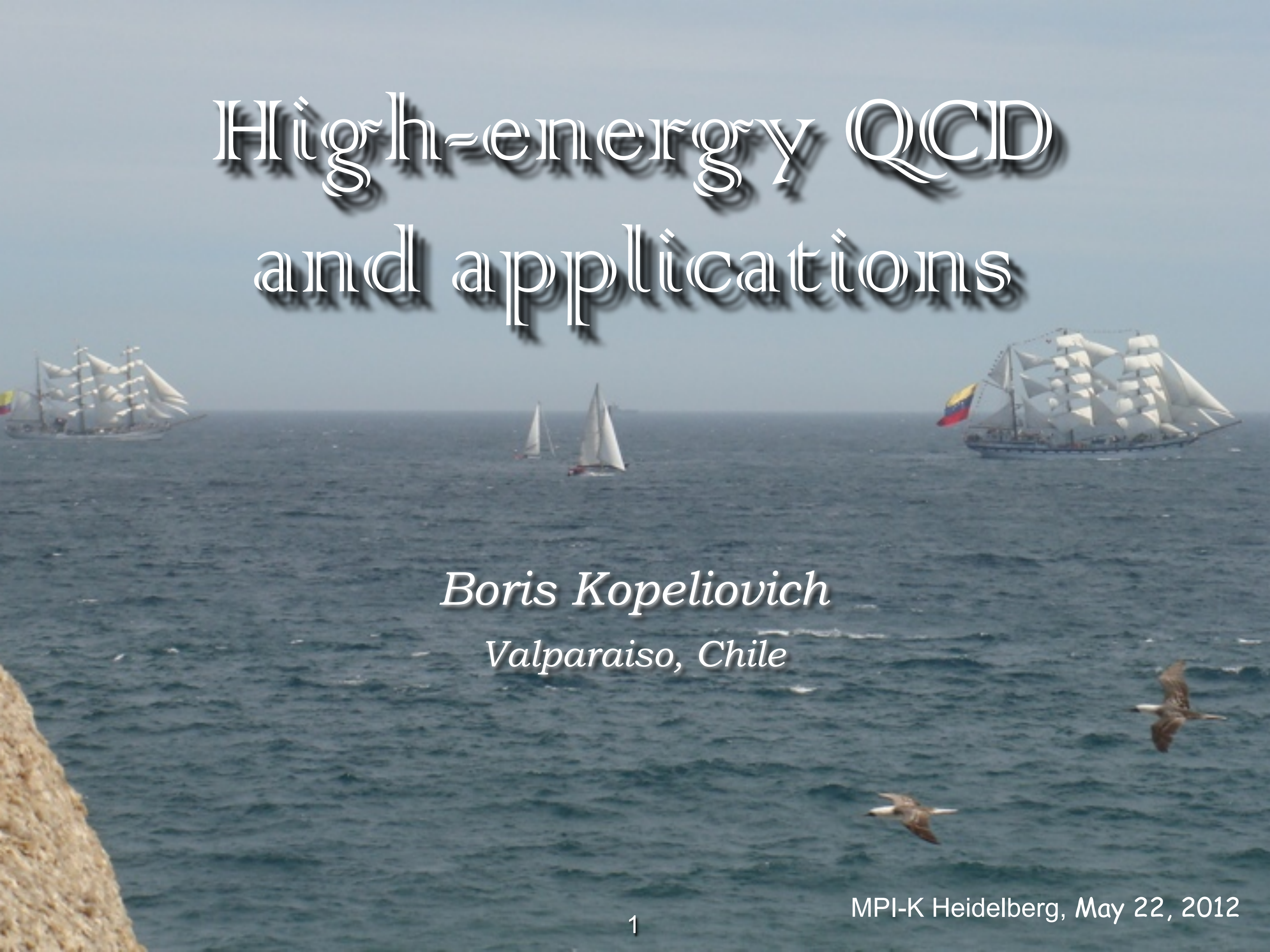

High-energy QCD and applications

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Valparaiso, Chile

MPI-K Heidelberg, May 22, 2012

Outline

- Introduction: QCD vs QED
- Exclusive reactions, Color Transparency
- Confinement, string model, hadronization
- High-energy hadronic collisions: QCD vs Regge
- DIS: small x physics
- Drell-Yan processes
- Diffraction: soft and hard
- Diffractive Higgs production
- Nuclear shadowing
- Color glass condensate / saturation
- Optimistic and pessimistic conclusions

QCD versus QED

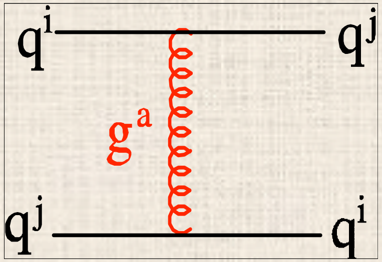
similarities and differences

Similarities:

✓ The color charged quarks emit and absorb gluons in the same way as the electrically charged leptons emit and absorb photons;

✓ the gluon and the photon are massless;

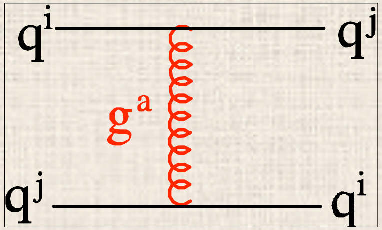
✓ the gluon and the photon have spin 1. Quarks are spin-1/2 point particles, very much like electrons.



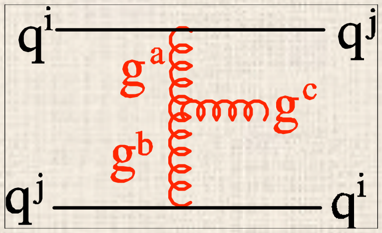
QCD versus QED

Differences:

✓ Radiation of a photon does not change the charge of the electron, while a gluon can change the quark's color. Gluons carry unbalanced color charges;



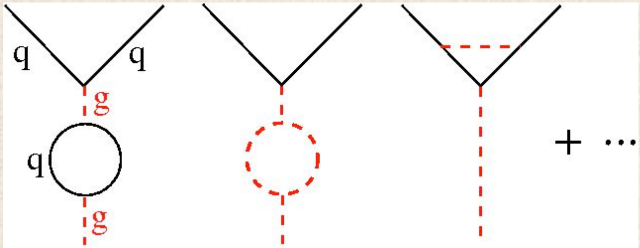
✓ the response of gluons to color charge, as measured by the QCD coupling constant, is much more vigorous than the response of photons to electric charge;



✓ gluons, unlike photons, interact directly with one another.

Renormalized QCD coupling

$$\beta_{QCD}(\alpha_s) \equiv \mu^2 \frac{d\alpha_s(\mu^2)}{d\mu^2} = -\beta_0 \frac{\alpha_s^2}{4\pi} + O(\alpha_s^3)$$



$$\beta_0 = 11 - \frac{2}{3} n_f > 0$$

(unless n_f >15)

