Neutrino Decoherence

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Outline

- Quantum Decoherence and Neutrino Oscillation
- The Layer Structure
- Neutrino Decoherence
- Experimental Potentials

Neutrino Oscillation (Quantum Coherence)

- Quantum coherence: Superposition of quantum states
- Neutrino oscillation:
  superposition of mass eigenstates in the measurement (flavor) basis

Density Matrix formalism (QM)

$$|\nu_\alpha\rangle = \sum_j U_{\alpha j} |\nu_j\rangle$$

U: PMNS matrix

$$\rho(t) = e^{iHt}\rho(0)e^{-iHt} = \sum_{jk} U_{j\delta} U^*_{k\delta}\, e^{-i(H_j - H_k)t} |\nu_j\rangle\langle\nu_k|$$

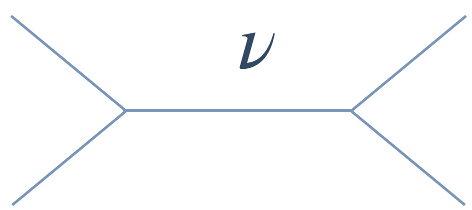
$$P_{\alpha\beta} = Tr\left(|\nu_\alpha\rangle\langle\nu_\beta| \rho(t) \right)$$

Coherent

S-Matrix formalism (QFT)

$$|\nu_\alpha\rangle = \sum_j U_{\alpha j}|\nu_j\rangle$$

$iA_{\alpha\beta} =$



$$\supset \sum_j U^*_{\alpha j} U_{\beta j} \int \frac{d^4p}{(2\pi)^4} \frac{\not{p} + m_j}{p^2 - m_j^2 + i\epsilon} e^{-ipx}$$

$$P_{\alpha\beta} = |A_{\alpha\beta}|^2$$

Density Matrix formalism (QM)

$$|\nu_\alpha\rangle = \sum_j U_{\alpha j}|\nu_j\rangle$$

$$\rho(t) = e^{iHt}\rho(0)e^{-iHt} = \sum_{jk} U_{j\delta}U^*_{k\delta}\, e^{-i(H_j - H_k)t}\,|\nu_j\rangle\langle\nu_k|$$

$$P_{\alpha\beta} = Tr\left(|\nu_\alpha\rangle\langle\nu_\beta|\rho(t)\right)$$

Decoherent

Consider Wavepackets

→ Not completely coherent to start with

- Some possible causes:
  - Wavepacket seperation ($v_j \neq v_k$)   [1001.4815], [2104.05806], …
  - Mass state identification   [0905.1903]

Consider an Open Quantum System

→ Lose information to the environment

- Some possible causes:
  - Interactions (matter effect, neutrino decay, …)   [1803.04438], …
  - Quantum gravity   [hep-ph/0002053], [2007.00068], …
  - Cosmological   [1704.06139]

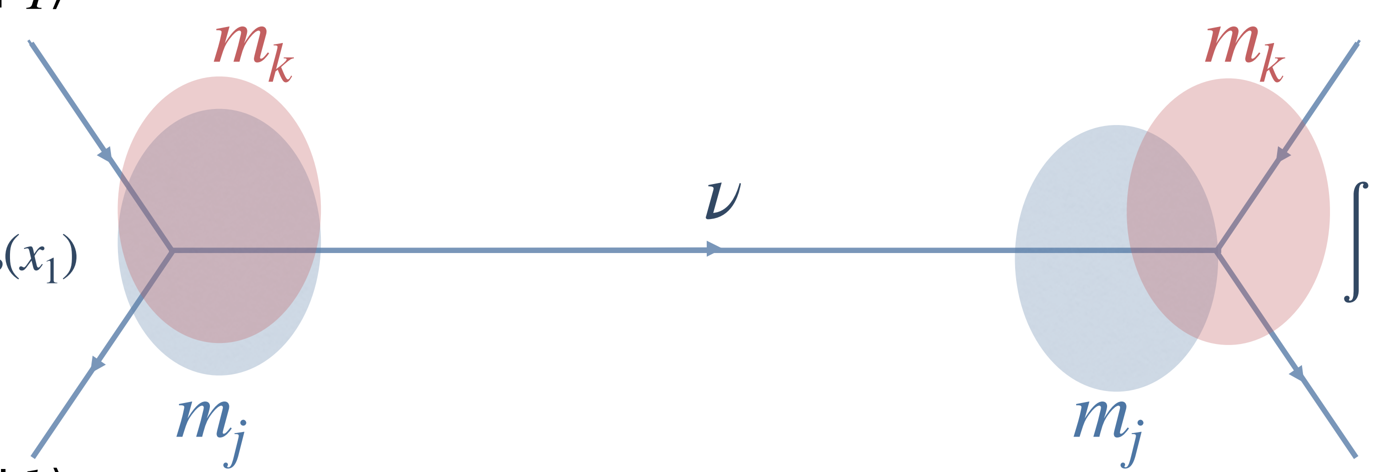
Decoherent

S-Matrix formalism (QFT) — Consider Wavepackets

$|P_i\rangle = \int [dq] f_{pi}(q,t) |q\rangle$

$|D_i\rangle = \int [dq'] f_{di}(q',t) |q'\rangle$

$iA_{\alpha\beta} = \int d^4x_1 g_P(x_1)$ ... $\int d^4x_2 g_D(x_2)$



$|P_f\rangle = \int [dk] f_{pf}(k,t) |k\rangle$

$|D_f\rangle = \int [dk'] f_{df}(k',t) |k'\rangle$

Density Matrix formalism (QM) — Open Quantum System

Lindblad Equation: $$\frac{\partial \rho}{\partial t} = -i[H,\rho] - \mathscr{D}[\rho]$$

$$\mathscr{D}[\rho] = -\frac{1}{2}\sum_k^{N^2-1}\left([V_k, \rho V_k^\dagger] + [V_k\rho, V_k^\dagger]\right) \equiv \left(D_{\mu\nu}\rho^\nu\right)\lambda^\mu$$

Gell-Mann matrices

9×9 entries

Decoherent from the Layer Structure

Consider Wavepackets + QFT description

→ Not completely coherent to start with

Layer 1

State Decoherence

Consider an Open (Quantum) System

→ Lose information to the environment

Layer 1 + Layer 2

Phase Decoherence

The Layer Structure

Measurement layer (layer 3)

- Kinematic variables : $\{ T_0 , \mathbf{L}_0 , \mathbf{P}_0 \}$ or $\{T_0 , L , E_0 , \Omega \}$ $\xrightarrow{\text{Consistent emission}}$ $\{L_0 , E_0 , \Omega\}_{T_0=0}$
- Uncertainties : Those from lower layers

$\uparrow \mathscr{L}\mathscr{M}\mathscr{O}^2$

Physical layer (layer 2)

- Kinematic variables : $\{T , \mathbf{L} , \mathbf{P}\}$
- Uncertainties : Macroscopic + Those from layer 1

$\uparrow \mathscr{L}\mathscr{M}\mathscr{O}^1$ $\qquad$ $\uparrow \mathscr{L}\mathscr{M}\mathscr{O}^{\bar{1}}$

Phase space layer (layer 1) — Fock space

- Kinematic variables : $\{x , p\} \xrightarrow{\text{on shell}} \{x , \mathbf{p}\}$
- Uncertainties : Microscopic

Phase space layer (layer 1) — Wigner space

- Kinematic variables : $\{\bar{t} , \bar{\mathbf{x}} , \bar{\mathbf{p}}\}$ (on shell)
- Uncertainties : Microscopic

The Layer Structure

- The layer moving operator:

$$\mathscr{LMO}^i = \int d^n x_i \int d^m p_i \, W_i(x_i, p_i; x_{i+1}, p_{i+1}),$$

$$\text{such that} \quad B_{i+1}(x_{i+1}, p_{i+1}) = \mathscr{LMO}^i B_i(x_i, p_i)$$

