

Precision measurement of CP violation in mixing at LHCb

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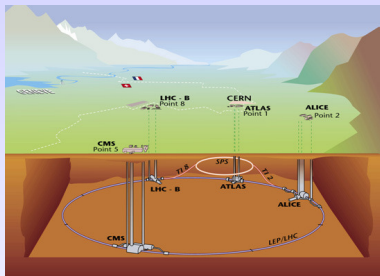
PT
FS

FOR PRECISION TESTS
OF FUNDAMENTAL
SYMMETRIES

- The LHC and the LHCb detector
- Introduction to CP violation
- \mathcal{CP} in interference of B_S^0 decay and mixing
- ϕ_S in $B_S^0 \rightarrow J/\psi \phi$
- Challenges of precision measurement
- Conclusions



The Large Hadron Collider (LHC)



- The LHC accelerates protons in both directions until they collide at four intersection points with a central mass system energy of 8 TeV (7 TeV in 2010-2011).
- The four main experiments located at the intersection points are: ATLAS, CMS, ALICE and **LHCb**


CP violation in mixing at LHCb

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Outline

The LHC

The LHCb experiment

Introduction to 

Challenge of precision measurement

Results

Ongoing studies

Nuisance CP asymmetries

Conclusions

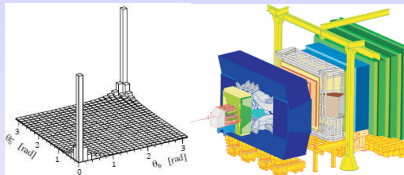
The LHCb experiment



LHCb

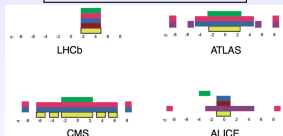
- Forward single arm spectrometer
- Acceptance: 15-300(250) mrad
- Copious source of b, c in the forward region

$b\bar{b}$ angular correlation



LHCb detector

Unique eta coverage



- ECAL
- HCAL
- Tracking
- Muon
- Hadron PID
- Counters

Studies

CP measurements require precision:

- Time dependent analysis need **good time resolution**
- Flavour tagging needs **particle IDentification**
- **High statistics, purity and efficiency** needed to reach SM predictions.

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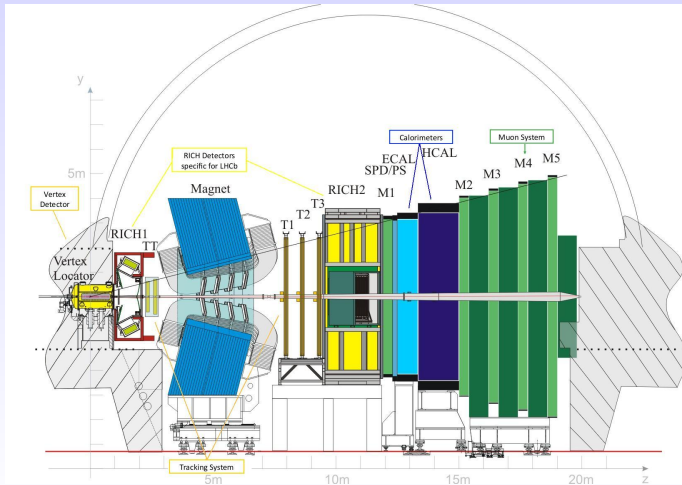
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The LHCb detector

LHCb was built precisely for this purpose!



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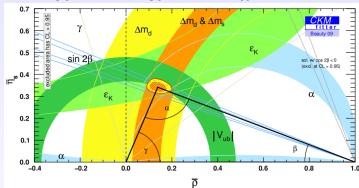
Short introduction to \mathcal{CP}

The only source of CP violation in the SM is the **Kobayshi-Maskawa mechanism**, which predicts the existence of a phase factor in the 3x3 CKM matrix:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

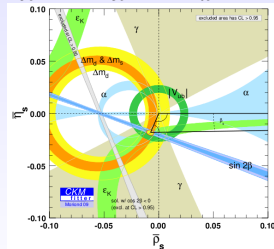
The request that the CKM matrix is unitary leads to relations between the elements:

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$



The height of the triangle depends on the value of the imaginary phase.

$$V_{ub}^* V_{us} + V_{cb}^* V_{cs} + V_{tb}^* V_{ts} = 0$$



In this talk I will focus on \mathcal{CP} in interference of B_s^0 decay and mixing.