Self-interaction of gluons leads to anti-screening

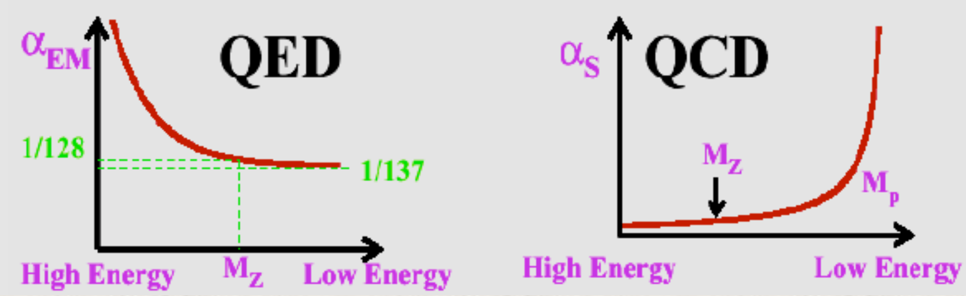
First order solution

$$\alpha_s(Q) = \frac{4\pi}{\beta_0 \ln \frac{Q^2}{\Lambda^2}}$$

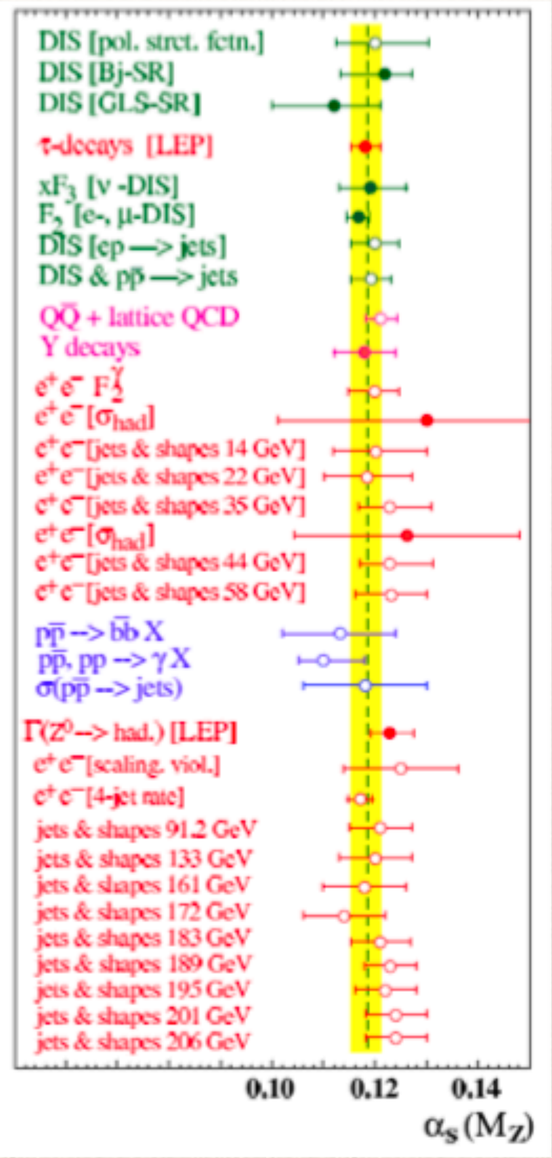
Λ is not given by theory, but by experiment

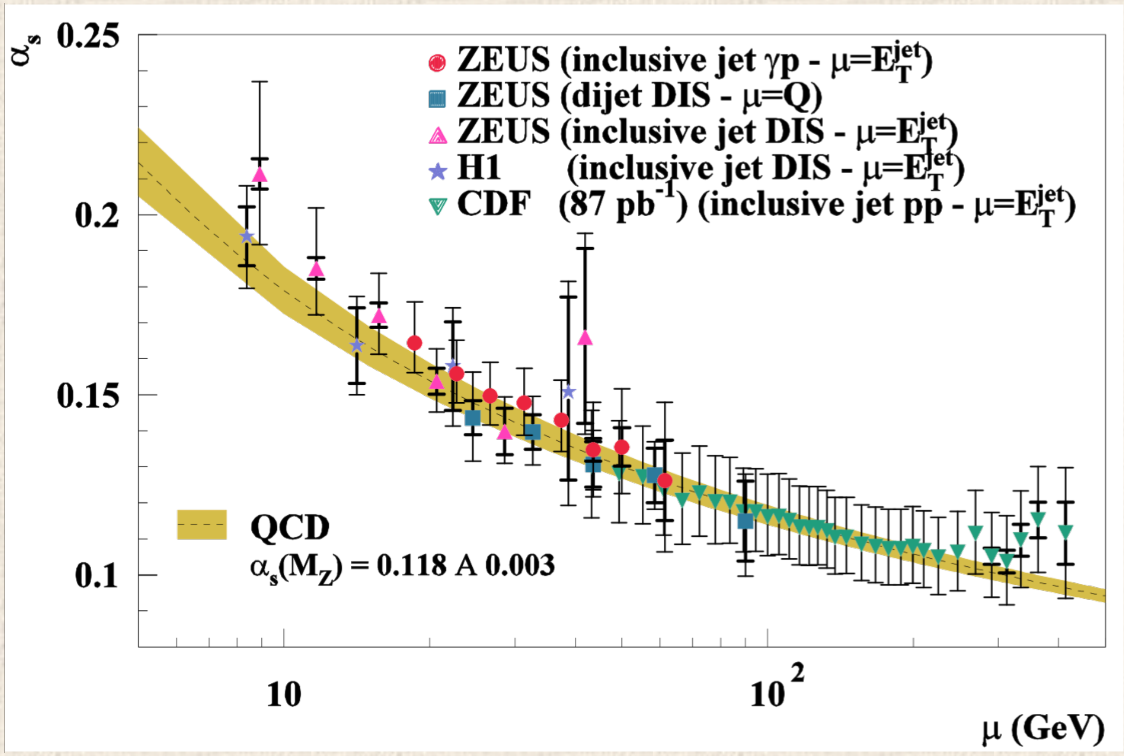
In QED the sign is opposite: screening

$$\beta_{QED}(\alpha_{em}) = \frac{1}{3\pi}\alpha_{em}^2$$



Measurements of the running coupling





$$\Lambda = 211\,MeV$$

Evidences for colored quarks

* In the baryon decuplet Δ^{++} state is $u\uparrow u\uparrow u\uparrow$ with angular momentum $\ell=0$ i.e. 3 fermions in the same state.

The wave function symmetric in space, spin and flavour $\rightarrow$ violates **Pauli Exclusion Principle**

One has to introduce a new Degree of Freedom, **Color**

Now the 3 fermions are in different states:

$u\uparrow$ (red) $u\uparrow$ (blue) $u\uparrow$ (green)

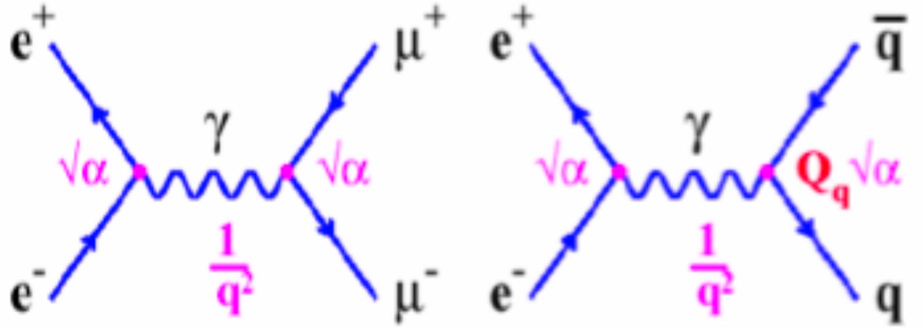
Evidences for colored quarks

* e^+e^- **annihilation**

Direct evidence for the existence of colour comes from e^+e^- Annihilation.

★ **Compare** $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to q\bar{q}$:

$$R_\mu = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



$$R = \begin{cases} \frac{2}{3} \; N_c & (u,d,s) \\ \frac{10}{9} \; N_c & (u,d,s,c) \\ \frac{11}{9} \; N_c & (u,d,s,c,b) \end{cases}$$

