On the Vacuum Alignment Problem in Flavour Models

Martin Holthausen

based on
MH, Michael A. Schmidt
JHEP 1201 (2012) 126 , arXiv: 1111.1730 &
MH, Manfred Lindner, Michael A. Schmidt
in preparation

IMPRS-PTFS Seminar
24.4.2012 MPIK

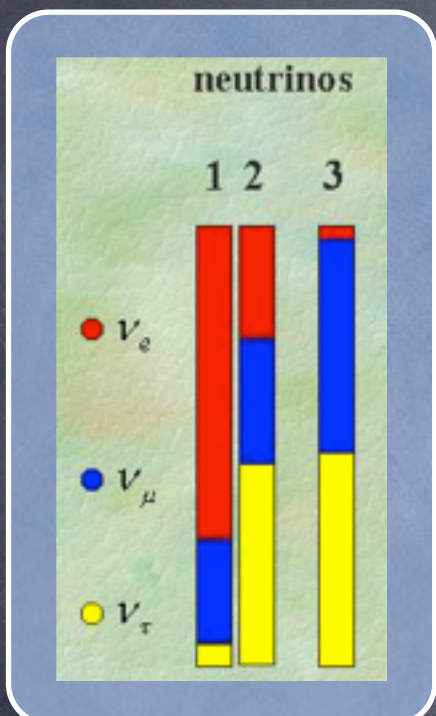
INTERNATIONAL MAX PLANCK RESEARCH SCHOOL
PT FS
FOR PRECISION TESTS OF FUNDAMENTAL SYMMETRIES

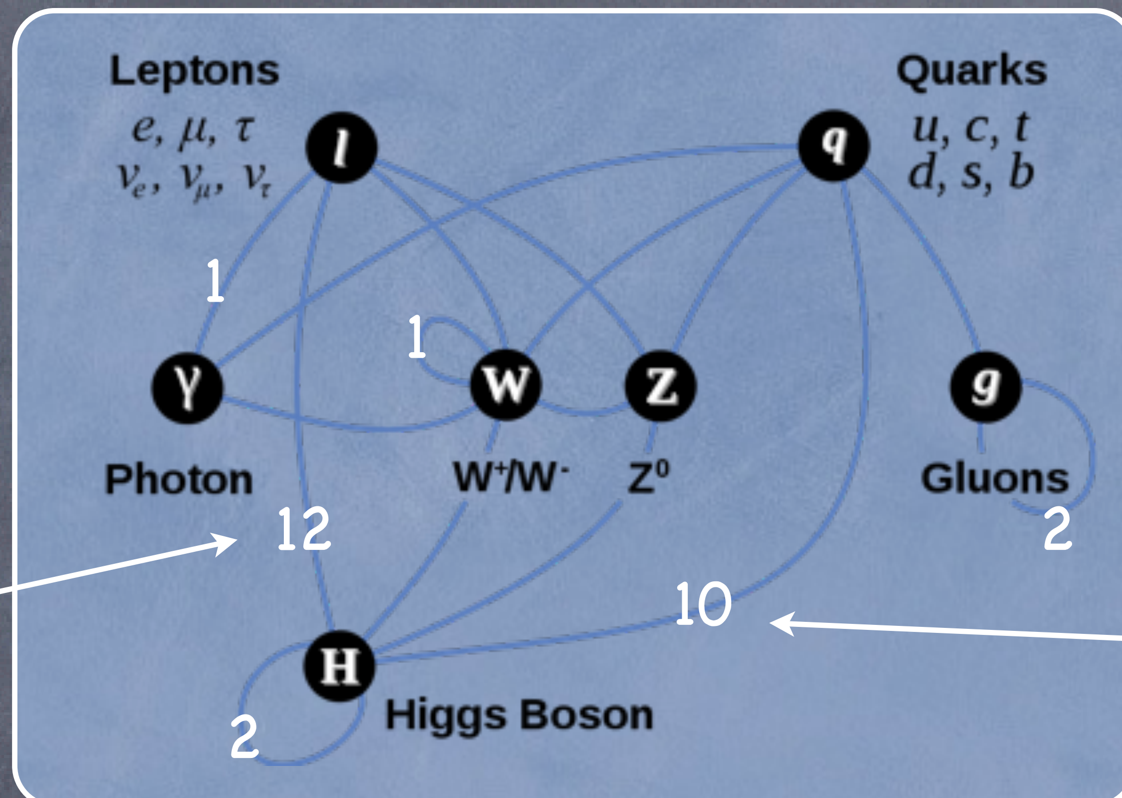
Outline

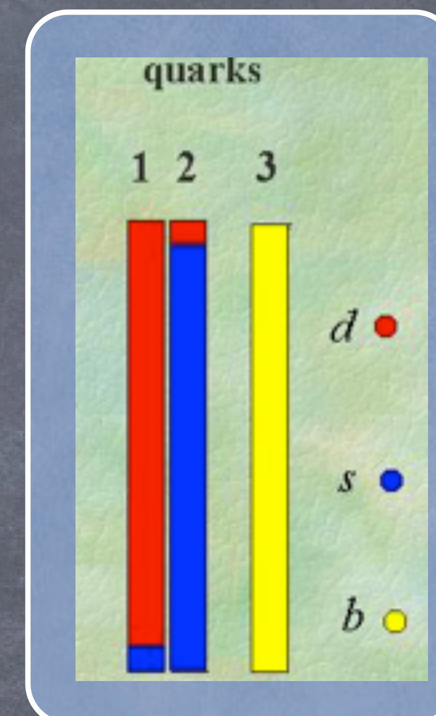
- Motivation of Flavour Symmetries
- Vacuum Alignment Problem
- Solution of the Vacuum Alignment Problem from Group Theory
  - General Conditions
  - Explicit Model based on $Q_8 \rtimes A_4$
- Conclusions

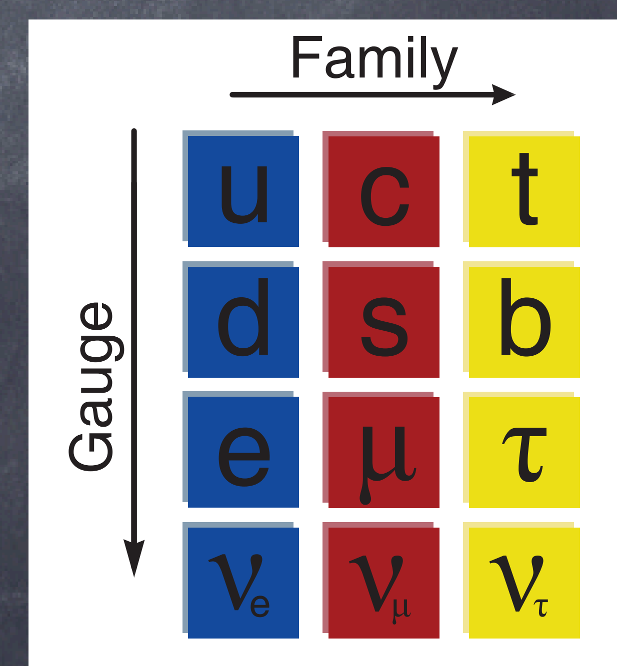
Why Flavour Symmetry?

in SM(+Majorana neutrinos) there are a total of 28 Parameters









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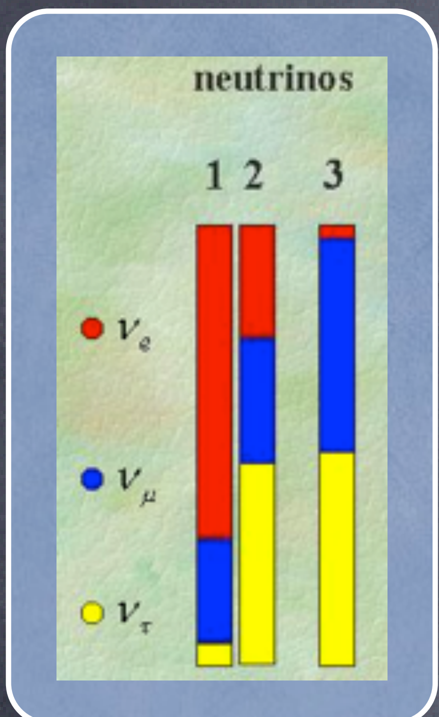
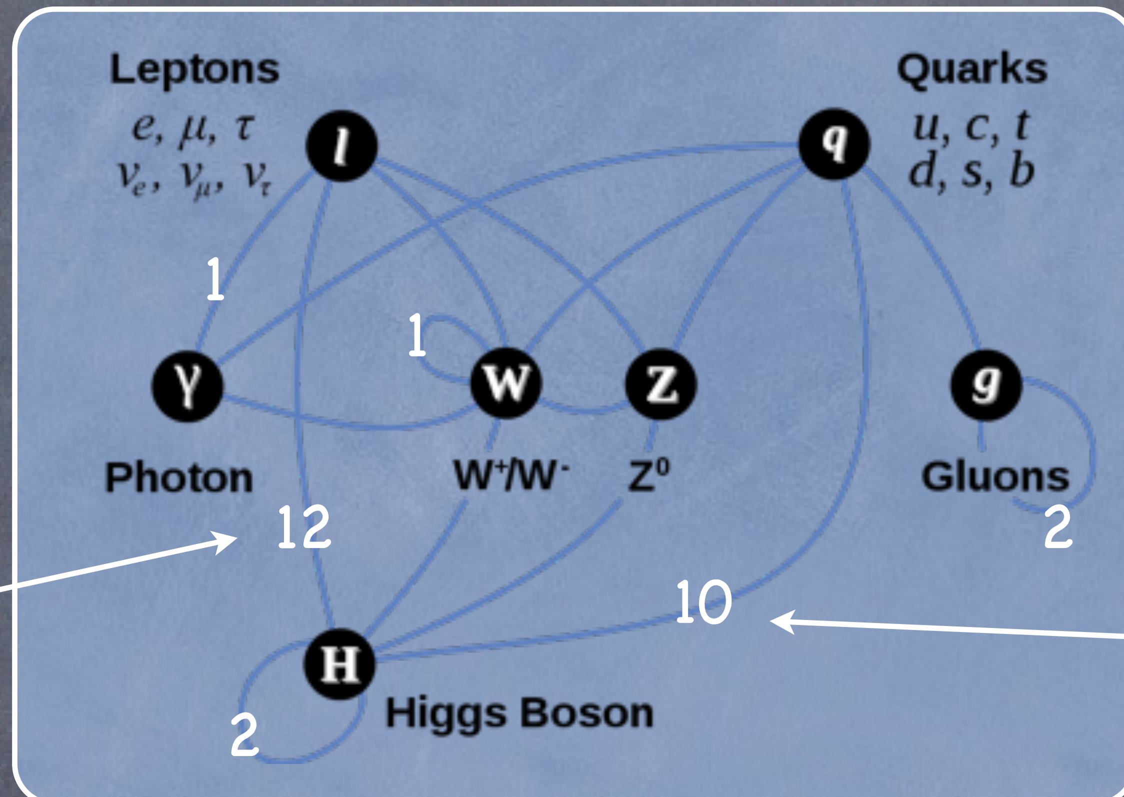
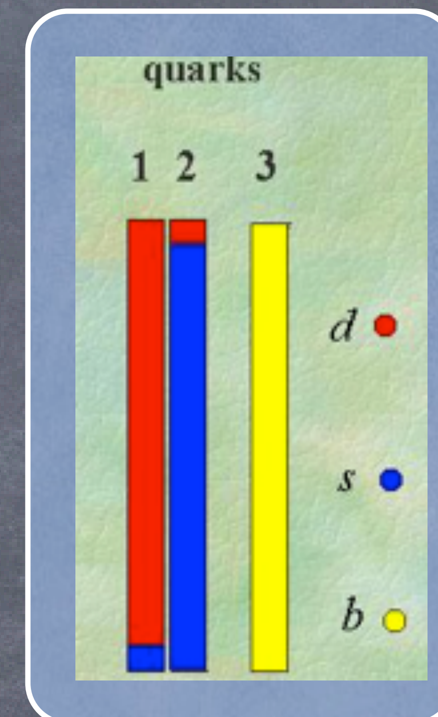
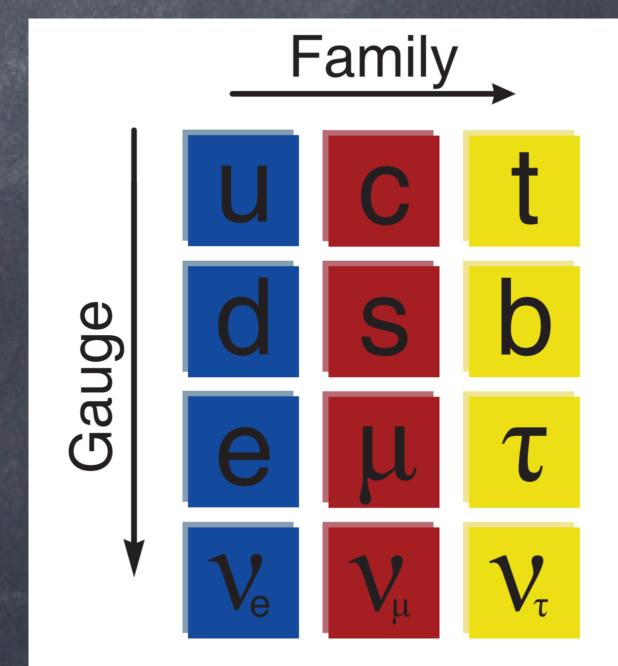






- most of them stem from interactions with the Higgs field, i.e. flavour parameters



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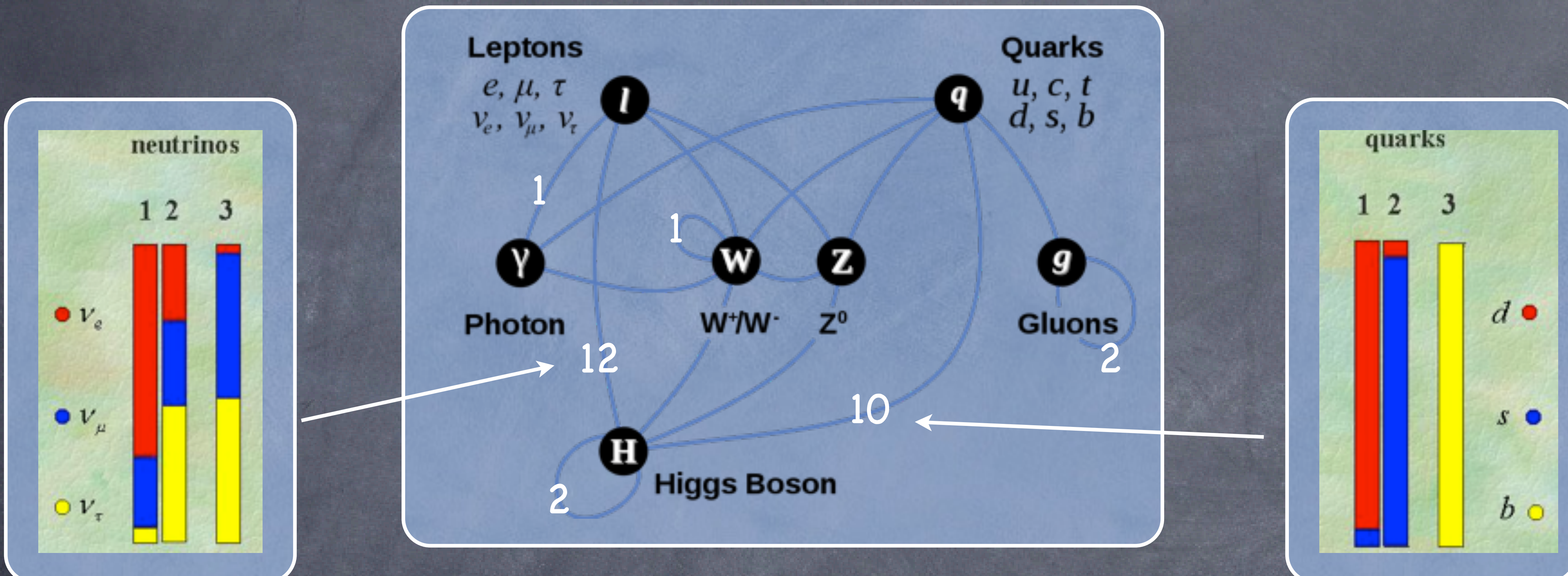

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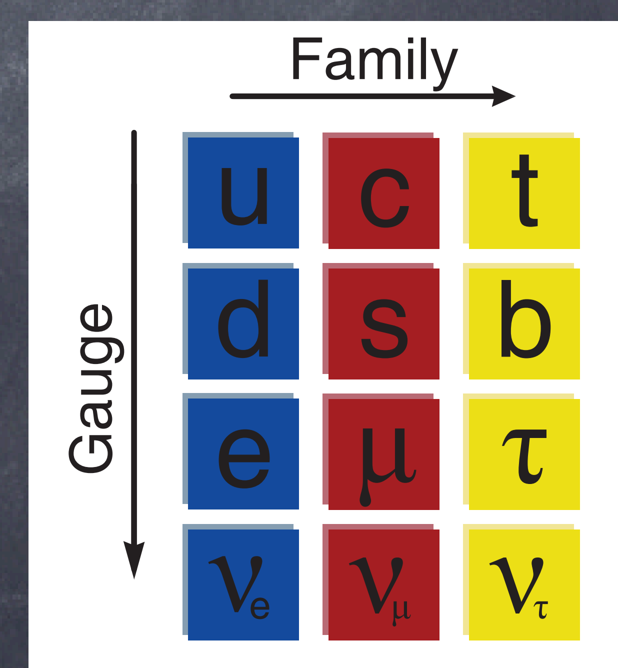
- other interactions tightly constrained by symmetry principles



