Nonequilibrium QFT approach to leptogenesis

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INTERNATIONAL MAX PLANCK RESEARCH SCHOOL PT FS FOR PRECISION TESTS OF FUNDAMENTAL SYMMETRIES

Based on arXiv:1211.2140 in collaboration with M. Garny, A. Hohenegger, A. Kartavtsev, and D. Mitrouskas

Outline

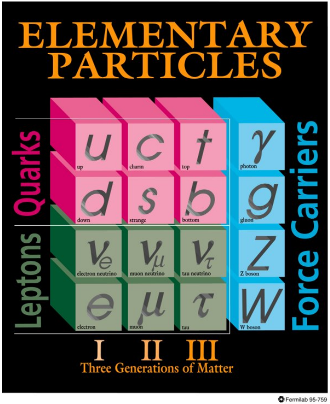
- Baryogenesis
- Leptogenesis
- Boltzmann approach to leptogenesis
- Nonequilibrium QFT approach to leptogenesis
- Conclusion

Baryogenesis

- The Earth is made of matter
- The Sun is made of matter
- The Milky Way is made of matter
- The Universe is made of matter

The Universe contains almost only matter, and no antimatter!

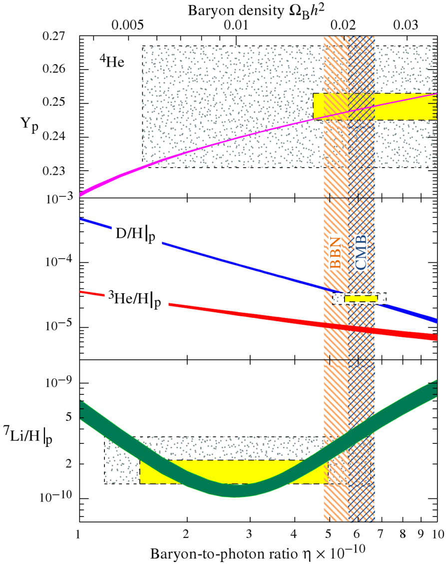
The Laws of Nature are *almost* the same for matter and for antimatter!



Baryon-to-photon ratio

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \sim 6 \times 10^{-10}$$

$$t_{\mathrm{BBN}} \sim 1 \text{ minute}$$
$$t_{\mathrm{CMB}} \sim 380\,000 \text{ years}$$



[astro-ph/0601514]

Sakharov conditions

Two different attitudes:

- The Universe contains initially more particles than antiparticles
  $\longrightarrow$ no need to produce the asymmetry
- The Universe was initially matter-antimatter **symmetric**, and the observed asymmetry must be dynamically produced

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Baryon number violation	$\leftrightarrow$ sphalerons
C and CP violation	$\leftrightarrow$ chirality, CKM matrix
out of equilibrium dynamics	$\leftrightarrow$ expanding Universe

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Beyond SM physics:

$\longrightarrow$ Electroweak baryogenesis, Affleck-Dine baryogenesis, **leptogenesis**, ...

Leptogenesis

Leptogenesis: particular model of baryogenesis

- **very simple model**: SM particles + n (at least 2) heavy, gauge-singlet right-handed neutrinos N_i
- RH neutrinos are unstable
- main decay channels:
  - lepton-Higgs pair
  - antilepton-antiHiggs pair
- CP violation $\rightarrow$ decay rates are unequal
- in an expanding Universe $\rightarrow$ production of **lepton asymmetry**
- EW sphaleron processes transfer part of the lepton asymmetry to the baryon sector

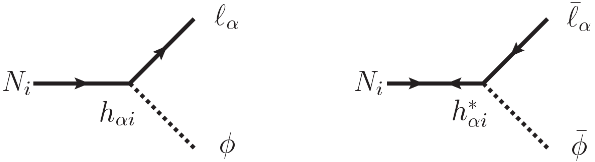
$\Rightarrow$ Production of a net baryon asymmetry!