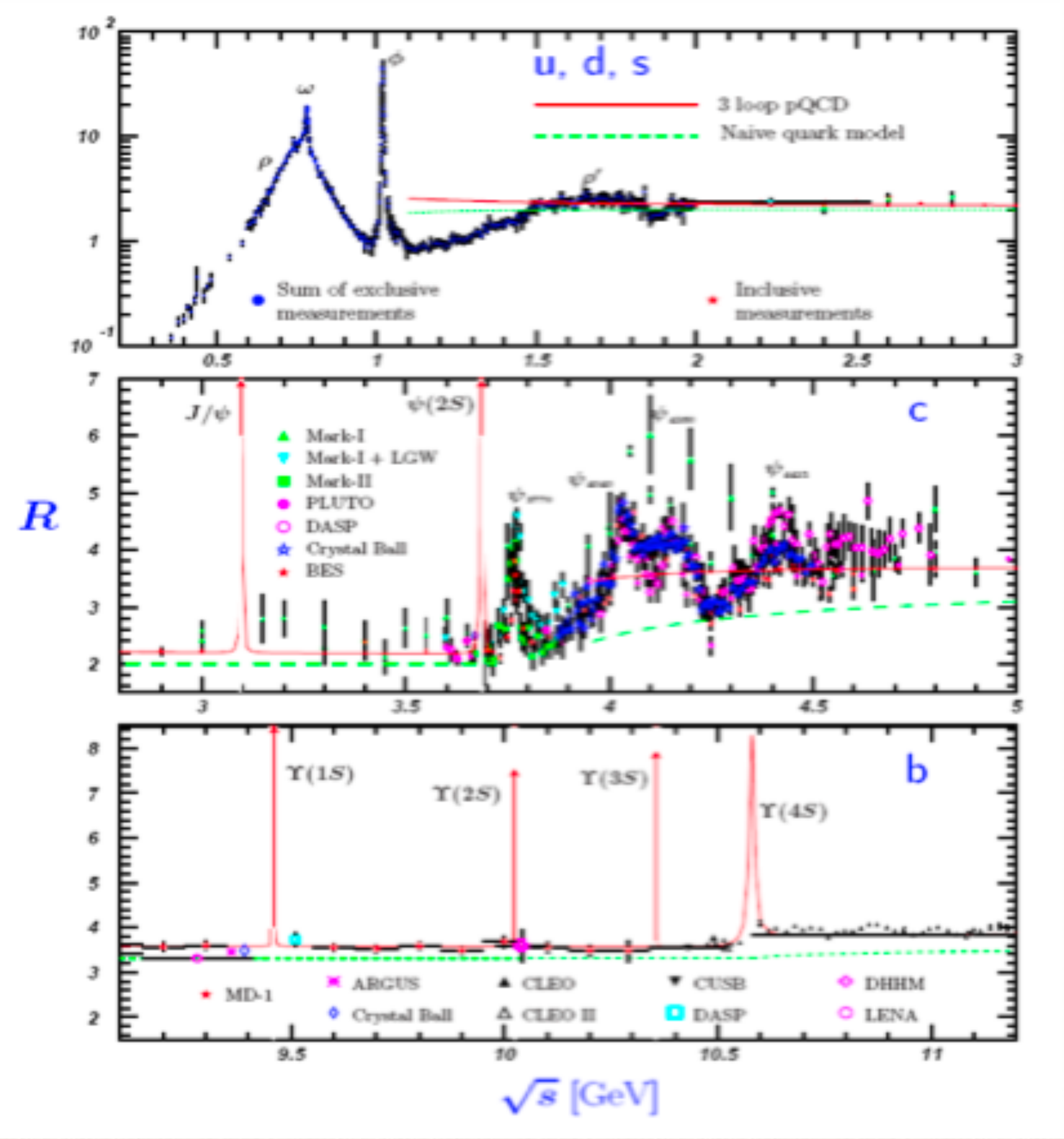
Evidences for colored quarks

e^+e^- **annihilation**

$R = 2$

$R = \frac{10}{3}$

$R = \frac{11}{3}$

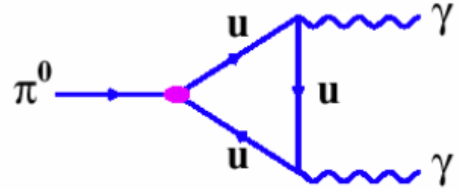


Evidences for colored quarks

Colour: experimental evidence

★ $\pi^0 \to \gamma\gamma$ decay rate

Need colour to explain $\pi^0 \to \gamma\gamma$ decay rate.



$$\Gamma(\pi^0 \to \gamma\gamma) \propto N^2_{colour}$$

$$\text{EXPT}: \quad N_{colour} = 2.99 \pm 0.12$$

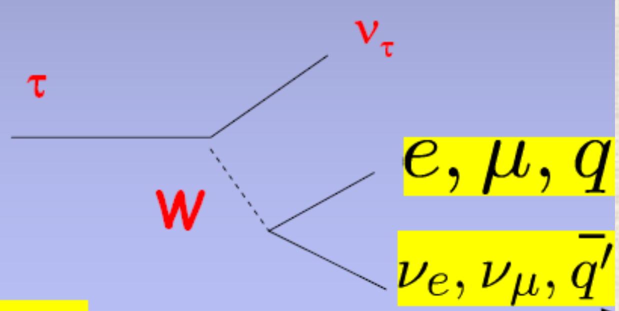
Triangle Diagram
Each color contributes one amplitude. Three colors changes the decay rate by 9.

τ decays: branching fractions

1/5 $e\nu_e$

1/5 $\mu\nu_\mu$

3 · 1/5 hadrons



Each quark in three colour states

Evidences for color dynamics

So far the color was just a new quantum number for quarks that distinguishes them. Are there any direct evidences that the color color is reponsible for strong interactions?

* **Color Transparency**

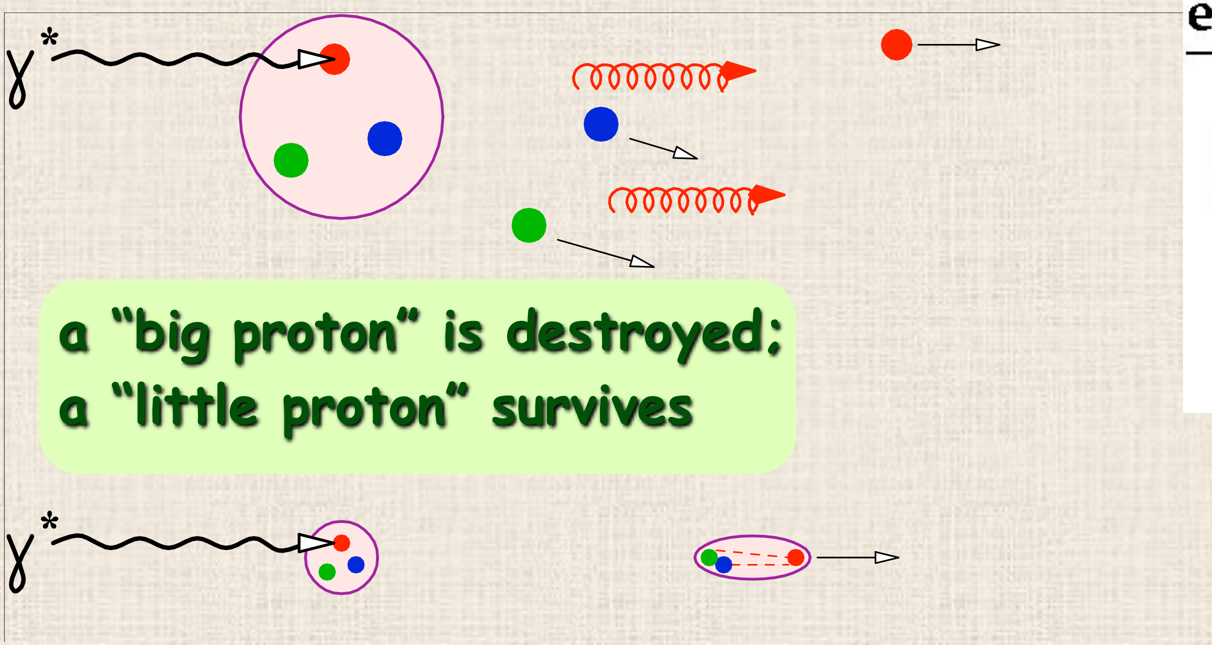
How can a colorless hadron interact?
- Only via its color-dipole moment

A point-like colorless object cannot interact with external color fields. Therefore the cross section of a small color dipole of transverse size r_T vanishes as

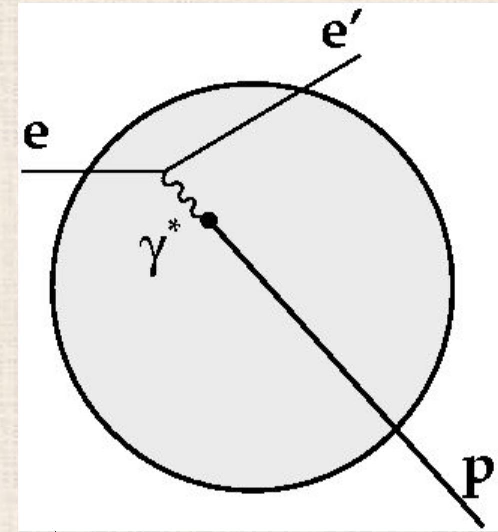
$$\sigma\left(r_T\right) \propto r_T^2$$

