Proposed antimatter gravity measurement with an antihydrogen beam

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Available online 15 December 2007

Abstract

The principle of the equivalence of gravitational and inertial mass is one of the cornerstones of general relativity. Considerable efforts have been made and are still being made to verify its validity. A quantum-mechanical formulation of gravity allows for non-Newtonian contributions to the force which might lead to a difference in the gravitational force on matter and antimatter. While it is widely expected that the gravitational interaction of matter and of antimatter should be identical, this assertion has never been tested experimentally. With the production of large amounts of cold antihydrogen at the CERN Antiproton Decelerator, such a test with neutral antimatter atoms has now become feasible. For this purpose, we have proposed to set up the AEGIS experiment at

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CERN/AD, whose primary goal will be the direct measurement of the Earth's gravitational acceleration on antihydrogen with a classical Moiré deflectometer.

1. Introduction

General relativity and quantum mechanics are two of the very foundations of modern physics. Yet, as was pointed out early on by Wigner [1], the two are to some degree incompatible. The most obvious discrepancy is the dynamic spacetime geometry of general relativity, which clashes with the fixed-background approach of quantum field theory. In order to repair some of these perceived contradictions and to properly incorporate gravity into the Standard Model, some scientists actively pursue a unification of gravity with the other fundamental interactions within what has been called a "Theory of Everything" [2].

A hypothetical quantum theory of gravitation necessarily constitutes a departure from the Einsteinian view of gravity as a geometric phenomenon. As in other quantum field theories, the interaction is mediated by exchange particles. The spins of these exchange bosons as well as the signs of the charges to which they couple determine whether a force is repulsive or attractive. Generally speaking, even-spin particles create an attractive force between all types of charges, whereas the exchange of odd-spin particles leads to a repulsive force between like charges. Hence, the formulation of a quantum theory of gravitation automatically brings about the possibility of different types of exchange particles as well as negative mass charge. Attempts at formulating a quantum gravity have mainly been hampered by the fact that such a theory is non-renormalizable, though a renormalization within the framework of an effective field theory may turn out to be feasible [3].

When constructing such a theory, ordinary "Newtonian" gravity is associated with a massless tensor (spin-2) exchange boson, as the force has an infinite range and is always attractive. In addition to this tensor part, gravity could have scalar (spin-0) and/or vector (spin-1) components. Unlike the tensor and scalar parts, a vector component would lead to a repulsive force acting between like charges. Such a force would thus produce a dramatic effect on antimatter particles in the Earth's gravitational field and constitute a violation of the weak equivalence principle (also called the universality of free fall).

There are a number of arguments against "anti-gravity," *i.e.* a tensor-type gravitational interaction with opposite sign for antimatter [4–7]. The most intuitive of these is Morrison's argument [8], which elegantly demonstrates that such a phenomenon would violate conservation of energy. Quantitative limits on possible anti-tensor gravity effects can be obtained, among others, from estimates of the effects of virtual antiparticles in ordinary matter [9,10] or the absence of changes in the cyclotron frequency of antiprotons ($\bar{p}$) confined in a Penning trap [11]. Most, if not all, of these arguments do not apply to more elaborate models involving vector and scalar gravitons. If the hypothetical vector and scalar charges, as well as their masses (and thus the ranges of the interactions) are carefully chosen, such contributions can be strongly suppressed in ordinary matter. Finally, it should be stated that the problem of the gravitational interaction of antimatter is completely independent from the – equally fascinating – question of matter–antimatter symmetry (CPT), as CPT invariance merely dictates the equality of the inertial masses of particle and antiparticle pairs, but places no restriction on the gravitational masses.

Unlike ordinary matter [12], the behavior of antimatter particles in a gravitational field has never been tested experimentally. Two attempts, at Stanford [13] and CERN's Low-Energy Antiproton Ring [14] were thwarted by the overwhelming effect of stray electric and magnetic fields upon the electrically charged test particles. The recent production of copious amounts of cold antihydrogen ($\overline{H}$) at CERN's Antiproton Decelerator (AD) [15,16] has paved the way for high-precision gravity experiments with neutral antimatter. We have proposed the AEGIS experiment (Antimatter Experiment: Gravity, Interferometry, Spectroscopy), to be performed at CERN/AD, in order to address this important question.

