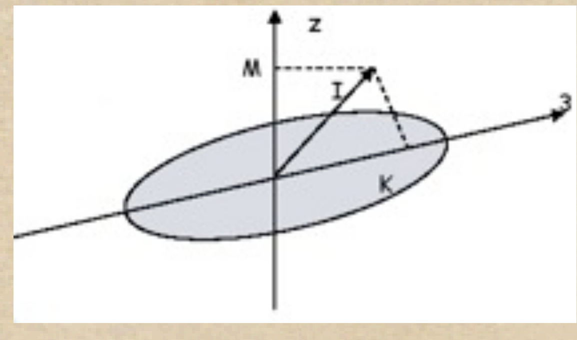
- So from now on we will work in deformed basis and using deformed wave function obtained by solution of Schroedinger Equation with deformed Woods-Saxon potential, the single particle wave can be written as

$$\left|\tau\rho_\tau\right\rangle = \sum_{N_d n_z} [b^{(+)}_{Nn_z\Omega_\tau} \left|N_d n_z \Lambda_\tau, \Omega_\tau = \Lambda_\tau + 1/2\right\rangle \left|\Sigma = 1/2\right\rangle$$

$$+ b^{(-)}_{Nn_z\Omega_\tau} \left|N_d n_z \Lambda_\tau + 1, \Omega_\tau = \Lambda_\tau + 1 - 1/2\right\rangle \left|\Sigma = -1/2\right\rangle]$$

Deformed Harmonic Oscillator w.f. Basis

Ground states



- So from now on we will work in deformed basis and using deformed wave function obtained by solution of Schroedinger Equation with deformed Woods-Saxon potential, the single particle wave can be written as

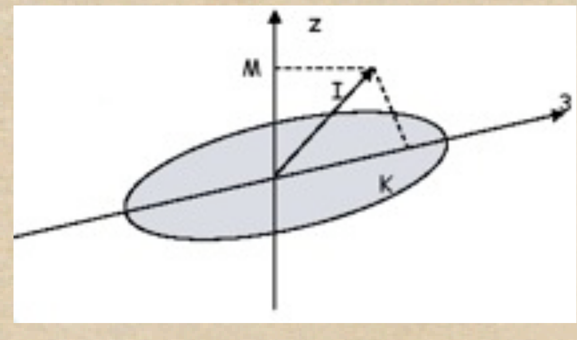
$$\left|\tau\rho_\tau\right\rangle = \sum_{N_d n_z} [b^{(+)}_{Nn_z\Omega_\tau} \left|N_d n_z \Lambda_\tau, \Omega_\tau = \Lambda_\tau + 1/2\right\rangle \left|\Sigma = 1/2\right\rangle$$

$$+ b^{(-)}_{Nn_z\Omega_\tau} \left|N_d n_z \Lambda_\tau + 1, \Omega_\tau = \Lambda_\tau + 1 - 1/2\right\rangle \left|\Sigma = -1/2\right\rangle]$$

Deformed Harmonic Oscillator w.f. Basis

spin w.f. basis

Ground states



- So from now on we will work in deformed basis and using deformed wave function obtained by solution of Schroedinger Equation with deformed Woods-Saxon potential, the single particle wave can be written as

$$\left|\tau\rho_\tau\right\rangle = \sum_{N_d n_z} [b^{(+)}_{N n_z \Omega_\tau} \left|N_d n_z \Lambda_\tau, \Omega_\tau = \Lambda_\tau + 1/2\right\rangle \left|\Sigma = 1/2\right\rangle$$

$$+ b^{(-)}_{N n_z \Omega_\tau} \left|N_d n_z \Lambda_\tau + 1, \Omega_\tau = \Lambda_\tau + 1 - 1/2\right\rangle \left|\Sigma = -1/2\right\rangle]$$

Expansion Coefficient

Deformed Harmonic Oscillator w.f. Basis

spin w.f. basis

Ground states

- As we all know, nuclear pairing plays important role in nuclei, so the initial and final ground states can be described as:

$$|0\rangle = \prod_{\rho_\tau} \alpha_{\rho_\tau} |vaccum\rangle$$

- Quasi-particle operator is defined as

$$\begin{pmatrix} \alpha^\dagger_{\rho_\tau} \\ \tilde{\alpha}_{\rho_\tau} \end{pmatrix} = \begin{pmatrix} u_{\rho_\tau} & v_{\rho_\tau} \\ -v_{\rho_\tau} & u_{\rho_\tau} \end{pmatrix} \begin{pmatrix} c^\dagger_{\rho_\tau} \\ \tilde{c}_{\rho_\tau} \end{pmatrix}$$