* Quasielastic electron scattering off nuclei $A(e,e'p)A'$

The nucleus acts as a color filter



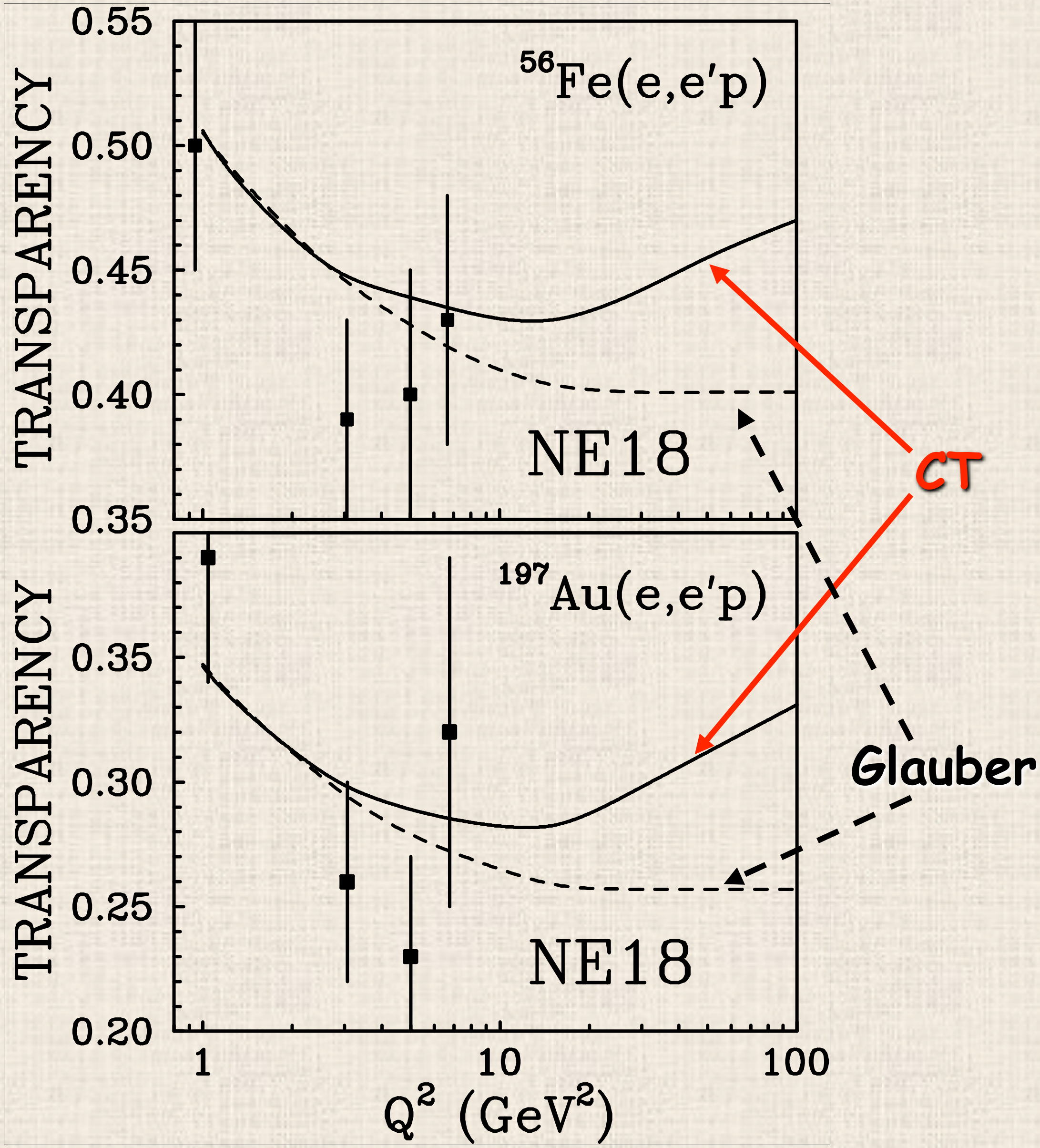
a "big proton" is destroyed;
a "little proton" survives



The recoil "proton" has a reduced size and experiences weaker final state interactions in the nucleus. Therefore, due to CT it should escape the nucleus with a higher probability, than is suggested by the Glauber model.

Unfortunately, the experiment NE18 at SLAC was not successful in hunting for CT

$$\mathrm{Tr} = \frac{\sigma(\mathbf{eA} \to \mathbf{e'pA^*})}{\mathbf{Z}\sigma(\mathbf{ep} \to \mathbf{e'p})}$$



Even data with a higher statistics would be unable to discriminate between the two models at

$$Q^2 < 10\,\mathrm{GeV}^2$$

Why these experiments failed to detect a signal of CT

Even if a small-size partonic state is produced, eventually it becomes a proton. How long does it take to form the proton wave function?

$$l_f < \frac{2E_p}{m_{p^*}^2 - m_p^2} \approx 0.4\, fm \times E_p(GeV)$$

$$E_p = \frac{Q^2 + 2m_p^2}{2m_p}$$

is the recoil proton energy.

At the maximal virtuality Q^2=8GeV^2 the formation length l_f=2fm is of the order of the mean inter-nucleon spacing in a nucleus.

Thus, the proton is too slow to keep the initial small size, and attenuates with the mean proton cross section.

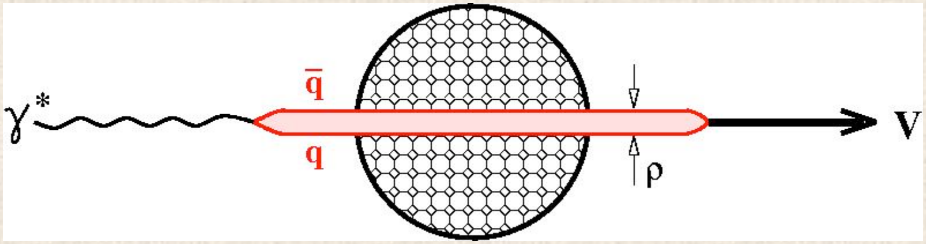
$$Tr = \frac{1}{A}\int d^2b \int_{-\infty}^{\infty} dz\, \rho_A(b,z) \exp\left[-\sigma_{in}^{pN} \int_z^{\infty} dz' \rho_A(b,z')\right]$$

(Glauber model)

Trying just to increase E_p one has to go to higher Q^2, and the cross section drops down dramatically.

* Diffractive virtual photoproduction of vector mesons off nuclei

The value of Q^2 can be large, but it does not correlate with the hadron energy.



Not a vector meson, but a quark-antiquark fluctuation of the photon propagates through the nucleus.

