Unified Emergence of Energy Scales and Cosmic Inflation

Jonas Rezacek
Max-Planck-Institut für Kernphysik

Based on 2012.09706
(Jisuke Kubo, Jeff Kuntz, Manfred Lindner, J. R., Philipp Saake, Andreas Trautner)

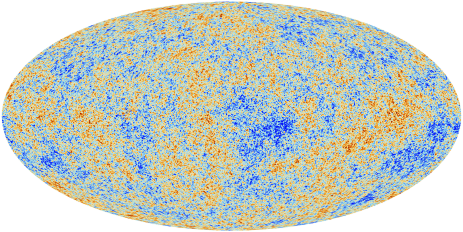
25th May 2021
IMPRS Seminar

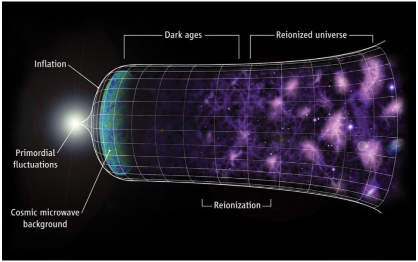
Max-Planck-Institut für Kernphysik Heidelberg

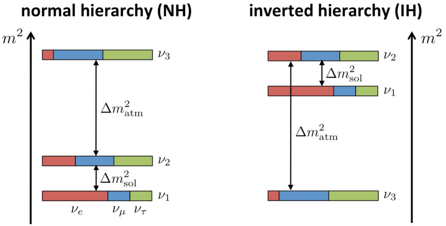
Introduction and Motivation

Open puzzles in cosmology and particles physics

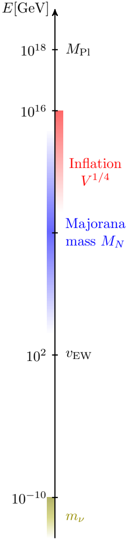
- Big Bang problems: horizon, flatness, monopoles
- Neutrino mass
- Higgs mass naturalness







Energy scales in high-energy physics



- $M_{\rm Pl}$ and m_h are known $\rightarrow$ **gauge hierarchy problem**

$$m_h^2 = m_{h,0}^2 + \delta m_h^2, \qquad \delta m_h^2 = f(g_i) M^2 \gg m_{h,0}^2$$

- Inflation scale constrained by **tensor-to-scalar ratio**

$$V^{1/4} \lesssim 10^{16}\,\text{GeV}$$ [Planck 2018]

- Light active neutrinos

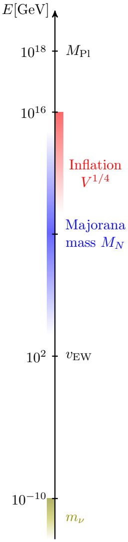
$$\sum_i m_{\nu_i} < 0.12\,\text{eV}$$ [Planck 2018]

- **Seesaw mechanism:** adding ν_R with Majorana mass M_N

$$m_\nu \simeq -m_D M_N^{-1} m_D^T$$

- Dark matter?

Energy scales in high-energy physics



- M_{Pl} and m_h are known $\rightarrow$ **gauge hierarchy problem**

$$m_h^2 = m_{h,0}^2 + \delta m_h^2, \qquad \delta m_h^2 = f(g_i) M^2 \gg m_{h,0}^2$$

- Inflation scale constrained by **tensor-to-scalar ratio**

$$V^{1/4} \lesssim 10^{16}\,\text{GeV} \qquad \text{[Planck 2018]}$$

- Light active neutrinos

$$\sum_i m_{\nu_i} < 0.12\,\mathrm{eV} \qquad \text{[Planck 2018]}$$

- **Seesaw mechanism:** adding ν_R with Majorana mass M_N

$$m_\nu \simeq -m_D M_N^{-1} m_D^T$$

- Dark matter?