- u and v are BCS coefficients which can be derived from the BCS equation, these coefficients determine the structure of the initial and final ground states

Intermediate states

- The intermediate states are constructed using pn-QRPA formalism, by exciting a quasi-neutron to a quasi-proton
- The phonon operator can be defined as

$$Q_K^{m\dagger} = \sum_i (X_{i,K}^m A_i^\dagger(K^\pi) - Y_{i,K}^m \tilde{A}_i(K^\pi))$$

- Intermediate states in lab system defined as

Intermediate states

- The intermediate states are constructed using pn-QRPA formalism, by exciting a quasi-neutron to a quasi-proton
- The phonon operator can be defined as

$$Q_K^{m\dagger} = \sum_i (X_{i,K}^m A_i^\dagger(K^\pi) - Y_{i,K}^m \tilde{A}_i(K^\pi)) \quad A_i^\dagger(K^\pi) = \alpha_{\rho_p}^\dagger \tilde{\alpha}_{\rho_n}^\dagger (\Omega_n - \Omega_p = K, \pi_p \pi_n = \pi)$$

- Intermediate states in lab system defined as

Intermediate states

- The intermediate states are constructed using pn-QRPA formalism, by exciting a quasi-neutron to a quasi-proton
- The phonon operator can be defined as

$$Q_K^{m\dagger} = \sum_i (X_{i,K}^m A_i^\dagger(K^\pi) - Y_{i,K}^m \tilde{A}_i(K^\pi)) \quad A_i^\dagger(K^\pi) = \alpha_{\rho_p}^\dagger \tilde{\alpha}_{\rho_n}^\dagger (\Omega_n - \Omega_p = K, \pi_p \pi_n = \pi)$$

- Intermediate states in lab system defined as

$$|JM(K),m\rangle = \sqrt{\frac{3}{16\pi^2}}[D_{MK}^J(\varphi,\theta,\psi)Q_K^{m\dagger} + (-1)^{J+K} D_{M-K}^J(\varphi,\theta,\psi)Q_{-K}^{m\dagger}]|RPA\rangle (K \neq 0)$$

$$|JM(K),m\rangle = \sqrt{\frac{3}{8\pi^2}} D_{MK}^J(\varphi,\theta,\psi)Q_K^{m\dagger}|RPA\rangle (K = 0)$$

QRPA Formalism

- Using variational method we can get the QRPA(Quasi-particle Random Phase Approximation) equation

$$\begin{pmatrix} \mathtt{A}(K) & \mathtt{B}(K) \\ -\mathtt{B}(K) & -\mathtt{A}(K) \end{pmatrix} \begin{pmatrix} X_K^m \\ Y_K^m \end{pmatrix} = \omega_K^m \begin{pmatrix} X_K^m \\ Y_K^m \end{pmatrix}$$

- With

$$\mathtt{A}_{ij}(K) = \langle RPA | [A_i, [H, A_j^\dagger]] | RPA \rangle$$

$$B_{ij}(K) = \langle RPA | [A_i, [H, \tilde{A}_j]] | RPA \rangle$$

QRPA Formalism

- The Hamiltonian has the form

$$H = H_0 + H_{\text{int}}$$

- $H_0 = \sum_{\tau,\rho_\tau} \varepsilon_{\rho_\tau} c^\dagger_{\rho_\tau} c_{\rho_\tau}$ (single particle energy)
- $H_{\text{int}} = \frac{1}{2} \sum_{pnp'n'} V_{pnp'n'} c^\dagger_p c^\dagger_n c_{n'} c_{p'}$ (residual two-body interaction)