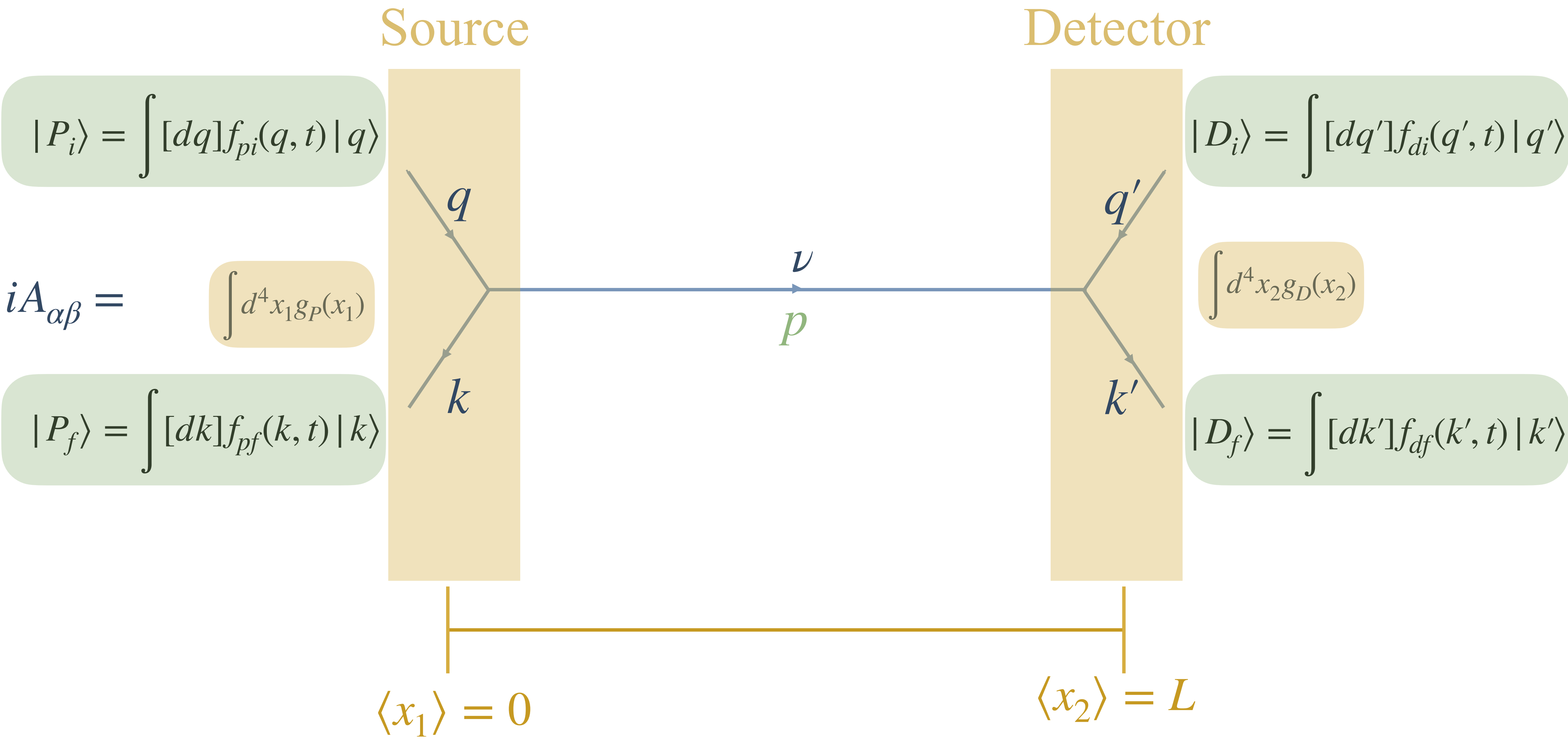
- Flavor transition probability:

$$P_3(T_0, \mathbf{L}_0, \mathbf{P}_0) = \mathscr{LMO}^2 P_2(T, \mathbf{L}, \mathbf{P}) = \mathscr{LMO}^2 \{A_2^*(T, \mathbf{L}, \mathbf{P})\, A_2(T, \mathbf{L}, \mathbf{P})\}$$

$$= \mathscr{LMO}^2 \{\mathscr{LMO}^1 A_1^*(x', p')\, \mathscr{LMO}^1 A_1(\mathrm{x}, \mathrm{p})\}$$

$$= \mathscr{LMO}^2 \{\mathscr{LMO}^{\bar{1}} P_{\bar{1}}(\bar{\mathbf{x}}, \bar{\mathbf{p}})\} \,.$$

Layer 1: QFT Transition Amplitude σ_p σ_x



$$\mathscr{L}\mathscr{M}\mathscr{O}^1 = \int d^4x \int d^4p\, \overset{\sigma_p}{F(p;P)}\, \overset{\sigma_x}{G(x;L)}, \qquad A_1(x,p) = \sum_j U^*_{\alpha j} U_{\beta j} \frac{\not p + m_j}{p^2 - m_j^2 + i\epsilon} e^{-ipx}$$

$$A_2(T,\mathbf{L},\mathbf{P}) \simeq \sum_j U^*_{\alpha j} U_{\beta j} e^{-iE_jT + i\mathbf{v}_j\mathbf{P}_jT} \int d^3p \int d^3x\, e^{i\mathbf{p}\mathbf{x}} F_j(\mathbf{p};\mathbf{P}_j) G_j(\mathbf{x};\mathbf{L}_j) = \sum_j U^*_{\alpha j} U_{\beta j} e^{-iE_jT + i\mathbf{v}_j\mathbf{P}_jT} \hat{\Phi}_j(\mathbf{L}_j, \mathbf{P}_j)$$

Layer $\bar{1}$: Wigner Transition Probability σ_p σ_x

Bridge of QM to statistical phase space

$$\mathscr{L}\mathscr{M}\mathscr{O}^1 A_1^*(x', p')\, \mathscr{L}\mathscr{M}\mathscr{O}^1 A_1(\mathrm{x}, \mathrm{p}) = \mathscr{L}\mathscr{M}\mathscr{O}^{\bar{1}} P_{\bar{1}}(\bar{\mathbf{x}}, \bar{\mathbf{p}})\,.$$

$$(x, x') \to (\bar{x} = \frac{1}{2}(x + x'), \Delta x = x - x')$$

$$\mathscr{L}\mathscr{M}\mathscr{O}^{\bar{1}} = \int d^3\bar{x} \int d^3\bar{p}, \qquad P_{\bar{1}jk} = \tilde{W}^G_{jk}(\bar{x}, \bar{p})\tilde{W}^F_{jk}(\bar{x}, \bar{p}),$$

$$\tilde{W}^G_{jk}(\bar{x}, \bar{p}) = \int d^3(\Delta x)\, e^{i\Delta x \bar{p}}\, G_j^*(\bar{x} - \frac{1}{2}\Delta x; \mathbf{L}_j)\, G_k(\bar{x} + \frac{1}{2}\Delta x; \mathbf{L}_k)$$

$$\tilde{W}^F_{jk}(\bar{x}, \bar{p}) = \int d^3(\Delta p)\, e^{i\Delta p \bar{x}}\, F_j^*(\bar{p} - \frac{1}{2}\Delta p; \mathbf{P}_j)\, F_k(\bar{p} + \frac{1}{2}\Delta p; \mathbf{P}_k)$$

Wigner quasi-probability distribution function

Layer 2: Physical Transition Probability σ_L σ_E

$$P_2(T,\mathbf{L},\mathbf{P}) = A_2^*(T,\mathbf{L},\mathbf{P})A_2(T,\mathbf{L},\mathbf{P}) = \mathscr{L}\mathscr{M}\mathscr{O}^{\bar{1}} P_{\bar{1}}(\bar{\mathbf{x}},\bar{\mathbf{p}})\,.$$

- Recall the fist layer weighting function: $W_1^j(\mathbf{x},\mathbf{p};T,\mathbf{L},\mathbf{P}) = F_j(\mathbf{p};\mathbf{P}_j)G_j(\mathbf{x};\mathbf{L}_j)$
- Relation between $\{T,\mathbf{L},\mathbf{P}\}$ and $\{\mathbf{P}_j,\mathbf{L}_j\}$: $|\mathbf{P}_j| \equiv E + \delta E_j$ and $\mathbf{L}_j = \mathbf{L} - \mathbf{v}_j T$