$\rightarrow$ **Dynamical generation of all scales**
$\rightarrow$ **All scales vanish at tree-level (classical scale invariance)**

Scale invariance

Scale-invariant Gravity

- Dynamical generation of $M_{\rm Pl} = \langle \phi \rangle$

$$\mathcal{L}_{\sf EH} = \sqrt{-g}\,\xi\phi^2 R \quad \rightarrow \quad \sqrt{-g}\,\xi\langle\phi^2\rangle R = \sqrt{-g}\,M_{\sf Pl}^2 R$$

- Incorporate inflation

Scale invariance

Scale-invariant Gravity

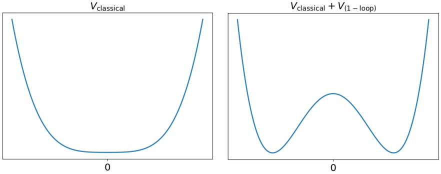
- Dynamical generation of $M_{\mathrm{Pl}} = \langle\phi\rangle$

$$\mathcal{L}_{\mathrm{EH}} = \sqrt{-g}\,\xi\phi^2 R \quad \rightarrow \quad \sqrt{-g}\,\xi\langle\phi^2\rangle R = \sqrt{-g}\,M_{\mathrm{Pl}}^2 R$$

- Incorporate inflation

Scale-invariant SM

- Only dimensionful parameter in the SM: μ_H
- Radiative corrections modify Higgs potential $\rightarrow$ EW symmetry breaking
  [Coleman, Weinberg '73]
- M_{Pl} and m_H exponentially separated and radiatively stable if: no intermediate scales [Meissner, Nicolai, hep-th/0612165]



Scale invariance

Scale-invariant Gravity

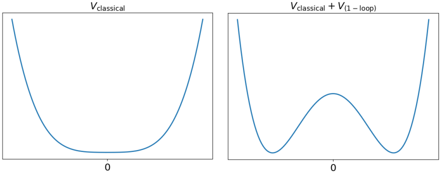
- Dynamical generation of $M_{\rm Pl} = \langle\phi\rangle$

$$\mathcal{L}_{\rm EH} = \sqrt{-g}\,\xi\phi^2 R \quad \to \quad \sqrt{-g}\,\xi\langle\phi^2\rangle R = \sqrt{-g}\,M_{\rm Pl}^2 R$$

- Incorporate inflation

Scale-invariant SM

- Only dimensionful parameter in the SM: μ_H
- Radiative corrections modify Higgs potential $\to$ EW symmetry breaking
  [Coleman, Weinberg '73]
- $M_{\rm Pl}$ and m_H exponentially separated and radiatively stable if:
  no intermediate scales [Meissner, Nicolai, hep-th/0612165]



How to unify both?

Contents

Contents

Classical scale invariance

$$g_{\mu\nu} \to e^{2\sigma} g_{\mu\nu} \quad \text{where} \quad \sigma = \text{const}\,, \qquad \Phi \to e^{-q[\Phi]\sigma}\Phi \quad \text{where} \quad \begin{cases} q[\varphi] = 1 \\ q[\psi] = 3/2 \\ q[A_\mu] = 0 \end{cases}$$

$\rightarrow$ **All model parameters dimensionless!**

Classical scale invariance

$$g_{\mu\nu} \to e^{2\sigma} g_{\mu\nu} \quad \text{where} \quad \sigma = \text{const}\,, \qquad \Phi \to e^{-q[\Phi]\sigma}\Phi \quad \text{where} \quad \begin{cases} q[\varphi] = 1 \\ q[\psi] = 3/2 \\ q[A_\mu] = 0 \end{cases}$$

$\to$ **All model parameters dimensionless!**

Building blocks

Gravity:

$$S_G = \int \mathrm{d}^4x\sqrt{-g}\left(-\beta\varphi^2\, R + \gamma\, R^2 + \kappa W_{\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta} + \text{(GB-term)}\right)$$

Scalars:

$$S_\phi = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi_i - \lambda^{ijkl}\phi_i\phi_j\phi_k\phi_l\right)$$

in the SM: $\mu_H = 0$

Fermions:

$$S_\phi = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{i}{2}\overline{\psi}\partial\!\!\!/\psi + y_i\phi^i\overline{\psi}\psi\right)$$