QRPA Formalism

- The Hamiltonian has the form

$$H = H_0 + H_{\text{int}}$$

- $H_0 = \sum_{\tau,\rho_\tau} \varepsilon_{\rho_\tau} c^\dagger_{\rho_\tau} c_{\rho_\tau}$ (single particle energy)
- $H_{\text{int}} = \frac{1}{2} \sum_{pnp'n'} V_{pnp'n'} c^\dagger_p c^\dagger_n c_{n'} c_{p'}$ (residual two-body interaction)

$$\mathrm{A}_{p_1 n_1 p_2 n_2}(K) = (E_{p_1} + E_{n_1})\delta_{p_1 p_2}\delta_{n_1 n_2} + g_{pp}[V_{p_1 \tilde{n}_1 p_2 \tilde{n}_2} u_{p_1} u_{n_1} u_{p_2} u_{n_2} + V_{\tilde{p}_1 n_1 \tilde{p}_2 n_2} v_{p_1} v_{n_1} v_{p_2} v_{n_2}]$$
$$-g_{ph}[V_{p_2 n_1 p_1 n_2} u_{p_1} v_{n_1} u_{p_2} v_{n_2} + V_{p_1 n_2 p_2 n_1} v_{p_1} u_{n_1} v_{p_2} u_{n_2}]$$

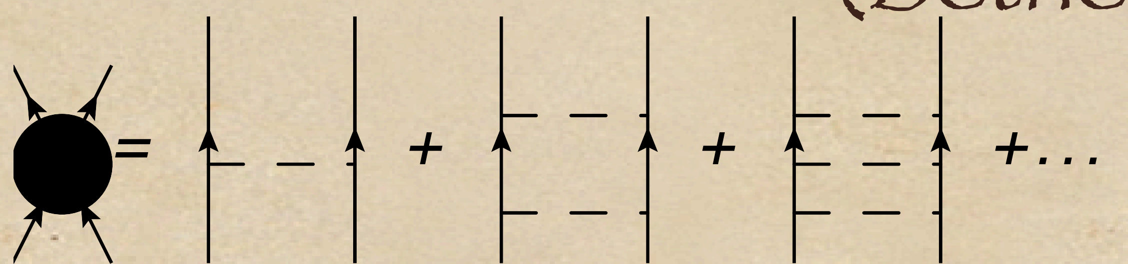
$$\mathrm{B}_{p_1 n_1 p_2 n_2}(K) = -g_{pp}[V_{p_2 \tilde{n}_2 p_1 \tilde{n}_1} v_{p_1} v_{n_1} u_{p_2} u_{n_2} + V_{\tilde{p}_1 n_1 \tilde{p}_2 n_2} u_{p_1} u_{n_1} v_{p_2} v_{n_2}]$$
$$-g_{ph}[V_{p_2 n_1 p_1 n_2} v_{p_1} u_{n_1} u_{p_2} v_{n_2} + V_{p_1 n_2 p_2 n_1} u_{p_1} v_{n_1} v_{p_2} u_{n_2}]$$

QRPA Formalism

- Choice of residual interaction: Realistic Interaction (less parameters)
- (Bethe-Goldstein Equation)
- From spherical basis to deformed basis

QRPA Formalism

- Choice of residual interaction: Realistic Interaction (less parameters)
- (Bethe-Goldstein Equation)



- From spherical basis to deformed basis

QRPA Formalism

- Choice of residual interaction: Realistic Interaction (less parameters)
- $$G = V + V \frac{Q}{W - H_0 + i\varepsilon} G$$ (Bethe-Goldstein Equation)
- From spherical basis to deformed basis

QRPA Formalism

- Choice of residual interaction: Realistic Interaction (less parameters)
- $G = V + V \dfrac{Q}{W - H_0 + i\varepsilon} G$ (Bethe-Goldstein Equation)

Bare nucleon-nucleon force

Projection operator

- From spherical basis to deformed basis

QRPA Formalism

- Choice of residual interaction: Realistic Interaction (less parameters)
- $G = V + V\dfrac{Q}{W - H_0 + i\varepsilon}G$ (Bethe-Goldstein Equation)