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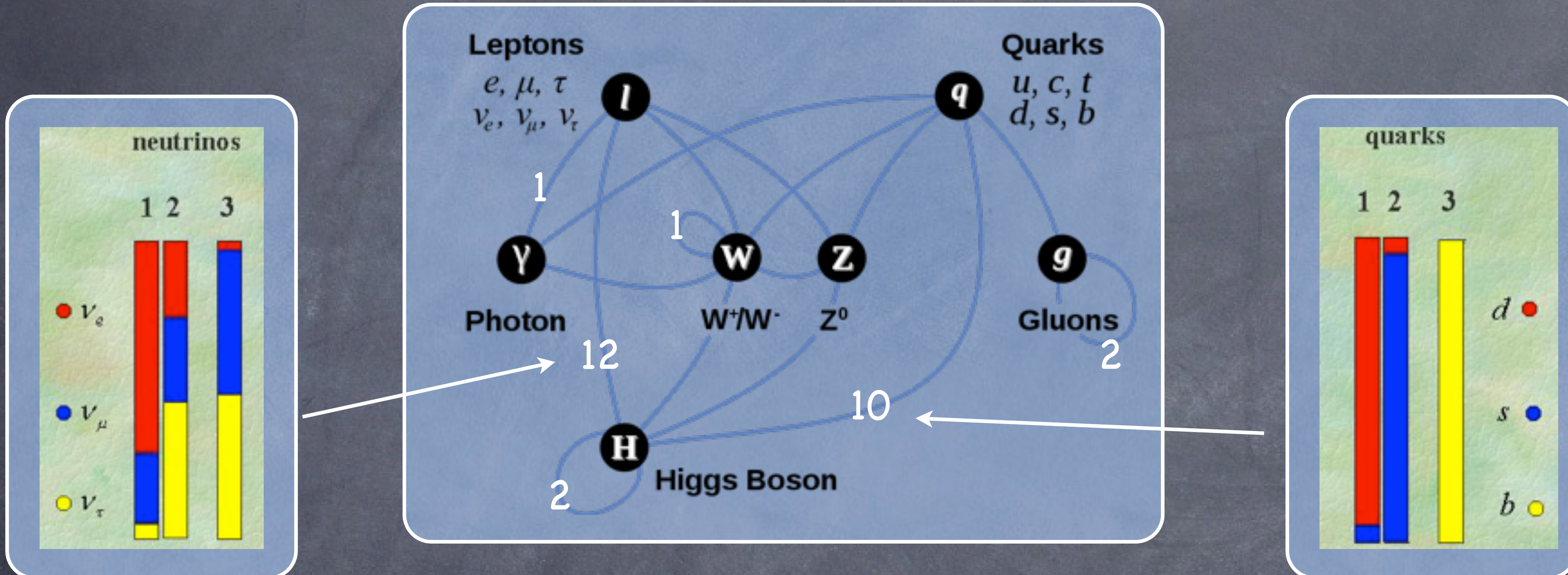

- most of them stem from interactions with the Higgs field, i.e. flavour parameters
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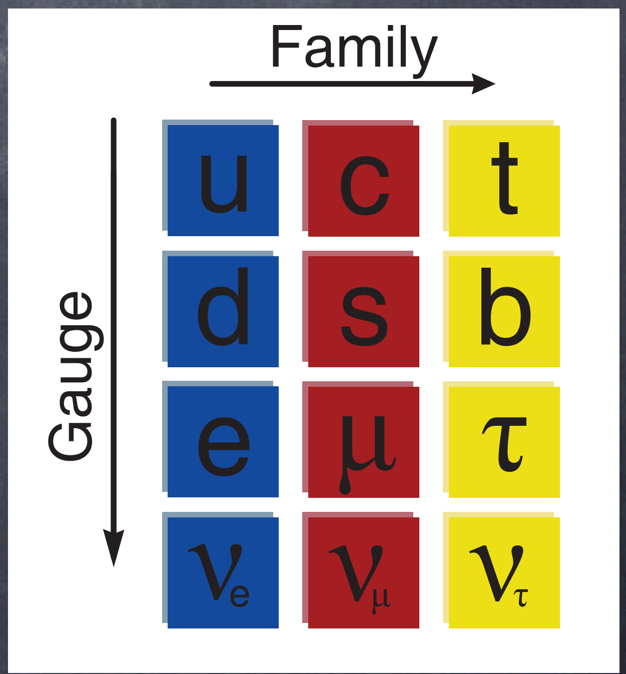
- quarks small mixings; leptons large mixings



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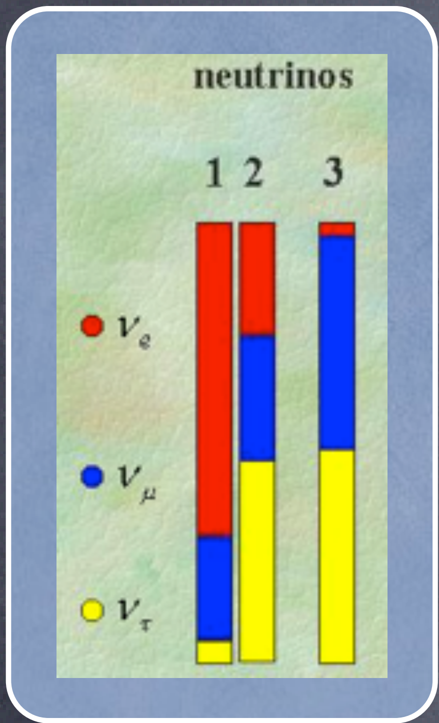
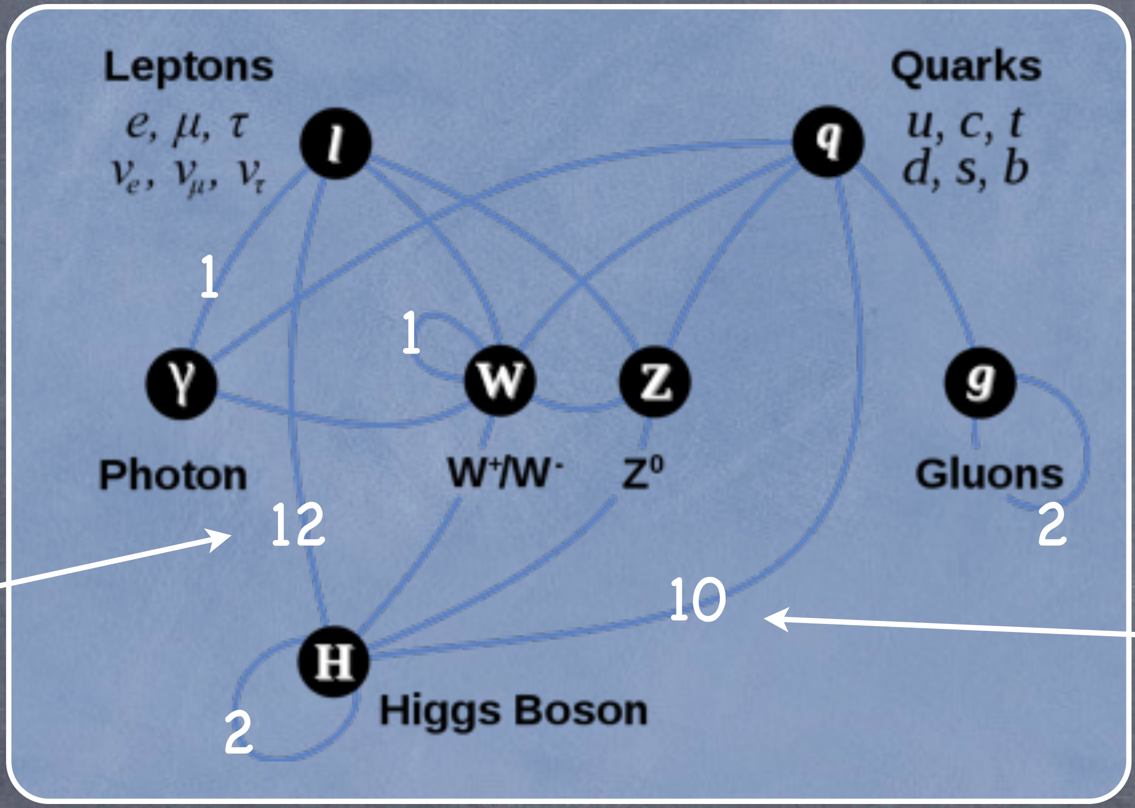
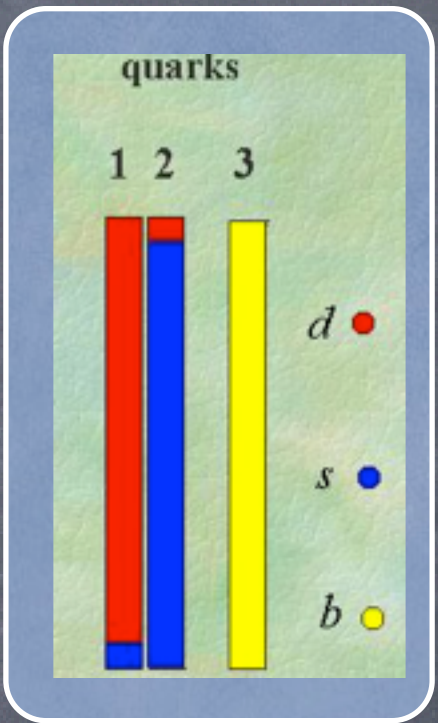
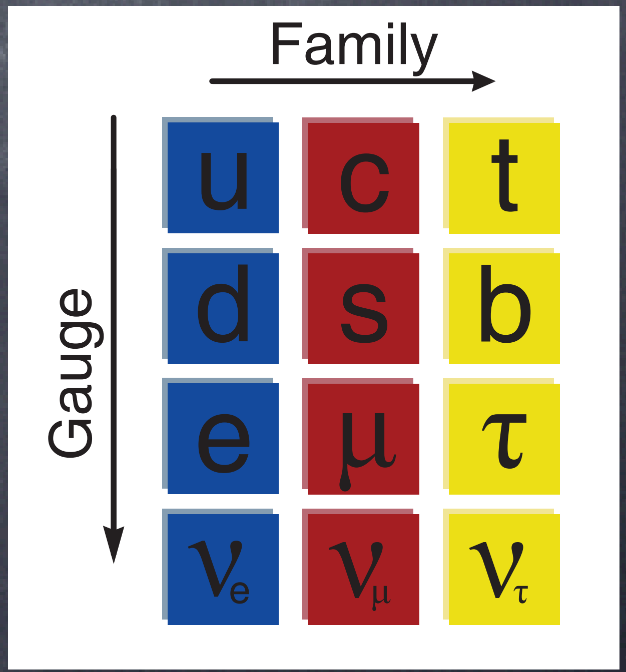


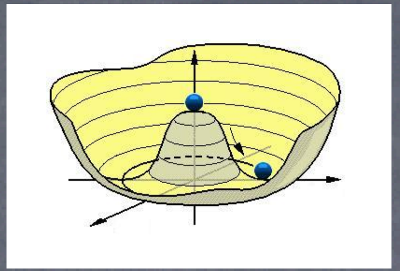




- most of them stem from interactions with the Higgs field, i.e. flavour parameters
- other interactions tightly constrained by symmetry principles
- quarks small mixings; leptons large mixings
- this talk only leptons, for quarks see next talk



Leptonic Mixing



- in SM there are three generations of leptons, two mass matrices

$$\mathcal{L} \supset -L^T Y_e e^c \tilde{H} + \frac{(Y_\nu)_{ij}}{\Lambda}(L_i H)(L_j H) + \text{h.c.} \quad \Rightarrow \quad \mathcal{L} \supset \frac{1}{2}\nu^T M_\nu \nu + e^T M_e e^c + \text{h.c.}$$

- after diagonalization of two mass matrices

$$V_e^T M_e M_e^\dagger V_e^* = \text{diag}(m_e^2, m_\mu^2, m_\tau^2) \qquad \text{and} \qquad V_\nu^T M_\nu V_\nu = \text{diag}(m_1, m_2, m_3)$$

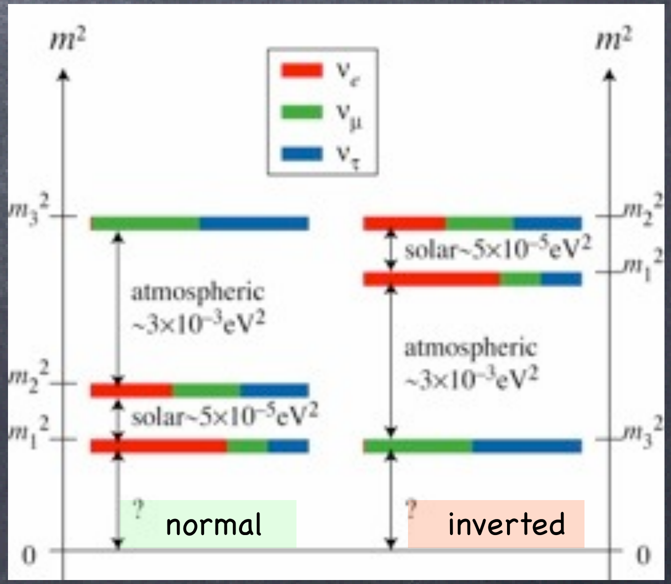
- flavour violation only in charged current interactions, analog of CKM

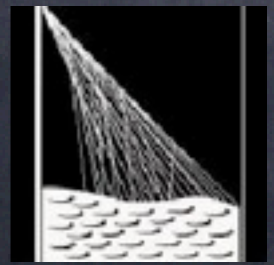
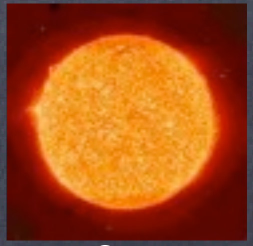
$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}\left[e^\dagger \sigma^\mu U_{PMNS}\nu\right] W_\mu^+ + \text{h.c.}$$

$$\boxed{U_{PMNS} = V_e^\dagger V_\nu}$$

($s_{ij} = \sin\theta_{ij}$ $c_{ij} = \cos\theta_{ij}$)

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$



= () x () x ()

$$\sin^2\theta_{23} = 0.52^{+0.06}_{-0.07}, \qquad \sin^2\theta_{13} = 0.02 - 0.03, \qquad \sin^2\theta_{12} = 0.312^{+0.017}_{-0.015}$$

Lepton mixing from discrete groups

G_f

$L \to \rho(g)L$

complete flavour group

residual symmetry of ($m_e m_e^+$) $e \to \rho(g_e)e$

$$\rho(g_e)^T M_e M_e^\dagger \rho(g_e)^* = M_e M_e^\dagger$$

$\nu \to \rho(g_\nu)$

residual symmetry of m_ν

$$\rho(g_\nu)^T M_\nu \rho(g_\nu) = M_\nu$$

$G_e=Z_3$

abelian

(Z3 smallest choice, but can also be continuous)

$G_\nu=Z_2 x Z_2$

abelian

(Z2xZ2 most general choice if mixing angles do not depend on masses & Majorana vs)

$$\Omega_e^\dagger \rho(g_e)\Omega_e = \rho(g_e)_{diag}$$

$$\Omega_\nu^\dagger \rho(g_\nu)\Omega_\nu = \rho(g_\nu)_{diag}$$

misaligned non-commuting symmetries lead to

[He, Keum, Volkas '06; Lam'07,'08; Altarelli,Feruglio'05]

$$U_{PMNS} = \Omega_e^\dagger \Omega_U$$

mixing matrix determined from symmetry up to interchanging of rows/columns and diagonal phase matrix

Lepton mixing from discrete groups

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complete flavour group $L \to \rho(g)L$

residual symmetry of ($m_e m_e^\dagger$) $e \to \rho(g_e)e$

$$\rho(g_e)^T M_e M_e^\dagger \rho(g_e)^* = M_e M_e^\dagger$$

$G_e = \langle T \rangle = Z_3$

$$\rho(T) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\Omega_e^\dagger \rho(g_e) \Omega_e = \rho(g_e)_{diag}$$

$\nu \to \rho(g_\nu)$ residual symmetry of m_ν

$$\rho(g_\nu)^T M_\nu \rho(g_\nu) = M_\nu$$

$G_\nu = \langle S,U \rangle = Z_2 \times Z_2$

$$\rho(S) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \rho(U) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\Omega_\nu^\dagger \rho(g_\nu) \Omega_\nu = \rho(g_\nu)_{diag}$$

misaligned non-commuting symmetries lead to

$$U_{PMNS} = \Omega_e^\dagger \Omega_U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin^2\theta_{12} = \frac{1}{3}, \quad \sin^2\theta_{23} = \frac{1}{2}, \quad \sin^2\theta_{13} = 0$$

[He, Keum, Volkas '06; Lam'07,'08; Altarelli,Feruglio'05]

mixing matrix determined from symmetry up to interchanging of rows/columns and diagonal phase matrix

tri-bimaximal mixing(TBM)

Lepton mixing from discrete groups

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complete flavour group

$L \to \rho(g)L$

residual symmetry of ($m_e m_e^\dagger$) $e \to \rho(g_e)e$

$$\rho(g_e)^T M_e M_e^\dagger \rho(g_e)^* = M_e M_e^\dagger$$

residual symmetry of m_ν $\nu \to \rho(g_\nu)$

$$\rho(g_\nu)^T M_\nu \rho(g_\nu) = M_\nu$$

$G_e = \langle T \rangle = Z_3$

$$\rho(T) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$G_\nu = \langle S \rangle = Z_2$

$$\rho(S) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\Omega_e^\dagger \rho(g_e) \Omega_e = \rho(g_e)_{diag}$$

$$\Omega_\nu^\dagger \rho(g_\nu) \Omega_\nu = \rho(g_\nu)_{diag}$$

misaligned non-commuting symmetries lead to

$$U_{PMNS} = U_{TBM} \begin{pmatrix} \cos\theta & 0 & e^{i\delta}\sin\theta \\ 0 & 1 & 0 \\ -e^{i\delta}\sin\theta & 0 & \cos\theta \end{pmatrix}$$

trimaximal mixing(TMM)

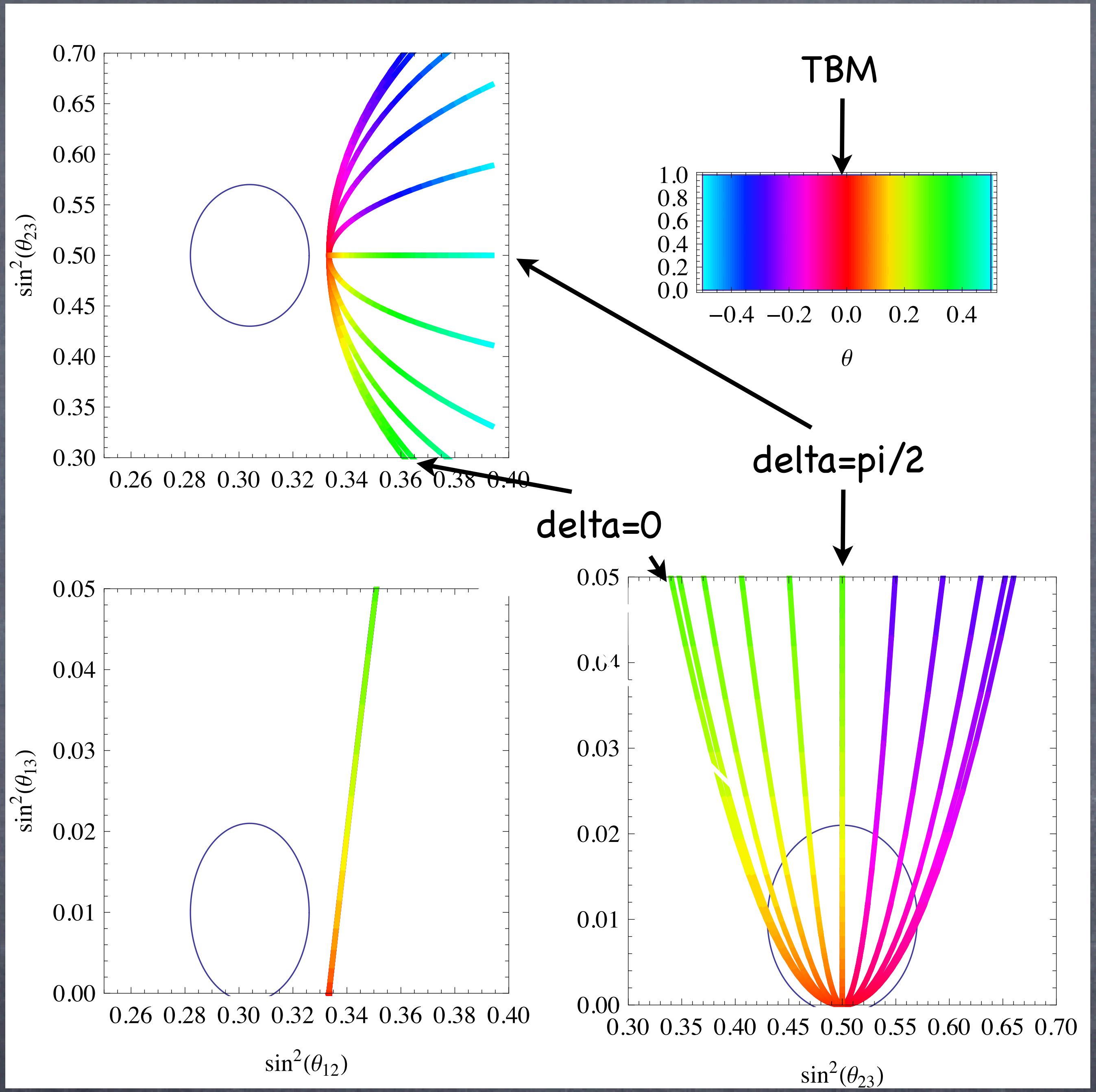
[Lin'10, Shimizu,Tanimoto, Watanabe'11,Luhn,King' 11, Hernandez,Smirnov 12,...]

mixing matrix determined from symmetry up to interchanging of rows/columns and diagonal phase matrix

Comparison to data

one year ago...