The primary scientific goal of AEGIS is the direct measurement of the Earth's local gravitational acceleration $\bar{g}$ on $\overline{H}$. In a first phase of the experiment, a gravity measurement with 1% relative precision will be carried out by observing the vertical displacement of the shadow image produced by an $\overline{H}$ beam as it traverses a Moiré deflectometer, the classical counterpart of a matter wave interferometer. In spite of its limited precision, this measurement will represent the first direct determination of the gravitational effect on antimatter.

The essential steps leading to the production of $\overline{H}$ and the measurement of $\bar{g}$ with AEGIS are the following:

– Production of positrons (e^+) from a Surko-type source and accumulator;
– Capture and accumulation of $\bar{p}$ from the AD in a cylindrical Penning trap;
– Production of positronium (Ps) by bombardment of a nanoporous material with an intense e^+ pulse;

– Excitation of the Ps to a Rydberg state with principal quantum number $n = 30 \ldots 40$;
– Recombination of $\overline{H}$ by resonant charge exchange between Rydberg Ps and cold $\overline{p}$;
– Formation of an $\overline{H}$ beam by Stark acceleration with inhomogeneous electric fields;
– Determination of $\overline{g}$ in a two-grating Moiré deflectometer coupled with a position-sensitive detector.

The feasibility of the first two points has been conclusively demonstrated by the ATHENA and ATRAP collaborations (see, in particular, [17,18]). In the following, we will discuss the remaining aspects of the proposed technique in more detail.

2. Method

2.1. Positronium production and excitation

In recent years, the potential for nanoporous insulator materials to be used as highly efficient Ps converters has been recognized, and the relevant formation mechanisms have been studied extensively [19]. When e^+ are implanted into such a material at kinetic energies ranging from several 100 eV to a few keV, they scatter off atoms and electrons (e^-) in the bulk and are slowed to eV energies within a few ps. With efficiencies ranging from 10% to 50%, the slow e^+ capture either bound e^- or those liberated in prior collisions and form Ps. These tend to accumulate in defects of the material due to the reduced dielectric strength and hence increased Ps binding energy. In the pores, Ps repeatedly bounces off the cavity walls and eventually approaches complete thermalization with the target material.

While some *ortho*-Ps are lost due to so-called pick-off annihilations of e^+ with the molecular e^- of the cavity walls, a sizable fraction diffuses out of the film at thermal energies. The overall *ortho*-Ps yield as well as the final velocity distribution depend upon the characteristics of the target material (in particular, its pore structure), the implantation depth, and the target temperature. Measurements using positron annihilation lifetime spectroscopy have shown that the *ortho*-Ps fraction (outside the sample) can reach 30% in silicon-based polymer materials cooled to 50 K [20]. In other experimental work, it was shown that the energy profile of Ps emitted from the surface of a silica film at room temperature followed a Maxwell–Boltzmann distribution and was compatible with the Ps being fully thermalized [21]. We are currently conducting experiments in order to determine the optimal converter material and e^+ energy in terms of *ortho*-Ps yield. Furthermore, we are investigating how well the emitted Ps is thermalized at very low target temperatures.

The photo-excitation of Ps to Rydberg states requires photon energies close to the binding energy of 6.8 eV. Laser systems at the corresponding wavelengths (≈ 180 nm) are not commercially available. We are therefore planning to perform a two-step excitation, from the ground state to the $n = 3$ state ($\lambda = 205$ nm), and then to the $n = 35$ Rydberg band ($\lambda \approx 1670$ nm). Two pulsed-laser systems, both of which are pumped by a Q-switched Nd:YAG laser (1064 nm, 4 ns, 140 mJ), are currently under development. Both systems must provide sufficient power to excite the emitted Ps within a few ns and must be geometrically matched to the expanding cloud. Furthermore, the bandwidths of the lasers must be tailored to the transition linewidth broadened due to the Doppler effect as well as level splitting due to the motional Stark effect and the linear and quadratic Zeeman effect (the latter three creating the Rydberg band).