The transverse size of the dipole is controled by the photon virtuality

$$\sigma(r_T) \propto r_T^2 \sim \frac{1}{Q^2}$$

The dipole lifetime, called coherence time (length) is

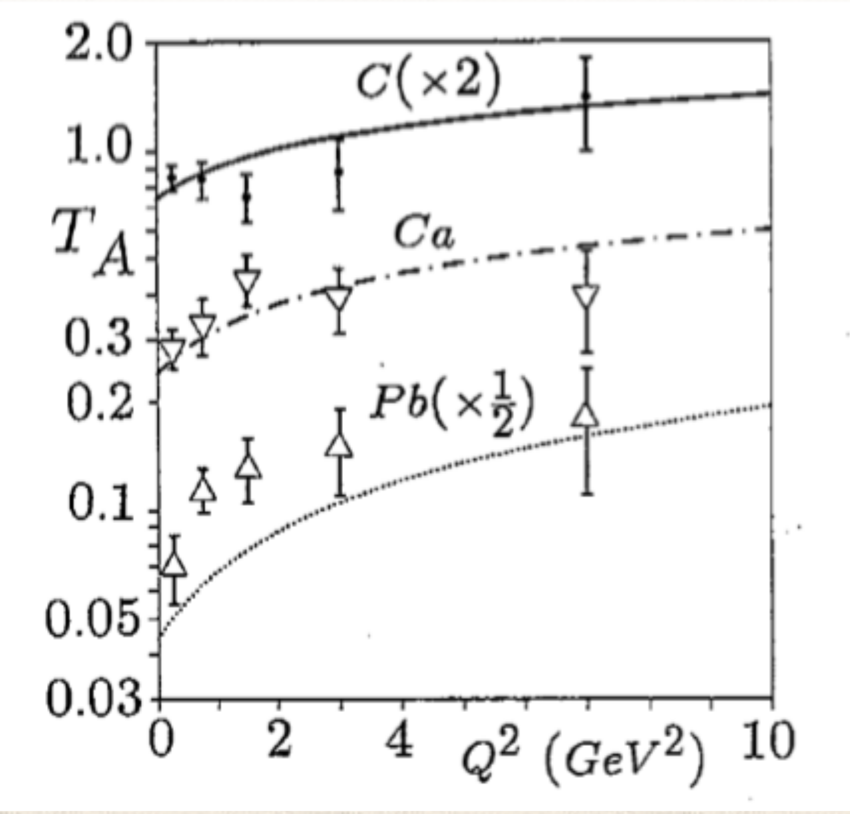
$$l_c = \frac{2E_{\gamma^*}}{Q^2}$$

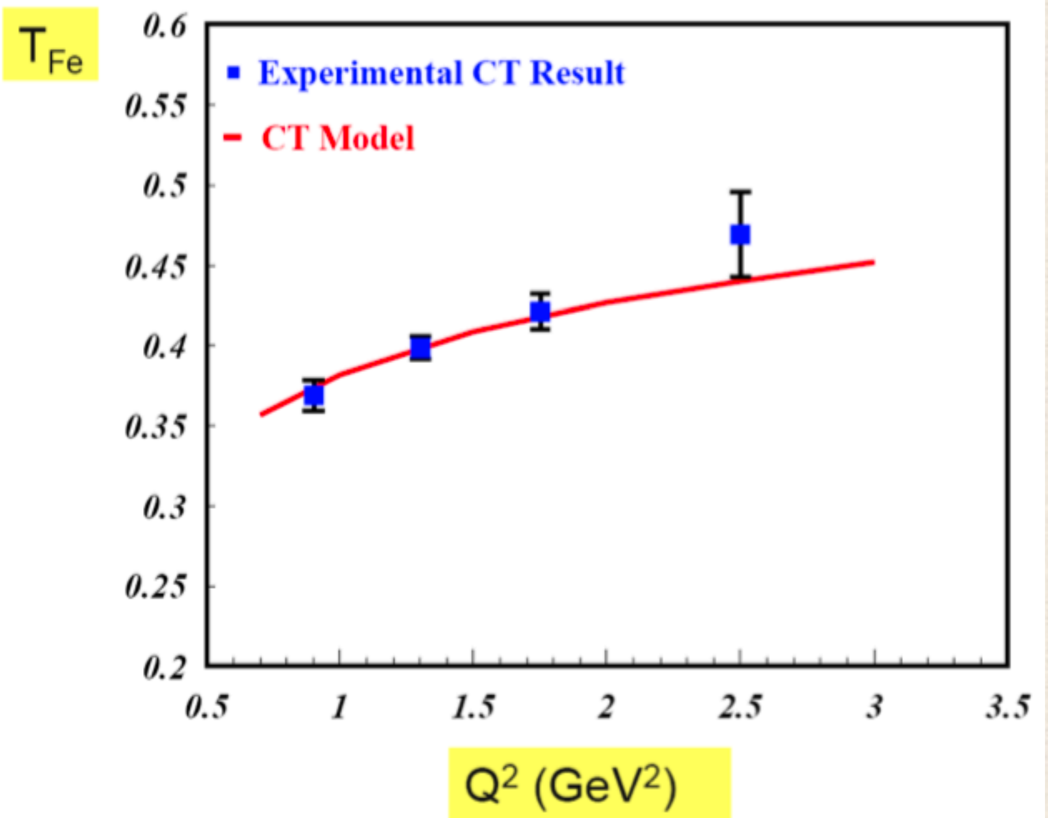
In this process the photon energy and virtuality vary independently.

CT should be at work: the A-dependence is expected to vary from $A^{1/3}$ at low Q^2 up to A at high Q^2

Successful experiments searching for CT in $\gamma^* \mathbf{A} \to \rho \mathbf{A}^*$

Nuclear tranparency $T_A = \dfrac{\sigma_A}{A\sigma_N}$: data versus theoretical predictions



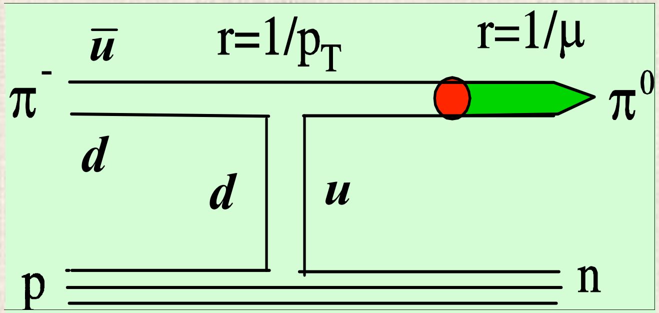


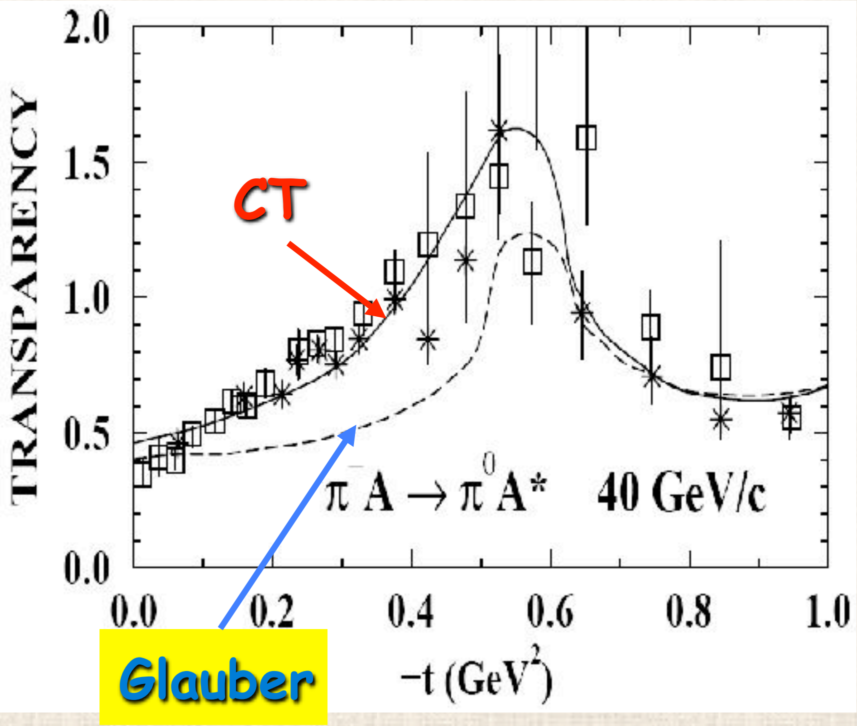
E665 experiment at Fermilab

Preliminary results from CLAS experiment at JLAB

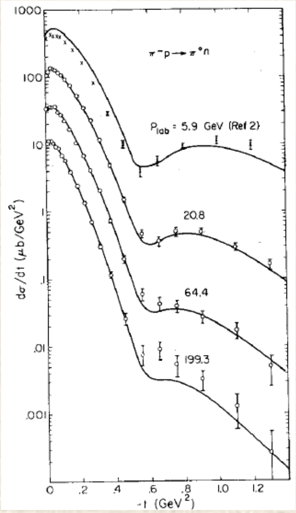
* Search for CT in quasi-free charge exchange pion scattering

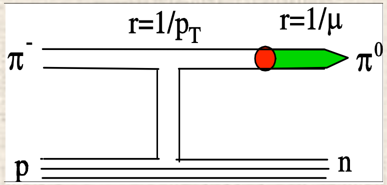
$$\pi^- \mathbf{A} \to \pi^0 \mathbf{A}^*$$

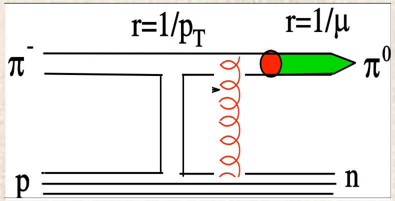




Pion formfactor suppresses large-size configurations in the pion wave function





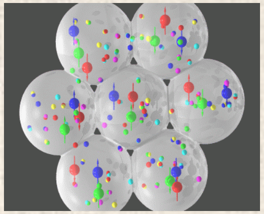


Experiment PROZA at Serpukhov

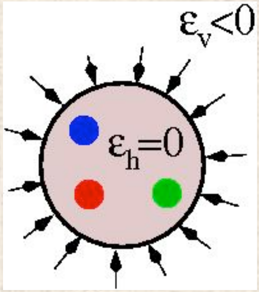
Interference of these graphs creates the minimum at $t=-0.6\,GeV^2$.