TBM allowed within 2σ



ellipses:(rough) 1σ experimental uncertainties

$$U_{PMNS} = U_{TBM} \begin{pmatrix} \cos\theta & 0 & e^{i\delta}\sin\theta \\ 0 & 1 & 0 \\ -e^{i\delta}\sin\theta & 0 & \cos\theta \end{pmatrix}$$

Comparison to data

today

after

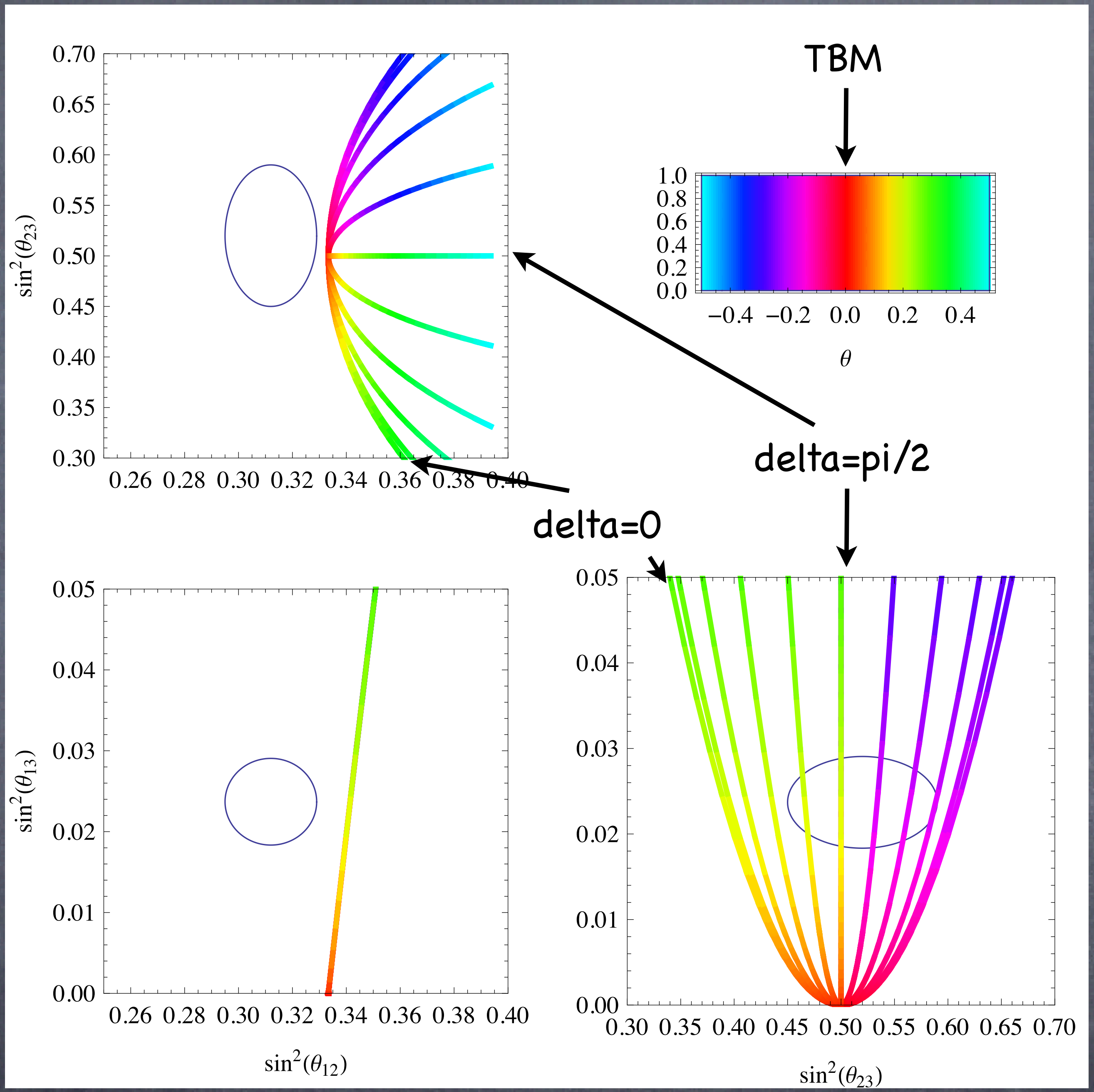
6/2011

8/2011

11/2011

3/2012

4/2012



ellipses:(rough) 1σ experimental uncertainties

TBM out

(or needs large NLO corrections)

TMM ok

$$U_{PMNS} = U_{TBM} \begin{pmatrix} \cos\theta & 0 & e^{i\delta}\sin\theta \\ 0 & 1 & 0 \\ -e^{i\delta}\sin\theta & 0 & \cos\theta \end{pmatrix}$$

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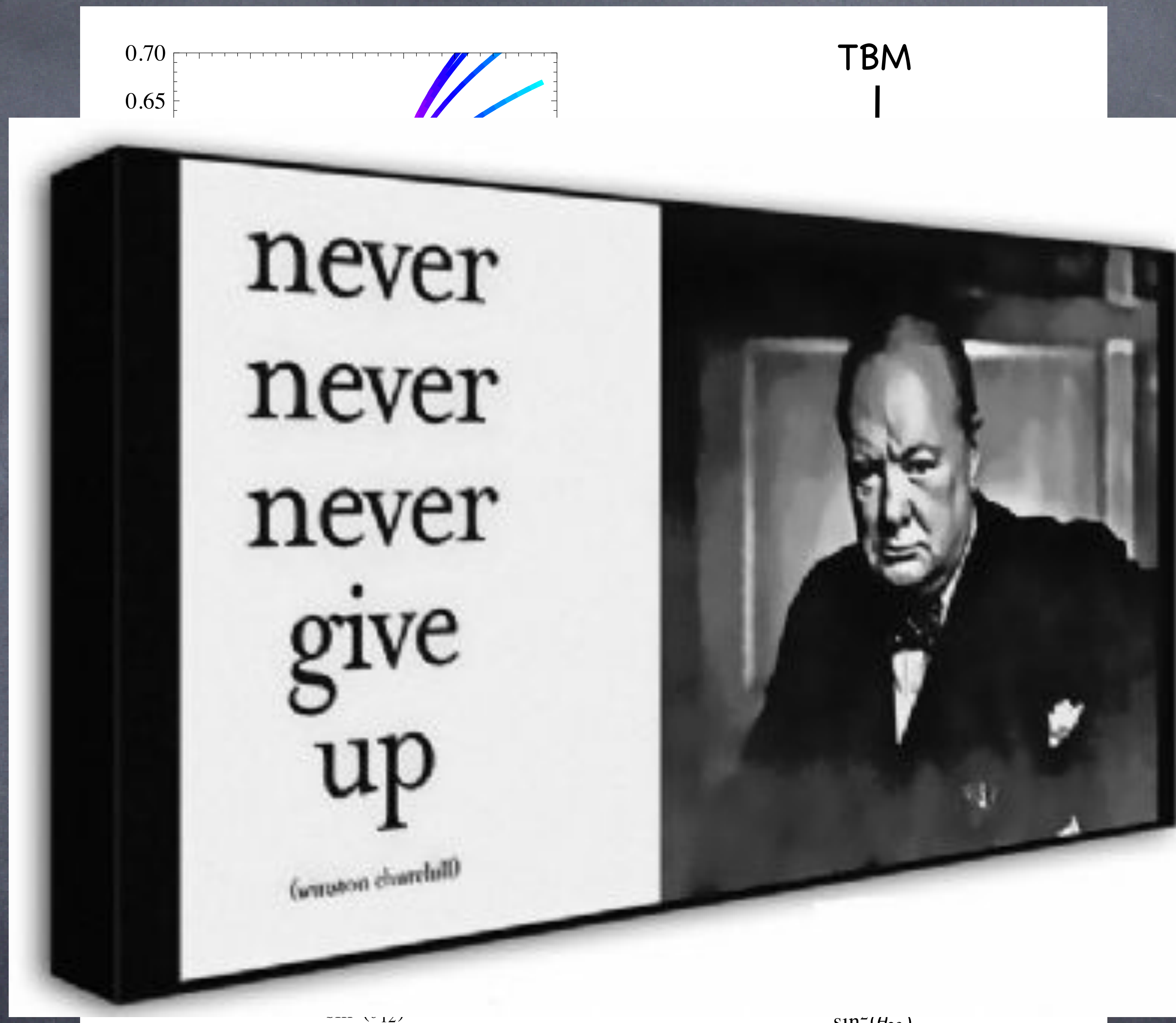

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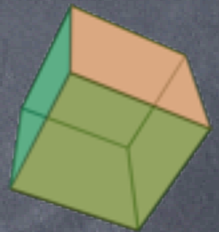
$$U_{PMNS} = U_{TBM} \begin{pmatrix} \cos\theta & 0 & e^{i\delta}\sin\theta \\ 0 & 1 & 0 \\ -e^{i\delta}\sin\theta & 0 & \cos\theta \end{pmatrix}$$

What is the Flavour Group ?

- we have seen which residual symmetries in the charged lepton and neutrino sector lead to interesting mixing patterns

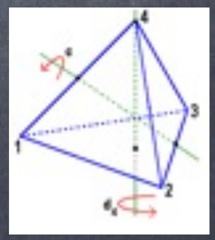
- if all residual symmetries are symmetries of the entire Lagrangian, in the case of tri-bimaximal mixing, we find the group

$$S_4 = \langle S, T, U | S^2 = T^3 = (ST)^3 = U^2 = (US)^2 = (UT)^2 = 1 \rangle$$



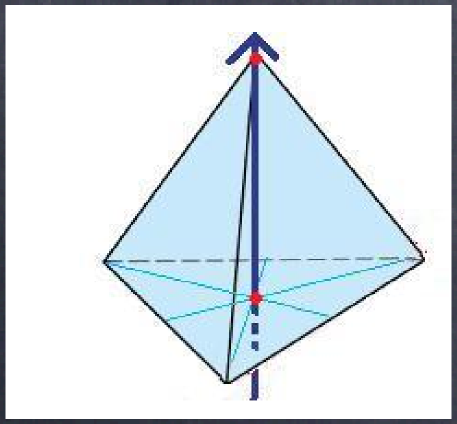
- for the case of $G_\nu = \langle S \rangle = Z_2$, the symmetry group is

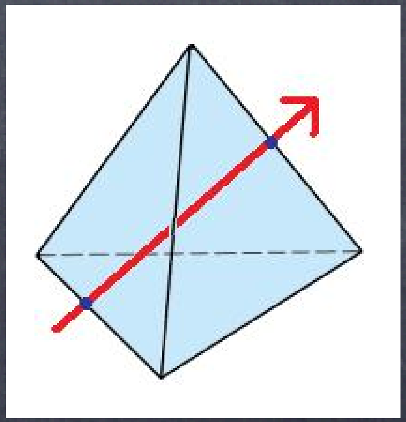
$$A_4 = \langle S, T | S^2 = T^3 = (ST)^3 = 1 \rangle$$



- models based on this symmetry often lead to TBM because of accidental symmetries

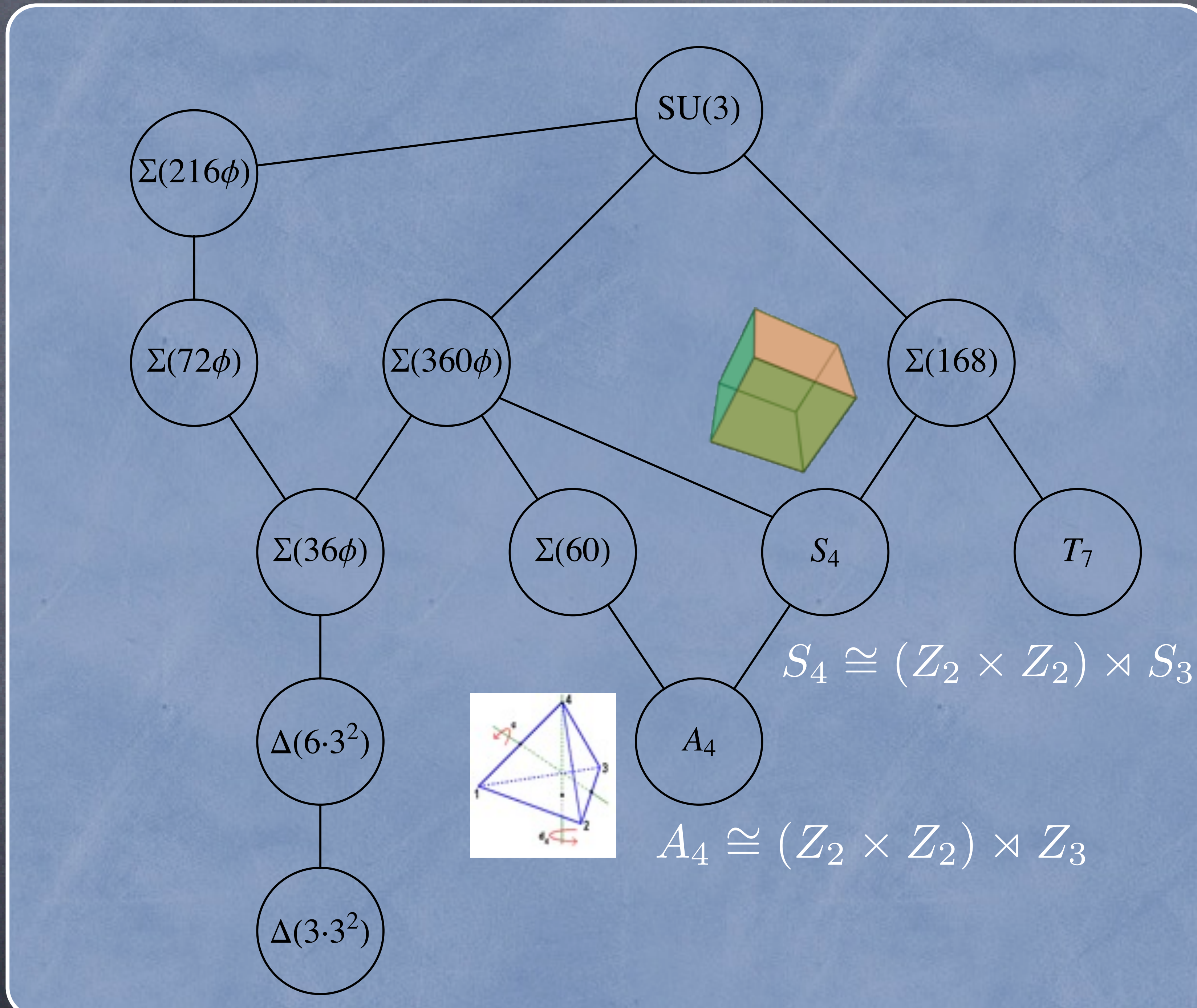
T





S

Other Candidate Groups



$S_4 \cong (Z_2 \times Z_2) \rtimes S_3$

$A_4 \cong (Z_2 \times Z_2) \rtimes Z_3$

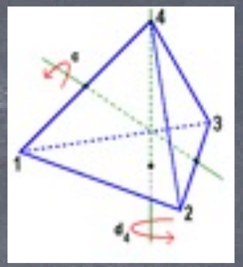
$T_7 \cong Z_7 \rtimes Z_3$

$T' \cong Z_2.A_4$

$\Delta(27) \cong (Z_3 \times Z_3) \rtimes Z_3$

[Merle,Zwicky 1110.4891]

A_4 Symmetry Group



A_4 is the smallest symmetry group that can lead to TBM mixing:

$$A_4 \cong (Z_2 \times Z_2) \rtimes Z_3 \cong \langle S, T | S^2 = T^3 = (ST)^3 = 1 \rangle$$

	S	T
$\underline{\mathbf{1}}_\mathbf{1}$	1	1
$\underline{\mathbf{1}}_\mathbf{2}$	1	ω
$\underline{\mathbf{1}}_\mathbf{3}$	1	ω^2
$\underline{\mathbf{3}}_\mathbf{1}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

	1	T	T^2	S
$\underline{\mathbf{1}}_\mathbf{1}$	1	1	1	1
$\underline{\mathbf{1}}_\mathbf{2}$	1	ω	ω^2	1
$\underline{\mathbf{1}}_\mathbf{3}$	1	ω^2	ω	1
$\underline{\mathbf{3}}$	3	0	0	-1

1-d reps. correspond to reps. of Z_3

$\omega = e^{i2\pi/3}$

$$\underline{\mathbf{3}} \times \underline{\mathbf{3}} = \underline{\mathbf{1}}_\mathbf{1} + \underline{\mathbf{1}}_\mathbf{2} + \underline{\mathbf{1}}_\mathbf{3} + \underline{\mathbf{3}}_S + \underline{\mathbf{3}}_A$$

$$(ab)_{\underline{\mathbf{1}}_\mathbf{1}} = \frac{1}{\sqrt{3}}(a_1b_1 + a_2b_2 + a_3b_3)$$

$$(ab)_{\underline{\mathbf{1}}_\mathbf{2}} = \frac{1}{\sqrt{3}}(a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3) \qquad (ab)_{\underline{\mathbf{1}}_\mathbf{3}} = \frac{1}{\sqrt{3}}(a_1b_1 + \omega a_2b_2 + \omega^2 a_3b_3)$$

$$(ab)_{A,\underline{\mathbf{3}}} = \frac{1}{2}\begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix} \qquad (ab)_{S,\underline{\mathbf{3}}} = \frac{1}{2}\begin{pmatrix} a_2b_3 + a_3b_2 \\ a_3b_1 + a_1b_3 \\ a_1b_2 + a_2b_1 \end{pmatrix}$$

where $(a_1, a_2, a_3), (b_1, b_2, b_3) \sim \underline{\mathbf{3}}$.