An overview sketch of the planned laser setup is shown in Fig. 1. The first system is a dye laser whose optical cavity uses prisms as selective elements in order to produce the large bandwidth required. After up-conversion with a succession of second and third harmonic crystals, the laser will produce 205 nm radiation with pulse energies of ≈ 200 μJ. The second system combines an optical parametric generator with an amplification stage, both of which make use of commercially available periodically poled crystals (*i.e.* materials with birefringent layers in alternating orientation). In these non-linear media, the pump photons are down-converted to an idler and a signal photon (signal wavelength $1600 \ldots 1700$ nm, pulse energy 300 μJ). The wavelengths can be precisely controlled by adjusting the crystal temperature. Due to the broad bandwidth, the coherence time of the second system will be several orders of magnitude shorter than the pulse length. Nevertheless, excitation fractions in the Rydberg band of 30% or higher can be expected.

2.2. Antihydrogen recombination and beam formation

An $\overline{H}$ recombination scheme based on resonant charge exchange with Ps was first proposed almost twenty years ago [22]. The reaction proceeds according to the equation
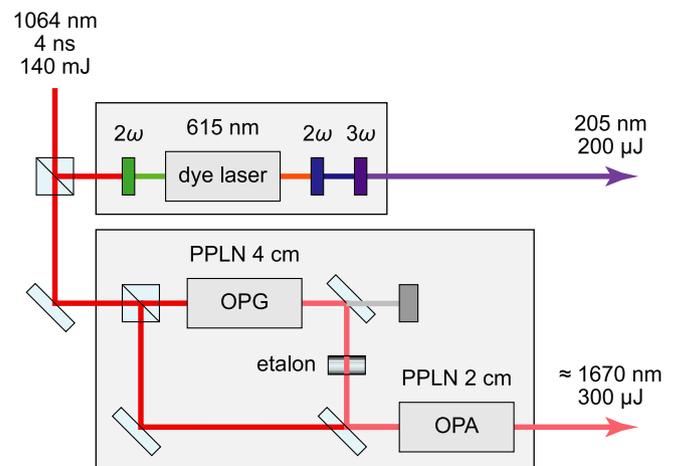
$$Ps^* + \overline{p} \rightarrow \overline{H}^* + e^- \tag{1}$$

Fig. 1. Laser system for the Rydberg excitation of Ps prior to $\overline{H}$ recombination.

where the star denotes a highly excited Rydberg state. This reaction owes its appeal to the fact that the cross-section scales approximately with the fourth power of the principal quantum number. In addition, it creates $\overline{H}$ in a narrow and well-defined band of final states. Most importantly, $\overline{H}$ formed with $\bar{p}$ at rest is created with a velocity distribution dominated by the $\bar{p}$ temperature, hence the surrounding (cryogenic) environment [23]. This is in stark contrast to the rather high $\overline{H}$ temperature observed when using the nested-well technique pioneered by ATRAP and ATHE-NA [24,25]. Our proposed technique is conceptually similar to a charge exchange technique based on Rydberg cesium [26] which has been successfully demonstrated by ATRAP [27].

The principle is illustrated in Fig. 2. The Ps emitted from the porous insulator material are excited to Rydberg states. They then traverse a Penning trap region in which several 10^4 $\bar{p}$ have been accumulated and stored. The charge exchange cross-section is very large ($\approx 10^7$ Å^2 for $n = 35$) and reaches a maximum when the e$^+$ and $\bar{p}$ relative velocities are matched. Taking into account the corresponding kinetic energy, as well as a smaller contribution due to converted internal energy, $\overline{H}$ is created at velocities of $25\dots80$ m s^{-1}.

While neutral atoms are not sensitive (to first order) to constant electric fields, they do experience a force when their electric dipole moment is exposed to an electric-field gradient. Since the dipole moment scales approximately with the square of the principal quantum number, Rydberg atoms are especially amenable to being manipulated in this way. This technique is related to the splitting of spectral lines due to the presence of external electric fields (the Stark effect) and is therefore called Stark acceleration.