Type-I seesaw Lagrangian

SM + n heavy right-handed gauge-singlet Majorana neutrinos:

$$\mathcal{L} = \mathcal{L}_{SM} + \bar{N}_i i\partial\!\!\!/ N_i - \frac{1}{2} M_i \left(\bar{N}_i^c N_i + \bar{N}_i N_i^c \right) - h_{\alpha i} \bar{\ell}_\alpha \tilde{\phi} N_i - h^\dagger_{i\alpha} \bar{N}_i \tilde{\phi}^\dagger \ell_\alpha$$

- the mass matrix violates lepton number
- Yukawa couplings h generate the vertices

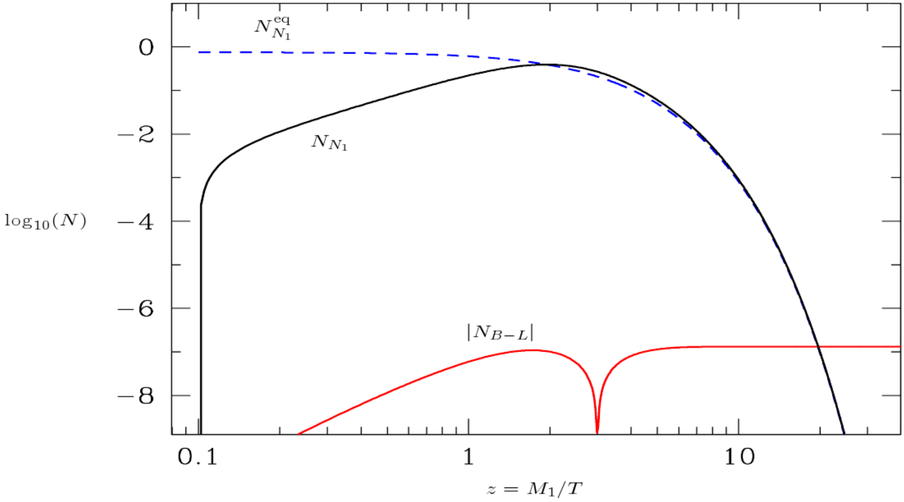


- gives **mass to active neutrinos** through the seesaw formula

$$m_\nu = -m_D M^{-1} m_D, \quad \text{where} \quad m_D = vh$$

- "heavy" means $\mathcal{O}(10^{10} - 10^{13})$ GeV
  - $\Rightarrow$ no hope to see these particles in future experiments

Evolution of the asymmetry



[hep-ph/0502169]

Transition amplitudes

Generation of the asymmetry:

	production	washout
N_i (inverse)decay	$\mathcal{O}\left(h^4\right)$	$\mathcal{O}\left(h^2\right)$
$\Delta L = 2$ scattering	$\mathcal{O}\left(h^6\right)$	$\mathcal{O}\left(h^4\right)$
top scattering	$\mathcal{O}\left(\lambda_t^2 h^4\right)$	$\mathcal{O}\left(\lambda_t^2 h^2\right)$
gauge scattering	$\mathcal{O}\left(g^2 h^4\right)$	$\mathcal{O}\left(g^2 h^2\right)$

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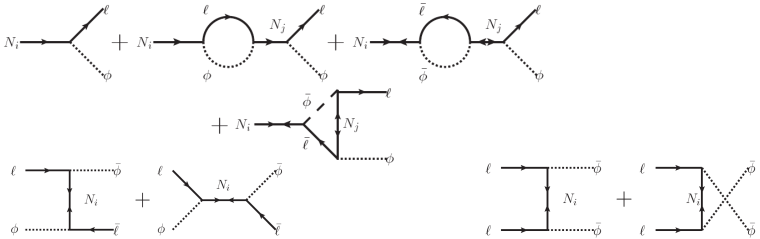


⇒ into the **Boltzmann** equation for the leptons and antileptons!

Boltzmann equation

$$\frac{df_\ell}{dt} = \frac{1}{2E_\ell} \sum_{\{a\ldots\},\{i\ldots\}} \int \frac{d^3p_a}{2E_a} \cdots \frac{d^3p_i}{2E_i} \cdots (2\pi)^4 \delta(\sum p_a - \sum p_i - p_\ell)$$
$$\times f_a \ldots (1 \pm f_i) \ldots (1 - f_\ell) \left|\mathcal{M}_{a+\ldots\to i+\ldots+\ell}\right|^2$$
$$- \frac{1}{2E_\ell} \sum_{\{a\ldots\},\{i\ldots\}} \int \frac{d^3p_a}{2E_a} \cdots \frac{d^3p_i}{2E_i} \cdots (2\pi)^4 \delta(\sum p_a + p_\ell - \sum p_i)$$
$$\times f_a \ldots f_\ell (1 \pm f_i) \ldots \left|\mathcal{M}_{a+\ldots+\ell\to i+\ldots}\right|^2$$

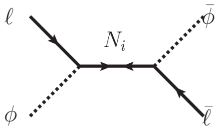
+ similar equation for antilepton

In **equilibrium**, $\frac{d(f_\ell - f_{\bar{\ell}})}{dt}$ is non zero if one uses the Feynman amplitudes!