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$B_s^0 - \bar{B}_s^0$ mixing and $B_s^0 \rightarrow J/\psi \phi$ decay

$B_s^0 - \bar{B}_s^0$ mixing

Time development of the mixing described by phenomenological Schrodinger eq:

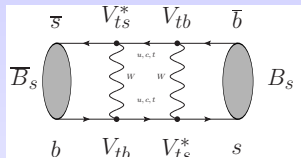
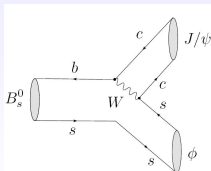
$$i \frac{d}{dt} \begin{pmatrix} B_s \\ \bar{B}_s \end{pmatrix} = (M - \frac{i}{2} \Gamma) \begin{pmatrix} B_s \\ \bar{B}_s \end{pmatrix}$$

Diagonalizing it in terms of mass eigenstates:

$$i \frac{d}{dt} (B_L) = (M_L - \frac{i}{2} \Gamma_L) (B_L)$$

$$i \frac{d}{dt} (B_H) = (M_H - \frac{i}{2} \Gamma_H) (B_H)$$

$B_s^0 \rightarrow J/\psi \phi$ decay



Phenomenological mixing parameters:

- $\Delta \Gamma = \Gamma_L - \Gamma_H$
 $\Delta M = M_L - M_H$
- Mixing phase:
 $\phi_M = 2 \arg (V_{ts} V_{tb}^*)$
- Decay dominated by tree level
- Decay phase: $\phi_D = \arg (V_{cs} V_{cb}^*) \approx 0$

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CP in interference of B_s^0 decay and mixing

- The interference between B_s^0 decays to $J/\psi\phi$ with or without $B_s^0 - \bar{B}_s^0$ oscillation allows the measurement of ϕ_s , via CP violation.

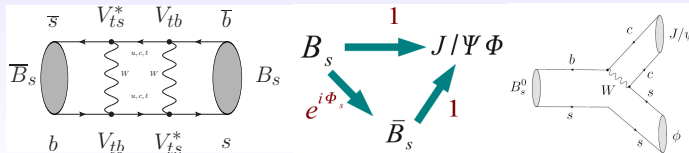
- The SM prediction is very precise:

$$\phi_s^{SM} = -2\beta_s = -2 \arg(-V_{ts}V_{tb}^*/V_{cs}V_{cb}^*) = (-0.0363 \pm 0.0016) \text{ rad}$$

[J.Charles et al., Phys.Rev. D84, 033005 (2011)]

- ϕ_s sensitive to New Physics (eg. 4th generation in the box):

$$\phi_s \rightarrow \phi_s^{SM} + \phi_s^{NP}$$

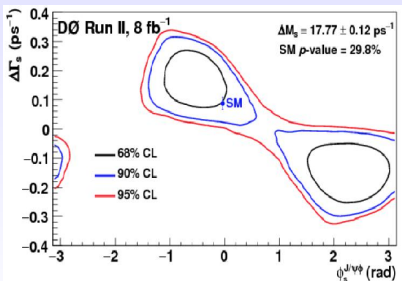


ϕ_s , the \mathcal{CP} phase, a bit of history

Tevatron experiments have looked to CP violation in $B_s^0 \rightarrow J/\psi \phi$, deriving confidence intervals for ϕ_s .

Originally they found a combined $\sim 2\sigma$ deviation from $-2\beta_s$.

Deviation has decreased with more data, but $\sigma(\phi_s^{exp})$ still much larger than ϕ_s^{SM} .