Bare nucleon-nucleon force

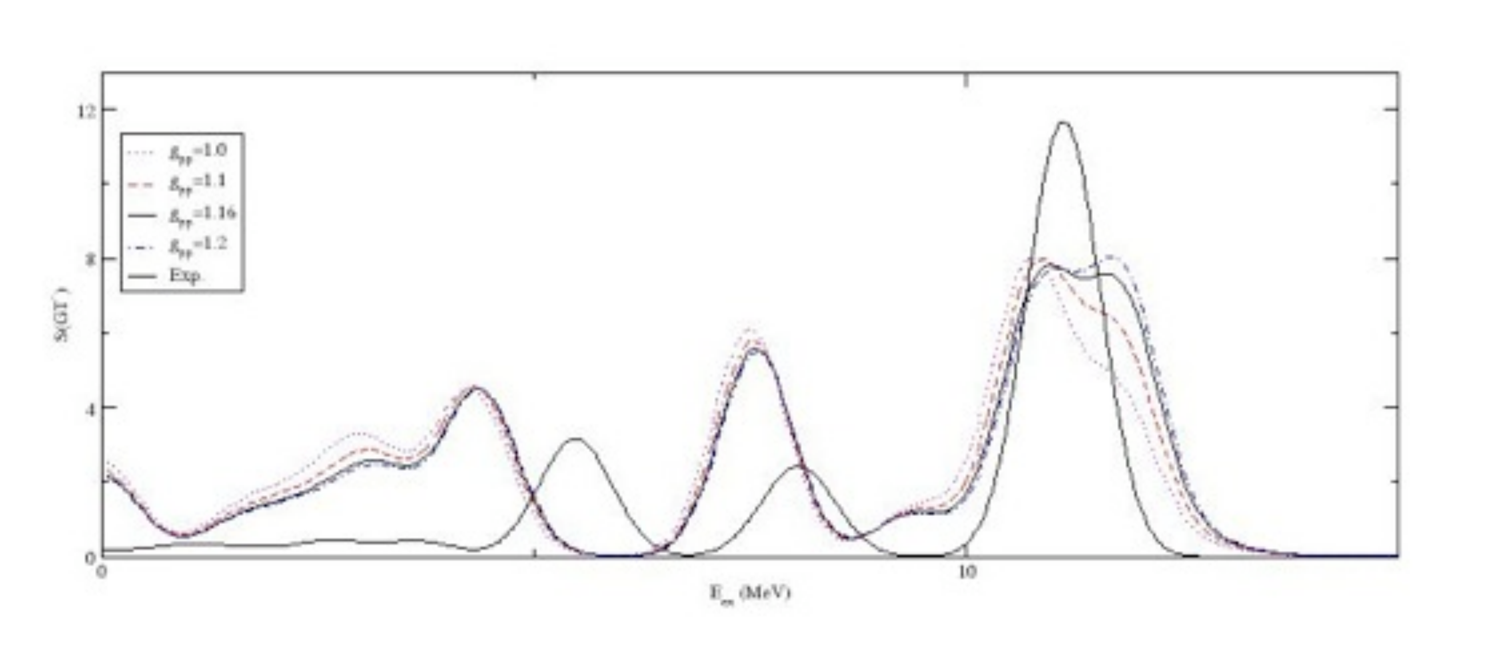
Projection operator

- From spherical basis to deformed basis

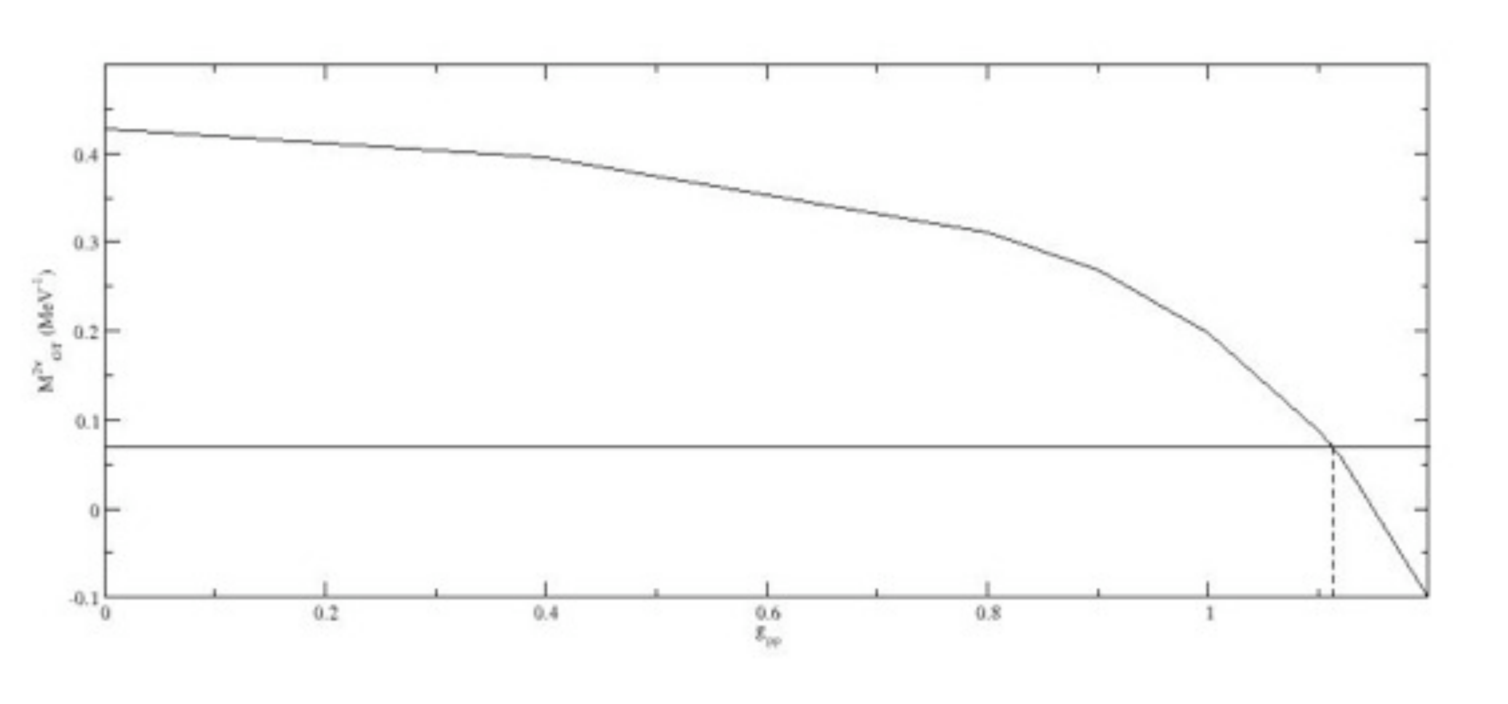
$$V_{p\tilde{n},p'\tilde{n}'} = -2\sum_{J}\sum_{\eta_p\eta_n}\sum_{\eta_{p'}\eta_{n'}} F^{JK}_{p\eta_p n\eta_n} F^{JK}_{p'\eta_{p'}n'\eta_{n'}} G(\eta_p\eta_n\eta_{p'}\eta_{n'},J)$$

$$V_{pn,p'n'} = 2\sum_{J}\sum_{\eta_p\eta_n}\sum_{\eta_{p'}\eta_{n'}} F^{JK'}_{p\eta_p\tilde{n}\eta_n} F^{JK'}_{p'\eta_{p'}\tilde{n}'\eta_{n'}} G(\eta_p\eta_n\eta_{p'}\eta_{n'},J)$$

Choice of Parameters



Particle-hole strength (**g_{ph}=1.16**)



particle-particle strength (**g_{pp}=1.11**)

Calculation of 0νNME

- Final Expression of the nuclear matrix element