On a nucleus the minimum is shifted to larger $|t|$.

Why don't we see quarks? Bags, strings…



Gluonic condensate in vacuum pushes the energy density below the perturbative level, $\varepsilon_v < 0$.



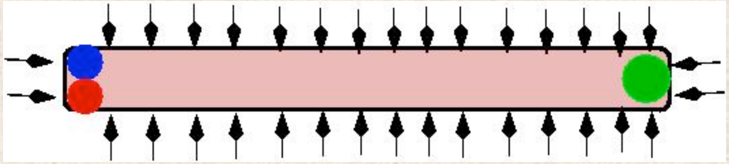
If the energy density inside the hadron is higher (e.g. perturbative), vacuum tries to squeeze the hadron.

"Quarks, Neutrinos… All those damn particles you can't see. That's what drove me to drink. But now I can see them!"

However, the chromo-electromagnetic energy, $\frac{1}{2}(E^2+H^2)$, rises leading to an equilibrium.

Is this confinement? What happens if a quark is knocked out with a high momentum?

Due to the same properties of the QCD vacuum the chromo electric flux is squeezed into a tube of a constant cross section.



Usually the transverse size is not important, so the tube may be treated as a one-dimensional **string**.

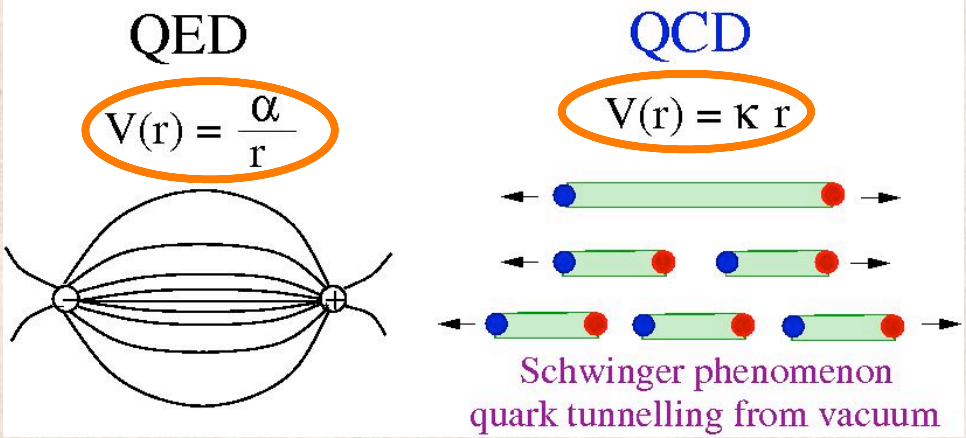
$$\pi r^2 = \frac{g^2}{8\kappa}$$

$$\kappa = \frac{1}{2\pi\alpha'_R} \approx 1\frac{GeV}{fm}$$

The string tension κ is the energy density per unit of length. It can be calculated on the lattice, but is easily related to the universal slope of Regge trajectories α'_R=0.9 GeV^{-2} (see below).

This energy is sufficient for creation of a couple of constituent quarks via tunneling from vacuum

One can hardly stretch a string longer than 1fm, it breaks up to pieces.



The Schwinger phenomenon and existence of light quarks are the main reason why we don't see free quarks and gluons (color screening).

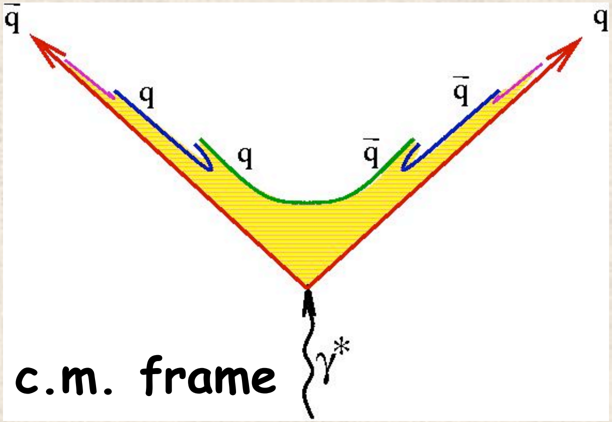
Hadronization

The probability of string break up over time T

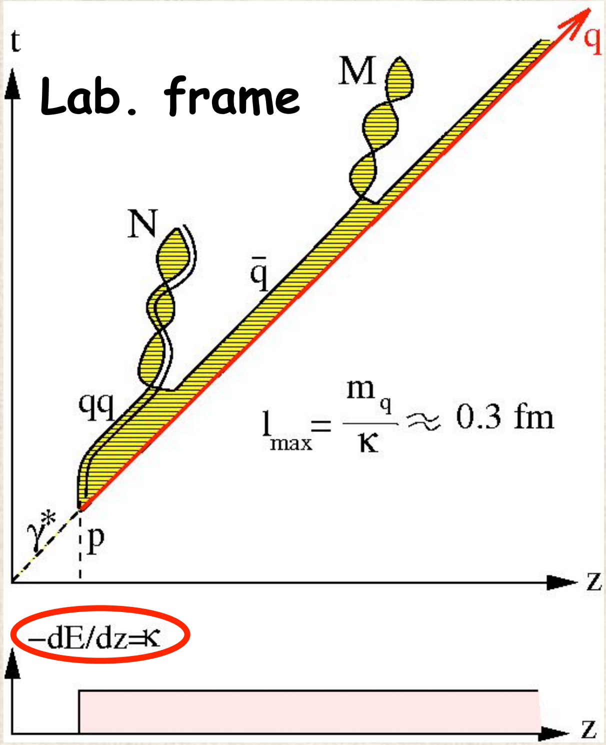
$$P(t) = 1 - \exp\left[w \int_0^T dt L(t) \right]$$

where the probability density to create a quark pair over unit of time and unit of length

$$w = \left(\frac{\kappa\, r}{\pi}\right)^2 \exp\left(-\frac{2\pi\, m_q^2}{\kappa}\right) \approx 2\, fm^{-2}$$