An A_4 Prototype model

- (A_4, Z_4) charge assignments: $L \sim (3,i)$, $e^c \sim (1_1,-i)$, $\mu^c \sim (1_2,-i)$, $\tau^c \sim (1_3,-i)$, $\chi \sim (3,1)$, $\Phi \sim (3,-1)$, $\xi \sim (1,-1)$
- auxiliary Z_4 separates neutral and charged lepton sectors at LO

$$y_e(\ell\chi)_{\underline{\mathbf{1}}_1} e^c + y_\mu(\ell\chi)_{\underline{\mathbf{1}}_3} \mu^c + y_\tau(\ell\chi)_{\underline{\mathbf{1}}_2} \tau^c$$

$\langle\chi\rangle \sim (1,1,1)$ A_4 $\langle\Phi\rangle \sim (1,0,0)$

$$(LL)_{\underline{\mathbf{3}}}\phi + (LL)_{\underline{\mathbf{1}}_1}\xi$$

$Z_3 = \langle T \rangle$ $Z_2 = \langle S \rangle$

$$M_E \sim \begin{pmatrix} y_e & y_\mu & y_\tau \\ y_e & \omega y_\mu & \omega^2 y_\tau \\ y_e & \omega^2 y_\mu & \omega y_\tau \end{pmatrix} \quad \xleftrightarrow{\text{TBM}} \quad M_\nu \sim \begin{pmatrix} \tilde{a} & 0 & 0 \\ 0 & \tilde{a} & \tilde{d} \\ 0 & \tilde{d} & \tilde{a} \end{pmatrix}$$

(symmetry U accidental)

Vacuum alignment crucial!

[e.g. Ma,Rajasekaran'01, Babu, Ma, Valle '03, Altarelli,Feruglio, '05,'06]

An A_4 Prototype model

- (A_4,Z_4) charge assignments: $L\sim(3,i)$, $e^c\sim(1_1,-i)$, $\mu^c\sim(1_2,-i)$, $\tau^c\sim(1_3,-i)$, $\chi\sim(3,1)$, $\Phi\sim(3,-1)$, $\xi\sim(1,-1)$
- auxiliary Z_4 separates neutral and charged lepton sectors at LO

$$y_e(\ell\chi)_{\underline{\mathbf{1}}_{\mathbf{1}}}e^c + y_\mu(\ell\chi)_{\underline{\mathbf{1}}_{\mathbf{3}}}\mu^c + y_\tau(\ell\chi)_{\underline{\mathbf{1}}_{\mathbf{2}}}\tau^c$$

A_4

$\langle\chi\rangle \sim (1,1,1)$

$$(LL)_{\underline{\mathbf{3}}}\phi + (LL)_{\underline{\mathbf{1}}_{\mathbf{1}}}\xi + (LL)_{\underline{\mathbf{1}}_{\mathbf{3}}}\tilde{\xi}$$

$\langle\Phi\rangle \sim (1,0,0)$

$Z_3 = \langle T\rangle$

$Z_2 = \langle S\rangle$

$$M_E \sim \begin{pmatrix} y_e & y_\mu & y_\tau \\ y_e & \omega y_\mu & \omega^2 y_\tau \\ y_e & \omega^2 y_\mu & \omega y_\tau \end{pmatrix}$$

TMM

$$\tan 2\theta \sim \frac{\sqrt{3}c}{2a-c}$$

$$M_\nu \sim \begin{pmatrix} \tilde{a}+\tilde{c} & 0 & 0 \\ 0 & \tilde{a}+\omega\tilde{c} & \tilde{d} \\ 0 & \tilde{d} & \tilde{a}+\omega^2\tilde{c} \end{pmatrix}$$

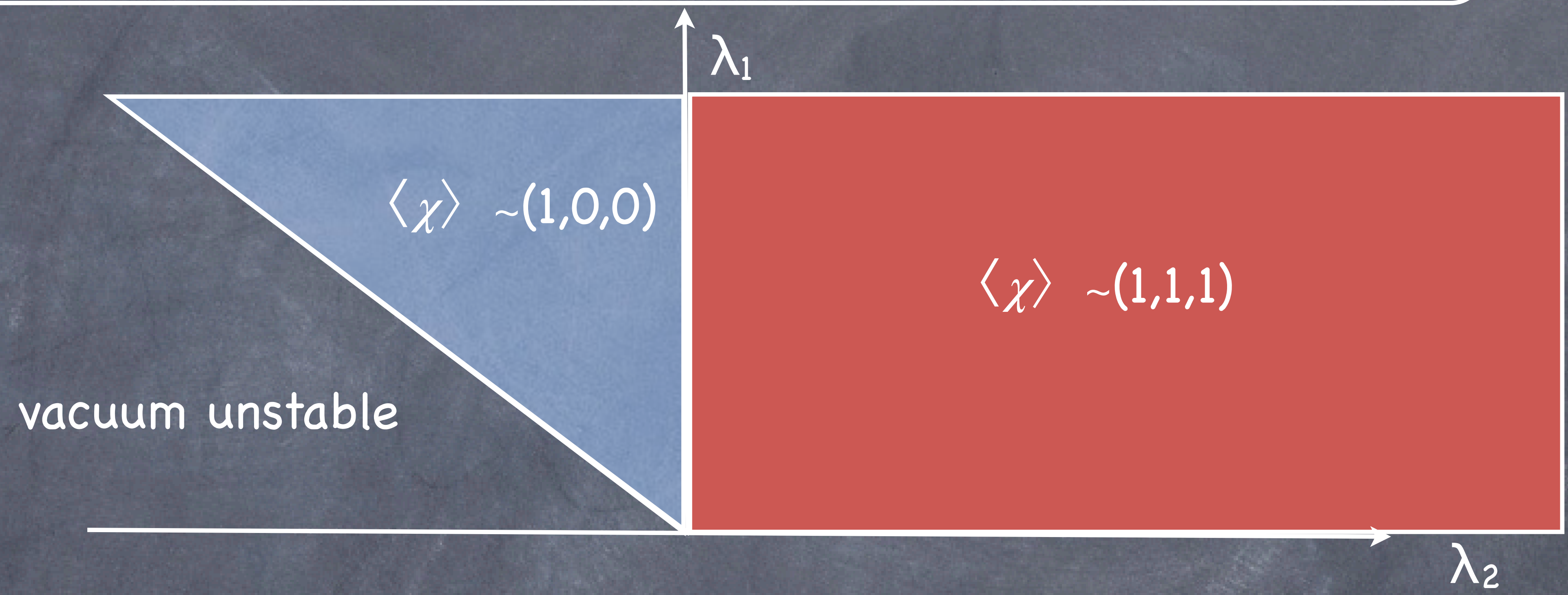
(symmetry U broken)

Vacuum alignment crucial!

[e.g. Ma,Rajasekaran'01, Babu, Ma, Valle '03, Altarelli,Feruglio, '05,'06]

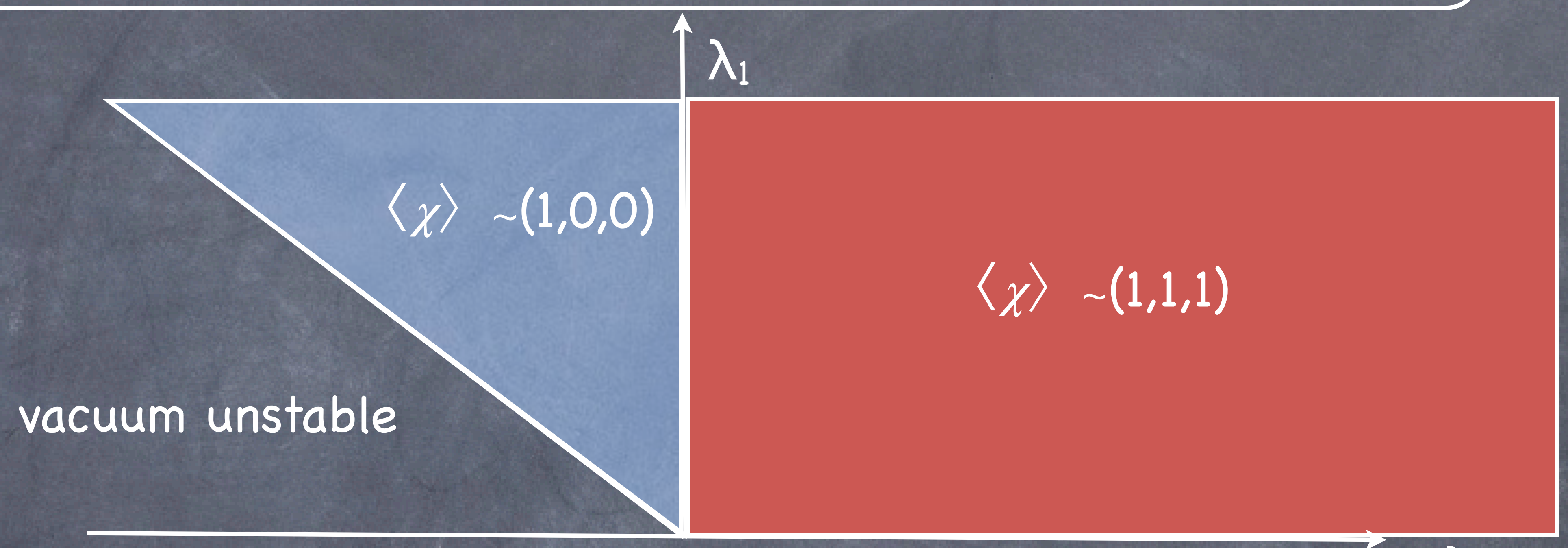
Can Vacuum Alignment be realised?

$$V_\chi = m_0^2\,(\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{1}}} + \lambda_1\,(\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{1}}}(\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{1}}} + \lambda_2\,(\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{2}}}(\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{3}}}$$



Can Vacuum Alignment be realised?

$$V_\chi = m_0^2\,(\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{1}}} + \lambda_1\,(\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{1}}}(\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{1}}} + \lambda_2\,(\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{2}}}(\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{3}}}$$



Effect of breaking to Z_2 in another sector can be included by adding:

$$V_{soft,Z_2} = m_A^2\chi_1^2 + m_B^2\chi_2^2 + m_C^2\chi_2\chi_3$$

Minimization conditions then give:

$$0 = \left[\frac{\partial V}{\partial \chi_1}\right]_{\chi_i = v'} = \frac{2}{\sqrt{3}}\left(m_0^2 + \sqrt{3}m_A^2\right)\, v' + 4\lambda_1 v'^3$$

$$0 = \left[\frac{\partial}{\partial \chi_2}V - \frac{\partial}{\partial \chi_3}V\right]_{\chi_i = v'} = 2\,m_B^2\,v'$$

$$0 = \left[\frac{\partial}{\partial \chi_1}V - \frac{\partial}{\partial \chi_3}V\right]_{\chi_i = v'} = \left(2\,m_A^2 - m_C^2\right)\, v'$$

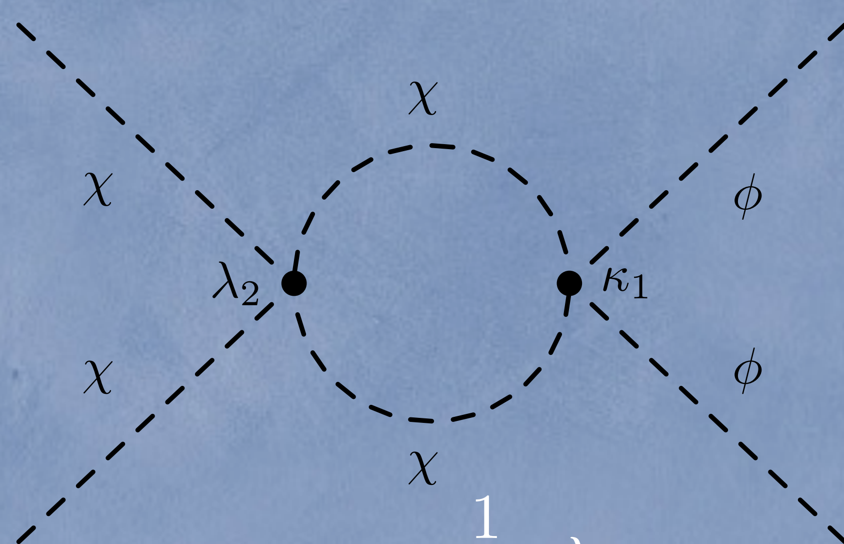
- This thus requires $m_A = m_B = m_C = 0$, i.e. all non-trivial contractions between Φ and χ have to vanish in the potential.

Can Vacuum Alignment be realized?

- To get the correct vacuum alignment, one thus needs to fine-tune the couplings

$$V_{\rm mix}(\chi,\phi) = \kappa_{\underline{\mathbf{3}}_{\mathbf{1}}}(\phi\phi)_{\underline{\mathbf{3}}_{\mathbf{1}}}(\chi\chi)_{\underline{\mathbf{3}}_{\mathbf{1}}} + \left(\kappa_{\underline{\mathbf{1}}_{\mathbf{2}}}(\phi\phi)_{\underline{\mathbf{1}}_{\mathbf{2}}}(\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{3}}} + \text{h.c.}\right) + \rho_{\underline{\mathbf{3}}_{\mathbf{1}}}\phi(\chi\chi)_{\underline{\mathbf{3}}_{\mathbf{1}}}$$

- even if one sets the couplings to zero, they will be generated at one-loop level



$$\kappa_{\underline{\mathbf{1}}_{\mathbf{2}}} \sim \frac{1}{16\pi^2}\lambda_2\kappa_1$$

$\kappa_1(\phi\phi)_{\underline{\mathbf{1}}_{\mathbf{1}}}(\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{1}}}$

flavour conserving

- one needs a symmetry to enforce $V=V_\Phi(\Phi)+V_\chi(\chi)+(\Phi\Phi)_1(\chi\chi)_1$.

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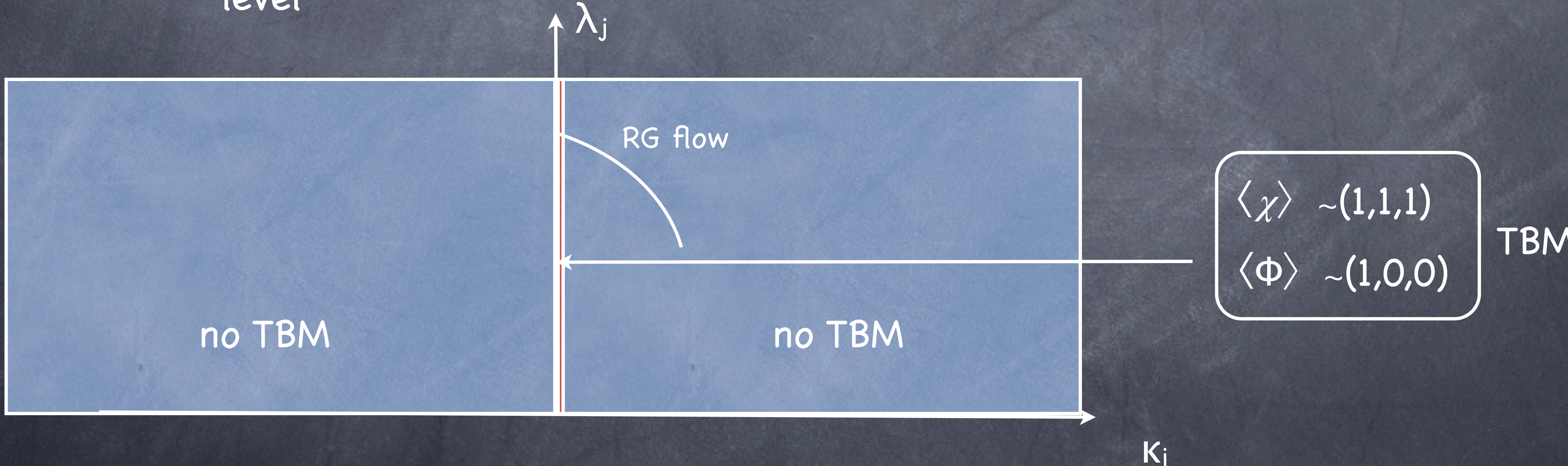

- for a natural model the realizes vacuum alignment, we need to have a finite portion of parameter space in which TBM vacuum is realized

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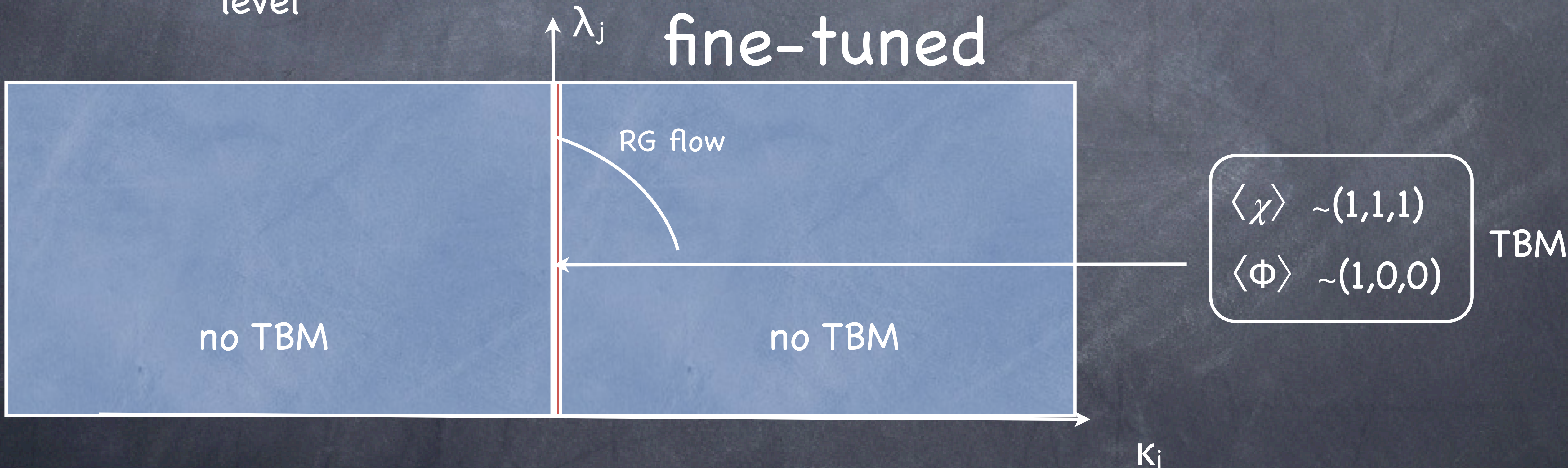

