IMPRS Seminar:

Scale generation, hierarchy and conformal symmetry

Philipp Saake

Max Planck Institute for Nuclear Physics,
Heidelberg University

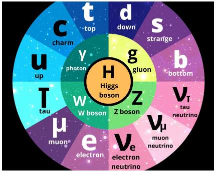
24.01.2022

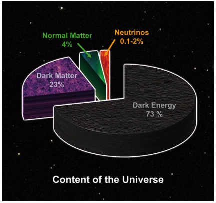
Standard Model and beyond

The Standard Model (SM) is

- Extremely successful,
  e.g. QED $g-2$: $g/2 = 1.00115965218085(76)$

Problems of the SM:





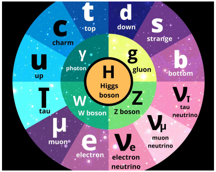
[Smf; Dmf]

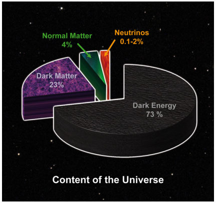
Standard Model and beyond

The Standard Model (SM) is

- Extremely successful,
  e.g. QED $g-2$: $g/2 = 1.00115965218085(76)$
- Closed, Complete, Self-consistent

Problems of the SM:





[Smf; Dmf]

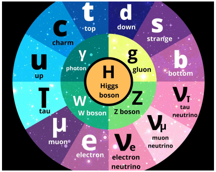
Standard Model and beyond

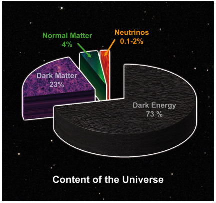
The Standard Model (SM) is

- Extremely successful,
  e.g. QED $g-2$: $g/2 = 1.00115965218085(76)$
- Closed, Complete, Self-consistent

Problems of the SM:

- **Gauge hierarchy problem**: $m_{h,0} \ll M$





[Smf; Dmf]

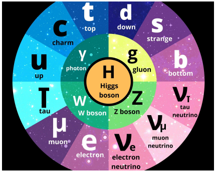
Standard Model and beyond

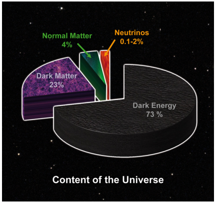
The Standard Model (SM) is

- Extremely successful,
  e.g. QED $g-2$: $g/2 = 1.00115965218085(76)$
- Closed, Complete, Self-consistent

Problems of the SM:

- **Gauge hierarchy problem**: $m_{h,0} \ll M$
- Neutrino mass: $0 < \sum_i m_{\nu_i} < 0.12\,\text{eV}$



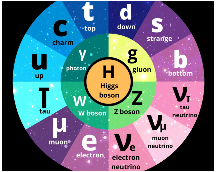


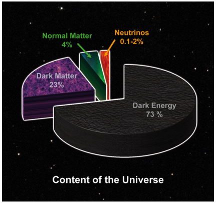
[Smf; Dmf]

Standard Model and beyond

The Standard Model (SM) is

- Extremely successful,
  e.g. QED $g-2$: $g/2 = 1.00115965218085(76)$
- Closed, Complete, Self-consistent





[Smf; Dmf]

Problems of the SM:

- **Gauge hierarchy problem**: $m_{h,0} \ll M$
- Neutrino mass: $0 < \sum_i m_{\nu_i} < 0.12\,\text{eV}$
- Dark matter & Dark Energy

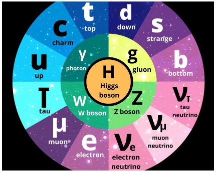
Standard Model and beyond

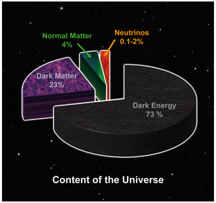
The Standard Model (SM) is

- Extremely successful,
  e.g. QED $g-2$: $g/2 = 1.00115965218085(76)$
- Closed, Complete, Self-consistent

Problems of the SM:

- **Gauge hierarchy problem**: $m_{h,0} \ll M$
- Neutrino mass: $0 < \sum_i m_{\nu_i} < 0.12\,\mathrm{eV}$
- Dark matter & Dark Energy
- Matter-Antimatter asymmetry





[Smf; Dmf]