The Model

$$\frac{\mathcal{L}_{\rm CW}}{\sqrt{-g}} = \frac{1}{2} g^{\mu\nu}\partial_\mu S \partial_\nu S + \frac{1}{2} g^{\mu\nu}\partial_\mu \sigma \partial_\nu \sigma - \frac{1}{4}\lambda_S S^4 - \frac{1}{4}\lambda_\sigma \sigma^4 - \frac{1}{4}\lambda_{s\sigma} S^2 \sigma^2$$

$$\frac{\mathcal{L}_{\rm GR}}{\sqrt{-g}} = -\frac{1}{2}(\beta_S S^2 + \beta_\sigma \sigma^2 + \beta_H H^\dagger H)\, R + \gamma\, R^2 + \kappa W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta}$$

$$\frac{\mathcal{L}_{\rm SM}}{\sqrt{-g}} = \mathcal{L}_{\rm SM}\big|_{\mu_H=0} - \frac{1}{4}(\lambda_{HS} S^2 + \lambda_{H\sigma}\sigma^2) H^\dagger H$$

$$\frac{\mathcal{L}_{N\chi}}{\sqrt{-g}} = \frac{i}{2}\overline{N_R}\partial\!\!\!/ N_R - \left(\frac{1}{2} y_M S \overline{N_R}(N_R)^c + y_\nu \bar{L}\tilde{H} N_R + \text{h.c.}\right)$$

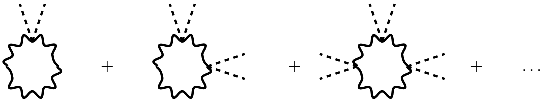
1. breaking of scale-invariance by Coleman-Weinberg mechanism ($\langle S\rangle = v_S$)
2. identifcation of $M_{\rm Pl}$ and inflation
3. SM interactions
4. type-I seesaw, inducing Higgs mass

Contents

Dimensional transmutation

Mechanisms for dynamical generation of scales

- Coleman-Weinberg mechanism [Coleman, Weinberg '73]



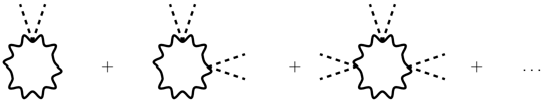
e.g. massless sQED $$V_{\text{eff}}(\varphi) = \frac{\lambda}{4!}\varphi^4 + 3\frac{(g\varphi)^4}{64\pi^2}\left[\log\left(\frac{(g\varphi)^2}{\mu^2}\right) - \frac{5}{6}\right]$$

$$\mathcal{O}(\lambda) \sim \mathcal{O}(g^4) \quad \rightarrow \quad \langle\varphi\rangle \neq 0$$

Dimensional transmutation

Mechanisms for dynamical generation of scales

- Coleman-Weinberg mechanism [Coleman, Weinberg '73]

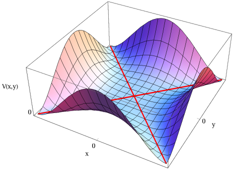


e.g. massless sQED $$V_{\text{eff}}(\varphi) = \frac{\lambda}{4!}\varphi^4 + 3\frac{(g\varphi)^4}{64\pi^2}\left[\log\left(\frac{(g\varphi)^2}{\mu^2}\right) - \frac{5}{6}\right]$$

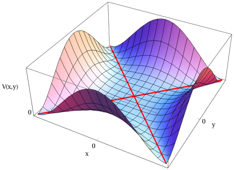
$$\mathcal{O}(\lambda) \sim \mathcal{O}(g^4) \quad \rightarrow \quad \langle\varphi\rangle \neq 0$$

Approximation tool for multi-scalar potential: **Gildener-Weinberg approach**

[Gildener, Weinberg '76]



SSB of scale invariance



- Desired flat direction $(S \neq 0, \sigma = 0)$ for

$$V_{\text{tree}}(S, \sigma) = \frac{1}{4}\left(\lambda_S S^4 + \lambda_\sigma \sigma^4 + \lambda_{S\sigma} S^2 \sigma^2\right)$$

$$\lambda_S \ll \lambda_{S\sigma} \quad \text{and} \quad \lambda_S \ll \lambda_\sigma$$