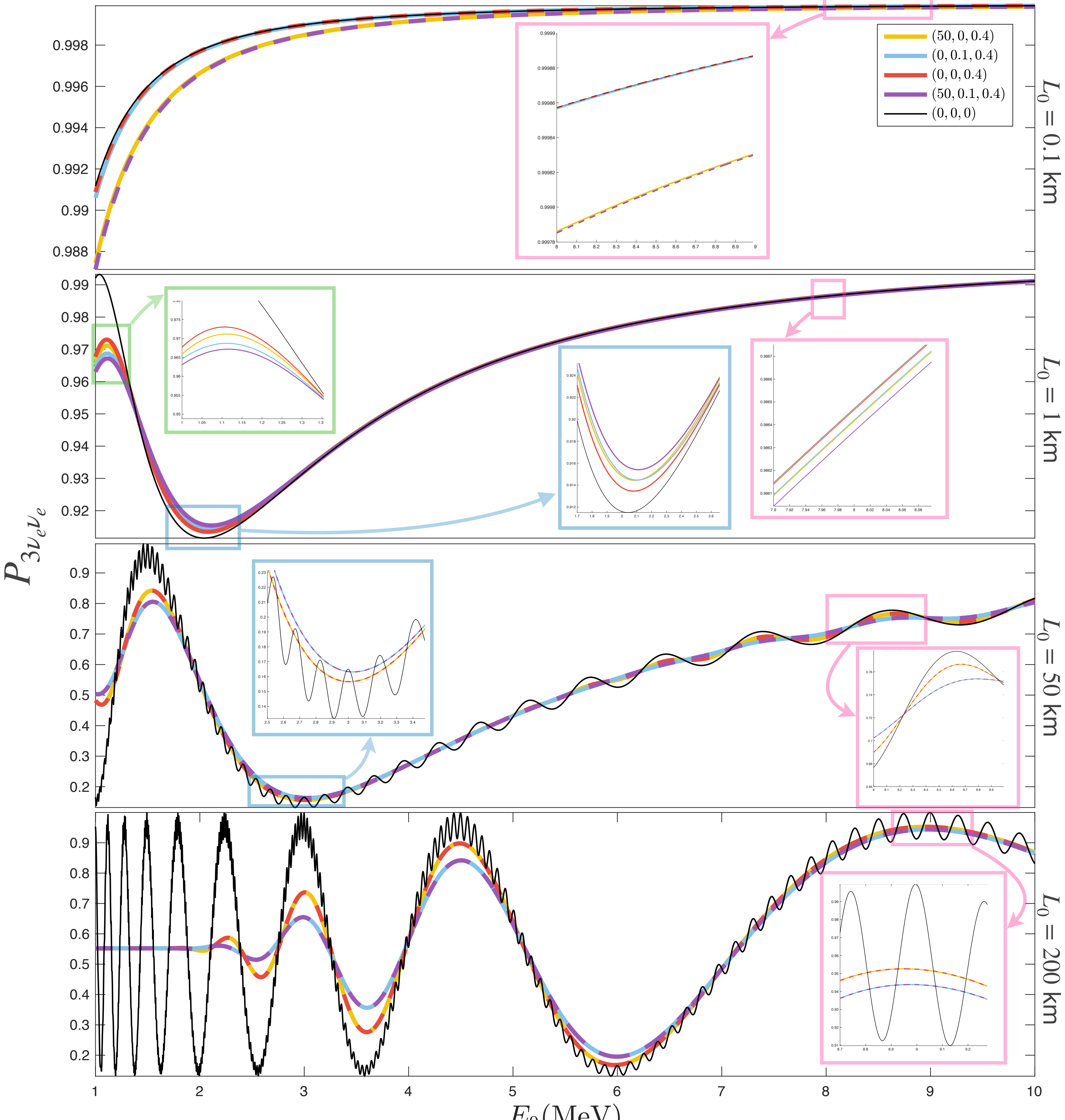
  Energy budget from the external particles , i.e. relativistic neutrinos
- $\delta E_j \neq 0$ and $\mathbf{v}_j \neq 1$ results from neutrinos caring mass:

$$\mathbf{P}_j \simeq \mathbf{P} - \vec{\xi}_j \frac{m_j^2}{2E}, = \vec{\xi}_p E - \vec{\xi}_j \frac{m_j^2}{2E}, \text{ and } \mathbf{v}_j = \frac{\mathbf{P}_j}{E_j} \simeq \vec{\xi}_p \left(1 - \frac{m_j^2}{2E^2}\right)$$

- Physical transition probability : $P_2(T,\mathbf{L},\mathbf{P}) \simeq \sum_{jk} U_{\alpha j}^* U_{\beta j} U_{\alpha k}^* U_{\beta k}\, e^{i\frac{\Delta m_{jk} L}{2E}}\, e^{-S_x^2 - S_p^2}$

  With (state) decoherence terms: $S_x = \dfrac{\Delta m_{jk}\sigma_x}{2\sqrt{2}\Delta E}$ and $S_p = \dfrac{\Delta m_{jk}\sigma_p L}{2\sqrt{2}\Delta E^2} = \dfrac{L}{L_{jk}^{coh}}$, $\Delta^2 = 1 + \sigma_p^2\sigma_x^2$
- $$\mathscr{L}\mathscr{M}\mathscr{O}^2 = \int dL \int dE \int dT \int d\Omega\, \underbrace{H_L(L;L_0)}_{\sigma_L}\, \underbrace{H_E(E;E_0)}_{\sigma_E}\, H_T(T;T_0)\, H_\Omega(\Omega;\Omega_0)$$

Layer 3: Measurement Transition Probability



$$P_3(T_0, \mathbf{L}_0, \mathbf{P}_0) = \mathscr{L}\mathscr{M}\mathscr{O}^2 P_2(T, \mathbf{L}, \mathbf{P})$$

σ_x → Subdominant & absorbed into σ_L

	σ_L	σ_E	σ_p
	X	0	X
	0	X	X
	0	0	X
	X	X	X
	0	0	0

- Dominance:

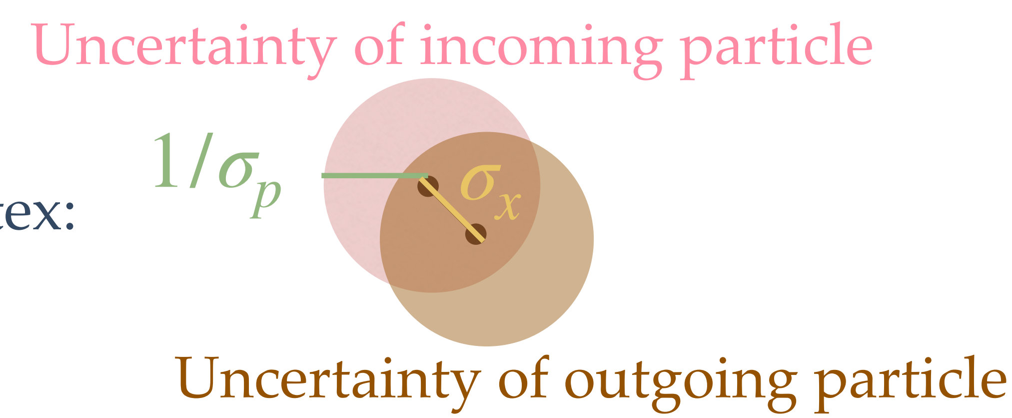
Short Distance $\rightarrow \sigma_L$

Long Distance $\rightarrow \sigma_E \, , \sigma_p$

Uncertainties

- Possible contributions to each uncertainties:
  - σ_p : external particle WP size
  - σ_x : non central collisions
  - σ_E : energy resolution, reconstruction model
  - σ_L : production profile, reactor core size, decay length, space fluctuation (e.g. quantum gravity)

Uncertainty @ vertex:



Neutrino Decoherence

- Operation definition of transition probability:

Neutrino oscillation experiment

$$P_{3jk}(X_3) = \frac{\int dX_2\, H(X_2;X_3)\,\Gamma_{2jk}(X_2;X_3)}{\sqrt{\int dX_2\, H(X_2;X_3)\,\Gamma_{2jj}(X_2;X_3)}\sqrt{\int dX_2\, H(X_2;X_3)\,\Gamma_{2kk}(X_2;X_3)}},$$

Neutrino mass measuring experiment

$$\simeq \frac{\int dX_2 \Gamma_\alpha^{\text{pro.}}(X_2)\, P_{2\alpha\to\beta}(X_2;X_3)\, \sigma_\beta^{\text{det.}}(X_2)\, H(X_2;X_3)}{\int dX_2 \Gamma_\alpha^{\text{pro.}}(X_2)\sigma_\beta^{\text{det.}}(X_2) H(X_2;X_3)}$$

$$\Gamma_{2jk}(X_2;X_3) \to \Gamma_\alpha^{\text{pro.}}(X_2)\sigma_\beta^{\text{det.}}(X_2)P_{2jk}(X_2;X_3)$$