[D0 Prelim, S. Burdin]

Note the **2-fold ambiguity**

Decay rates invariant under transform:

$$\phi_s \iff \pi - \phi_s \quad \Delta\Gamma_s \iff -\Delta\Gamma_s$$

$$\delta_{\parallel} \iff 2\pi - \delta_{\parallel} \quad \delta_{\perp} \iff -\delta_{\perp}$$

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CP Asymmetry :

$$A_{CP} = \frac{\Gamma(\bar{B}_s(t) \rightarrow (J/\psi \phi)_{CP}) - \Gamma(B_s(t) \rightarrow (J/\psi \phi)_{CP})}{\Gamma(\bar{B}_s(t) \rightarrow (J/\psi \phi)_{CP}) + \Gamma(B_s(t) \rightarrow (J/\psi \phi)_{CP})}$$

What we measure is:

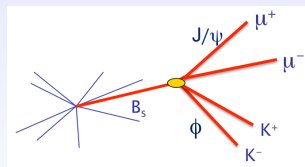
$$A_{CP} = -\eta_{CP} \cdot \sin(\phi_s) \cdot \sin(\Delta m_s t)$$

Decay time: fast B_s^0 oscillation needs to be resolved ($\Delta m_s = 17.63 \text{ps}^{-1}$);

Flavour tagging: to separate B_s^0 from \bar{B}_s^0

Mass: to separate signal from background;

!!! $B_s^0 \rightarrow J/\psi \phi$ is **not a CP eigenstate**.



Separation of CP eigenstates

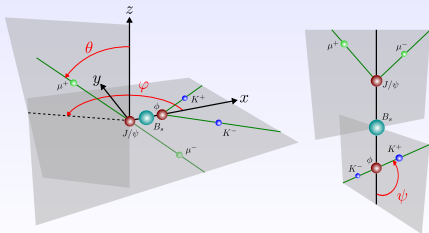
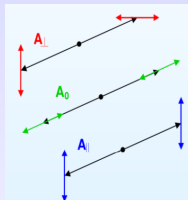
$$B_s^0 \rightarrow J/\psi (\rightarrow \mu^+ \mu^-) \phi (\rightarrow K^+ K^-)$$

Pseudoscalar to **vector mesons** ($J^{PC} = 1^{--}$) decay: final states CP odd and CP even.

$$\begin{aligned} L = 0, 2 & \quad CP = (-1)^L = +1 \\ L = 1 & \quad CP = (-1)^L = -1 \end{aligned}$$

Three polarisation amplitudes and phases:

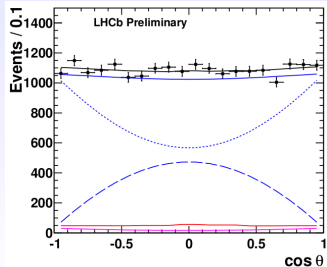
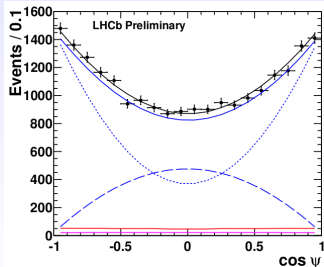
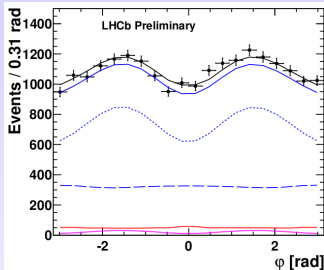
- $|A_0|^2, |A_{\parallel}|^2, \delta_0, \delta_{\parallel}$ (CP -even)
- $|A_{\perp}|^2, \delta_{\perp}$ (CP -odd)



Angular analysis in θ, ϕ, ψ to separate $CP = \pm 1$ states and extract ϕ_s .