- Coleman-Weinberg potential in background $\sigma = 0$:

$$U_{\text{eff}}(S, R, \sigma) = V_{\text{tree}}(S, \sigma) + U_{\text{(1-loop)}}(S, R)$$

- Stationary condition for $\sigma = 0$

$$\left.\frac{\partial U_{\text{eff}}}{\partial S}\right|_{S=v_S, R=0} = 0 \quad \Rightarrow \quad v_S = v_S(\mu)$$

Generated scales

- By dimensional transmutation $\langle S \rangle = v_S \neq 0$
- **Planck mass**

$$M_{\rm Pl} \approx v_S \sqrt{\beta_S}$$

For inflation $\beta_S \sim 10^{(2-3)} \Rightarrow v_S \sim 10^{(16-17)}$ GeV

- **Majorana masses**

$$m_N = y_M \, v_S \qquad \text{(neutrino option)}$$

Generated scales

- By dimensional transmutation $\langle S \rangle = v_S \neq 0$
- **Planck mass**

$$M_{\mathrm{Pl}} \approx v_S \sqrt{\beta_S}$$

For inflation $\beta_S \sim 10^{(2-3)} \Rightarrow v_S \sim 10^{(16-17)}$ GeV

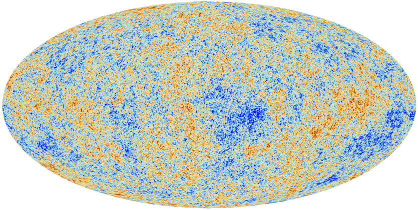
- **Majorana masses**

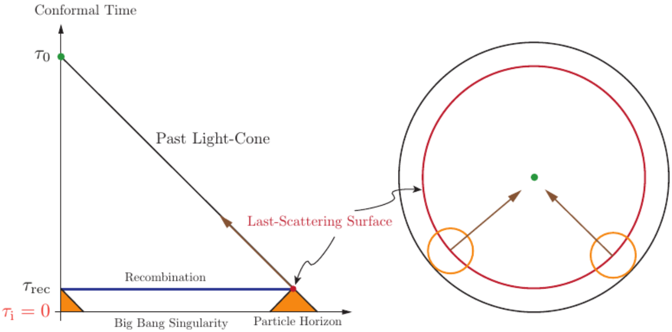
$$m_N = y_M \, v_S \qquad \text{(neutrino option)}$$

- **Higgs portal** has to be suppressed $\lambda_{HS} \ll 1$

Contents

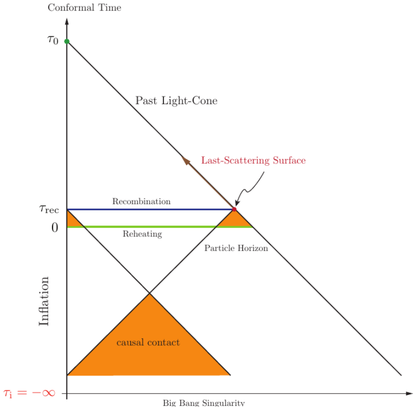
CMB in the big bang picture





[Figure taken from Baumann, 0907.5424]

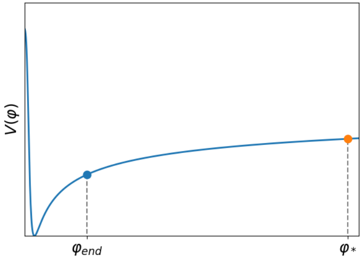
CMB in the inflation picture



[Figure taken from Baumann, 0907.5424]