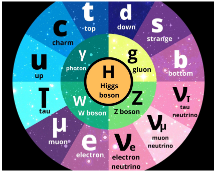
Standard Model and beyond

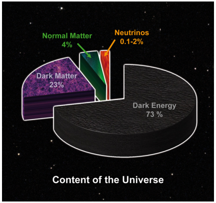
The Standard Model (SM) is

- Extremely successful,
  e.g. QED $g-2$: $g/2 = 1.00115965218085(76)$
- Closed, Complete, Self-consistent

Problems of the SM:

- **Gauge hierarchy problem**: $m_{h,0} \ll M$
- Neutrino mass: $0 < \sum_i m_{\nu_i} < 0.12\,\text{eV}$
- Dark matter & Dark Energy
- Matter-Antimatter asymmetry
- Inflation





[Smf; Dmf]

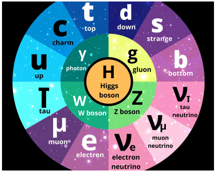
Standard Model and beyond

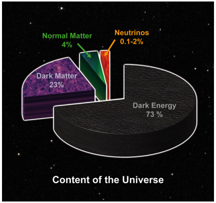
The Standard Model (SM) is

- Extremely successful,
  e.g. QED $g-2$: $g/2 = 1.00115965218085(76)$
- Closed, Complete, Self-consistent

Problems of the SM:

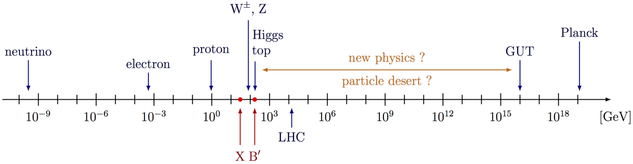
- **Gauge hierarchy problem**: $m_{h,0} \ll M$
- Neutrino mass: $0 < \sum_i m_{\nu_i} < 0.12\,\text{eV}$
- Dark matter & Dark Energy
- Matter-Antimatter asymmetry
- Inflation
- ...





[Smf; Dmf]

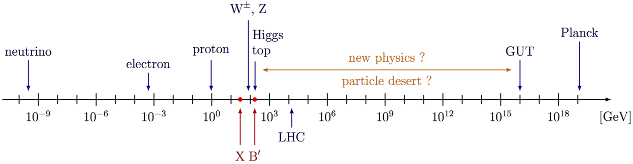
Scales in high energy particle physics



[Higa]

- **One** fundamental scale in SM: $\langle v_H \rangle \simeq 246$ GeV [Ams+08]

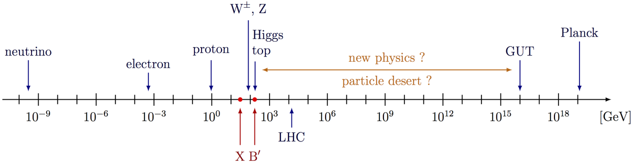
Scales in high energy particle physics



[Higa]

- **One** fundamental scale in SM: $\langle v_H \rangle \simeq 246$ GeV [Ams+08]
  - Gives particles mass via Higgs mechanism ⇒ masses depend on $\langle v_H \rangle$

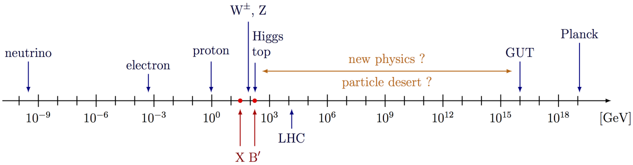
Scales in high energy particle physics



[Higa]

- **One** fundamental scale in SM: $\langle v_H \rangle \simeq 246$ GeV [Ams+08]
    - Gives particles mass via Higgs mechanism $\Rightarrow$ masses depend on $\langle v_H \rangle$
- Only **one** scale in Gravity: $M_{\mathrm{Pl}} \simeq 1.2 \times 10^{19}$ GeV

Scales in high energy particle physics

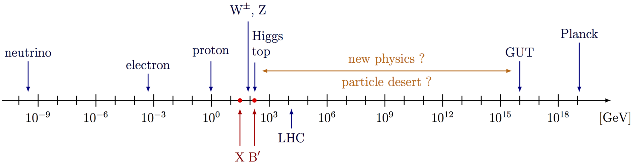


[Higa]