Recently, Stark acceleration has been successfully demonstrated by one of the AEGIS groups with (ordinary) hydrogen after excitation to the $n = 22$, 23, 24 states [28,29]. In these experiments, accelerations of 2×10^8 m s^{-2} were achieved using the electrode geometry shown in Fig. 3. A hydrogen beam traveling at 700 m s^{-1} was stopped within 5 μs over a distance of only 1.8 mm. We
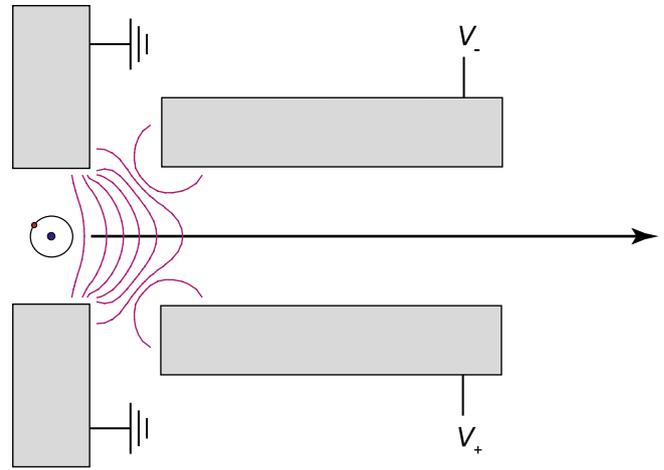

Fig. 3. Electrode geometry and resulting equipotential lines (magenta) as employed for Stark acceleration of Rydberg atoms. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

intend to use a similar electrode configuration, adapted to the cylindrical geometry of a Penning trap, to accelerate the created $\overline{H}$ atoms to about 400 m s^{-1} in the direction of the deflectometer apparatus. Prior to the beam formation, remaining $\bar{p}$ can be transferred back to the accumulation trap in order to be reused in the next cycle.

2.3. Gravity measurement

In matter wave interferometers of the Mach–Zehnder type [30,31], three identical gratings are placed at equal distances L from each other. The first two gratings produce an interference pattern at the location of the third. That pattern has the same period d as the gratings, and its position perpendicular to the diffracted particle beam can be determined precisely by displacing the third grating and recording the overall transmission with a particle detector. Under the influence of gravity, the interference pattern is vertically displaced (it "falls") by a distance

$$\delta x = -gT^2, \tag{2}$$

where g is the local gravitational acceleration and T is the time of flight L/v between each pair of gratings of a particle beam traveling at velocity v.

Unlike atomic fountain interferometers, such a device does not necessitate trapped atoms. Furthermore, neither spatial nor temporal coherence of the incoming particle beam are required. Due to these unique features, an anti-matter gravity experiment based on a Mach–Zehnder interferometer was proposed some ten years ago [32]. However, interactions between (anti)-matter waves and material gratings can lead to a number of decoherence effects in matter wave diffraction: quenching of metastable states; deflection of Rydberg states in field gradients; transitions between sub-levels of Rydberg states; and annihilation of anti-atoms on the grating. Furthermore, the technique places a very stringent limit on the acceptable beam divergence,
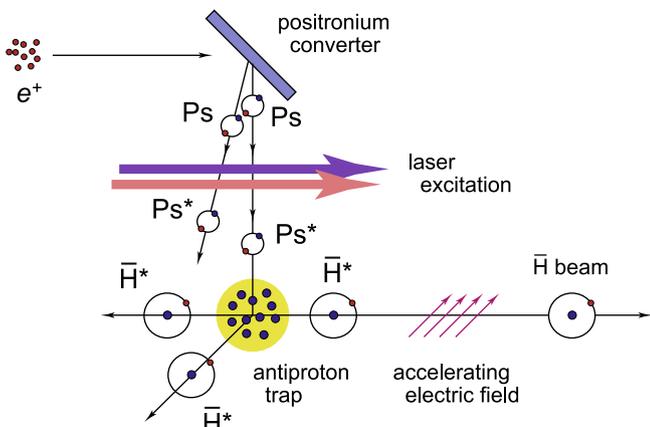

Fig. 2. Proposed method for $\overline{H}$ recombination and subsequent acceleration.

which must be smaller than the diffraction angle $\phi = \lambda_{dB}/d$, where λ_{dB} is the de Broglie wavelength of the matter wave. All of these limitations can be alleviated by increasing the grating period relative to the de Broglie wavelength. At the point where $d^2/L \gg \lambda_{dB}$, diffraction no longer occurs.