Slow-roll inflation

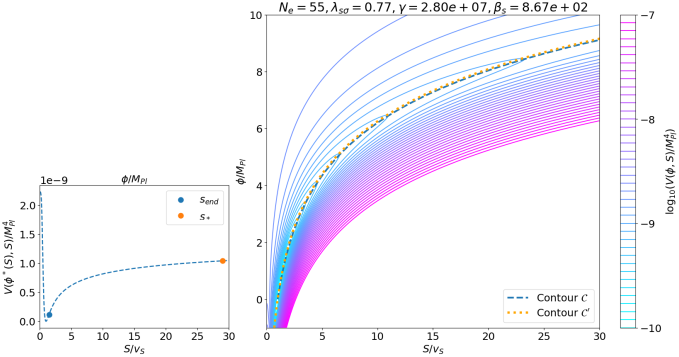
$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{M_{\mathrm{Pl}}}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] , \qquad \mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$



$$\frac{\ddot{a}}{a} = -\frac{1}{6}\left(\rho_\phi + 3p_\phi\right) = -\frac{\rho_\phi}{6}\left(1 + 3w_\phi\right) > 0 \qquad \Rightarrow \qquad \boxed{w_\phi < -\frac{1}{3}}$$

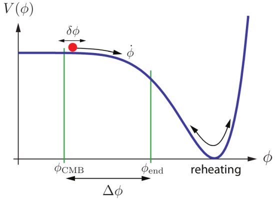
$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \qquad \Rightarrow \qquad \boxed{V(\phi) \gg \frac{1}{2}\dot{\phi}^2}$$

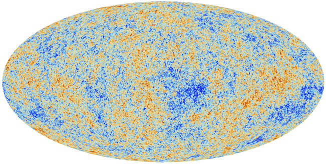
Valley approximation



- Slow-roll satisfied along valley
- Flat potentials natural in scale-invariant models

Can we constrain inflation models with the CMB?

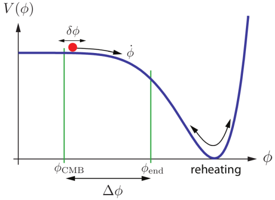


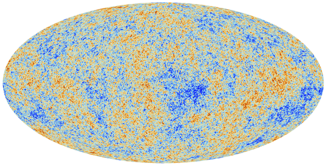


- **Inflation:** primordial quantum fluctuations seed structure
- CMB 2-point correlation for temperature T constrains primordial power spectrum

$$C^{TT}(k) \rightarrow \Delta^{\text{primordial}}(k)$$

Can we constrain inflation models with the CMB?





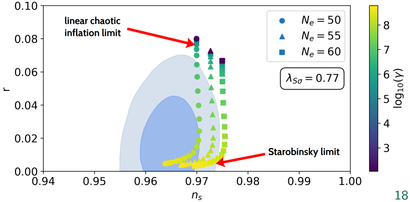
- **Inflation:** primordial quantum fluctuations seed structure
- CMB 2-point correlation for temperature T constrains primordial power spectrum

$$C^{TT}(k) \to \Delta^{\text{primordial}}(k)$$

- Inflationary CMB observables

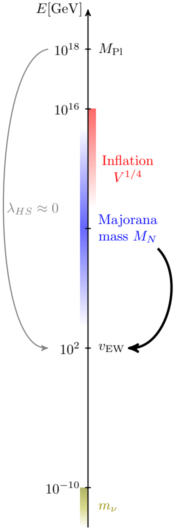
n_s scalar spectral-tilt (scale dependence)

r tensor-to-scalar ratio



Contents

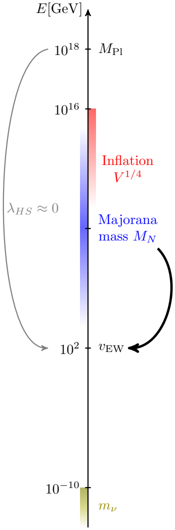
How to connect the Planck and EW scale?



- New approach to hierarchy problem: **Neutrino Option**

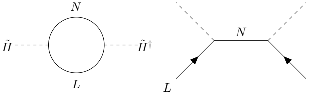
  [Brivio, Trott, 1703.10924]

How to connect the Planck and EW scale?

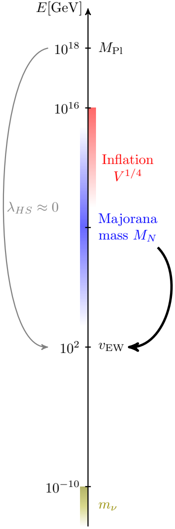


- New approach to hierarchy problem: **Neutrino Option**
  [Brivio, Trott, 1703.10924]
- Higgs potential is radiatively generated

$$\mu_H = 0\,(\text{tree level}), \quad \Delta\mu_H^2 \sim -\frac{y_\nu^2 m_N^2}{16\pi^2}$$



How to connect the Planck and EW scale?