Double counting problem

BE equations + *naive* Feynman amplitudes are **not consistent** with the third Sakharov condition
$\longrightarrow$ unstable particles as in/out-state

- some processes are counted twice



- can be solved if one neglects quantum statistical factors:
  $$(1 \pm f_i)(1 \pm f_j) \ldots \to 1$$
- substraction of the real intermediate state from the s-channel $\left|\mathcal{M}_{\ell\phi\to\bar{\ell}\bar{\phi}}\right|^2$:
  $\longrightarrow$ so-called **RIS-subtraction**

First principles approach

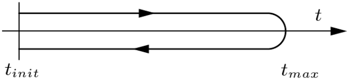
- RIS-subtraction is an **unsatisfactory** solution
- derive Boltzmann-like equations from **first principles**
  - $\longrightarrow$ free of double counting problem
  - $\longrightarrow$ include thermal masses
  - $\longrightarrow$ include thermal amplitudes
  - $\longrightarrow$ include thermal width
- keep track of the approximations done
  - $\longrightarrow$ range of validity of the resulting equations

$\Rightarrow$ Nonequilibrium quantum field theory approach!

Basics of nonequilibrium QFT

Lepton propagator on CTP:

$$S_{\alpha\beta}(x,y) =< T_{\mathcal{C}} \ell_\alpha(x) \bar{\ell}_\beta(y) >$$

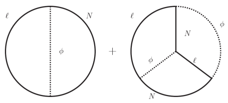


Closed **T**ime **P**ath

Schwinger-Dyson equation

$$\hat{S}^{-1}(x,y) = \hat{S}_0^{-1}(x,y) - \hat{\Sigma}(x,y)\ , \qquad \Sigma_{\alpha\beta}(x,y) = -\frac{\delta i\Gamma_2}{\delta S_{\beta\alpha}(y,x)}$$

$\Gamma_2 =$ 2PI effective action $=$



$\rightarrow$ doubling of the d.o.f.: $\hat{S}(x,y) = \underbrace{\hat{S}_F(x,y)}_{\text{statistical}} - \frac{i}{2}\mathrm{sign}_{\mathcal{C}}(x^0 - y^0)\underbrace{\hat{S}_\rho(x,y)}_{\text{spectral}}$

Kadanoff-Baym equations

Kadanoff-Baym equations

$$i\partial\!\!\!/_x \hat{S}_F(x,y) = \int_0^{x^0} d^4z \hat{\Sigma}_\rho(x,z)\hat{S}_F(z,y) - \int_0^{y^0} d^4z \hat{\Sigma}_F(x,z)\hat{S}_\rho(z,y)$$

$$i\partial\!\!\!/_x \hat{S}_\rho(x,y) = \int_{y^0}^{x^0} d^4z \hat{\Sigma}_\rho(x,z)\hat{S}_\rho(z,y)$$

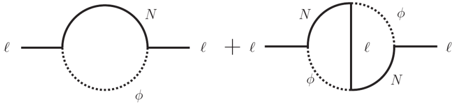
- equivalent to the Schwinger-Dyson equation
- **exact** equations
- only Gaussian initial conditions
- self-energies are functional of the propagators
  → need a loop expansion
- extremely difficult to solve numerically
  → **approximations** needed!