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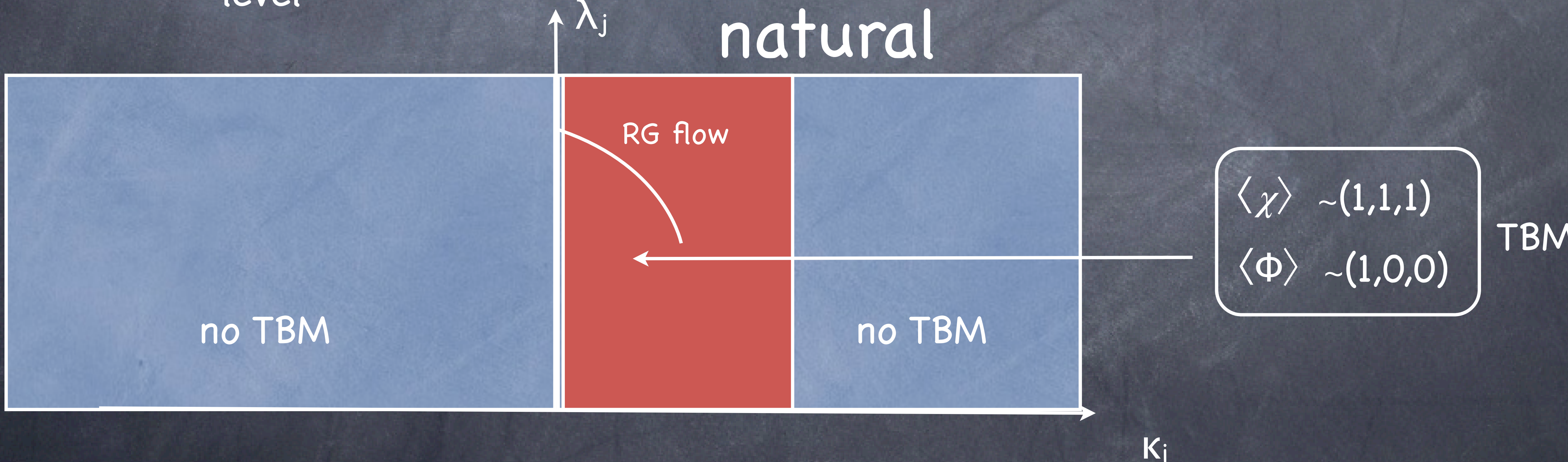

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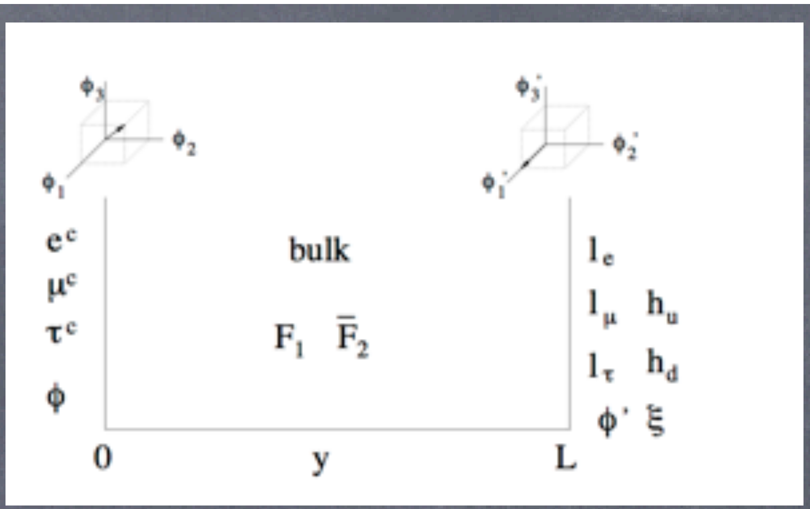
Solutions in the Literature

In models with extra dimensions(ED), it is possible to locate the various fields at different locations in the ED, thereby forbidding the cross-couplings.

Altarelli, Feruglio 2005



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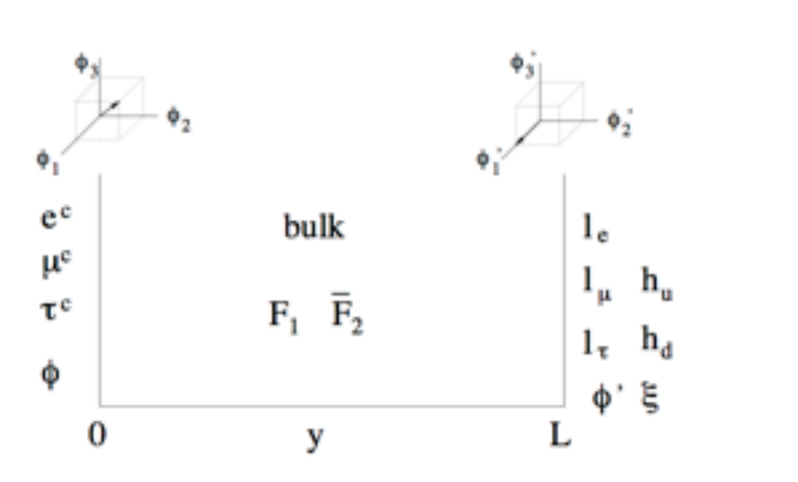

In SUSY, one has to introduce a continuous R-symmetry and additional fields with R-charge 2(driving fields). These fields enter the superpotential only linearly and allow the vacuum alignment.

Altarelli, Feruglio 2006

Field	φ_T	φ_S	ξ	$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A_4	3	3	1	1	3	3	1
Z_3	1	ω	ω	ω	1	ω	ω
$U(1)_R$	0	0	0	0	2	2	2

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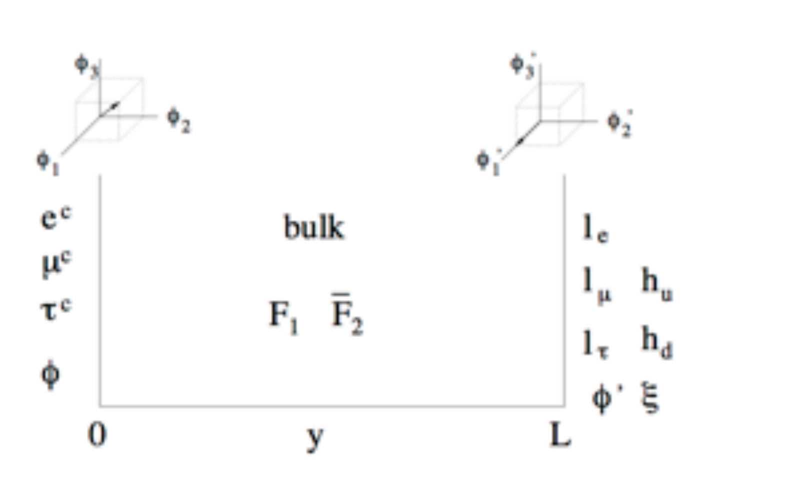

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- leptons transform only under A_4 subgroup
- if one takes $\Phi \sim 16$, vacuum alignment possible as $V=V(\Phi)+V(\chi)+(\Phi\ \Phi)_1(\chi\chi)_1$
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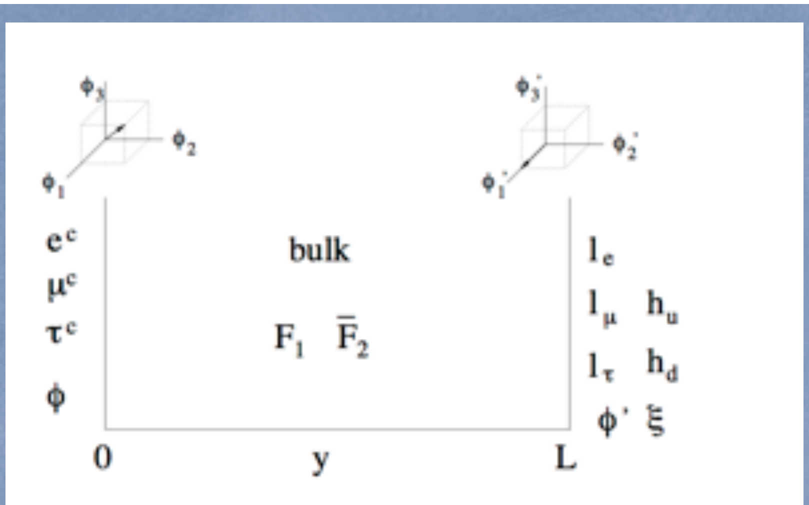

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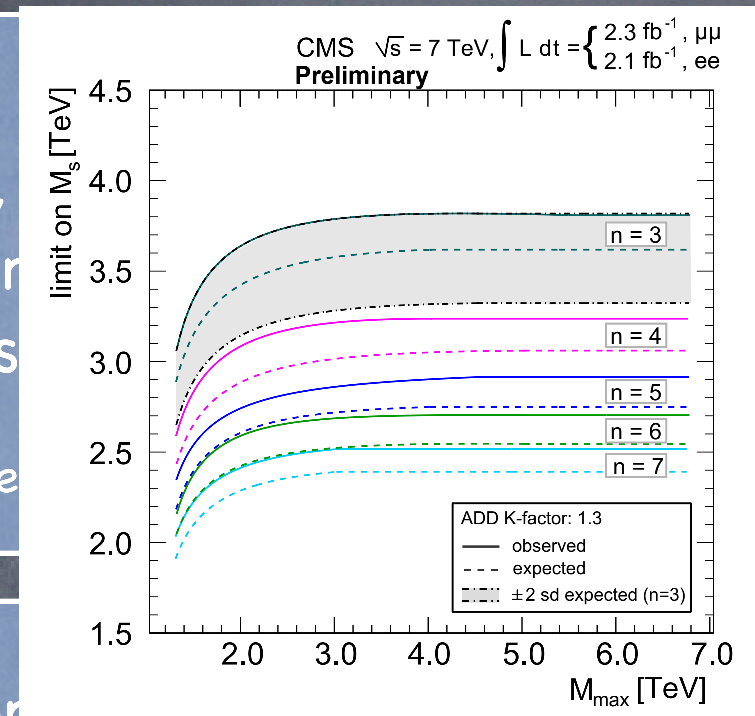

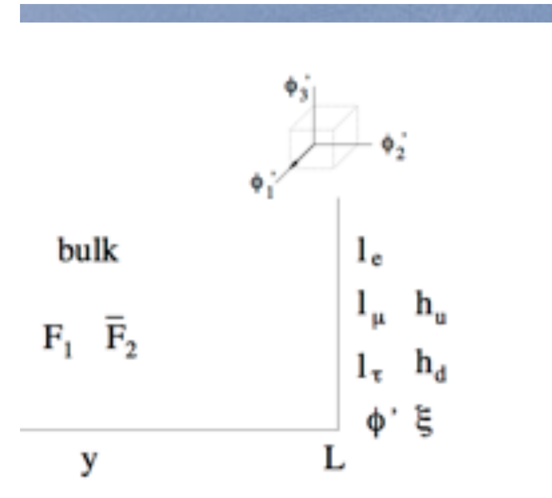

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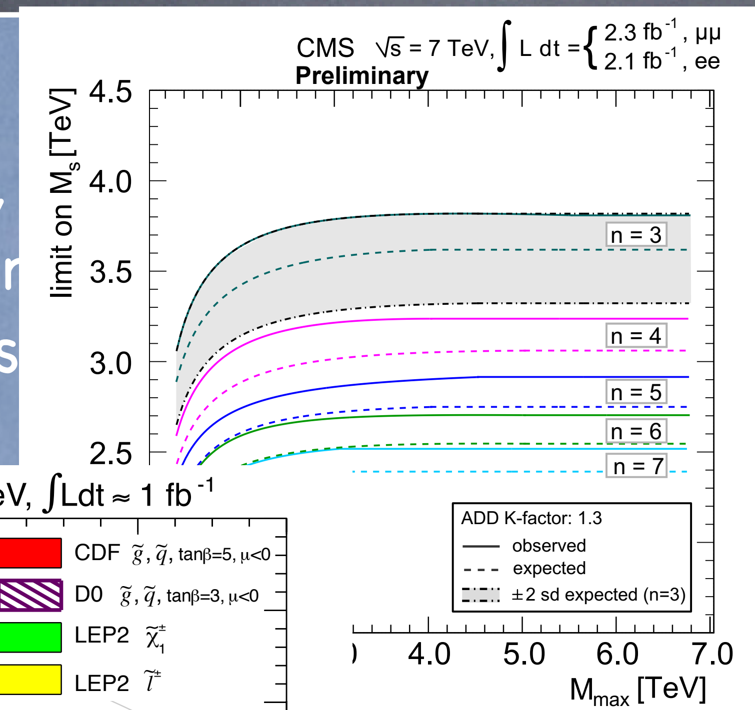

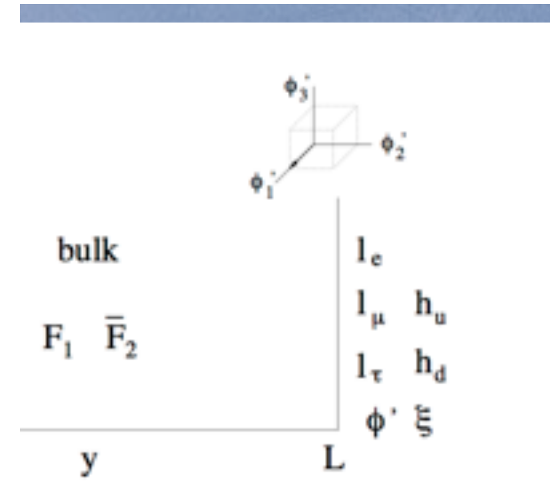

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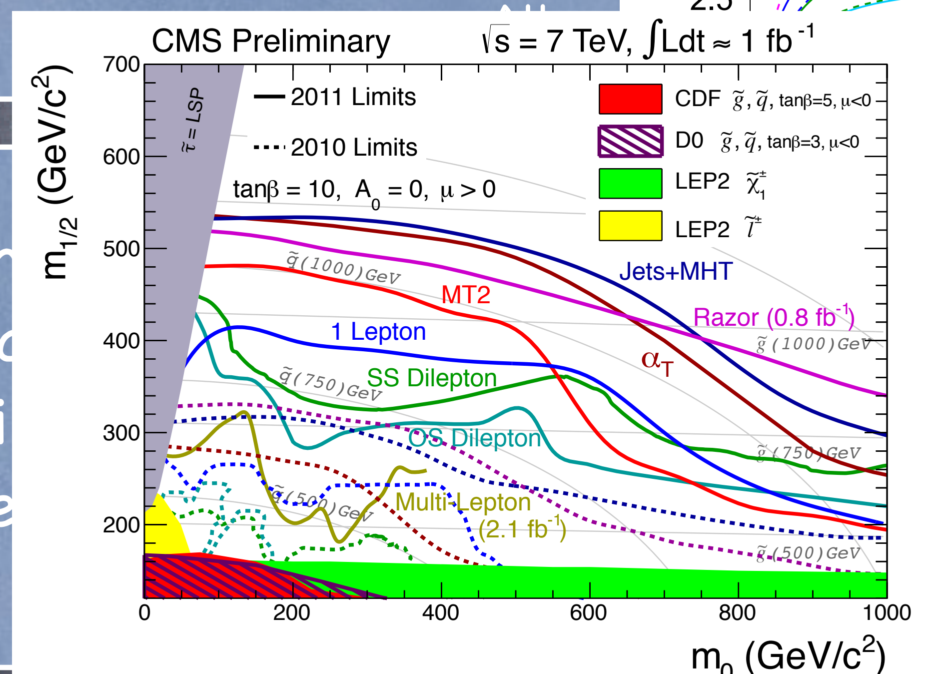

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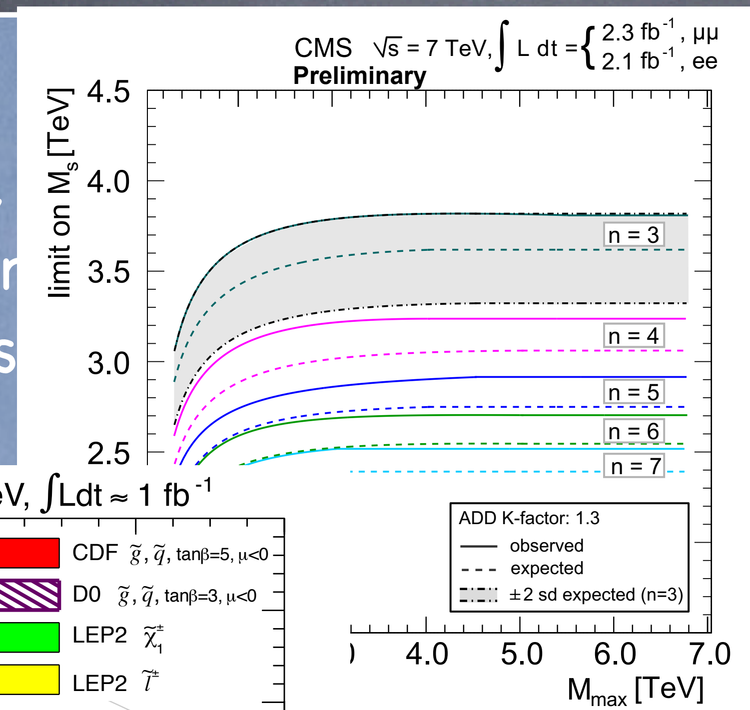