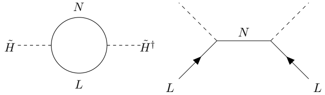


- New approach to hierarchy problem: **Neutrino Option**

  [Brivio, Trott, 1703.10924]

- Higgs potential is radiatively generated

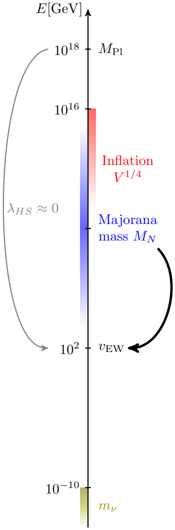
$$\mu_H = 0 \,(\text{tree level}), \quad \Delta\mu_H^2 \sim -\frac{y_\nu^2 m_N^2}{16\pi^2}$$



- Type-I seesaw mechanism

$$m_\nu \simeq y_\nu^2 v_h^2 / m_N \sim 0.1\,\text{eV}$$

How to connect the Planck and EW scale?

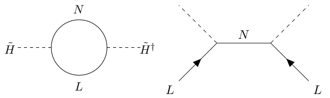


- New approach to hierarchy problem: **Neutrino Option**

  [Brivio, Trott, 1703.10924]

- Higgs potential is radiatively generated

$$\mu_H = 0\,(\text{tree level}), \quad \Delta\mu_H^2 \sim -\frac{y_\nu^2 m_N^2}{16\pi^2}$$



- Type-I seesaw mechanism

$$m_\nu \simeq y_\nu^2 v_h^2 / m_N \sim 0.1\,\text{eV}$$

- Correct Higgs mass and active neutrino masses scale obtained for

$$m_N \sim 10^7\,\text{GeV}, \quad y_\nu \sim 10^{-4}$$

  [Brivio, Trott, 1809.03450]

- Embedding in scale-invariant theory ($m_N = y_m v_S$)

  [Brdar et al, 1807.11490]

Neutrino option

- **Another contribution to the Higgs mass**

$$\lambda_{HS} S^2 (H^\dagger H) \to \lambda_{HS} v_S^2 (H^\dagger H)$$

- Assume $\lambda_{HS} \ll 1$ at tree level

$$\{\lambda_{HS}, y_M\} \sim 0 \quad \text{stable under renormalization group}$$

- λ_{HS} not fine-tuned to special value!

Neutrino option

- **Another contribution to the Higgs mass**

$$\lambda_{HS} S^2 (H^\dagger H) \to \lambda_{HS} v_S^2 (H^\dagger H)$$

- Assume $\lambda_{HS} \ll 1$ at tree level

$$\{\lambda_{HS}, y_M\} \sim 0 \quad \text{stable under renormalization group}$$

- λ_{HS} not fine-tuned to special value!
- **Majorana Yukawa coupling fixed by Planck scale and inflation**

$$y_M = \frac{m_N}{v_S} \simeq \frac{m_N \beta_S^{1/2}}{M_{\text{Pl}}} \simeq 10^{-10} \left(\frac{\beta_S}{10^3}\right)^{1/2}$$

- $y_M \to 0$ technically natural ($\text{U}(1)_{B-L}$ restored) ['t Hooft '80]

Summary & conclusion

- Classically scale invariant model with dynamical generation of all scales
- Extended scalar sector for Coleman-Weinberg-type breaking
- VEV $v_S = 10^{16-17}$ GeV generates Planck scale $M_{\mathrm{Pl}} \approx \beta_S^{1/2} v_S$
- Inflation predictions consistent with Planck observations
- Majorana mass scale $M_N = y_N v_S \sim 10^7$ GeV
- Higgs mass realized by neutrino option (+ light active neutrinos)
- Dark matter production possible without spoiling neutrino option

Thank you!