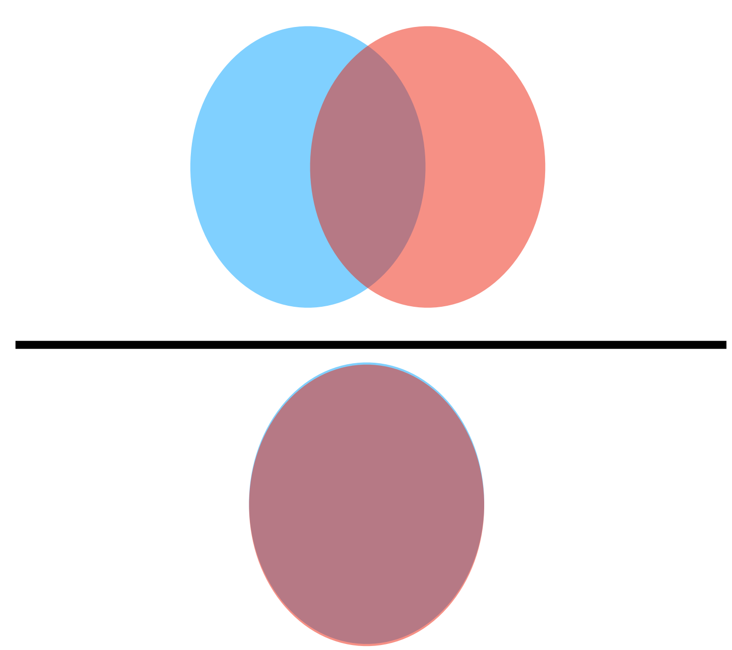
Automatic $\sum_\alpha P_{3\alpha\to\beta} = 1$

Neutrino Decoherence

- State Decoherence and Phase Decoherence

State decoherence term $\left(S_{jk}\right)$ × Phase decoherence term $\left(\Phi_{jk}\right)$

$$P_{3jk}(X_3) = \frac{\int dX_2 H \,|\Gamma_{2jk}|}{\sqrt{\int dX_2\, H\, \Gamma_{2jj}}\sqrt{\int dX_2\, H\, \Gamma_{2kk}}} \times \frac{\int dX_2\, H\, \Gamma_{2jk}}{\int dX_2 H \,|\Gamma_{2jk}|}$$



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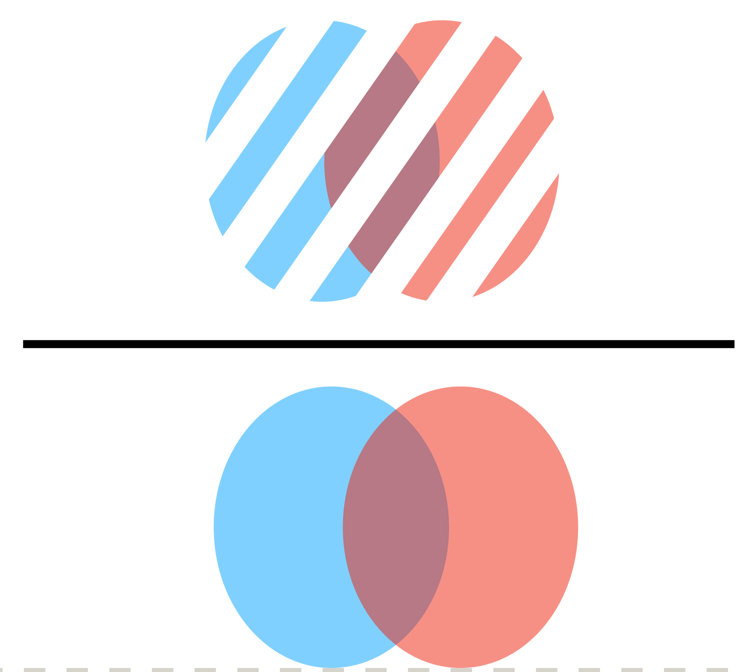
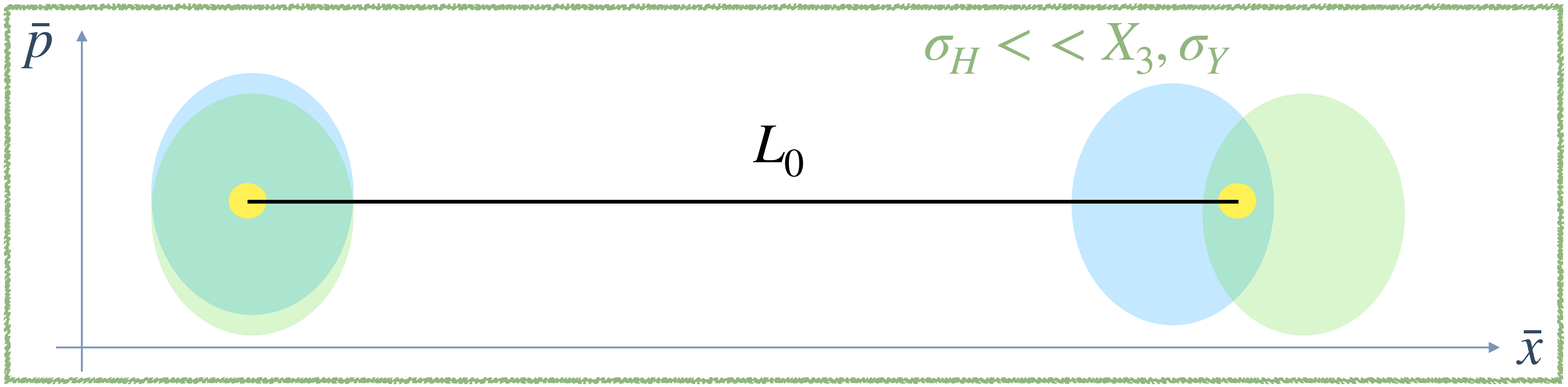


State Decoherence and Phase Decoherence

- SD term: $$S_{3jk}(X_3) = \frac{\int dX_2 Y_{jk}}{\sqrt{\int dX_2 Y_{jj}}\sqrt{\int dX_2 Y_{kk}}} \simeq S_{2jk}(X_2)\Big|_{X_2=X_3}$$



$S_{2jk}(X_2)$

Washout effect at the Wigner layer $(\bar{1})$

$$Y_{jk} = H|\Gamma_{2jk}| = H e^{-i\theta_{jk}} \frac{\int d\bar{x}\int d\bar{p}\,\Gamma_{1jk}(\bar{x},\bar{p};X_2)}{\int d\bar{x}\int d\bar{p}\,|\Gamma_{1jk}(\bar{x},\bar{p};X_2)|} \int d\bar{x}\int d\bar{p}\,|\Gamma_{1jk}(\bar{x},\bar{p};X_2)|$$