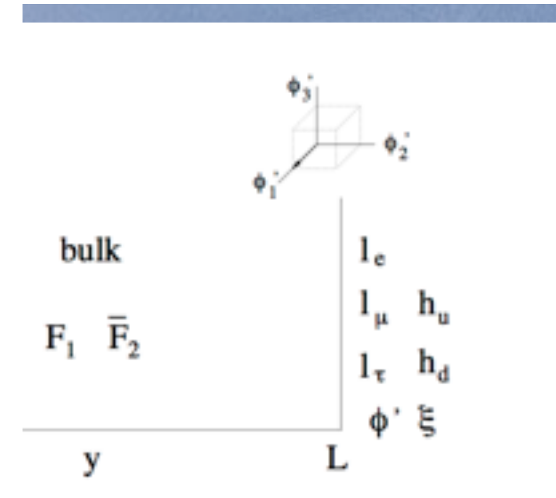

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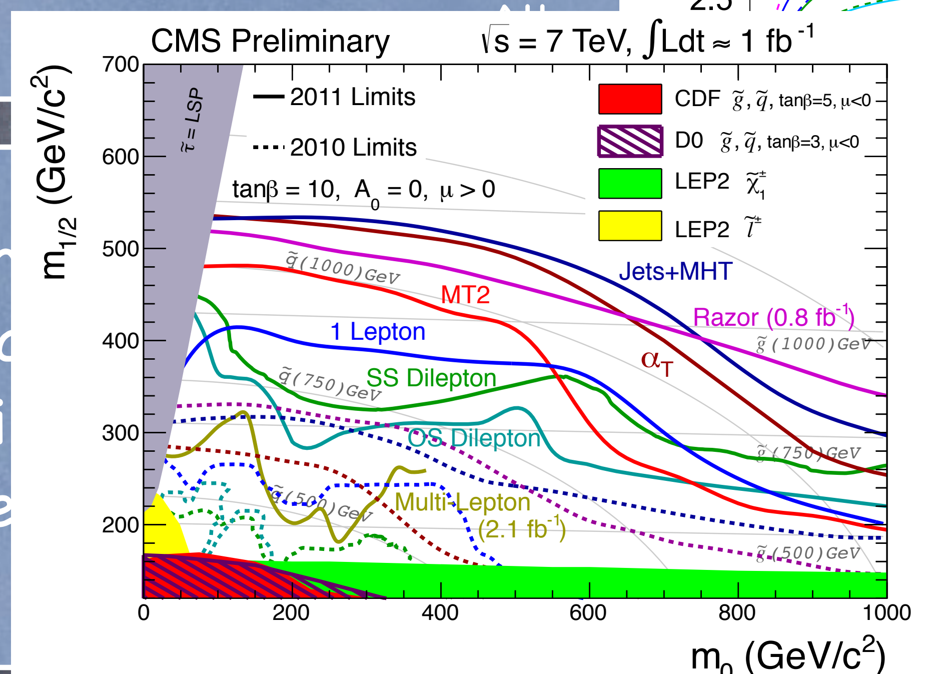

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2006

Babu and Gab ... the properties
- leptons t
- if one ta ... $_1(\chi\chi)_1$
- neutrino

- Model is fine-tuned/needs special UV completion: different mass entries in neutrino mass matrix stem from operators of very different mass dimensions $(ll)_3\Phi^4+(ll)_1$
- non-minimal(size: 15552), needs large representations

Pro ... ions

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In SUSY, one
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Babu and Gab ... the properties

- leptons t
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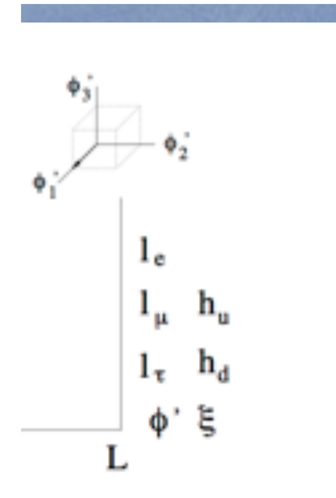
TABLE II. The character table for $(S_3 \times S_3 \times S_3 \times S_3) \rtimes A_4$.

	1	8	24	32	16	12	54	108	81	72	144	216	96	216	216	108	432	432	648	1296	⋯
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	⋯
$1'$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	⋯
$1''$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	⋯
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	−1	−1	−1	−1	−1	⋯
$\tilde{1}$	1	1	1	1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	1	−1	−1	⋯
$\tilde{1}'$	1	1	1	1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	1	−1	−1	⋯
$\tilde{1}''$	1	1	1	1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	1	−1	−1	⋯
$\tilde{3}$	3	3	3	3	3	−3	3	−3	3	−3	−3	3	−3	3	−3	−1	−1	−1	1	1	⋯
4	4	4	4	4	4	2	0	−2	−4	2	2	0	2	0	−2	0	0	0	0	0	⋯
$4'$	4	4	4	4	4	2	0	−2	−4	2	2	0	2	0	−2	0	0	0	0	0	⋯
$4''$	4	4	4	4	4	2	0	−2	−4	2	2	0	2	0	−2	0	0	0	0	0	⋯
$\tilde{4}$	4	4	4	4	4	−2	0	2	−4	−2	−2	0	−2	0	2	0	0	0	0	0	⋯
$\tilde{4}'$	4	4	4	4	4	−2	0	2	−4	−2	−2	0	−2	0	2	0	0	0	0	0	⋯
$\tilde{4}''$	4	4	4	4	4	−2	0	2	−4	−2	−2	0	−2	0	2	0	0	0	0	0	⋯
6	6	6	6	6	6	0	−2	0	6	0	0	−2	0	−2	0	2	2	2	0	0	⋯
$6'$	6	6	6	6	6	0	−2	0	6	0	0	−2	0	−2	0	−2	−2	−2	0	0	⋯
8	8	5	2	−1	−4	6	4	2	0	3	0	1	−3	−2	−1	0	0	0	0	0	⋯
$8'$	8	5	2	−1	−4	6	4	2	0	3	0	1	−3	−2	−1	0	0	0	0	0	⋯
$8''$	8	5	2	−1	−4	6	4	2	0	3	0	1	−3	−2	−1	0	0	0	0	0	⋯
$\tilde{8}$	8	5	2	−1	−4	−6	4	−2	0	−3	0	1	3	−2	1	0	0	0	0	0	⋯
$\tilde{8}'$	8	5	2	−1	−4	−6	4	−2	0	−3	0	1	3	−2	1	0	0	0	0	0	⋯
$\tilde{8}''$	8	5	2	−1	−4	−6	4	−2	0	−3	0	1	3	−2	1	0	0	0	0	0	⋯
16	16	−8	4	−2	1	0	0	0	0	0	0	0	0	0	0	4	−2	1	0	0	⋯
$16'$	16	−8	4	−2	1	0	0	0	0	0	0	0	0	0	0	4	−2	1	0	0	⋯
$16''$	16	−8	4	−2	1	0	0	0	0	0	0	0	0	0	0	4	−2	1	0	0	⋯
24	24	6	−3	−3	6	12	4	0	0	0	−3	−2	3	1	0	4	1	−2	2	−1	⋯
$24'$	24	6	−3	−3	6	12	4	0	0	0	−3	−2	3	1	0	−4	−1	2	−2	1	⋯
$\widetilde{24}$	24	6	−3	−3	6	−12	4	0	0	0	3	−2	−3	1	0	4	1	−2	−2	1	⋯
$\widetilde{24}'$	24	6	−3	−3	6	−12	4	0	0	0	3	−2	−3	1	0	−4	−1	2	2	−1	⋯
$24''$	24	15	6	−3	−12	6	−4	−6	0	3	0	−1	−3	2	3	0	0	0	0	0	⋯
$\widetilde{24}''$	24	15	6	−3	−12	−6	−4	6	0	−3	0	−1	3	2	−3	0	0	0	0	0	⋯
32	32	−4	−4	5	−4	8	0	0	0	−4	2	0	−1	0	0	0	0	0	0	0	⋯
$32'$	32	−4	−4	5	−4	8	0	0	0	−4	2	0	−1	0	0	0	0	0	0	0	⋯
$32''$	32	−4	−4	5	−4	8	0	0	0	−4	2	0	−1	0	0	0	0	0	0	0	⋯
$\widetilde{32}$	32	−4	−4	5	−4	−8	0	0	0	4	−2	0	1	0	0	0	0	0	0	0	⋯
$\widetilde{32}'$	32	−4	−4	5	−4	−8	0	0	0	4	−2	0	1	0	0	0	0	0	0	0	⋯
$\widetilde{32}''$	32	−4	−4	5	−4	−8	0	0	0	4	−2	0	1	0	0	0	0	0	0	0	⋯
48	48	−24	12	−6	3	0	0	0	0	0	0	0	0	0	0	−4	2	−1	0	0	⋯
$\widetilde{48}$	48	12	−6	−6	12	0	−8	0	0	0	0	4	0	−2	0	0	0	0	0	0	⋯

	972	144	288	576	432	1296	864	288	432	864	144	288	576	432	1296	864	288	432	864
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$1'$	1	ω	ω	ω	ω	ω	ω	ω	ω	ω	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2
$1''$	1	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω	ω	ω	ω	ω	ω	ω	ω	ω
3	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\tilde{1}$	1	1	1	1	−1	1	−1	1	−1	−1	1	1	1	−1	1	−1	1	−1	−1
$\tilde{1}'$	1	ω	ω	ω	$-\omega$	ω	$-\omega$	ω	$-\omega$	$-\omega$	ω^2	ω^2	ω^2	$-\omega^2$	ω^2	$-\omega^2$	ω^2	$-\omega^2$	$-\omega^2$
$\tilde{1}''$	1	ω^2	ω^2	ω^2	$-\omega^2$	ω^2	$-\omega^2$	ω^2	$-\omega^2$	$-\omega^2$	ω	ω	ω	$-\omega$	ω	$-\omega$	ω	$-\omega$	$-\omega$
$\tilde{3}$	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	1	1	1	−1	−1	1	1	1	−1	1	1	1	−1	−1	1	1	1	−1



$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
1	3	3	1
ω	1	ω	ω
0	2	2	2

Group extensions and Vacuum alignment

- To solve the vacuum alignment problem, we extend the flavour group H [e.g. the successful groups H=A_4,T_7,S_4,T′ or $\Delta(27)$].
- we require the following:
  - lepton structure should be same -> irreps of H should be promoted to irreps of G, we therefore need a surjective homomorphism $\xi : G \to H$ such that $\rho^G \equiv \rho^H \circ \xi$, $l \sim \underline{\mathbf{3}}^G$, $\chi \sim \underline{\mathbf{3}}^G$
  - there should be an irrep Φ, the product Φ^n should contain a $\underline{\mathbf{3}}^G$
- renormalizable scalar potential should be of form: $V=V(\Phi)+V(\chi)+(\Phi\Phi)_1(\chi\chi)_1$.

$\boxed{(l\chi)e^c}$ $\qquad$ $\boxed{(ll)_3\Phi^n+(ll)_1\Phi^n}$

$$\begin{array}{ccc} & G & \\ \swarrow_{\langle\chi\rangle \sim (1,1,1)} & & \searrow^{\langle\Phi^n\rangle \sim (1,0,0)} \\ Z_3=\langle T\rangle & \xleftrightarrow[\text{TBM}]{} & Z_2=\langle S\rangle \end{array}$$

Scan for Small Groups

- using the computer algebra system GAP and its SmallGroups catalogue, we have checked all groups with size smaller than 1000 (11,758,814 groups) and we have found a number of candidates:

- no candidates for T_7 or $\Delta(27)$, maybe because here 3 is complex and there are more couplings that have to be forbidden (also smaller number of possible extensions)

- all candidates in list have non-trivial centre(=elements that commute with all other elements), not necessary true for all groups(see e.g. $(S_3)^4 \rtimes A_4$ studied in Babu/Gabriel 2010)

Subgroup H	Order of G	GAP	Structure Description	$Z(G)$
A_4	96	204	$Q_8 \rtimes A_4$	Z_2
	288	860	$T' \rtimes A_4$	Z_2
	384	617, 20123	$((Z_2 \times Q_8) \rtimes Z_2) \rtimes A_4$	Z_2
	576	8273	$(Z_2.S_4) \rtimes A_4$	Z_2
	768	1083945	$(Z_4.Z_4^2) \rtimes A_4$	Z_4
		1085279	$((Z_2 \times Q_{16}) \rtimes Z_2) \rtimes A_4$	Z_2
S_4	192	1494	$Q_8 \rtimes S_4$	Z_2
	384	18133, 20092	$(Z_2 \times Q_8) \rtimes S_4$	Z_2
		20096	$((Z_4 \times Z_2) \rtimes Z_2) \rtimes S_4$	Z_4
	576	8282	$T' \rtimes S_4$	Z_2
		8480	$(Z_3 \times Q_8) \rtimes S_4$	Z_6
	768	1086052, 1086053	$((Z_2 \times Q_8) \rtimes Z_2) \rtimes S_4$	Z_2
	960	11114	$(Z_5 \times Q_8) \rtimes S_4$	Z_{10}
T'	192	1022	$Q_8 \rtimes T'$	Z_2^2
	648	533	$\Delta(27) \rtimes T'$	Z_3
	768	1083573, 1085187	$((Z_2 \times Q_8) \rtimes Z_2) \rtimes T'$	Z_2^2

Groups of the Structure $G \simeq N \rtimes H$, H is subgroup of G

Scan for Small Groups

- using the computer algebra system GAP and its SmallGroups catalogue, we have checked all groups with size smaller than 1000 (11,758,814 groups) and we have found a number of candidates:

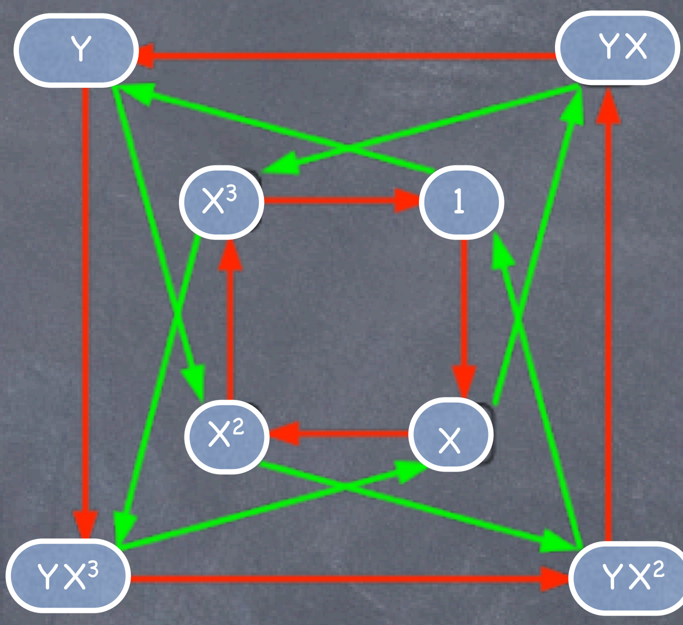
- no candidates for T_7 or $\Delta(27)$, maybe because here 3 is complex and there are more couplings that have to be forbidden (also smaller number of possible extensions)

- all candidates in list have non-trivial centre(=elements that commute with all other elements), not necessary true for all groups(see e.g. $(S_3)^4 \rtimes A_4$ studied in Babu/Gabriel 2010)

Quotient Group H	Order of G	GAP	Structure Description
A_4	96	201	$Z_2.(Z_2^2 \times A_4)$
	144	127	$Z_2.(A_4 \times S_3)$
	192	1017	$Z_2.(D_8 \times A_4)$
S_4	96	67, 192	$Z_4.S_4$
	144	121, 122	$Z_6.S_4$
	192	187, 963	$Z_8.S_4$
	192	987, 988	$Z_2.((Z_2^2 \times A_4) \rtimes Z_2)$
	192	1483,1484	$Z_2.(Z_2^2 \times S_4)$
	192	1492	$Z_2.((Z_2^4 \rtimes Z_3) \rtimes Z_2)$
T'	192	1007	$Z_2^2.(Z_2^2 \times A_4)$

Groups for which H is not a subgroup of G

Smallest Group



The smallest candidate group that contains A_4 as a subgroup is the semidirect product of the quaternion group Q_8

$$\langle X, Y | X^4 = 1,\ X^2 = Y^2,\ Y^{-1}XY = X^{-1} \rangle$$

with A_4

$$\langle S, T | S^2 = T^3 = (ST)^3 = 1 \rangle$$

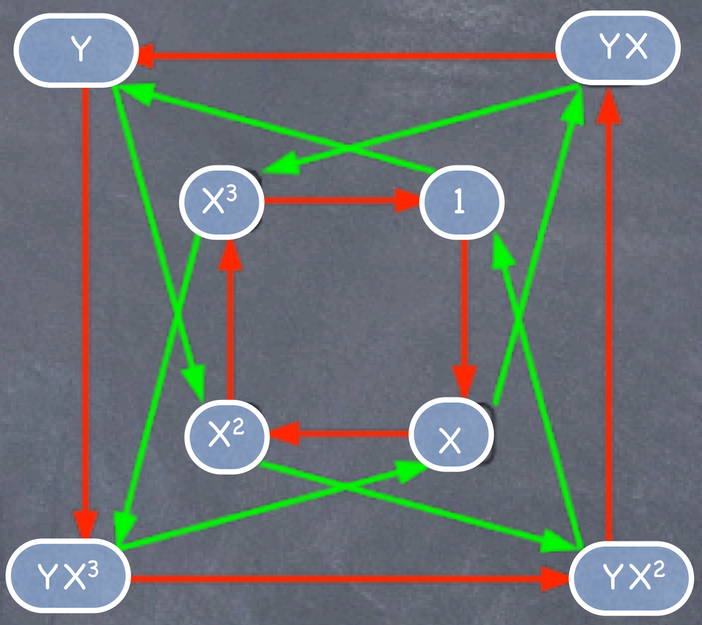
defined by the additional relations

$$SXS^{-1} = X, \qquad SYS^{-1} = Y^{-1}, \qquad TXT^{-1} = YX, \qquad TYT^{-1} = X\ .$$

Representations:

		1	T	SYX	SY	Y^2	T^2	TY	S	SX	X	$STYT$
	$\underline{\mathbf{1}}_{\mathbf{1}}$	1	1	1	1	1	1	1	1	1	1	1
unfaithful A_4 reps	$\underline{\mathbf{1}}_{\mathbf{2}}$	1	ω	1	1	1	ω^2	ω	1	1	1	ω^2
for leptons, χ	$\underline{\mathbf{1}}_{\mathbf{3}}$	1	ω^2	1	1	1	ω	ω^2	1	1	1	ω
	$\underline{\mathbf{3}}_{\mathbf{1}}$	3	.	-1	-1	3	.	.	-1	-1	3	.
	$\underline{\mathbf{3}}_{\mathbf{2}}$	3	.	3	-1	3	.	.	-1	-1	-1	.
	$\underline{\mathbf{3}}_{\mathbf{3}}$	3	.	-1	3	3	.	.	-1	-1	-1	.
	$\underline{\mathbf{3}}_{\mathbf{4}}$	3	.	-1	-1	3	.	.	3	-1	-1	.
	$\underline{\mathbf{3}}_{\mathbf{5}}$	3	.	-1	-1	3	.	.	-1	3	-1	.
faithful rep for Φ	$\underline{\mathbf{4}}_{\mathbf{1}}$	4	1	.	.	-4	1	-1	.	.	.	-1
	$\underline{\mathbf{4}}_{\mathbf{2}}$	4	ω^2	.	.	-4	ω	$-\omega^2$	.	.	.	$-\omega$
	$\underline{\mathbf{4}}_{\mathbf{3}}$	4	ω	.	.	-4	ω^2	$-\omega$	.	.	.	$-\omega^2$