The resulting device is the so-called Moiré deflectometer, in which diffraction on the gratings is replaced by a (classical) shadow pattern of those particles that converge onto the third grating. Interestingly, the gross characteristics of the interferometer are retained [33], in particular, the vertical displacement of the interference pattern according to Eq. (2). A three-grating Moiré deflectometer has been used to measure the local gravitational acceleration to a relative precision of 2×10^{-4} with a beam of argon atoms traveling at an average velocity of 750 m s^{-1} [33]. In departing from the three-grating deflectometer, we intend to replace the third grating by a position-sensitive silicon strip detector (see Fig. 4). Thereby the overall transmission of the apparatus is increased by the inverse of the grid's open fraction (roughly a factor of three).

The value of g is extracted from the primary observables (time of flight T and vertical displacement of the fringe pattern δx) in the following way, as illustrated by Monte-Carlo simulations performed by us: First, the ensemble of all $\overline{H}$ annihilation events on the detector is plotted as a function of T, as shown in Fig. 5(a). These events are binned in symmetric classes of T^2, one of which is shown shaded in dark blue in the figure. Secondly, the vertical displacement δx of the fringe pattern is extracted for each of the count classes, as illustrated in Fig. 5(b). Thirdly, the vertical displacement for all count classes is plotted against the mean time of flight in the class. A quadratic fit to that graph, as shown in Fig. 5(c), will then yield g. In these simulations, a grating period of 80 µm was used, and a finite detector resolution of 10 µm was taken into account.

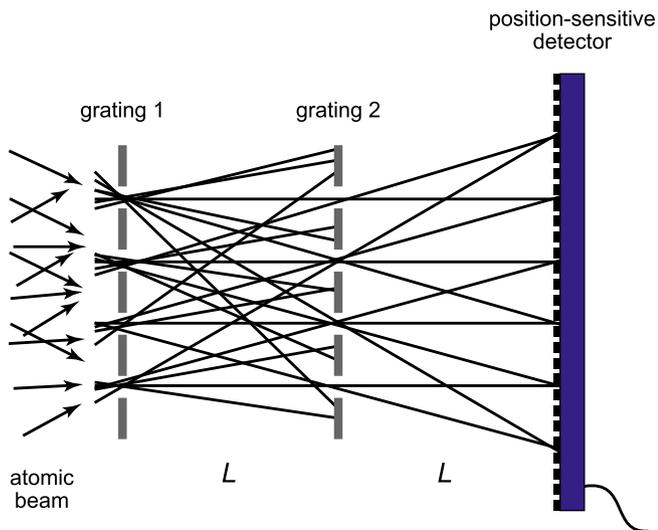Our simulations have shown that in order to perform a measurement of $\bar{g}$ to 1% relative precision, about 10^5 $\overline{H}$
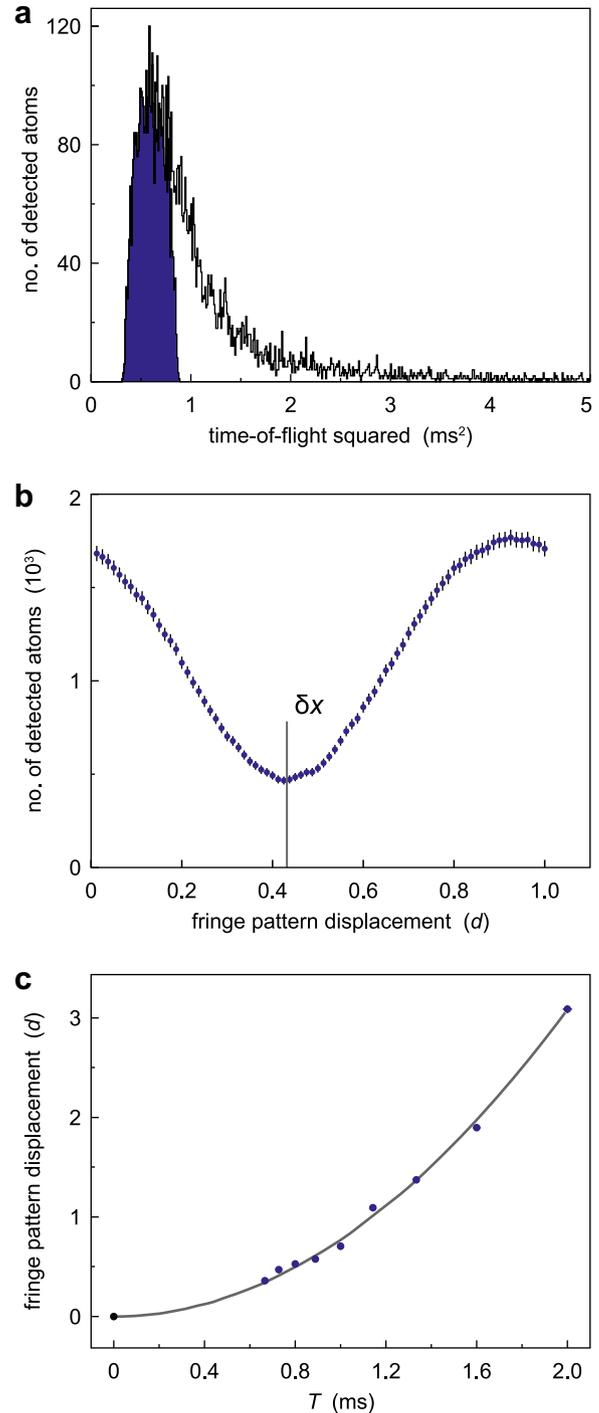