$B_s^0 \rightarrow J/\psi\phi$ angular and proper time distributions

- clear separation of CP even and CP odd states
- different shapes in angular distributions



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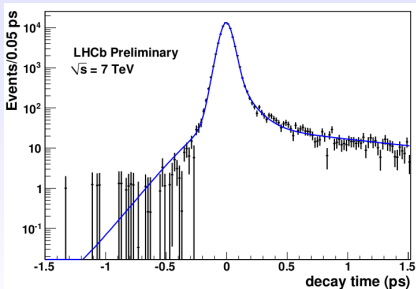
Conclusions

Decay time resolution

Fast B_s^0 oscillation need to be resolved! Time resolution affect the observed A_{CP} :

$$A_{CP} \approx -\eta_{CP} \cdot \sin(\phi_s) \cdot D_{\sigma_{ct}} \cdot \sin(\Delta m_s t) \quad D_{\sigma_{ct}} \approx \exp[-(\Delta m_s \sigma_{ct})^2/2]$$

We need good proper time resolution σ_{ct} w.r.t. sinusoid period of oscillation $\approx 350\text{fs}$, and excellent knowledge of σ_{ct} in data.



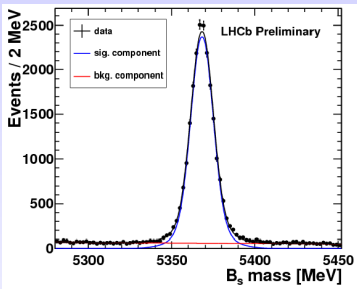
[LHCb-CONF-2012-002]

How to determine σ_{ct} in data?

!! IDEA

- Reconstructing fake B_s^0 with zero lifetime;
- Using prompt J/ψ bkg plus 2 random tracks: $\tau = 0 \pm \sigma_{ct}$
- Decay time resolution $\sigma_{ct} \approx 45\text{fs}$

$B_s^0 \rightarrow J/\psi \phi$ in LHCb

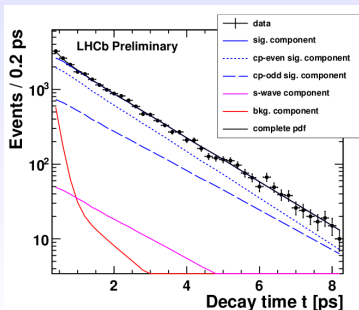


[LHCb-CONF-2012-002]

- $\mathcal{L} = 1 \text{ fb}^{-1}$
- Very pure sample:
 - ≈ 21200 signal candidates.
 - background $\mathcal{O}(\%)$
- World's largest $B_s^0 \rightarrow J/\psi \phi$ dataset!

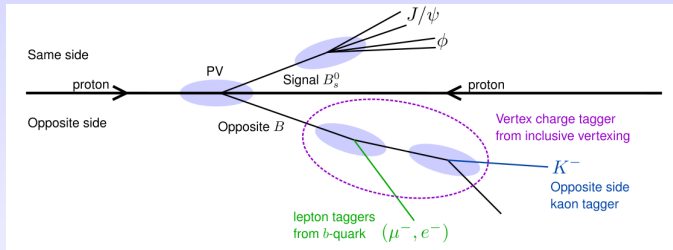
- Different lifetimes for CP odd and CP even components

$$\Delta\Gamma_s = \Gamma_L - \Gamma_H \approx \Gamma_{CP\text{ odd}} - \Gamma_{CP\text{ even}}$$



Flavour tagging - B_s^0 or \bar{B}_s^0 ?

Tagging: determine flavour of decaying B_s^0 -meson.