Smallest Group



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with A_4

$$\langle S, T | S^2 = T^3 = (ST)^3 = 1\rangle$$

defined by the additional relations

$$SXS^{-1} = X, \qquad SYS^{-1} = Y^{-1}, \qquad TXT^{-1} = YX, \qquad TYT^{-1} = X\ .$$

Representations:

$$\underline{\mathbf{3}}_\mathbf{i} \times \underline{\mathbf{3}}_\mathbf{i} = \underline{\mathbf{1}}_\mathbf{1} + \underline{\mathbf{1}}_\mathbf{2} + \underline{\mathbf{1}}_\mathbf{3} + \underline{\mathbf{3}}_{\mathbf{i}\,S} + \underline{\mathbf{3}}_{\mathbf{i}\,A}$$

$$\underline{\mathbf{3}}_\mathbf{i} \times \underline{\mathbf{3}}_\mathbf{j} = \sum_{\substack{k=1 \\ k\neq i,j}}^{5} \underline{\mathbf{3}}_\mathbf{k} \qquad (i \neq j)$$

$$\underline{\mathbf{3}}_\mathbf{i} \times \underline{\mathbf{4}}_\mathbf{j} = \underline{\mathbf{4}}_\mathbf{1} + \underline{\mathbf{4}}_\mathbf{2} + \underline{\mathbf{4}}_\mathbf{3}$$

$$\underline{\mathbf{4}}_\mathbf{1} \times \underline{\mathbf{4}}_\mathbf{1} = \underline{\mathbf{1}}_{\mathbf{1}\,S} + \underline{\mathbf{3}}_{\mathbf{1}\,A} + \underline{\mathbf{3}}_{\mathbf{2}\,S} + \underline{\mathbf{3}}_{\mathbf{3}\,S} + \underline{\mathbf{3}}_{\mathbf{4}\,S} + \underline{\mathbf{3}}_{\mathbf{5}\,A}$$

$$\underline{\mathbf{4}}_\mathbf{1} \times \underline{\mathbf{4}}_\mathbf{2} = \underline{\mathbf{1}}_{\mathbf{2}\,S} + \underline{\mathbf{3}}_{\mathbf{1}\,A} + \underline{\mathbf{3}}_{\mathbf{2}\,S} + \underline{\mathbf{3}}_{\mathbf{3}\,S} + \underline{\mathbf{3}}_{\mathbf{4}\,S} + \underline{\mathbf{3}}_{\mathbf{5}\,A}$$

		S	T	X	Y
A_4 reps	$\underline{\mathbf{1}}_\mathbf{1}$	1	1	1	1
	$\underline{\mathbf{1}}_\mathbf{2}$	1	ω	1	1
	$\underline{\mathbf{1}}_\mathbf{3}$	1	ω^2	1	1
	$\underline{\mathbf{3}}_\mathbf{1}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
Φ	$\underline{\mathbf{4}}_\mathbf{1}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$

faithful representation Φ is what we were looking for.
$(\Phi\ \Phi)$ only contains non-trivial contraction of the A_4 subgroup.

The model

particle	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$Q_8 \rtimes A_4$	Z_4
ℓ	1	2	-1/2	$\underline{\mathbf{3}}_{\mathbf{1}}$	i
$e^c + \mu^c + \tau^c$	1	1	1	$\underline{\mathbf{1}}_{\mathbf{1}} + \underline{\mathbf{1}}_{\mathbf{2}} + \underline{\mathbf{1}}_{\mathbf{3}}$	$-$i
H	1	2	1/2	$\underline{\mathbf{1}}_{\mathbf{1}}$	1
χ	1	1	0	$\underline{\mathbf{3}}_{\mathbf{1}}$	1
ϕ_1	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	1
ϕ_2	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	-1

The model

particle	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$Q_8 \rtimes A_4$	Z_4
ℓ	1	2	-1/2	$\underline{\mathbf{3}}_{\mathbf{1}}$	i
$e^c + \mu^c + \tau^c$	1	1	1	$\underline{\mathbf{1}}_{\mathbf{1}} + \underline{\mathbf{1}}_{\mathbf{2}} + \underline{\mathbf{1}}_{\mathbf{3}}$	$-$i
H	1	2	1/2	$\underline{\mathbf{1}}_{\mathbf{1}}$	1
χ	1	1	0	$\underline{\mathbf{3}}_{\mathbf{1}}$	1
ϕ_1	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	1
ϕ_2	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	-1

$$\langle \chi \rangle = (v', v', v')^T,$$

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}}(a, a, b, -b)^T,$$

$$\langle \phi_2 \rangle = \frac{1}{\sqrt{2}}(c, c, d, -d)^T$$

$$\langle (\phi_1 \phi_2)_{\underline{\mathbf{3}}_{\mathbf{1}}} \rangle = \frac{1}{2}(bc - ad, 0, 0)^T$$

$$\langle (\phi_1 \phi_2)_{\underline{\mathbf{1}}_{\mathbf{1}}} \rangle = \frac{1}{2}(ac + bd)$$

The model

particle	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$Q_8 \rtimes A_4$	Z_4
ℓ	1	2	-1/2	$\underline{\mathbf{3}}_{\mathbf{1}}$	i
$e^c + \mu^c + \tau^c$	1	1	1	$\underline{\mathbf{1}}_{\mathbf{1}} + \underline{\mathbf{1}}_{\mathbf{2}} + \underline{\mathbf{1}}_{\mathbf{3}}$	$-$i
H	1	2	1/2	$\underline{\mathbf{1}}_{\mathbf{1}}$	1
χ	1	1	0	$\underline{\mathbf{3}}_{\mathbf{1}}$	1
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$$\langle \chi \rangle = (v', v', v')^T,$$

VEVs:

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$$\langle (\phi_1 \phi_2)_{\underline{\mathbf{1}}_{\mathbf{1}}} \rangle = \frac{1}{2}(ac + bd)$$

LO charged lepton masses:

$$\mathcal{L}_e^{(5)} = y_e (\ell \chi)_{\underline{\mathbf{1}}_{\mathbf{1}}} e^c \tilde{H}/\Lambda + y_\mu (\ell \chi)_{\underline{\mathbf{1}}_{\mathbf{3}}} \mu^c \tilde{H}/\Lambda + y_\tau (\ell \chi)_{\underline{\mathbf{1}}_{\mathbf{2}}} \tau^c \tilde{H}/\Lambda + \text{h.c.}\ ,$$

$$\Big\downarrow$$

$$\boxed{M_E \sim \begin{pmatrix} y_e & y_\mu & y_\tau \\ y_e & \omega y_\mu & \omega^2 y_\tau \\ y_e & \omega^2 y_\mu & \omega y_\tau \end{pmatrix}}$$

The model

particle	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$Q_8 \rtimes A_4$	Z_4
ℓ	1	2	-1/2	$\underline{\mathbf{3}}_{\mathbf{1}}$	i
$e^c + \mu^c + \tau^c$	1	1	1	$\underline{\mathbf{1}}_{\mathbf{1}} + \underline{\mathbf{1}}_{\mathbf{2}} + \underline{\mathbf{1}}_{\mathbf{3}}$	$-$i
H	1	2	1/2	$\underline{\mathbf{1}}_{\mathbf{1}}$	1
χ	1	1	0	$\underline{\mathbf{3}}_{\mathbf{1}}$	1
ϕ_1	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	1
ϕ_2	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	-1

VEVs:

$$\langle\chi\rangle = (v', v', v')^T,$$

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}(a, a, b, -b)^T,$$

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LO neutral lepton masses:

$$\mathcal{L}_\nu^{(7)} = x_a(\ell H \ell H)_{\underline{\mathbf{1}}_{\mathbf{1}}} (\phi_1\phi_2)_{\underline{\mathbf{1}}_{\mathbf{1}}}/\Lambda^3 + x_d(\ell H \ell H)_{\underline{\mathbf{3}}_{\mathbf{1}}} \cdot (\phi_1\phi_2)_{\underline{\mathbf{3}}_{\mathbf{1}}}/\Lambda^3 + \text{h.c.}\ .$$

$$M_E \sim \begin{pmatrix} y_e & y_\mu & y_\tau \\ y_e & \omega y_\mu & \omega^2 y_\tau \\ y_e & \omega^2 y_\mu & \omega y_\tau \end{pmatrix}$$

$\longleftrightarrow$ TBM

$$M_\nu \sim \begin{pmatrix} \tilde{a} & 0 & 0 \\ 0 & \tilde{a} & \tilde{d} \\ 0 & \tilde{d} & \tilde{a} \end{pmatrix}$$

(symmetry U accidental)

The model

particle	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$Q_8 \rtimes A_4$	Z_4
ℓ	1	2	-1/2	$\underline{\mathbf{3}}_{\mathbf{1}}$	i
$e^c + \mu^c + \tau^c$	1	1	1	$\underline{\mathbf{1}}_{\mathbf{1}} + \underline{\mathbf{1}}_{\mathbf{2}} + \underline{\mathbf{1}}_{\mathbf{3}}$	$-$i
H	1	2	1/2	$\underline{\mathbf{1}}_{\mathbf{1}}$	1
χ	1	1	0	$\underline{\mathbf{3}}_{\mathbf{1}}$	1
ϕ_1	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	1
ϕ_2	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	-1

$$\langle \chi \rangle = (v', v', v')^T ,$$

VEVs:

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}}(a, a, b, -b)^T ,$$

$$\langle \phi_2 \rangle = \frac{1}{\sqrt{2}}(c, c, d, -d)^T$$

$$\langle (\phi_1 \phi_2)_{\underline{\mathbf{3}}_{\mathbf{1}}} \rangle = \frac{1}{2}(bc - ad, 0, 0)^T$$

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$$\mathcal{L}_e^{(5)} = y_e (\ell\chi)_{\underline{\mathbf{1}}_{\mathbf{1}}} e^c \tilde{H}/\Lambda + y_\mu (\ell\chi)_{\underline{\mathbf{1}}_{\mathbf{3}}} \mu^c \tilde{H}/\Lambda + y_\tau (\ell\chi)_{\underline{\mathbf{1}}_{\mathbf{2}}} \tau^c \tilde{H}/\Lambda + \text{h.c.} ,$$

LO neutral lepton masses:

$$\mathcal{L}_\nu^{(7)} = x_a (\ell H \ell H)_{\underline{\mathbf{1}}_{\mathbf{1}}} (\phi_1\phi_2)_{\underline{\mathbf{1}}_{\mathbf{1}}} /\Lambda^3 + x_d (\ell H \ell H)_{\underline{\mathbf{3}}_{\mathbf{1}}} \cdot (\phi_1\phi_2)_{\underline{\mathbf{3}}_{\mathbf{1}}} /\Lambda^3 + \text{h.c.} .$$

- additional 4_1 necessary to get correct symmetry breaking (otherwise only breaking to A_4)
- same # of d.o.f. as in case of complex triplet and singlet, no additional driving fields necessary
- low flavour symmetry breaking scale possible, testable

Scalar Potential & Vacuum Alignment

The most general scalar potential invariant under the flavour symmetry is given by

$$V(\chi,\phi_1,\phi_2)=V_\chi(\chi)+V_\phi(\phi_1,\phi_2)+V_{\rm mix}(\chi,\phi_1,\phi_2)$$

with

$$\boxed{\begin{aligned}V_\phi(\phi_1,\phi_2)=&\mu_1^2(\phi_1\phi_1)_{\underline{\mathbf{1}}_\mathbf{1}}+\alpha_1(\phi_1\phi_1)^2_{\underline{\mathbf{1}}_\mathbf{1}}+\sum_{i=2,3}\alpha_i(\phi_1\phi_1)_{\underline{\mathbf{3}}_\mathbf{i}}\cdot(\phi_1\phi_1)_{\underline{\mathbf{3}}_\mathbf{i}}\\&+\mu_2^2(\phi_2\phi_2)_{\underline{\mathbf{1}}_\mathbf{1}}+\beta_1(\phi_2\phi_2)^2_{\underline{\mathbf{1}}_\mathbf{1}}+\sum_{i=2,3}\beta_i(\phi_2\phi_2)_{\underline{\mathbf{3}}_\mathbf{i}}\cdot(\phi_2\phi_2)_{\underline{\mathbf{3}}_\mathbf{i}}\\&+\gamma_1(\phi_1\phi_1)_{\underline{\mathbf{1}}_\mathbf{1}}(\phi_2\phi_2)_{\underline{\mathbf{1}}_\mathbf{1}}+\sum_{i=2,3,4}\gamma_i(\phi_1\phi_1)_{\underline{\mathbf{3}}_\mathbf{i}}\cdot(\phi_2\phi_2)_{\underline{\mathbf{3}}_\mathbf{i}}\\V_\chi(\chi)=&\mu_3^2(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}}+\rho_1(\chi\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}}+\lambda_1(\chi\chi)^2_{\underline{\mathbf{1}}_\mathbf{1}}+\lambda_2(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{2}}(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{3}}\\V_{\rm mix}(\chi,\phi_1,\phi_2)=&\zeta_{13}(\phi_1\phi_1)_{\underline{\mathbf{1}}_\mathbf{1}}(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}}+\zeta_{23}(\phi_2\phi_2)_{\underline{\mathbf{1}}_\mathbf{1}}(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}}\end{aligned}}$$

- Potential has an accidental symmetry $[(Q_8\rtimes A_4)\times A_4]\times Z_4$
  - invariant under independent transformations of Φ and χ
- note that couplings such as $\chi\cdot(\phi_1\phi_2)_{\underline{\mathbf{3}}_\mathbf{1}}$ are forbidden by the auxiliary Z_4 symmetry that separates the charged and neutral lepton sectors

Scalar Potential & Vacuum Alignment

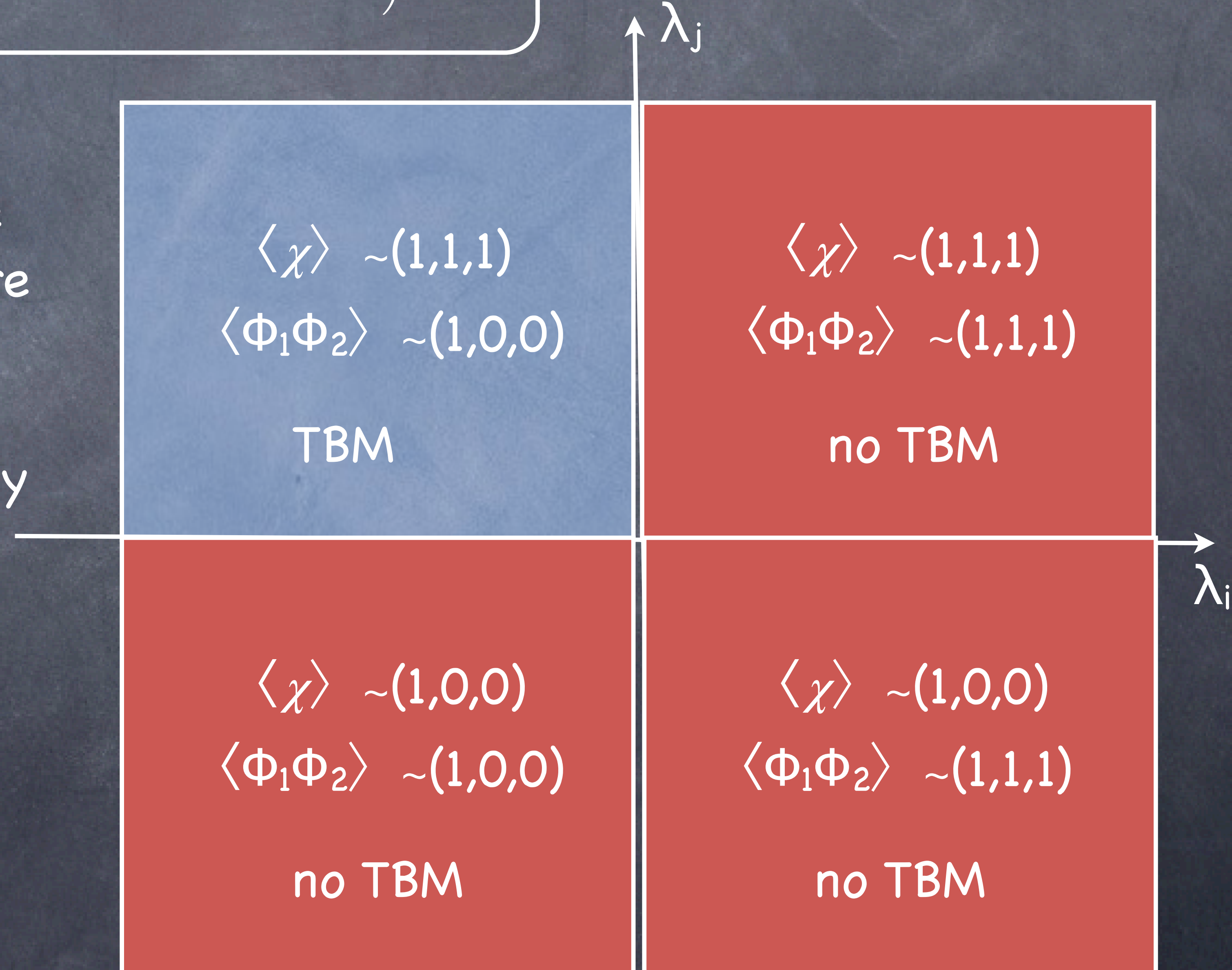
Minimum Conditions

$$a\left(\alpha_+\left(a^2+b^2\right)+\alpha_-\left(a^2-b^2\right)+\gamma_+\left(c^2+d^2\right)+\gamma_-\left(c^2-d^2\right)+U_1\right)+\Gamma bcd=0$$
$$b\left(\alpha_+\left(a^2+b^2\right)-\alpha_-\left(a^2-b^2\right)+\gamma_+\left(c^2+d^2\right)-\gamma_-\left(c^2-d^2\right)+U_1\right)+\Gamma acd=0$$
$$c\left(\beta_+\left(c^2+d^2\right)+\beta_-\left(c^2-d^2\right)+\gamma_+\left(a^2+b^2\right)+\gamma_-\left(a^2-b^2\right)+U_2\right)+\Gamma abd=0$$
$$d\left(\beta_+\left(c^2+d^2\right)-\beta_-\left(c^2-d^2\right)+\gamma_+\left(a^2+b^2\right)-\gamma_-\left(a^2-b^2\right)+U_2\right)+\Gamma abc=0$$
$$v'\left(4\sqrt{3}\lambda_1 v'^2+3\rho_1 v'+2\mu_3^2+\zeta_{13}(a^2+b^2)+\zeta_{23}(c^2+d^2)\right)=0$$

with

$$\xi_+=\frac{\xi_1}{2},\xi_-=\frac{\xi_2+\xi_3}{2\sqrt{3}}\ \text{for}\ \xi=\alpha,\beta$$
$$\gamma_+=\frac{\sqrt{3}\gamma_1+\gamma_4}{4\sqrt{3}},\quad \gamma_-=\frac{\gamma_2+\gamma_3}{4\sqrt{3}}\quad \text{and}\quad \Gamma=\frac{\gamma_4}{\sqrt{3}}$$