Quantum Boltzmann equation

- **Wigner transform** $\leftrightarrow$ Fourrier transform wrt relative coordinates
- **Gradient expansion** $\leftrightarrow$ Expansion in slow relative to fast time-scales (H/T)
- **Quasiparticle approximation** $\leftrightarrow$ Narrow width approximation
- **Kadanoff-Baym ansatz** $\leftrightarrow$ One-particle distribution function

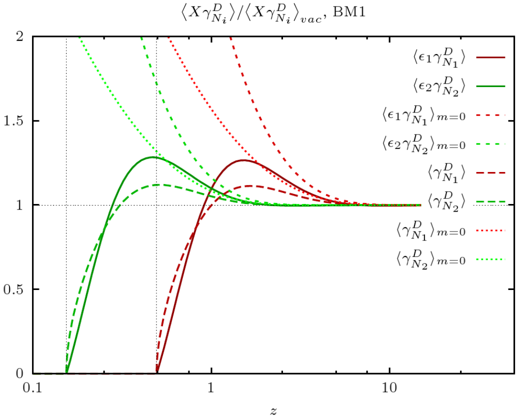
Quantum Boltzmann equation

$$\frac{d}{dt}(n_\ell(t)-n_{\bar{\ell}}(t)) = g_w \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\Big[\Big(\underbrace{\Sigma_{\ell<}(t,p)(1-f_\ell^p)}_{\text{gain term}} + \underbrace{\Sigma_{\ell>}(t,p)f_\ell^p}_{\text{loss term}}\Big)S_{\ell\rho}(t,p)\Big]$$

$\Sigma_\ell =$ 

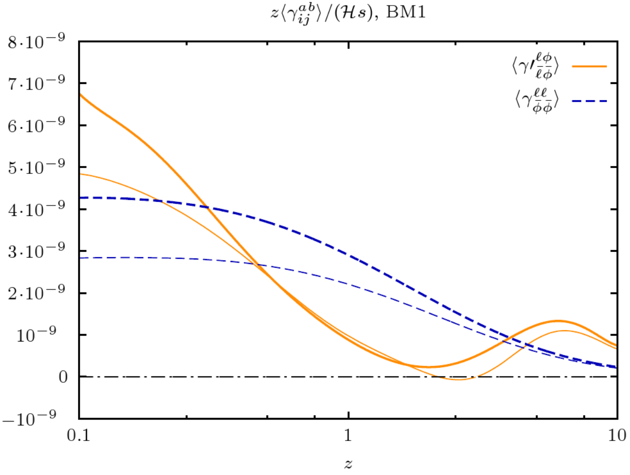
Results: RH neutrino decay

- $z = M_1/T$
- $\epsilon_i = \dfrac{\left|\mathcal{M}_{N_i \to \ell\phi}\right|^2 - \left|\mathcal{M}_{N_i \to \bar{\ell}\bar{\phi}}\right|^2}{\left|\mathcal{M}_{N_i \to \ell\phi}\right|^2 + \left|\mathcal{M}_{N_i \to \bar{\ell}\bar{\phi}}\right|^2}$
- $\langle \ldots \gamma^D_{N_i} \rangle$: reaction density
  - $\sim$ thermally averaged amplitudes



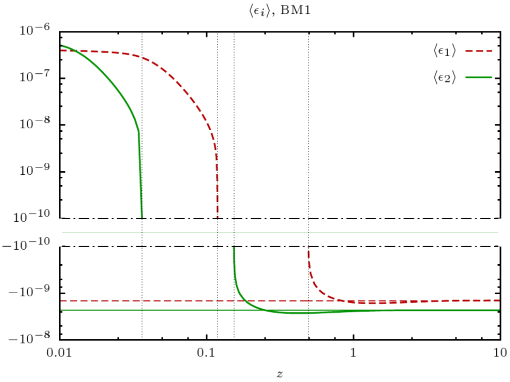
$\Rightarrow$ thermal enhancement partially compensated by thermal masses!

Results: scattering process



Results: Higgs decay

- due to thermal masses the **phase space** for heavy neutrino **shrinks** at high temperature
- at even higher temperature the Higgs decay becomes **kinematically allowed**



Conclusion

- Nonequilibrium QFT is the appropriate tool to study leptogenesis
- lead to consistent set of equations
  - $\longrightarrow$ free of double counting problem
  - $\longrightarrow$ include thermal masses
  - $\longrightarrow$ include thermal corrections to the amplitudes
- total amplitudes are not very sensitive to thermal corrections
  - $\longrightarrow$ tree level
- CP-violating parameter is more sensitive to thermal corrections
  - $\longrightarrow$ loop corrections
- inclusion of top scattering (work in progress)
- inclusion of gauge scattering (more complicated . . .)

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Thank you for your attention!