- tagging efficiency $\epsilon_{tag} = (32.99 \pm 0.33)\%$
- wrong tag probability $\omega_{tag} = (36.81 \pm 0.18 \pm 0.74)\%$
- effective tagging power $\epsilon_{tag} (1 - 2\omega_{tag})^2 = (2.29 \pm 0.07 \pm 0.26)\%$

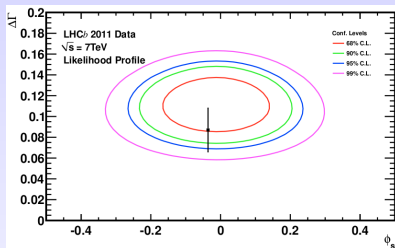
$$A_{CP} \approx -\eta_{CP} \cdot (1 - 2\omega_{tag}) \cdot \sin(\phi_s) \cdot D_{\sigma ct} \cdot \sin(\Delta m_s t)$$

There are still other ingredients needed:

- Proper time acceptance.
- Angular acceptances.
- Mass distribution modelling.
- Background composition and acceptances.
- S-wave modelling.
- ...

$B_s^0 \rightarrow J/\psi\phi$ Results

$\phi_s - \Delta\Gamma_s$ profile Likelihood contour plot



[LHCb-CONF-2012-002]

$$\Gamma_s = 0.6580 \pm 0.0054 \text{ (stat.)} \pm 0.0066 \text{ (syst.) } ps^{-1}$$

$$\Delta\Gamma_s = 0.116 \pm 0.018 \text{ (stat.)} \pm 0.006 \text{ (syst.) } ps^{-1}$$

$$\phi_s = -0.001 \pm 0.101 \text{ (stat.)} \pm 0.027 \text{ (syst.) } rad$$

Two-fold ambiguity resolved in a different measurement!

Solution with positive $\Delta\Gamma_s$ is preferred at 4.7σ .

[LHCb-PAPER-2011-028, arXiv:1202.4717[hep-ex]]

Systematic Uncertainties

Source	Γ_s [ps ⁻¹]	$\Delta\Gamma_s$ [ps ⁻¹]	A_{\perp}^2	A_0^2	F_S	δ_{\parallel}	δ_{\perp}	δ_s	ϕ_s
Description of background	0.0010	0.004	-	0.002	0.005	0.04	0.04	0.06	0.011
Angular acceptances	0.0018	0.002	0.012	0.024	0.005	0.12	0.06	0.05	0.012
t acceptance model	0.0062	0.002	0.001	0.001	-	-	-	-	-
z and momentum scale	0.0009	-	-	-	-	-	-	-	-
Production asymmetry ($\pm 10\%$)	0.0002	0.002	-	-	-	-	-	-	0.008
CPV mixing & decay ($\pm 5\%$)	0.0003	0.002	-	-	-	-	-	-	0.020
Fit bias	-	0.001	0.003	-	0.001	0.02	0.02	0.01	0.005
Quadratic sum	0.0066	0.006	0.013	0.024	0.007	0.13	0.07	0.08	0.027

- The dominant contribution for Γ_s from the **proper time acceptance**.
- The dominant contribution for ϕ_s from the **CP in mixing and decay**.
- In view of more data, with the goal of separating the observed ϕ_s from the SM prediction to see if New Physics is playing a role, the measurement must be very precise.
- Systematic uncertainty need to be reduced!

There are several effects that may affect the the CP asymmetry:

- $B_s^0-\bar{B}_s^0$ production. A difference in the production rate of B and \bar{B} introduces a **production asymmetry**
- Tagging efficiency. A different probability to tag B and \bar{B} may cause a **tagging efficiency asymmetry**.
- Wrong-tag probability. A different probability for a wrong tag for B and \bar{B} can be parametrized by **wron tag asymmetry**.
- Additional CP violation in mixing and/or decay. Can be parametrized by a parameter λ .

In the past this systematic uncertainties due to nuisance “CP ”asymmetries were:

- **production asymmetry** (ν_p) \rightarrow performing a toy study, which includes a $\nu_p = 10\%$ and fitting a decay model without the nuisance asymmetry. The bias in the parameter of interested is the resulting systematic uncertainty;
- **additional CP violation in mixing and/or decay** \rightarrow fitting for it in data to get a feeling of the magnitude and then using the same procedure as before with $|\lambda|^2 = 1 \pm 5\%$;
- **tagging/wrong tag asymmetries** \rightarrow already covered by the uncertainties on the tagging calibration parameters.