- **One** fundamental scale in SM: $\langle v_H \rangle \simeq 246$ GeV [Ams+08]
  - Gives particles mass via Higgs mechanism ⇒ masses depend on $\langle v_H \rangle$
- Only **one** scale in Gravity: $M_{\text{Pl}} \simeq 1.2 \times 10^{19}$ GeV

⇒ Problem for unification/ Quantum Gravity

Scales in high energy particle physics



[Higa]

- **One** fundamental scale in SM: $\langle v_H \rangle \simeq 246$ GeV [Ams+08]
  - Gives particles mass via Higgs mechanism $\Rightarrow$ masses depend on $\langle v_H \rangle$
- Only **one** scale in Gravity: $M_{\mathrm{Pl}} \simeq 1.2 \times 10^{19}$ GeV

  $\Rightarrow$ Problem for unification/ Quantum Gravity
- New physics introduce intermediate scales $m_H \ll M \ll M_{\mathrm{Pl}}$

  $\Rightarrow$ Problematic corrections (e.g. via loops) to $m_H \overset{!}{\simeq} 125$ GeV [Higb]

Conformal symmetry and scale invariance

- Conformal symmetry/ group $=$
$\underbrace{\text{Lorentz generators} + \text{Translations}}_{\text{Poincaré}} + \underbrace{\text{Dilatations}}_{\text{Scale invariance}} + \text{SCT}$

Conformal symmetry and scale invariance

- Conformal symmetry/ group =
  $\underbrace{\text{Lorentz generators} + \text{Translations}}_{\text{Poincaré}} + \underbrace{\text{Dilatations}}_{\text{Scale invariance}} + \text{SCT}$
- Largest possible non-supersymmetric group of space-time symmetries

Conformal symmetry and scale invariance

- Conformal symmetry/ group =
  $\underbrace{\text{Lorentz generators} + \text{Translations}}_{\text{Poincaré}} + \underbrace{\text{Dilatations}}_{\text{Scale invariance}} + \text{SCT}$
- Largest possible non-supersymmetric group of space-time symmetries
- For a **unitary** and **renormalizable** QFT:

  Conformal invariance $\equiv$ classical scale invariance

Conformal symmetry and scale invariance

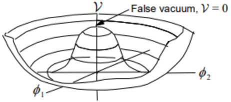
- Conformal symmetry/ group =
  $$\underbrace{\text{Lorentz generators} + \text{ Translations}}_{\text{Poincaré}} + \underbrace{\text{Dilatations}}_{\text{Scale invariance}} + \text{SCT}$$
- Largest possible non-supersymmetric group of space-time symmetries
- For a **unitary** and **renormalizable** QFT:
  $$\text{Conformal invariance} \equiv \text{classical scale invariance}$$
- Cl. scale invariance ≡ **no dimensionful** couplings at tree level in the Lagrangian $\mathcal{L}$
  $$\text{No } \left(m_i^2, \ldots\right), \qquad \text{but } (y_i,\ \lambda_i,\ g_i, \ldots) \in \mathcal{L}$$

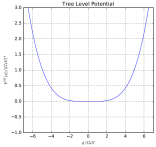
Conformal symmetry and scale invariance

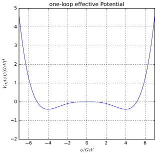
- Conformal symmetry/ group =
  $$\underbrace{\text{Lorentz generators} + \text{ Translations}}_{\text{Poincaré}} + \underbrace{\text{Dilatations}}_{\text{Scale invariance}} + \text{SCT}$$
- Largest possible non-supersymmetric group of space-time symmetries
- For a **unitary** and **renormalizable** QFT:
  $$\text{Conformal invariance} \equiv \text{classical scale invariance}$$
- Cl. scale invariance ≡ **no dimensionful** couplings at tree level in the Lagrangian $\mathcal{L}$
  $$\text{No } \left(m_i^2, \ldots\right), \qquad \text{but } \left(y_i,\ \lambda_i,\ g_i, \ldots\right) \in \mathcal{L}$$
- Extensions via (gauged) **scalars** [Hel+17]

Mechanisms

- Technicolor, Composite W and Z bosons, Top-quark condensates, Extra Dimensions, $\underbrace{\text{Higgs, Coleman-Weinberg}}_{\text{via spontaneous symmetry breaking (SSB)}}$