- eleven minimization conditions reduce to these 5 equations for 5 VEVs there is therefore generally a solution
- we have performed a numerical study to show that there is finite region of parameter space where the desired vacuum configuration is the global minimum



Higher Order Corrections

- NLO Corrections to vacuum potential

$$V^{(5)} = \sum_{L,M=1}^{2} \sum_{i,j=2}^{4} \frac{\delta_{ij}^{(LM)}}{\Lambda} \chi \cdot \left\{ (\phi_L \phi_L)_{\underline{\mathbf{3}}_{\mathbf{i}}} \cdot (\phi_M \phi_M)_{\underline{\mathbf{3}}_{\mathbf{j}}} \right\}_{\underline{\mathbf{3}}_{\mathbf{1}}} +$$

$$+ \frac{\chi^3}{\Lambda} \left(\delta_1^{(3)} \chi^2 + \delta_2^{(3)} (\phi_1 \phi_1)_{\underline{\mathbf{1}}_{\mathbf{1}}} + \delta_3^{(3)} (\phi_2 \phi_2)_{\underline{\mathbf{1}}_{\mathbf{1}}} \right)$$

$\delta_{ij}^{(LM)} = 0$ for $i \geq j$

- leads to shifts in VEVs

$$\langle \chi \rangle = (v' + \delta v'_1, v' + \delta v'_2, v' + \delta v'_2)^T,$$

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} (a + \delta a_1, a + \delta a_2, b + \delta a_3, -b + \delta a_4)^T,$$

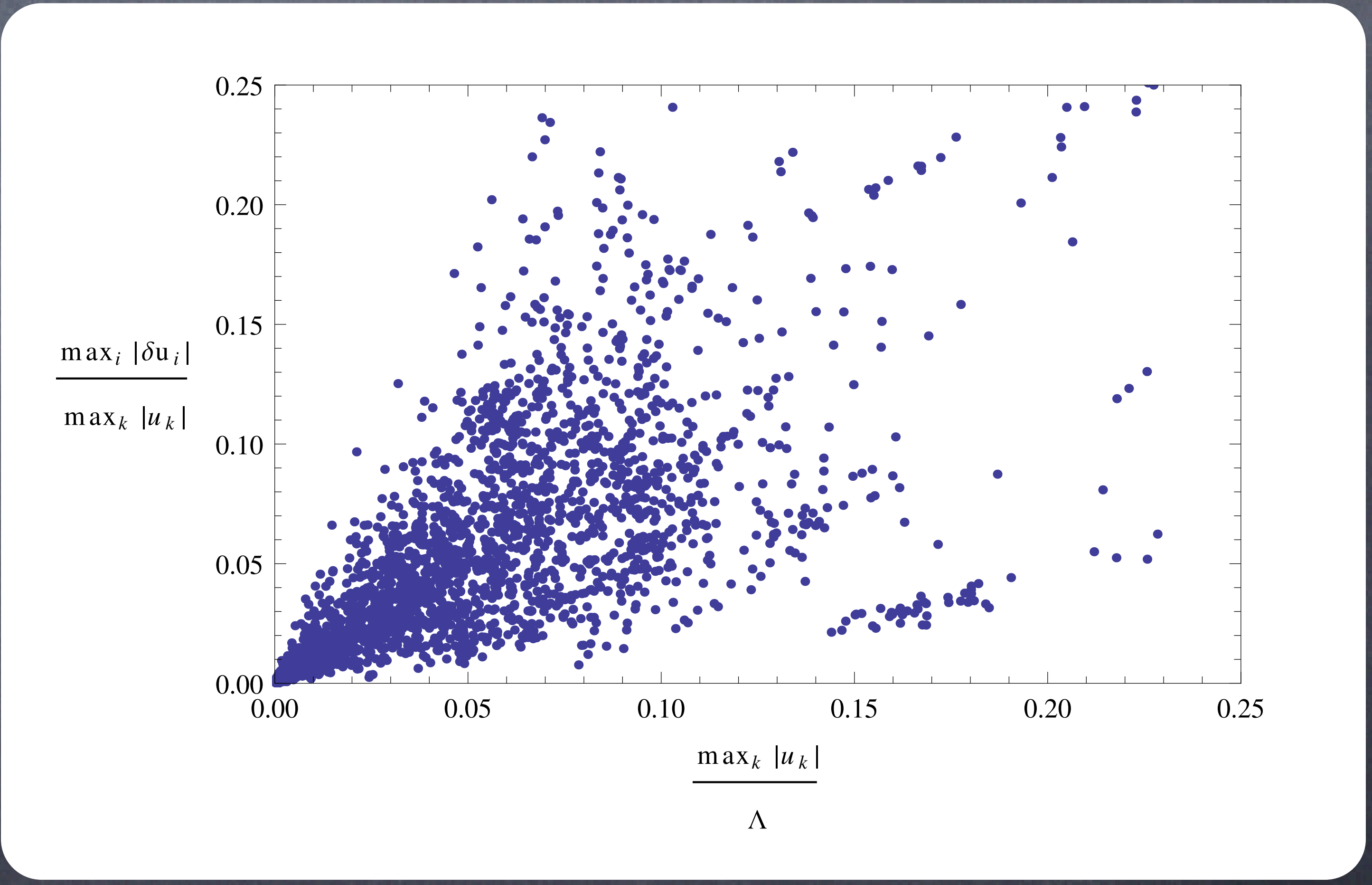
$$\langle \phi_2 \rangle = \frac{1}{\sqrt{2}} (c + \delta b_1, c + \delta b_2, d + \delta b_3, -d + \delta b_4)^T$$

- generic size of shifts

$$\frac{\delta u}{u} \sim \frac{u}{\Lambda}$$

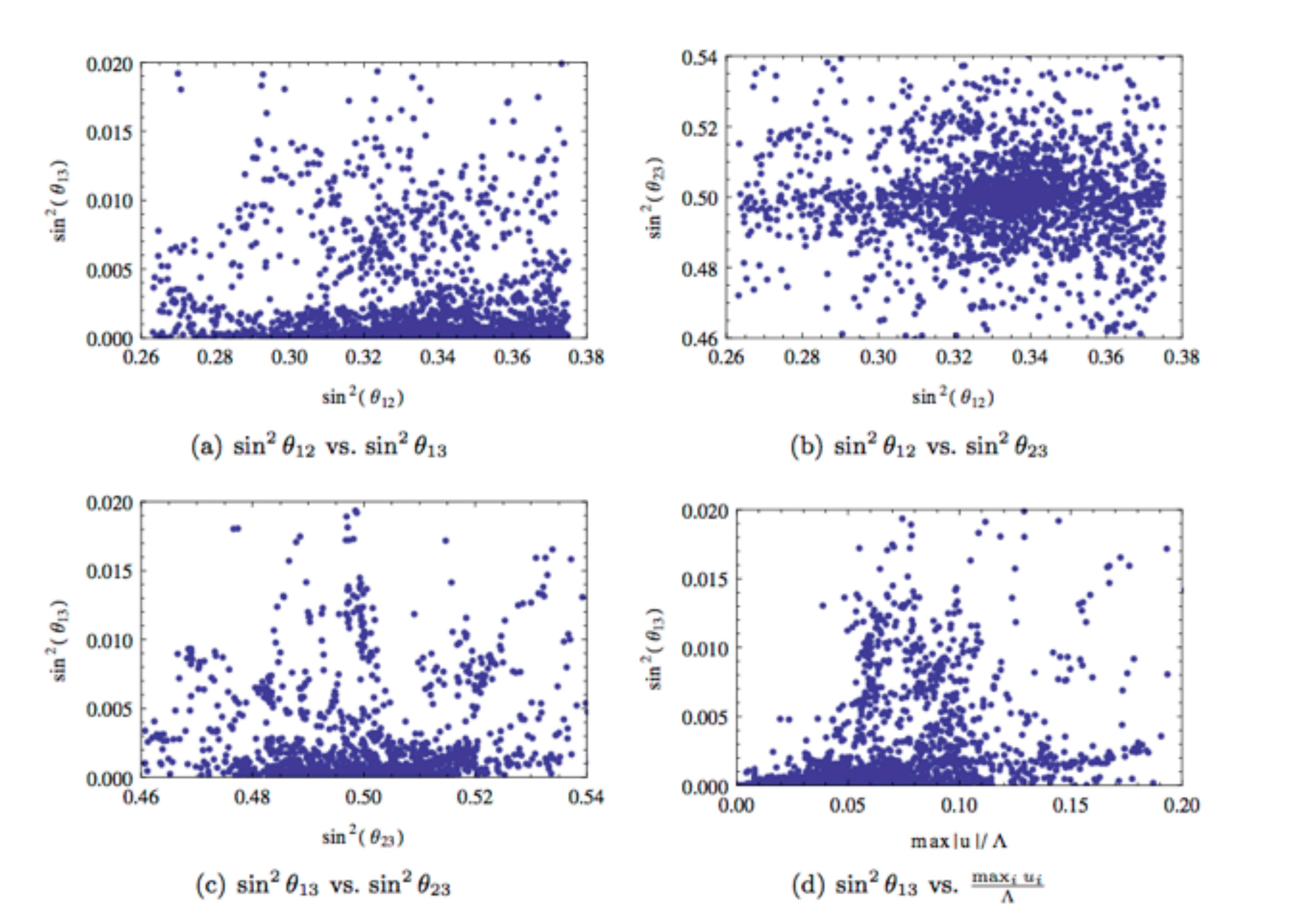
$$\langle \chi_2 \rangle - \langle \chi_3 \rangle = \mathcal{O}(1/\Lambda^2)$$

VEV alignment not destroyed!



generic size of shifts for scalar potential parameters of order one

Higher Order Corrections



- $\sin^2\Theta_{13}\approx.02$ as suggested by T2K can be accommodated at NLO
- or by introducing additional non-trivial singlet field $\xi \sim (1_2,i)$ giving trimaximal mixing[does not destroy VEV alignment]

[Lin'10, Shimizu,Tanimoto, Watanabe'11,Luhn,King'11]

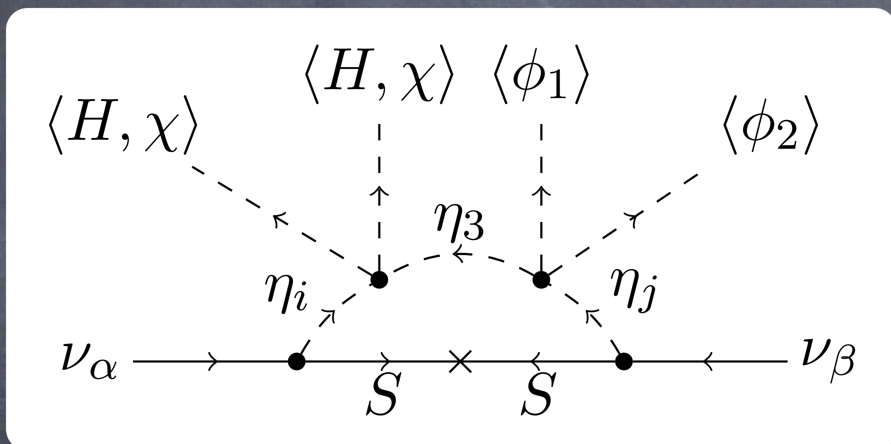
Flavour Breaking at the Electroweak Scale

- vacuum alignment mechanisms based on SUSY or extra dimensions require a high breaking scale
  - well motivated from see-saw, GUTs,...
  - hard to test outside of lepton sector, cannot observe driving fields etc.
  - predictive mixing schemes such as TBM ruled out
- one alternative: build models that break flavour symmetry at accessible scales, i.e. the electroweak scale. Would want:
  - some predictivity with regards to masses and mixings
  - explanation for smallness of neutrino masses
  - keep LFVs/FCNCs under control
  - dark matter from flavour symmetry?

[MH, M. Lindner, M. Schmidt, in preparation]

Flavour Breaking at the Electroweak Scale

- the complete laundry list can be achieved by a UV completion of the previous model, without introducing new symmetries. The only difference is that χ is an EW doublet, Φs singlet



fermion	$SU(2)_L$	$U(1)_Y$	$Q_8 \rtimes A_4$	Z_4
S	1	0	$\underline{\mathbf{3}}_{\mathbf{2}}$	-1
scalars	$SU(2)_L$	$U(1)_Y$	$Q_8 \rtimes A_4$	Z_4
η_1	2	1/2	$\underline{\mathbf{3}}_{\mathbf{5}}$	i
η_2	2	1/2	$\underline{\mathbf{3}}_{\mathbf{4}}$	i
η_3	2	1/2	$\underline{\mathbf{3}}_{\mathbf{5}}$	$-$i

- neutrino masses loop suppressed, flavour structure similar to trimaximal mixing

- LFVs kept small because effective LFV operator $L\sigma \cdot F e^c \tilde{H}/M^2 \sim (\underline{\mathbf{3}}_{\mathbf{1}}, 1)$ transforms as $(3_1,1)$.

  - need four flavon fields to generate invariant operator; therefore two mass insertions -> highly suppressed LFV rate

- Dark Matter stabilised by accidental Z_2: $\eta_i \to -\eta_i$

[MH, M. Lindner, M. Schmidt, in preparation]

Mathematica Package Discrete

We have developed a Mathematica Package that can be used to facilitate model building using discrete groups. It has the features:

- has access to groups catalogue of GAP, which contains all groups one would ever want to use

Initialization

```
In[8]:= Needs["Discrete`ModelBuildingTools`"];
```

```
In[11]:= Group = MBloadGAPGroup["AlternatingGroup(4)"];
         starting GAP generating AlternatingGroup(4)...
         ...finished

         StructureDescription:A4

         Size of Group:12

         Number of irreps: 4

         Dimensions of irreps:
```

1	2	3	4
1	1	1	3

Character Table:

1	1	1	1
1	1	$e^{-\frac{2 i \pi}{3}}$	$e^{\frac{2 i \pi}{3}}$
1	1	$e^{\frac{2 i \pi}{3}}$	$e^{-\frac{2 i \pi}{3}}$
3	-1	0	0

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In[193]:=
```
χ = MBgetRepVector[Group, 4, χc]
L = MBgetRepVector[Group, 4, Lc]
```

Out[193]= $\{\{\}, \{\}, \{\}, \{\{\chi c1, \chi c2, \chi c3\}\}\}$

Out[194]= $\{\{\}, \{\}, \{\}, \{\{Lc1, Lc2, Lc3\}\}\}$

In[195]:=
```
MBmultiply[Group, χ, L]
```

Out[195]=
$$\Big\{\Big\{\Big\{\frac{Lc1\,\chi c1 + Lc2\,\chi c2 + Lc3\,\chi c3}{\sqrt{3}}\Big\}\Big\},$$
$$\Big\{\Big\{\frac{1}{6}\left(2\sqrt{3}\,Lc1\,\chi c1 - \left(3\,i + \sqrt{3}\right) Lc2\,\chi c2 - \left(-3\,i + \sqrt{3}\right) Lc3\,\chi c3\right)\Big\}\Big\},$$
$$\Big\{\Big\{\frac{1}{6}\left(2\sqrt{3}\,Lc1\,\chi c1 - \left(-3\,i + \sqrt{3}\right) Lc2\,\chi c2 - \left(3\,i + \sqrt{3}\right) Lc3\,\chi c3\right)\Big\}\Big\},$$
$$\{\{Lc3\,\chi c2,\ Lc1\,\chi c3,\ Lc2\,\chi c1\},\ \{Lc2\,\chi c3,\ Lc3\,\chi c1,\ Lc1\,\chi c2\}\}\Big\}$$

In[197]:=
```
MBmultiply[Group, {χ, χ, χ, L, L}][[1]]
```

Out[197]=
$$\Big\{\Big\{\left(Lc1^2 + Lc2^2 + Lc3^2\right) \chi c1\,\chi c2\,\chi c3\Big\},$$
$$\Big\{\frac{1}{3}\left(Lc2\,Lc3\,\chi c1 + Lc1\,Lc3\,\chi c2 + Lc1\,Lc2\,\chi c3\right)\left(\chi c1^2 + \chi c2^2 + \chi c3^2\right)\Big\},$$
$$\Big\{\frac{Lc1\,Lc3\,\chi c2\,\chi c3^2 + Lc2\,\chi c1\left(Lc3\,\chi c2^2 + Lc1\,\chi c1\,\chi c3\right)}{\sqrt{3}}\Big\},$$
$$\Big\{\frac{Lc2\,Lc3\,\chi c1\,\chi c3^2 + Lc1\,\chi c2\left(Lc3\,\chi c1^2 + Lc2\,\chi c2\,\chi c3\right)}{\sqrt{3}}\Big\},$$
$$\Big\{\frac{1}{6\sqrt{3}}\Big(Lc1\,Lc3\,\chi c2\left(-\left(-3\,i + \sqrt{3}\right)\chi c1^2 + 2\sqrt{3}\,\chi c2^2 - \left(3\,i + \sqrt{3}\right)\chi c3^2\right) +$$
$$Lc2\,\Big(Lc1\,\chi c3\left(-\left(3\,i + \sqrt{3}\right)\chi c1^2 - \left(-3\,i + \sqrt{3}\right)\chi c2^2 + 2\sqrt{3}\,\chi c3^2\right) +$$
$$Lc3\,\chi c1\left(2\sqrt{3}\,\chi c1^2 - \left(3\,i + \sqrt{3}\right)\chi c2^2 - \left(-3\,i + \sqrt{3}\right)\chi c3^2\right)\Big)\Big)\Big\},$$

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- reduce set covariants to a smaller set of independent covariants
- calculate flavon potentials

```
In[200]:= MBgetFlavonPotential[Group, χ, 4, λ]
          2
          3
          4
```

$$\text{Out[200]=}\ \lambda 3n1\ \chi c1\ \chi c2\ \chi c3 + \frac{\lambda 2n1\ (\chi c1^2 + \chi c2^2 + \chi c3^2)}{\sqrt{3}} + \lambda 4n1\ (\chi c1^4 + \chi c2^4 + \chi c3^4) + \frac{1}{3}\ \lambda 4n2\ (\chi c2^2\ \chi c3^2 + \chi c1^2\ (\chi c2^2 + \chi c3^2))$$

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- available at http://projects.hepforge.org/discrete/

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Thank you for your attention!