$$M = \sum_{K^\pi, ij} \sum_{m_i m_f} \left\langle 0_f^+ \middle| c_{p'}^\dagger c_{n'} \middle| K^\pi m_f \right\rangle \left\langle K^\pi m_f \middle| K^\pi m_i \right\rangle \left\langle K^\pi m_i \middle| c_p^\dagger c_n \middle| 0_i^+ \right\rangle \left\langle p(1), p'(2) \middle| \tau^+ \tau^+ O(12) \middle| n(1) n'(2) \right\rangle$$

$$= \sum_{K^\pi, ij} \sum_{m_i m_f} (u_p v_n X_{m_i}^{K^\pi} + v_p u_n Y_{m_i}^{K^\pi})(u_p v_n Y_{m_f}^{K^\pi} + v_p u_n X_{m_f}^{K^\pi}) \left\langle K^\pi m_f \middle| K^\pi m_i \right\rangle$$

$$\times \sum_{\eta_p \eta_n \eta_{p'} \eta_{n'} J} F_{p\eta_p p' \eta_{p'}}^{JM} F_{n\eta_n n' \eta_{n'}}^{JM} \left\langle \eta_p \eta_{p'}, JM \middle| \tau^+ \tau^+ O(12) \middle| \eta_n \eta_{n'}, JM \right\rangle$$