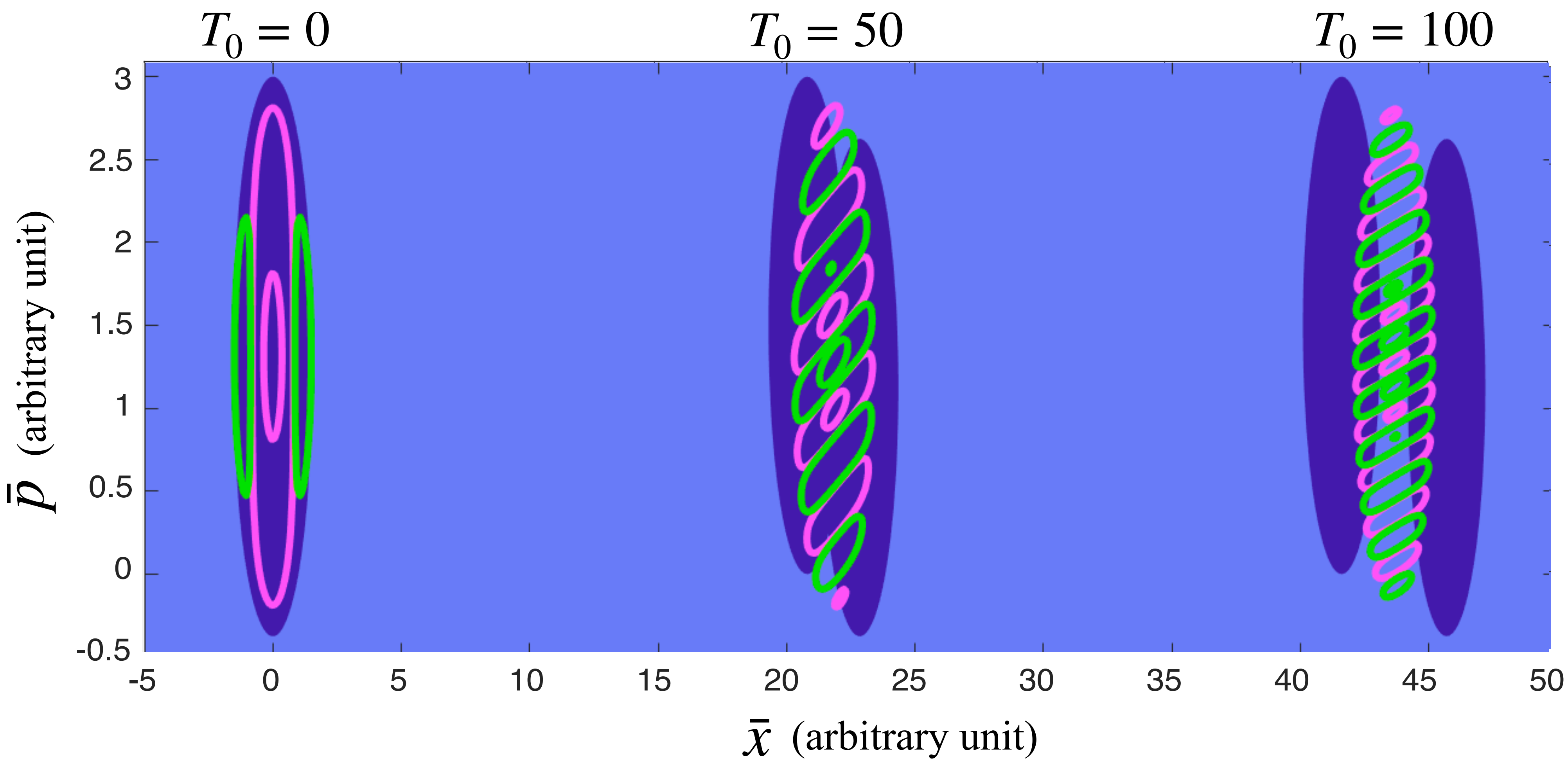
- PD term: $$\Phi_{jk}(X_3) = \frac{\int dX_2 Y_{jk}(X_2;X_3) e^{i\theta_{jk}(X_2)}}{\int dX_2 Y_{jk}(X_2;X_3)}$$

Washout effect at the physical layer (2)

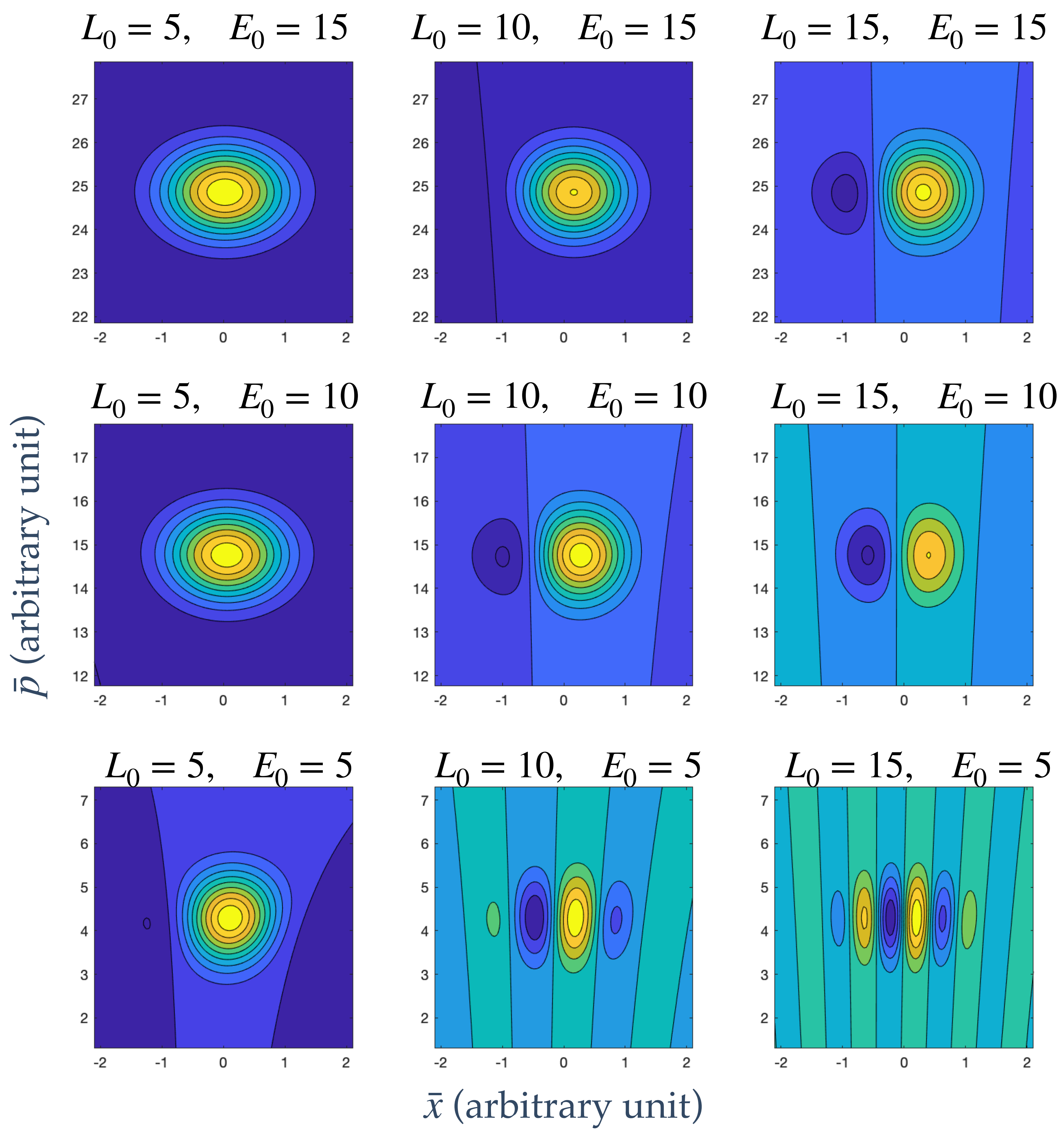
State Decoherence and Phase Decoherence

- Phase structure on the Wigner phase space layer $P_{\bar{1}}(\bar{x}, \bar{p}) = |P_{\bar{1}}(\bar{x}, \bar{p})| e^{i\eta_{jk}}$
  - Pulse neutrino emission : $\eta_{jk}(\bar{x}, \bar{p}; T, E)$
  - Continuous neutrino emission: $\eta_{jk}(\bar{x}, \bar{p}; \mathbf{L}, E)$
- Phase structure on the Physical phase space layer $P_2(\mathbf{x}, \mathbf{p}) = |P_2(\mathbf{x}, \mathbf{p})| e^{i\theta_{jk}}$
  - Pulse neutrino emission : $\theta_{jk}(T, \mathbf{L}, E)$
  - Continuous neutrino emission: $\theta_{jk}(\mathbf{L}, E) \simeq -\dfrac{i\Delta m_{jk}^2 L}{2E}$