Fig. 5. Analysis of deflectometer data (Monte Carlo simulation): (a) detector events are binned according to time-of-flight count classes, (b) the vertical displacement of the fringe pattern is determined for each count class, and (c) a quadratic fit to the plot of vertical displacement versus mean time of flight yields the local gravitational acceleration g.

atoms at a temperature of 100 mK will be required. This could be achieved within about 2–3 weeks of data taking, assuming the AD beam is shared among four experiments. The grids and detector must be kept precisely aligned for the entire data taking period, which can be achieved by an auxiliary laser beam. The zero position of the vertical

Fig. 4. Principle sketch of the Moiré deflectometry technique with two identical gratings and a position-sensitive detector.

displacement, (*i.e.* in the absence of gravity) is most conveniently obtained by performing a calibration measurement with the gratings and detector rotated by 90° about the beam axis.

3. Conclusion and outlook

In this paper, we have presented the layout of a proposed experiment for the measurement of the gravitational acceleration of antimatter. The AEGIS design is based upon the broad experience gained with the ATHENA and ATRAP experiments at the AD, a series of ongoing tests and developments, as well as extensive simulations of critical processes (charge exchange production of $\overline{H}$, Stark acceleration and propagation through the Moiré deflectometer, resolution of the position-sensitive detector located at the end of the deflectometer). The proposed gravity measurement becomes feasible by merging in a single experimental apparatus technologies already demonstrated and including some reasonable additional development. The precision of the gravity measurement is mainly limited by the $\overline{H}$ temperature. Obtaining samples of anti-atoms at 100 mK is therefore an essential requirement for this experiment. Gravity measurements with even higher precision as well as competitive CPT tests through spectroscopy will necessitate the development of novel techniques to attain even colder $\overline{H}$ ensembles.

The AEGIS proto-collaboration is actively involved in the research and development which is required for overcoming the experimental difficulties in reaching these ambitious goals. The proposed design of our apparatus is modular and readily allows, in a future stage, the insertion of a magnetic trap for $\overline{H}$, which will be spatially separated from the region where the anti-atoms are produced. The spatial separation between the production and measurement regions, which is already a feature of the setup that we have presented here, differs significantly from the approach chosen by the other $\overline{H}$ experiments ATRAP [34] and ALPHA [35], which incorporate a trap that simultaneously confines charged and neutral particles. The experience gained in the first phase of AEGIS with the formation of an $\overline{H}$ beam will be used to optimize the design of a trapping system. In this second phase, trapped $\overline{H}$ can be laser-cooled to mK temperatures, at which spectroscopic CPT tests with a sensitivity competitive with current limits on CPT violations become feasible.

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