Calculation of 0vNME

- Final Expression of the nuclear matrix element

$$M = \sum_{K^\pi, ij} \sum_{m_i m_f} \left\langle 0_f^+ \middle| c_{p'}^\dagger c_{n'} \middle| K^\pi m_f \right\rangle \left\langle K^\pi m_f \middle| K^\pi m_i \right\rangle \left\langle K^\pi m_i \middle| c_p^\dagger c_n \middle| 0_i^+ \right\rangle \langle p(1), p'(2) | \tau^+ \tau^+ O(12) | n(1) n'(2) \rangle$$

$$= \sum_{K^\pi, ij} \sum_{m_i m_f} (u_p v_n X_{m_i}^{K^\pi} + v_p u_n Y_{m_i}^{K^\pi})(u_p v_n Y_{m_f}^{K^\pi} + v_p u_n X_{m_f}^{K^\pi}) \left\langle K^\pi m_f \middle| K^\pi m_i \right\rangle$$

$$\times \sum_{\eta_p \eta_n \eta_{p'} \eta_{n'} J} F_{p\eta_p p'\eta_{p'}}^{JM} F_{n\eta_n n'\eta_{n'}}^{JM} \left\langle \eta_p \eta_{p'}, JM \middle| \tau^+ \tau^+ O(12) \middle| \eta_n \eta_{n'}, JM \right\rangle \qquad \left\langle K^\pi m_f \middle| K^\pi m_i \right\rangle = f(X, Y, u, v)$$

Calculation of 0vNME

- Final Expression of the nuclear matrix element

$$M = \sum_{K^\pi, ij} \sum_{m_i m_f} \langle 0_f^+ | c_{p'}^\dagger c_{n'} | K^\pi m_f \rangle \langle K^\pi m_f | K^\pi m_i \rangle \langle K^\pi m_i | c_p^\dagger c_n | 0_i^+ \rangle \langle p(1), p'(2) | \tau^+ \tau^+ O(12) | n(1) n'(2) \rangle$$

$$= \sum_{K^\pi, ij} \sum_{m_i m_f} (u_p v_n X_{m_i}^{K^\pi} + v_p u_n Y_{m_i}^{K^\pi})(u_p v_n Y_{m_f}^{K^\pi} + v_p u_n X_{m_f}^{K^\pi}) \langle K^\pi m_f | K^\pi m_i \rangle$$

$$\times \sum_{\eta_p \eta_n \eta_{p'} \eta_{n'} J} F_{p\eta_p p'\eta_{p'}}^{JM} F_{n\eta_n n'\eta_{n'}}^{JM} \langle \eta_p \eta_{p'}, JM | \tau^+ \tau^+ O(12) | \eta_n \eta_{n'}, JM \rangle$$

$$\langle K^\pi m_f | K^\pi m_i \rangle = f(X, Y, u, v)$$

$$O(12) = H_F(r_{12}) + H_{GT}(r_{12})\sigma_{12} + H_T(r_{12}) S_{12}$$

$$r_{12} = |\vec{r}_1 - \vec{r}_2|, \sigma_{12} = \vec{\sigma}_1 \bullet \vec{\sigma}_2$$

$$S_{12} = 3(\vec{\sigma}_1 \bullet \hat{r}_{12})(\vec{\sigma}_2 \bullet \hat{r}_{12}) - \sigma_{12}$$

Calculation of 0νNME

- Final Expression of the nuclear matrix element

$$M = \sum_{K^\pi, ij}\sum_{m_i m_f} \left\langle 0_f^+ \middle| c_{p'}^\dagger c_{n'} \middle| K^\pi m_f \right\rangle \left\langle K^\pi m_f \middle| K^\pi m_i \right\rangle \left\langle K^\pi m_i \middle| c_p^\dagger c_n \middle| 0_i^+ \right\rangle \left\langle p(1), p'(2) \middle| \tau^+\tau^+ O(12) \middle| n(1) n'(2) \right\rangle$$

$$= \sum_{K^\pi, ij}\sum_{m_i m_f} (u_p v_n X_{m_i}^{K^\pi} + v_p u_n Y_{m_i}^{K^\pi})(u_p v_n Y_{m_f}^{K^\pi} + v_p u_n X_{m_f}^{K^\pi}) \left\langle K^\pi m_f \middle| K^\pi m_i \right\rangle$$