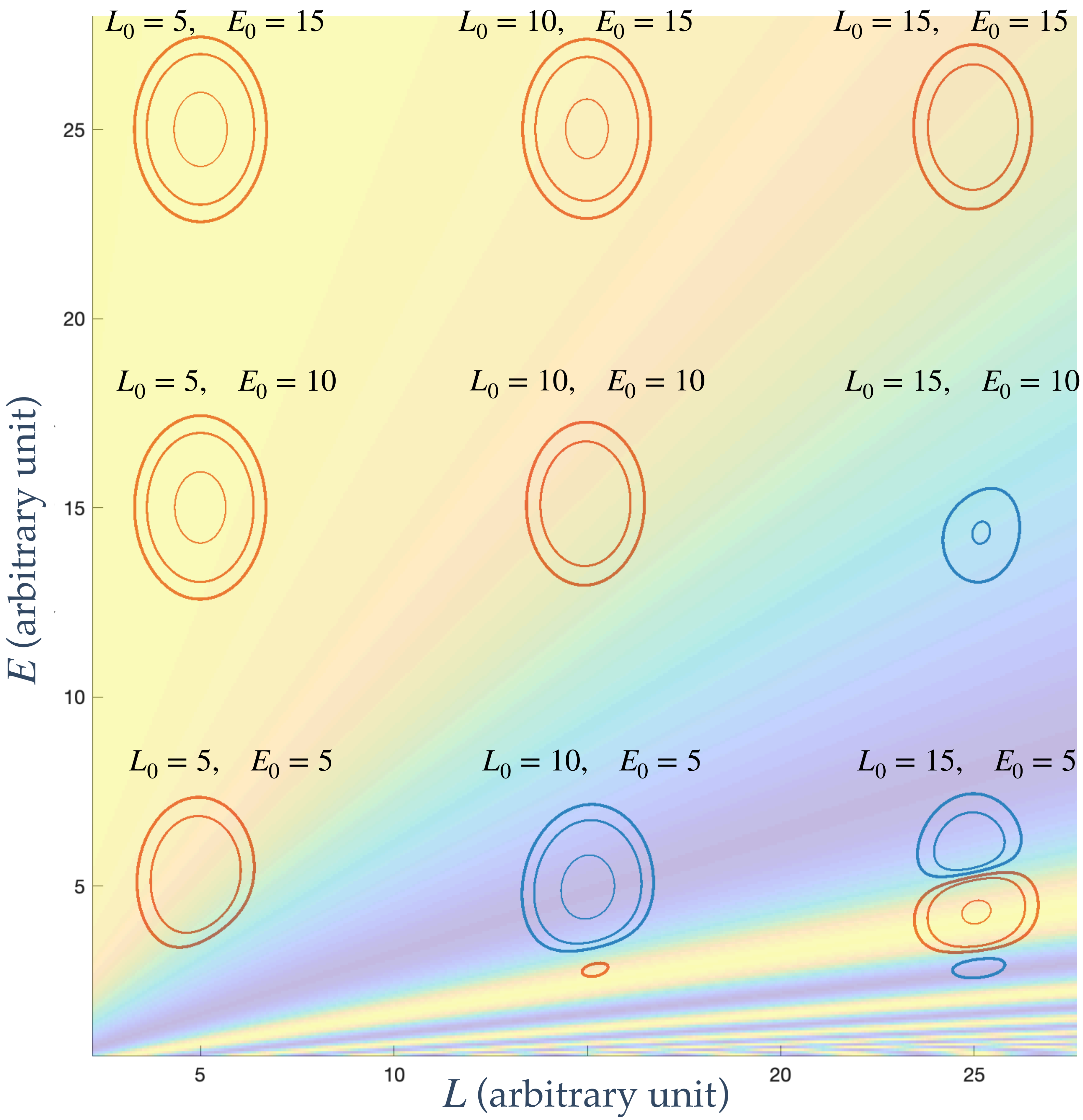
State Decoherence — Pulse neutrino emission



State Decoherence — Continuous neutrino emission



Phase Decoherence — Continuous neutrino emission

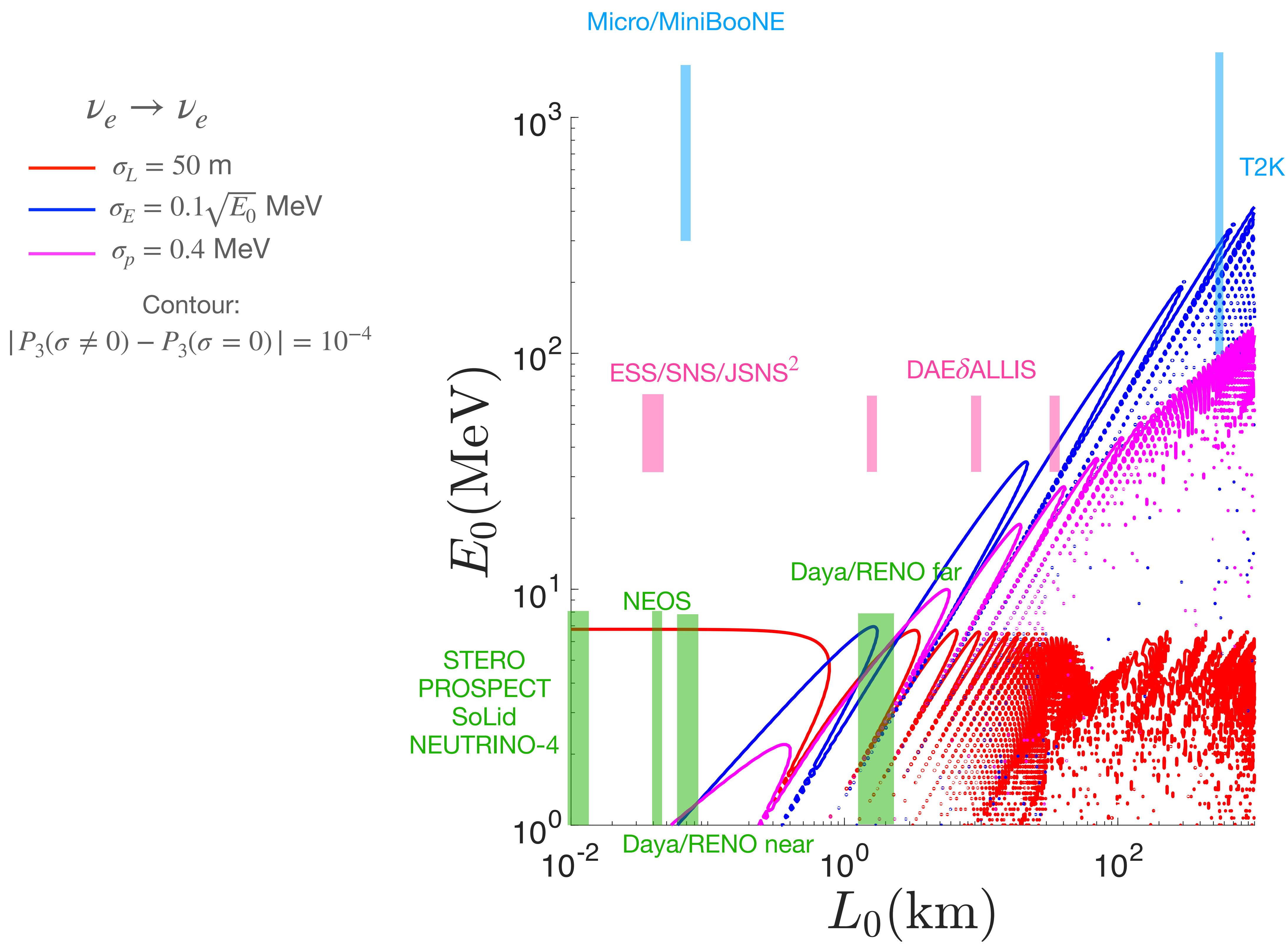


Experimental Potential

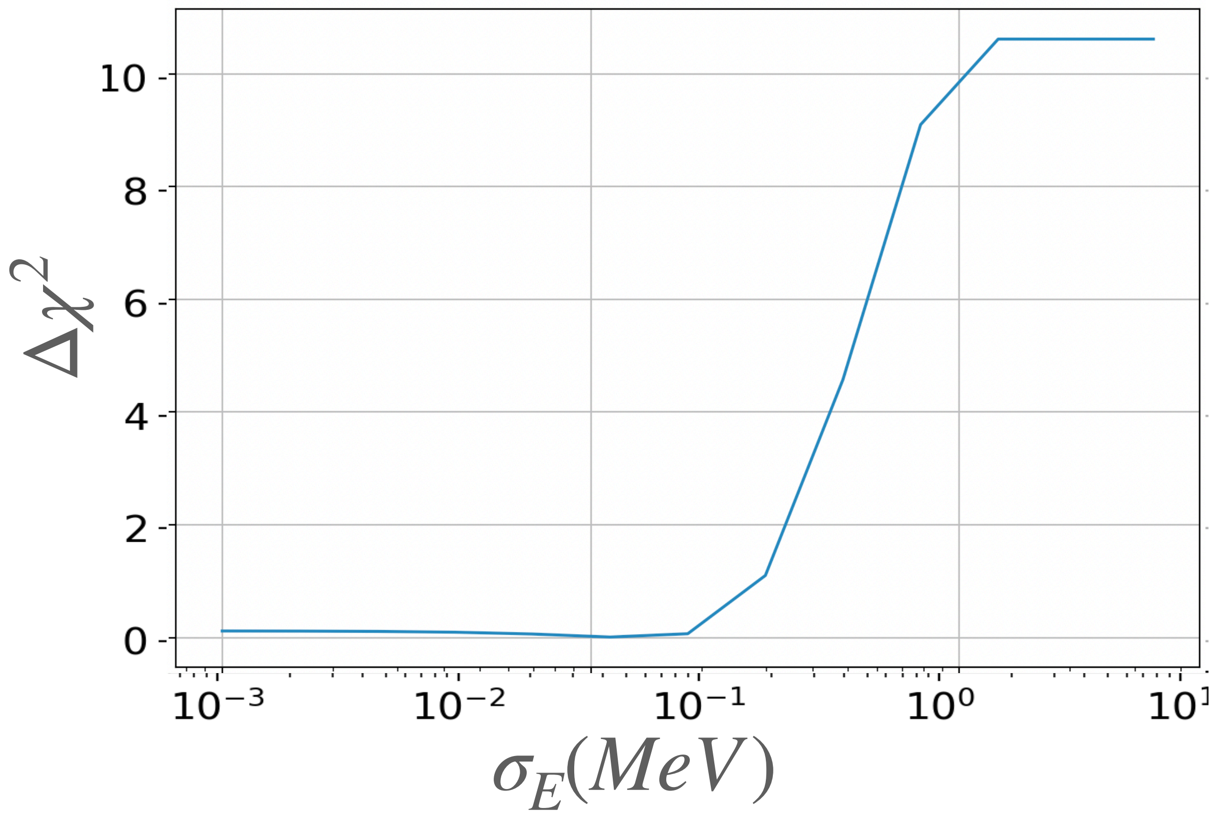
- Phase shift:
  - Asymmetry of the PDF w.r.t. the phase structure
    (If all PDFs are Gaussian, only σ_E is not symmetric)
  - Independent of other (smearing) decoherence effect
- Long distance: Effects of $\dfrac{\sigma_p}{\sqrt{1+\sigma_p\sigma_x}}$ and σ_E grows with distance
- Short distance: σ_L isn't affected by distance