The systematic uncertainties quoted are :

	Γ_s	$\Delta\Gamma_s$	ϕ_s
Additional CP in mix and/or decay	0.0003	0.002	0.020
Production asymmetry	0.0002	0.002	0.008
Total	0.0066	0.006	0.027

Can we fit directly in data for the additional \mathcal{CP} in mixing and/or decay?

Defining

$$\lambda = \frac{q}{p} \frac{\bar{A}_f}{A_f}$$

- q, p complex numbers that define B_s^0 mass eigenstates in terms of flavour eigenstates $|B_{H,L}\rangle = p |B_s^0\rangle \mp q |\bar{B}_s^0\rangle$
- \bar{A}_f and A_f the decay amplitudes.

Both the CP violation in B_s^0 mixing $|q/p| \neq 1$ or the CP violation in the decay (direct \mathcal{CP}) $|\bar{A}_f/A_f| \neq 1$ result in $|\lambda| \neq 1$.

Parametrizing λ as:

$$\lambda = |\lambda| e^{-i\phi_s}$$

and allowing for $|\lambda| \neq 1$ is it possible to fit for the \mathcal{CP} in mixing and/or decay.

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Fitting for the \mathcal{CP} in mixing and/or decay

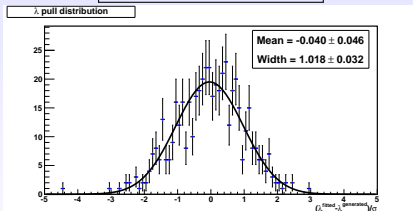
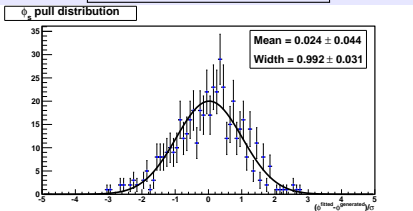
Allowing for $|\lambda| \neq 1$ is it possible to fit for the \mathcal{CP} in mixing and/or decay.

- Making toy studies to check if there are bias introduced fitting for $|\lambda|$;
- I generated the same statistics as in data and run over 500 toys.
- looking to the pull distribution for the parameter ϑ :

$$\frac{\vartheta_{\text{fitted}} - \vartheta_{\text{generated}}}{\sigma(\vartheta_{\text{fitted}})}$$

ϕ_s pull distribution

λ pull distribution



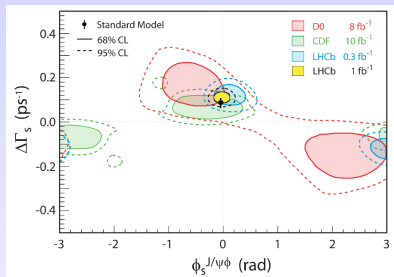
We can fit for the \mathcal{CP} in mixing and/or decay without any bias!

What about the other asymmetries?

- Included performing a toy study
- Adding a **tagging efficiency**, a **wrong tag probability** and a **production asymmetry** in generation
- fitting using a decay model without any asymmetry.

The bias in the parameter of interested is the resulting systematic uncertainty on the parameter:

	Γ_s	$\Delta\Gamma_s$	ϕ_s
Tagging efficiency asym	0.0002	0.003	0.007
Wrong tag asym	0.0003	0.002	0.006
Production asym	0.0001	0.001	0.005
All asimmetries	0.0003	0.002	0.007
\mathcal{CP} mix and/or decay	0.0003	0.002	0.020
Production asym	0.0002	0.002	0.008



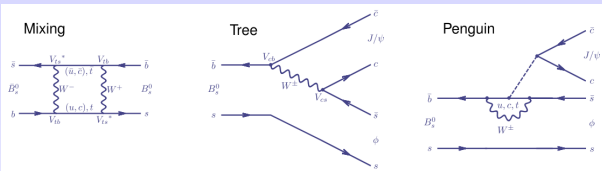
[LHCb-CONF-2012-002]