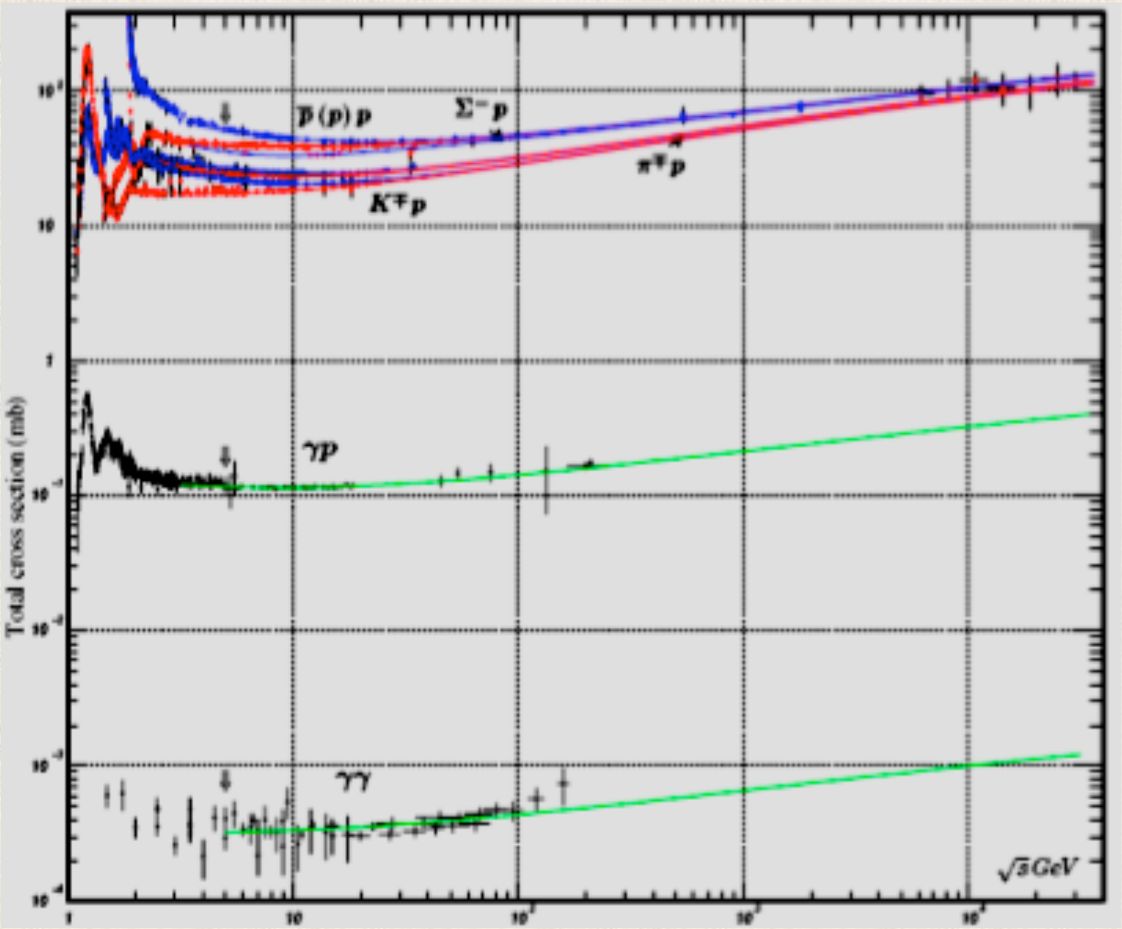
The string length L(t) is getting shorter after each break, which delays the next pair production. Therefore, hadron momenta rise in geometric progression, i.e. build a plateau in rapidity.



Nevertheless, the rate of **energy loss** is constant, like in pQCD

$$-\left.\frac{dE}{dz}\right|_{pQCD} = \frac{2\alpha_s}{3\pi} Q^2$$

Hints for the QCD dynamics from the basic features of hadronic collisions



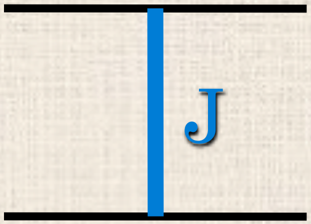
Total cross sections hardly depend on energy

$$\sigma_{tot} \approx Const$$

¿Why?

What does it tell us about the underlying dynamics?

Energy dependence of the scattering amplitude correlates with the spin of the particle exchanged in the cross channel.



$$A = \Pi_{\mu_1 \ldots \mu_J \nu_1 \ldots \nu_J}\, p_{\mu_1} \cdots p_{\mu_J}\, p'_{\nu_1} \cdots p'_{\nu_J} \propto s^J$$

$$\sigma_{tot}(s) = \frac{1}{s}\,\mathrm{Im}\,A(s) \propto s^{J-1}$$

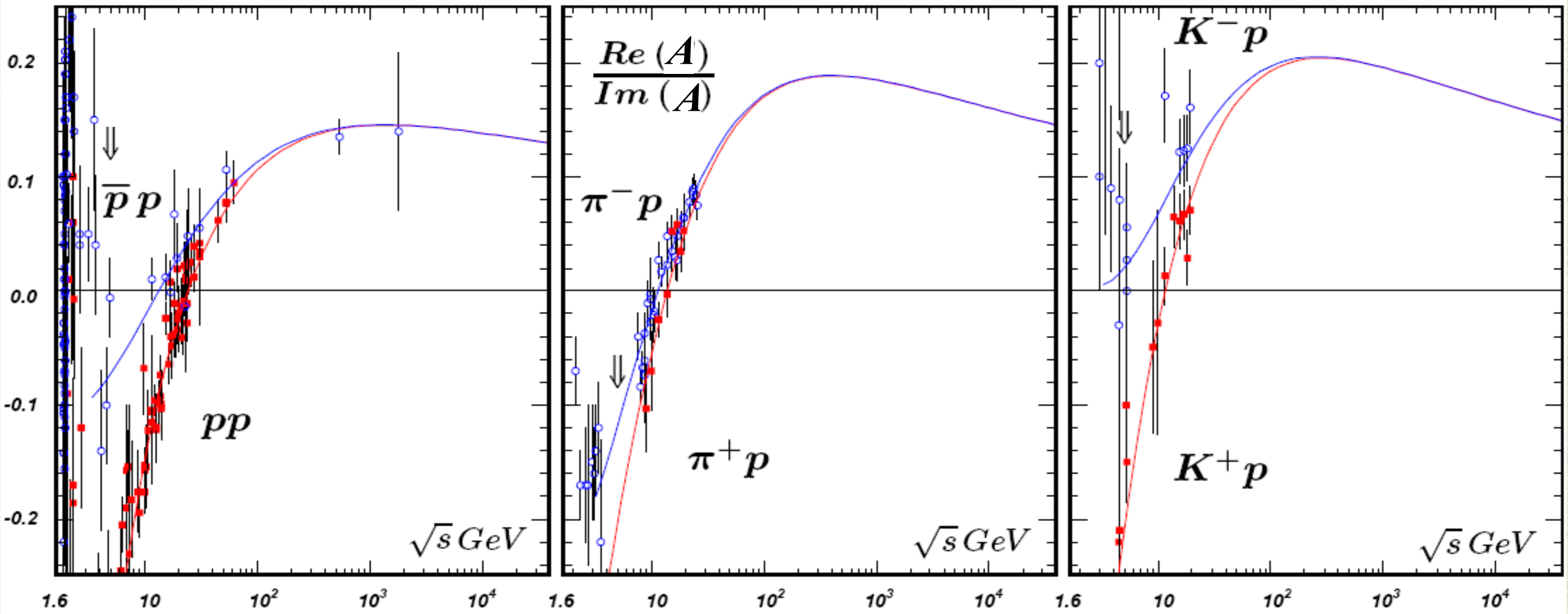
This observation is the key input for the Regge theory

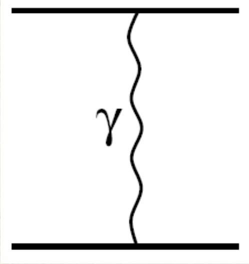
If gluons were spinless or had spin 2, the cross section would drop like 1/s, or rise as s. Neither of these complies with data. Therefore, the spin of the gluon is J=1.

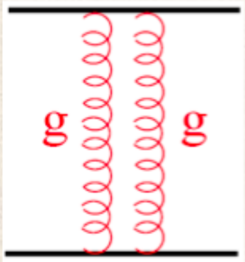
Still, the cross sections slowly rise with energy, and QCD helps to understand why. Experience with DIS and HERA data is also very informative.

Evidence for a non-Abelian dynamics

The forward elastic amplitude is nearly imaginary, ReA/ImA~1







ImA=0 **ReA=0**

The dominant Born graphs should be two-gluon exchange in order to comply with ReA/ImA~1.

This is possible only if gluons are colored

High-energy hadronic collisions: QCD vs Regge

The theory of Regge poles is a quite dormant topic. It does not seem to be taught very much anymore. In addition there is often found an attitude that **the subject is obsolete**, because it is identified so strongly with the prequark, pre-parton era of the S-matrix, dispersion-relations approach to strong interactions. **This point of view is just plain wrong.**

The Chew, Frautschi, Regge, et al. description of high energy behavior in terms of singularities in the complex angular momentum plane is completely general.

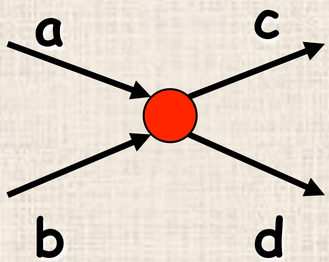
And the basic technique of Watson-Sommerfeld transform should be a **standard part of the training** in theoretical particle physics.