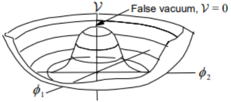


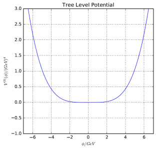


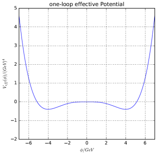


Mechanisms

- Technicolor, Composite W and Z bosons, Top-quark condensates, Extra Dimensions, $\underbrace{\text{Higgs, Coleman-Weinberg}}_{\text{via spontaneous symmetry breaking (SSB)}}$
- SSB: the vacuum $\langle v \rangle$ has less symmetry than the the Lagrangian $\mathcal{L}$



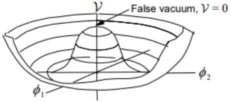


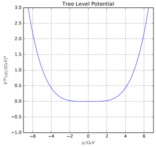


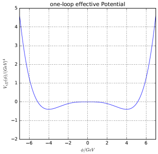
Mechanisms

- Technicolor, Composite W and Z bosons, Top-quark condensates, Extra Dimensions, $\underbrace{\text{Higgs, Coleman-Weinberg}}$

  via spontaneous symmetry breaking (SSB)
- SSB: the vacuum $\langle v \rangle$ has less symmetry than the the Lagrangian $\mathcal{L}$
- Higgs mechanism: V_H with mass $m_H \Rightarrow \langle v_H \rangle \neq 0$
  - $\rightarrow$ Fermion mass via couplings to the Higgs
  - $\rightarrow$ Bosons mass via Goldstone theorem



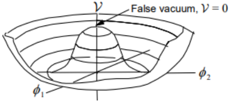


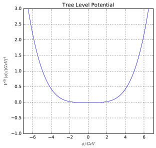


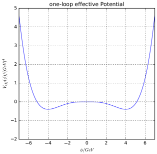
Mechanisms

- Technicolor, Composite W and Z bosons, Top-quark condensates, Extra Dimensions, $\underbrace{\text{Higgs, Coleman-Weinberg}}_{\text{via spontaneous symmetry breaking (SSB)}}$
- SSB: the vacuum $\langle v \rangle$ has less symmetry than the the Lagrangian $\mathcal{L}$
- Higgs mechanism: V_H with mass $m_H \Rightarrow \langle v_H \rangle \neq 0$
  - $\rightarrow$ Fermion mass via couplings to the Higgs
  - $\rightarrow$ Bosons mass via Goldstone theorem
- **Coleman-Weinberg** mechanism: not m_H but loop corrections induce SSB (radiatively) [CW73]

$$V_{\text{eff}}(\phi; \bar{\mu}) = \frac{1}{4!}\lambda\phi^4 + \frac{\lambda^2\phi^4}{256\pi^2}\left(\ln\left[\frac{\lambda\phi^2}{2\bar{\mu}^2}\right] - \frac{3}{2}\right)$$







Gap equation

- Generalized sph. coordinates for $\vec{\phi}$ via one radial (φ) and $n-1$ angular ($\vec{\vartheta}$) coordinates

Gap equation

- Generalized sph. coordinates for $\vec{\phi}$ via one radial (φ) and $n-1$ angular ($\vec{\vartheta}$) coordinates
- Symmetry breaking or criticality equations:

$$\begin{aligned} \text{radial} \;&:\; 0 \stackrel{!}{=} \kappa(\vec{\vartheta}_0;\varphi_0) + A(\vec{\vartheta}_0;\varphi_0) + \frac{1}{2}B(\vec{\vartheta}_0;\varphi_0) \\ \text{angular} \;&:\; 0 \stackrel{!}{=} \vec{\nabla}_\vartheta \left[\kappa(\vec{\vartheta};\varphi_0) + A(\vec{\vartheta};\varphi_0)\right]_{\vec{\vartheta}=\vec{\vartheta}_0} \end{aligned}$$

$$\Rightarrow \; f\left(\lambda_i(\mu), \vec{\vartheta}\right) \stackrel{!}{=} 0$$

Gap equation

- Generalized sph. coordinates for $\vec{\phi}$ via one radial (φ) and $n-1$ angular ($\vec{\vartheta}$) coordinates
- Symmetry breaking or criticality equations:

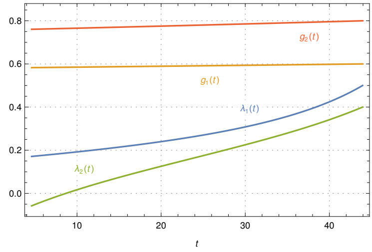
$$\begin{aligned} \text{radial} &: \ 0 \stackrel{!}{=} \kappa(\vec{\vartheta}_0;\varphi_0) + A(\vec{\vartheta}_0;\varphi_0) + \frac{1}{2}B(\vec{\vartheta}_0;\varphi_0) \\ \text{angular} &: \ 0 \stackrel{!}{=} \vec{\nabla}_\vartheta \left[\kappa(\vec{\vartheta};\varphi_0) + A(\vec{\vartheta};\varphi_0)\right]_{\vec{\vartheta}=\vec{\vartheta}_0} \end{aligned}$$

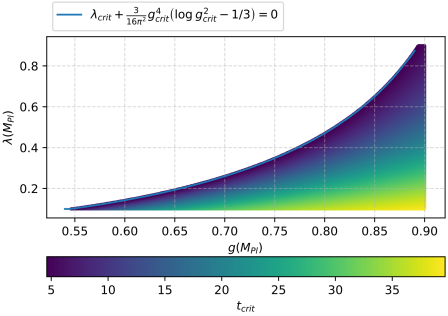
$$\Rightarrow \ f\left(\lambda_i(\mu), \vec{\vartheta}\right) \stackrel{!}{=} 0$$

$\Rightarrow$ Set the dimensionless coupling at, e.g. M_{Pl} and use RG-flow to evolve until both criticality equations are fulfilled

Example: massless QED

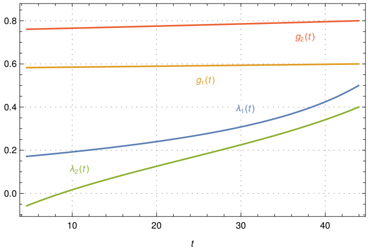
- Crit. equation: $\lambda + \frac{3g^4}{16\pi^2}\left(\ln(g^2) - 1/3\right) + \mathcal{O}(\lambda^2) \stackrel{!}{=} 0$

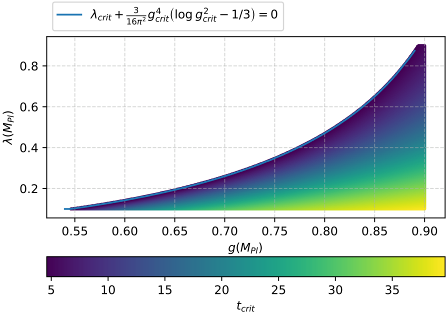




Example: massless QED

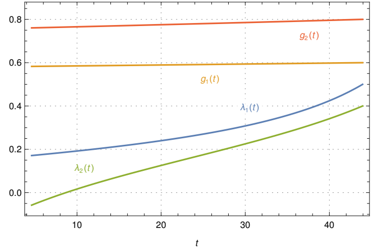
- Crit. equation: $\lambda + \frac{3g^4}{16\pi^2}\left(\ln(g^2) - 1/3\right) + \mathcal{O}(\lambda^2) \stackrel{!}{=} 0$
- **Dimensional transmutation**: $\lambda(\mu), g(\mu) \to g_{\text{crit}}, \phi_{\text{crit}}, \quad [\phi_{\text{crit}}] \neq 0$

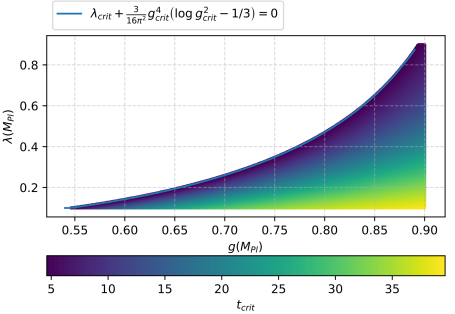




Example: massless QED

- Crit. equation: $\lambda + \frac{3g^4}{16\pi^2}\left(\ln(g^2) - 1/3\right) + \mathcal{O}(\lambda^2) \stackrel{!}{=} 0$
- **Dimensional transmutation**: $\lambda(\mu), g(\mu) \to g_{\text{crit}}, \phi_{\text{crit}}, \quad [\phi_{\text{crit}}] \neq 0$
- Field and gauge boson mass: $\frac{m_\phi^2}{m_A^2} = \frac{3g_{\text{crit}}^2}{8\pi^2}$ with $m_\phi^2 = \left.\frac{\partial^2 V_{\text{eff}}}{\partial\phi^2}\right|_{\phi=\phi_{\text{crit}}}$