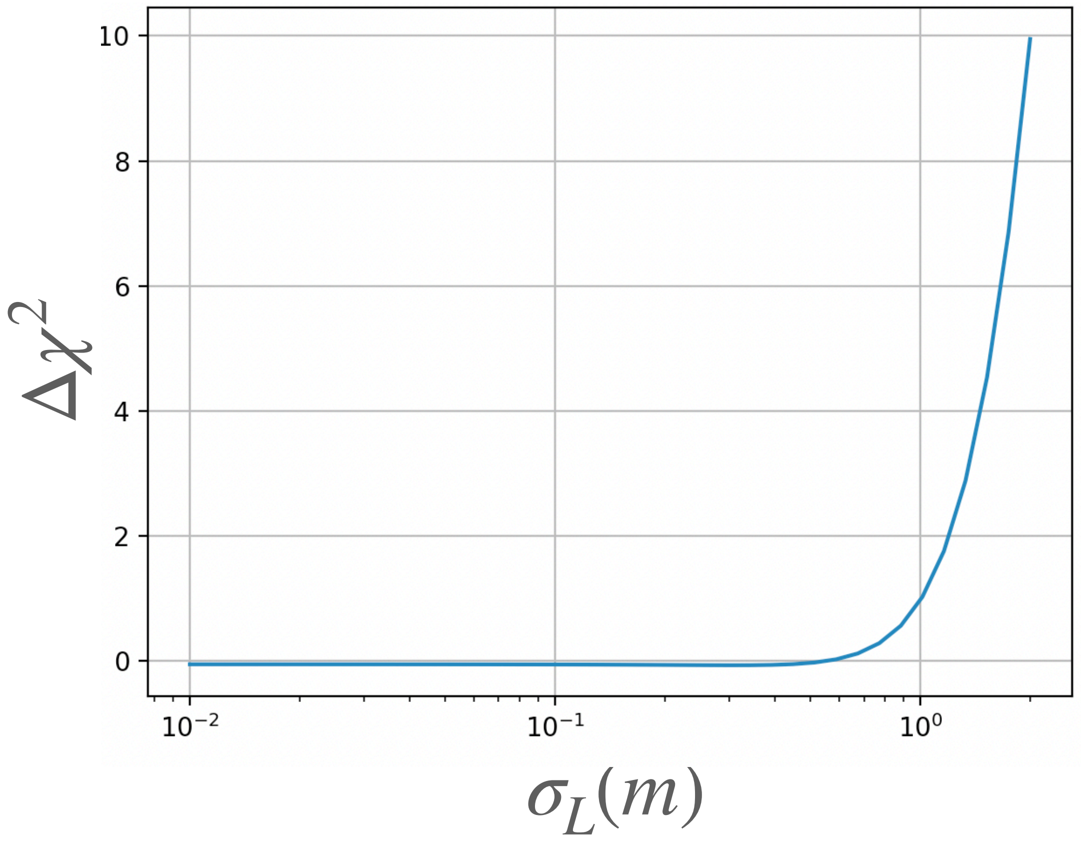
With the these differences, one can better identify the uncertainties through combined analysis of different experiments

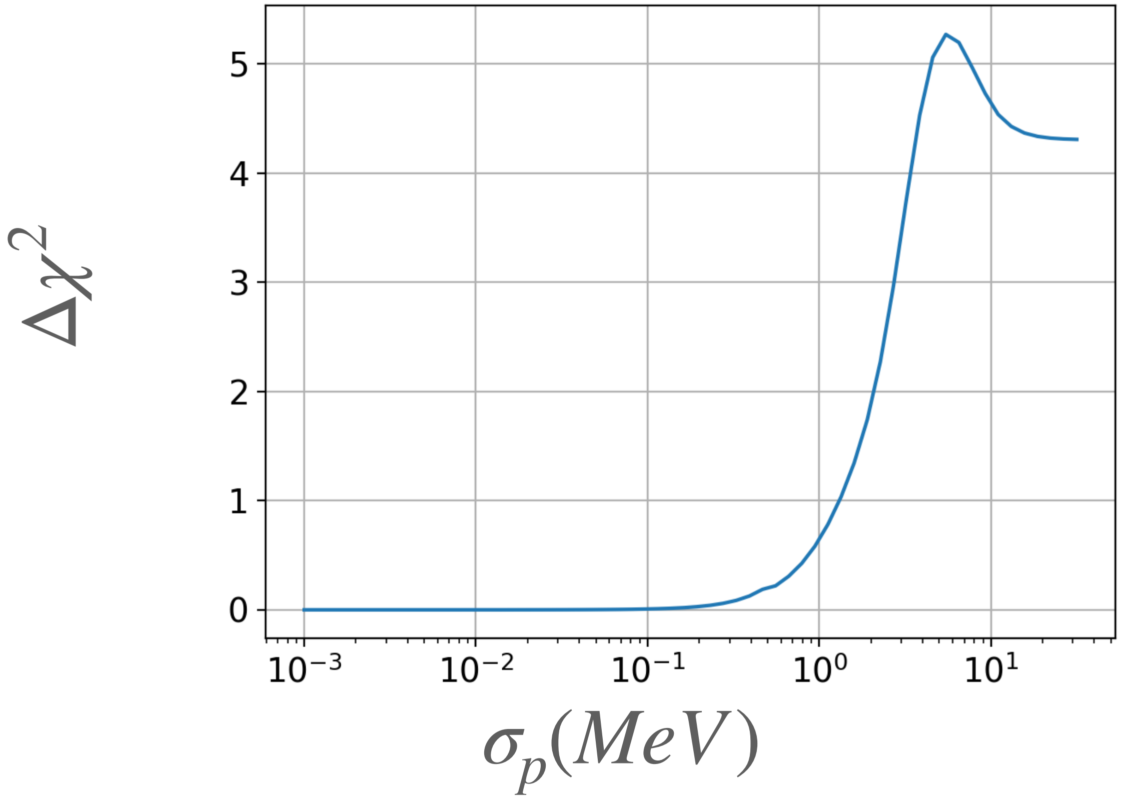
Sensitivity on the measurement layer



An example: RENO experiment







Future prospects

- Estimation of the sensitivity for each uncertainty for future experiments
- Develop (analysis, experimental) schemes to identify the uncertainties
- Consider specific mechanisms to contribute to the classified uncertainties

Conclusion

- Developed the layer structures which classifies uncertainties from microscopic to macroscopic, building on QFT calculation
- Classify decoherence into state decoherence and phase decoherence based on their causes and investigate their phenomenological properties
- State decoherence and phase decoherence and wash-out effects on the Wigner phase space and physical phase space respectively
- Showed sensitivity of decoherence effect for reactor neutrino experiments (not yet complete)