$$\times \sum_{\eta_p \eta_n \eta_{p'} \eta_{n'} J} F_{p\eta_p p'\eta_{p'}}^{JM} F_{n\eta_n n'\eta_{n'}}^{JM} \left\langle \eta_p \eta_{p'}, JM \middle| \tau^+\tau^+ O(12) \middle| \eta_n \eta_{n'}, JM \right\rangle$$

$$\left\langle K^\pi m_f \middle| K^\pi m_i \right\rangle = f(X, Y, u, v)$$

$$O(12) = H_F(r_{12}) + H_{GT}(r_{12})\sigma_{12} + H_T(r_{12})S_{12}$$

$$r_{12} = \left|\vec{r}_1 - \vec{r}_2\right|, \sigma_{12} = \vec{\sigma}_1 \bullet \vec{\sigma}_2$$

$$S_{12} = 3(\vec{\sigma}_1 \bullet \hat{r}_{12})(\vec{\sigma}_2 \bullet \hat{r}_{12}) - \sigma_{12}$$

$$H_I(r_{12}) = \frac{2}{\pi g_A^2}\frac{R}{r_{12}}\int_0^\infty \frac{\sin(q r_{12})}{q + \overline{\omega}_{K^\pi m_i m_f}} h_I(q^2) dq$$

$$\overline{\omega}_{m_i m_f}^{K^\pi} = (\omega_{m_i}^{K^\pi} - \omega_{1_i}^{K^\pi} + \omega_{m_f}^{K^\pi} - \omega_{1_f}^{K^\pi})/2$$

$$h_F(q^2) = -g_V^2(q^2) = g_V / (1 + q^2/\Lambda_V^2)^2$$

....

Calculation of 0vNME

- Final Expression of the nuclear matrix element

$$M = \sum_{K^\pi, ij}\sum_{m_i m_f} \langle 0_f^+ | c_{p'}^\dagger c_{n'} | K^\pi m_f \rangle \langle K^\pi m_f | K^\pi m_i \rangle \langle K^\pi m_i | c_p^\dagger c_n | 0_i^+ \rangle \langle p(1), p'(2) | \tau^+ \tau^+ O(12) | n(1) n'(2) \rangle$$

$$= \sum_{K^\pi, ij}\sum_{m_i m_f} (u_p v_n X_{m_i}^{K^\pi} + v_p u_n Y_{m_i}^{K^\pi})(u_p v_n Y_{m_f}^{K^\pi} + v_p u_n X_{m_f}^{K^\pi}) \langle K^\pi m_f | K^\pi m_i \rangle$$

$$\times \sum_{\eta_p \eta_n \eta_{p'} \eta_{n'} J} F_{p\eta_p p'\eta_{p'}}^{JM} F_{n\eta_n n'\eta_{n'}}^{JM} \langle \eta_p \eta_{p'}, JM | \tau^+ \tau^+ O(12) | \eta_n \eta_{n'}, JM \rangle$$

$$\langle K^\pi m_f | K^\pi m_i \rangle = f(X, Y, u, v)$$

$$O(12) = H_F(r_{12}) + H_{GT}(r_{12})\sigma_{12} + H_T(r_{12}) S_{12}$$

$$r_{12} = |\vec{r}_1 - \vec{r}_2|, \sigma_{12} = \vec{\sigma}_1 \bullet \vec{\sigma}_2$$

$$S_{12} = 3(\vec{\sigma}_1 \bullet \hat{r}_{12})(\vec{\sigma}_2 \bullet \hat{r}_{12}) - \sigma_{12}$$

$$H_I(r_{12}) = \frac{2}{\pi g_A^2} \frac{R}{r_{12}} \int_0^\infty \frac{\sin(q r_{12})}{q + \overline{\omega}_{K^\pi m_i m_f}} h_I(q^2) dq$$

$$\overline{\omega}_{m_i m_f}^{K^\pi} = \varpi(const.)$$

Realistic consideration

$$h_F(q^2) = -g_V^2(q^2) = g_V / (1 + q^2 / \Lambda_V^2)^2$$

....

Conclusion & Outlook

- The determination of the nuclear matrix element of double beta decay is as important as the measurement of the half lives to get the mass scale of the neutrino
- We have developed the pn-QRPA method which is suitable for the calculation of NME and extended it to the deformed case with the realistic residual interaction. We now have the QRPA solution in hand.
- The last step left is to transform available result from the spherical case to the deformed one and get more accurate value for heavy nuclei

Thank you !