- First direct observation of a non-zero value for $\Delta\Gamma_s$
- World's most precise measurement of CP violation in $B_s^0 \rightarrow J/\psi\phi$
- The measurement of ϕ_s is*:

$$\phi_s = -0.002 \pm 0.083 (stat) \pm 0.027 (syst) rad$$

* In combination with ϕ_s from $B_s^0 \rightarrow J/\psi\pi^+\pi^-$

Backup slides



- ▶ Mixing phase: $\phi_{mix} = \arg(V_{ts} V_{tb}^*)^2$
- ▶ $B_s^0 \rightarrow J/\psi \phi$ is a $b \rightarrow c\bar{c}s$ transition, Tree (T) and Penguin (P_q) terms:

$$\begin{aligned}
 A_{c\bar{c}s} &= V_{cs} V_{cb}^* (T + P_c) + V_{us} V_{ub}^* P_u + V_{ts} V_{tb}^* P_t \\
 &= V_{cs} V_{cb}^* (T + P_c - P_t) + V_{us} V_{ub}^* (P_u - P_t)
 \end{aligned}$$
- ▶ $V_{us} V_{ub}^*$ suppressed by $O(\lambda^2)$ WRT $V_{cs} V_{cb}^*$ so $(P_u - P_t)$ penguin pollution (δP) small
- ▶ This leaves $\phi_{decay} = \arg(V_{cs} V_{cb}^*)$

$$\begin{aligned}
 \phi_s &= \phi_{mix} - 2\phi_{decay} = \arg(V_{ts} V_{tb}^*)^2 - 2\arg(V_{cs} V_{cb}^*) + \delta P \\
 &= 2\arg\left[\frac{V_{ts} V_{tb}^*}{V_{cs} V_{cb}^*}\right] = -2\beta_s = -2\eta\lambda^2 - \eta\lambda^4 - O(\lambda^6)
 \end{aligned}$$

- Two solutions to decay rates in $B_s^0 \rightarrow J/\psi \phi$:

Solution I

$$\delta_{\parallel} - \delta_0$$

$$\delta_{\perp} - \delta_0$$

$$\delta_s - \delta_0$$

$$\phi_s$$

$$\Delta\Gamma_s$$

\Leftrightarrow

Solution II

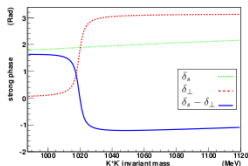
$$\delta_0 - \delta_{\perp}$$

$$\pi + \delta_0 - \delta_{\perp}$$

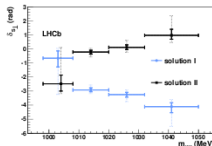
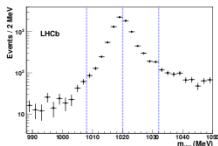
$$\delta_0 - \delta_s$$

$$\pi - \phi_s$$

$$-\Delta\Gamma_s$$



- P-wave phase (δ_{\perp}) increases rapidly across $\phi(1020)$ mass resonance, S-wave phase (δ_s) varies slowly
- Measuring $\delta_s - \delta_{\perp}$ in bins of $M(K^+K^-)$ resolves the ambiguity [arXiv:0908.3627 \[hep-ph\]](https://arxiv.org/abs/0908.3627)
- LHCb results, 0.37 fb^{-1} in 4 bins of $M(K^+K^-)$:



More about the pdf

Signs in blue are tag dependent and change for B_S^0

$$\begin{aligned}
 A_1 &= |A_0|^2 e^{-\Gamma_S t} \left[\cosh\left(\frac{\Delta\Gamma_S t}{2}\right) - \cos\phi_S \sinh\left(\frac{\Delta\Gamma_S t}{2}\right) \blacksquare \sin\phi_S \sin(\Delta m_S t) \right] \\
 A_2 &= |A_{\parallel}|^2 e^{-\Gamma_S t} \left[\cosh\left(\frac{\Delta\Gamma_S t}{2}\right) - \cos\phi_S \sinh\left(\frac{\Delta\Gamma_S t}{2}\right) \blacksquare \sin\phi_S \sin(\Delta m_S t) \right] \\
 A_3 &= |A_{\perp}|^2 e^{-\Gamma_S t} \left[\cosh\left(\frac{\Delta\Gamma_S t}{2}\right) + \cos\phi_S \sinh\left(\frac{\Delta\Gamma_S t}{2}\right) \blacksquare \sin\phi_S \sin(\Delta m_S t) \right] \\
 A_4 &= |A_{\parallel}| |A_{\perp}| e^{-\Gamma_S t} \left[-\cos(\delta_{\perp} - \delta_{\parallel}) \sin\phi_S \sinh\left(\frac{\Delta\Gamma_S t}{2}\right) \right. \\
 &\quad \blacksquare \cos(\delta_{\perp} - \delta_{\parallel}) \cos\phi_S \sin(\Delta m_S t) \blacksquare \sin(\delta_{\perp} - \delta_{\parallel}) \cos(\Delta m_S t) \left. \right] \\
 A_5 &= |A_0| |A_{\parallel}| e^{-\Gamma_S t} \cos(\delta_{\parallel} - \delta_0) \left[\cosh\left(\frac{\Delta\Gamma_S t}{2}\right) - \cos\phi_S \sinh\left(\frac{\Delta\Gamma_S t}{2}\right) \right. \\
 &\quad \blacksquare \sin\phi_S \sin(\Delta m_S t) \left. \right] \\
 A_6 &= |A_0| |A_{\perp}| e^{-\Gamma_S t} \left[-\cos(\delta_{\perp} - \delta_0) \sin\phi_S \sinh\left(\frac{\Delta\Gamma_S t}{2}\right) \right. \\
 &\quad \blacksquare \cos(\delta_{\perp} - \delta_0) \cos\phi_S \sin(\Delta m_S t) \blacksquare \sin(\delta_{\perp} - \delta_0) \cos(\Delta m_S t) \left. \right] \\
 A_7 &= |A_S|^2 e^{-\Gamma_S t} \left[\cosh\left(\frac{\Delta\Gamma_S t}{2}\right) + \cos\phi_S \sinh\left(\frac{\Delta\Gamma_S t}{2}\right) \blacksquare \sin\phi_S \sin(\Delta m_S t) \right] \\
 A_8 &= |A_S| |A_{\parallel}| e^{-\Gamma_S t} \left[-\sin(\delta_{\parallel} - \delta_S) \sin\phi_S \sinh\left(\frac{\Delta\Gamma_S t}{2}\right) \right. \\
 &\quad \blacksquare \sin(\delta_{\parallel} - \delta_S) \cos\phi_S \sin(\Delta m_S t) \blacksquare \cos(\delta_{\parallel} - \delta_S) \cos(\Delta m_S t) \left. \right] \\
 A_9 &= |A_S| |A_{\perp}| e^{-\Gamma_S t} \sin(\delta_{\perp} - \delta_S) \left[\cosh\left(\frac{\Delta\Gamma_S t}{2}\right) + \cos\phi_S \sinh\left(\frac{\Delta\Gamma_S t}{2}\right) \right. \\
 &\quad \blacksquare \sin\phi_S \sin(\Delta m_S t) \left. \right] \\
 A_{10} &= |A_S| |A_0| e^{-\Gamma_S t} \left[-\sin(\delta_0 - \delta_S) \sin\phi_S \sinh\left(\frac{\Delta\Gamma_S t}{2}\right) \right. \\
 &\quad \blacksquare \sin(\delta_0 - \delta_S) \cos\phi_S \sin(\Delta m_S t) \blacksquare \cos(\delta_0 - \delta_S) \cos(\Delta m_S t) \left. \right]
 \end{aligned}$$

CP violation in mixing at LHCb

F. Dordei

More theory

Ambiguity

More about the pdf