Back Up Slides

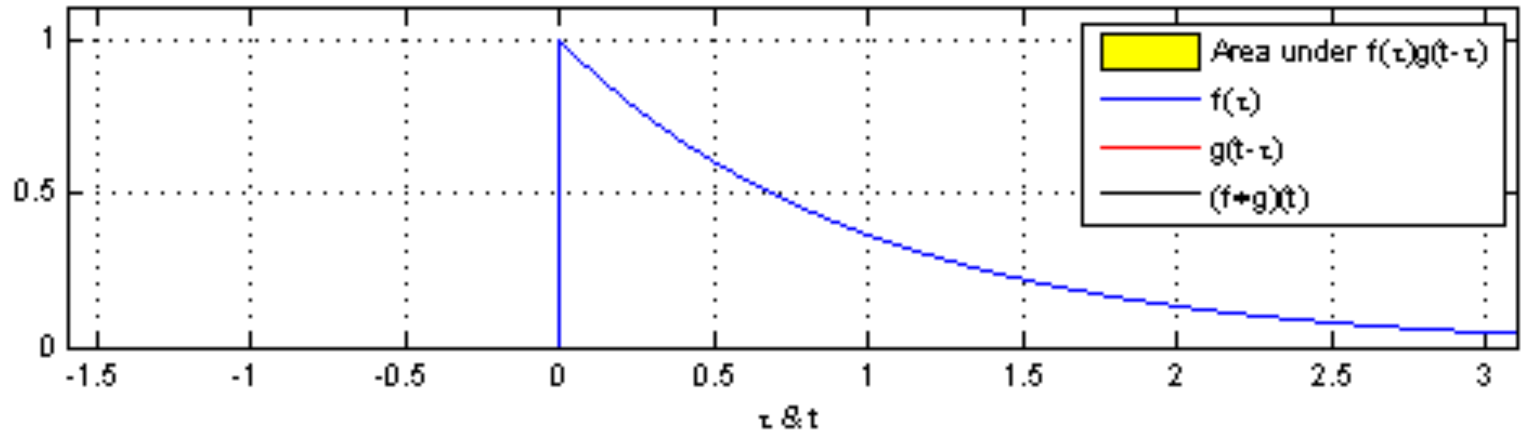
State Decoherence and Phase Decoherence

- Phase structure on the Wigner phase space layer $P_{\bar{1}}(\bar{x},\bar{p}) = |P_{\bar{1}}(\bar{x},\bar{p})| e^{i\eta_{jk}}$
  - Pulse neutrino emission : $\eta_{jk}(\bar{x},\bar{p};T,E) = -i\bar{p}T(\mathbf{v}_j - \mathbf{v}_k) - i\bar{x}(\mathbf{P}_j - \mathbf{P}_k)|_{T=T_0,E=E_0}$
  - Continuous neutrino emission: $\eta_{jk}(\bar{x},\bar{p};\mathbf{L},E) \simeq -\frac{i\Delta m_{jk}^2}{2E}((\mathbf{L}-\bar{x})\frac{\bar{p}}{E}+\bar{x})|_{\mathbf{L}=\mathbf{L}_0,E=E_0}$

- Phase structure on the Physical phase space layer $P_2(\mathbf{x},\mathbf{p}) = |P_2(\mathbf{x},\mathbf{p})| e^{i\theta_{jk}}$
  - Pulse neutrino emission : $\theta_{jk}(T,\mathbf{L},E) = -iT(E_j - E_k) + i\mathbf{L}(\mathbf{P}_j - \mathbf{P}_k)$
  - Continuous neutrino emission: $\theta_{jk}(\mathbf{L},E) \simeq -\frac{i\Delta m_{jk}^2 L}{2E}$

Uncertainties

- Convolution property:



$$G(L) = \int dx\, f(x) g(x - L)$$

$$\sigma_G \;\geq\; \sigma_f \text{ and } \sigma_g$$

- Fourier Transformation property: $F(p) = \int dx\, e^{ipx} f(x)$ $\quad \sigma_{xp} \uparrow \Leftrightarrow \sigma_x \downarrow$

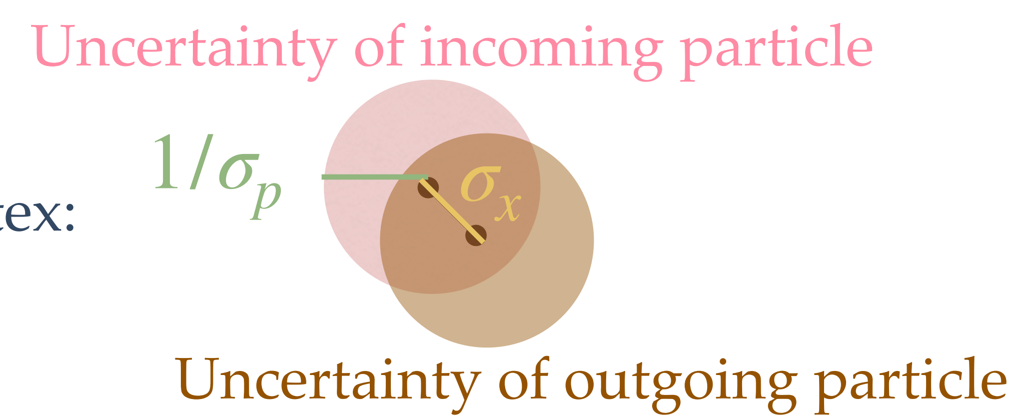
- Product of two PDF: $G'(x) = f(x)\, g(x)$ $\quad \sigma_{fg} \uparrow \leq \sigma_f, \sigma_g \uparrow$

  e.g. for two Gaussians: $\sigma_{fg}^2 = \dfrac{\sigma_f^2 \sigma_g^2}{\sigma_f^2 + \sigma_g^2} = \dfrac{\sigma_f^2}{\sigma_f^2 / \sigma_g^2 + 1}$

Uncertainties

- Possible contributions to each uncertainties:
  - σ_p : external particle WP size
  - σ_x : non central collisions
  - σ_E : energy resolution, reconstruction model
  - σ_L : production profile, reactor core size, decay length, space fluctuation (e.g. quantum gravity)

Uncertainty @ vertex:



- Properties of PDF
  - Convolution: $\sigma_{f \circledast g} \geq \sigma_f$ and σ_g
  - Fourier Transformation: $\sigma_{xp} \uparrow \Leftrightarrow \sigma_x \downarrow$
  - Product of two PDF: $\sigma_{fg} \uparrow \leq \sigma_f , \sigma_g \uparrow$

Uncertainties

- Layer $1 \to 2$ structure :

$$\int dp \int dx\, e^{i\mathbf{p}\mathbf{x}} F_j(\mathbf{p};\mathbf{P}_j) G_j(\mathbf{x};\mathbf{L}_j) = \int dp\, e^{i\mathbf{p}\mathbf{L}_j} F_j(\mathbf{p};\mathbf{P}_j) \tilde{G}_j(\mathbf{p};0;\mathbf{L}_j) = e^{i\mathbf{P}_j\mathbf{L}_j} \hat{\Phi}(\mathbf{L}_j,\mathbf{P}_j)$$

$$\sigma_p \qquad \sigma_x \uparrow \qquad\qquad \sigma_p^{eff} \downarrow \le \sigma_p\,,\ \sigma_{xp} \downarrow \qquad\qquad \sigma_p^{(2)} \uparrow$$

$$F_j(\mathbf{p};\mathbf{P}_j) = \Big(f_{pi}(q)M_q(q) \circledast f_{pf}(k)M_k(k)\Big) \times \Big(f_{di}(q')M_{q'}(q') \circledast f_{df}(k')M_{k'}(k')\Big)$$

Production of convolution of incoming & outgoing particles @ production / detection site

$$G_j(\mathbf{x};\mathbf{L}_j) = g_1(x_1) \circledast g_2(x_2)$$

Convolution of production and detection vertex uncertainties

- Layer $2 \to 3$ structure :

$$\int dL\, e^{i\frac{\Delta m_{jk} L}{2E_0}} H_L(L;L_0)\, \tilde{H}_E(L) \hat{\Phi}(\mathbf{L}_j,\mathbf{P}_j) = e^{i\frac{\Delta m_{jk} L_0}{2E_0}} D_{jk}(L_0)$$

$$\sigma_L^{eff} \le \sigma_L\,,\ \sigma_{EL}\,,\ \sigma_x^{(2)} \qquad\qquad \sigma_{L_0} \uparrow$$