Creating two scales via SSB

- Multiple scalars (often used Gildener-Weinberg approximation of *tree-level flat direction*)
  e.g.: $V_0 = \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_p(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$

Creating two scales via SSB

- Multiple scalars (often used Gildener-Weinberg approximation of *tree-level flat direction*)
  e.g.: $V_0 = \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_p(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$
- Gap equations allow for systematically analysis of SSB properties
  $\Rightarrow$ scalar-, gauge-, mixed symmetry breaking

Creating two scales via SSB

- Multiple scalars (often used Gildener-Weinberg approximation of *tree-level flat direction*)
  e.g.: $V_0 = \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_p(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$
- Gap equations allow for systematically analysis of SSB properties
  $\Rightarrow$ scalar-, gauge-, mixed symmetry breaking
- 1-loop masses of Φ_1, Φ_2 are highly complex $\rightarrow$ difficult analytical treatment of hierarchies in CW

Creating two scales via SSB

- Multiple scalars (often used Gildener-Weinberg approximation of *tree-level flat direction*)
  e.g.: $V_0 = \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_p(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$
- Gap equations allow for systematically analysis of SSB properties
  $\Rightarrow$ scalar-, gauge-, mixed symmetry breaking
- 1-loop masses of Φ_1, Φ_2 are highly complex $\rightarrow$ difficult analytical treatment of hierarchies in CW
  - Look at highly symmetric points: $\lambda_{1,0} = \lambda_{2,0}/2$, $g_{1,0} = g_{2,0}$
    $\Rightarrow$ find the naively expected $m_1^2/m_2^2 \propto \lambda_{p,0}$

Creating two scales via SSB

- Multiple scalars (often used Gildener-Weinberg approximation of *tree-level flat direction*)
  e.g.: $V_0 = \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_p(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$
- Gap equations allow for systematically analysis of SSB properties
  $\Rightarrow$ scalar-, gauge-, mixed symmetry breaking
- 1-loop masses of Φ_1, Φ_2 are highly complex $\rightarrow$ difficult analytical treatment of hierarchies in CW
  - Look at highly symmetric points: $\lambda_{1,0} = \lambda_{2,0}/2$, $g_{1,0} = g_{2,0}$
    $\Rightarrow$ find the naively expected $m_1^2/m_2^2 \propto \lambda_{p,0}$
- We expect: only spontaneously broken scale invariance may protect scale separation

But we do not know, yet ...

Summary & outlook

- Classically scale invariant model with dynamical scale generation via SSB
- Multiscalar Coleman-Weinberg (without approximations)
- Gap equations allow for intuitive description of SSB w.r.t. $\lambda_{i,0}, g_{i,0}$
- Incorporates the full RG-running
- Can relate 1-loop mass hierarchies (somewhat) analytically to $\lambda_{p,0}$
- A lot more to understand for more complex cases ...

Thank you for your attention!

[Ams+08] C. Amsler et al. "Review of Particle Physics". In: *Physics Letters B* 667.1 (2008). Review of Particle Physics, pp. 1–6. ISSN: 0370-2693. DOI: https://doi.org/10.1016/j.physletb.2008.07.018. URL: https://www.sciencedirect.com/science/article/pii/S0370269308008435.

[CW73] Sidney R. Coleman and Erick J. Weinberg. "Radiative Corrections as the Origin of Spontaneous Symmetry Breaking". In: *Phys. Rev. D* 7 (1973), pp. 1888–1910. DOI: 10.1103/PhysRevD.7.1888.

[Dmf] URL: https://www.hap-astroparticle.org/186.php.

[Hel+17] Alexander J. Helmboldt et al. "Minimal conformal extensions of the Higgs sector". In: *JHEP* 07 (2017), p. 113. DOI: 10.1007/JHEP07(2017)113. arXiv: 1603.03603 [hep-ph].

[Higa] URL: https://wiki.physik.uzh.ch/cms/latex:tikz.

[Higb] *Higgs Mass*. 2020. URL: https://pdg.lbl.gov/2020/listings/rpp2020-list-higgs-boson.pdf.

[Smf] URL: https://onheaven.co.in/wp-content/uploads/2021/07/Untitled-design-15-scaled.jpg.