James Bjorken

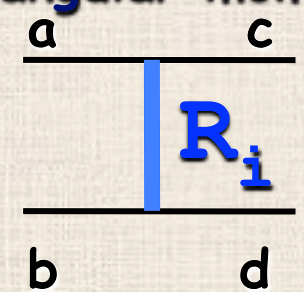
Regge theory for pedestrians

A brief survey of main results

The energy dependence of the amplitude is governed by poles (or cuts) in the complex angular momentum plane.



$$= \sum_i$$



$$A_i(t) = g_i^{ac}(t)\, g_i^{bd}(t)\, \xi_i(t) \left(\frac{s}{s_0}\right)^{\alpha_i(t)}$$

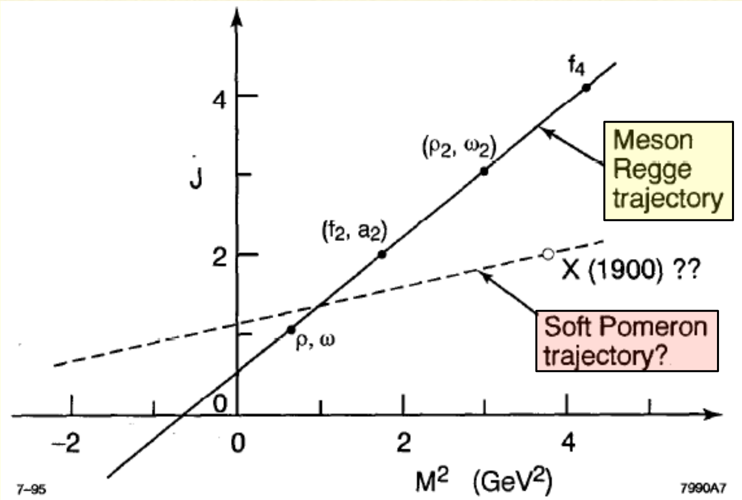
Energy dependence is given by the last factor, while the vertex functions $g_i(t)$, and the signature factor $\xi_i(t)$ depend only on t.

$$\xi_i(t) = \begin{cases} i + \mathrm{ctg}\left[\frac{\pi}{2}\alpha_i(t)\right] & s=-1 \\ -i + \mathrm{tg}\left[\frac{\pi}{2}\alpha_i(t)\right] & s=+1 \end{cases}$$

Each trajectory at t>0 passes through the states with either odd, or even spins. Signature, $s=(-1)^J$.

The energy dependence $\left(s/s_0\right)^{\alpha_i(t)}$ is controlled by the Regge trajectory $\alpha(t)$ which is nearly straight

$$\alpha(t) = \alpha(0) + \alpha' t$$



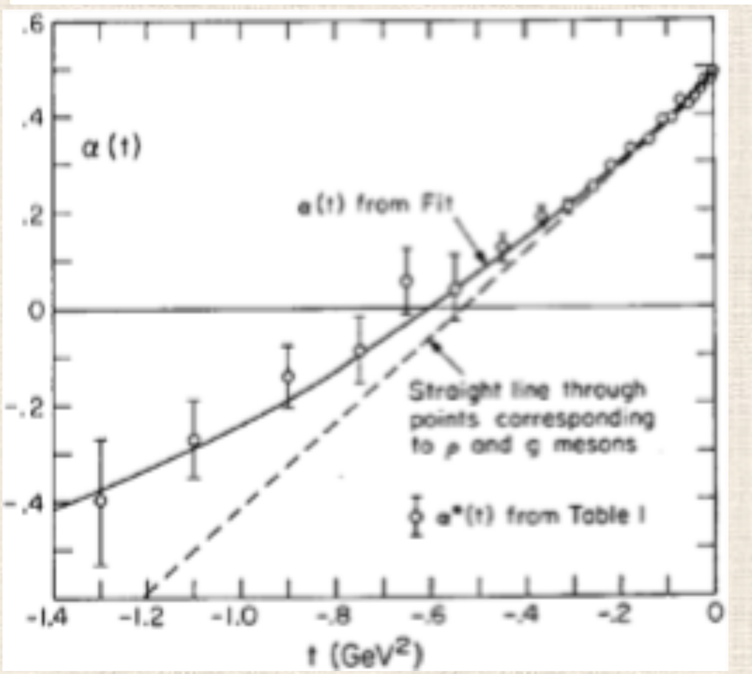
At high energies dominate Reggeons with highest intercept a(0).

The leading Reggeons contributing to pp elastic amplitude:

Pomeron: $\alpha_P(0) \approx 1.1;$ $\alpha'_P \approx 0.25 GeV^{-2}$

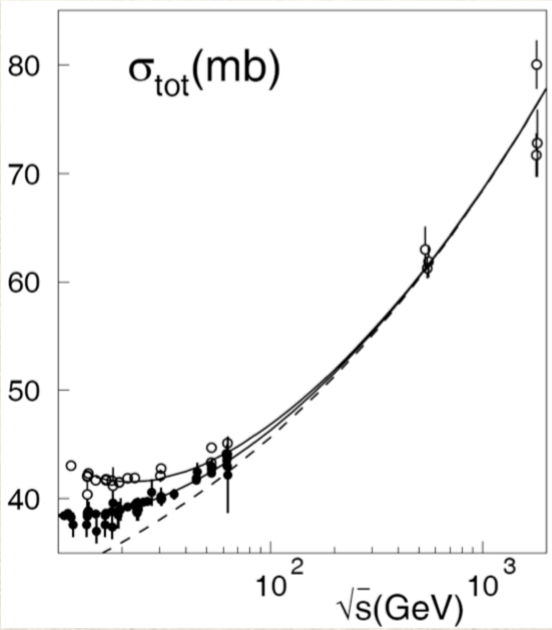
$$\alpha_\omega(0) \approx \alpha_f(0) \approx \alpha_\rho(0) \approx \alpha_{a_2}(0) \approx 0.5$$

$$\alpha'_R \approx 0.9 GeV^{-2}$$



The miracle of Regge theory:
a linear Regge trajectory bridges the low-energy physics of resonances ($t=M^2>0$) with high-energy scattering ($t<0$)

As far as the Pomeron intercept is above one, the total hadronic cross sections should rise with energy as $s^{a(0)-1}$ (postdiction).
Fits data well, although the Pomeron is not a Regge pole, either in QCD, or in data.



Elastic slope.

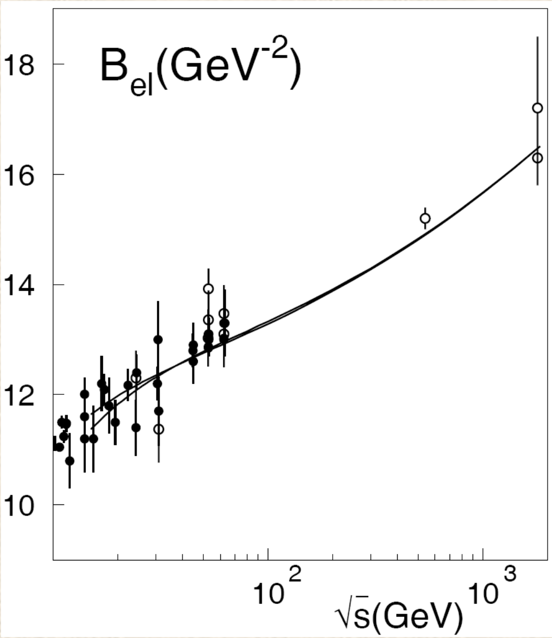
Parametrizing the residue function as

$$g_P^{pp}(t)\,g_P^{pp}(t) = g_P^{pp}(0)\,g_P^{pp}(0)\,e^{B_0 t/2},$$

we arrive at an energy dependent t-slope of elastic cross section,

$$A_i(t) = g_i^{ac}(t)\,g_i^{bd}(t)\,\xi_i(t)\left(\frac{s}{s_0}\right)^{\alpha_i(t)} \propto e^{B(s)t/2}$$

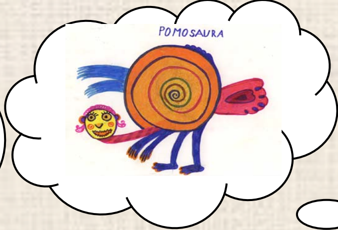
$$B_{el}(s) = B_0 + 2\alpha'_P \ln(s/s_0)$$

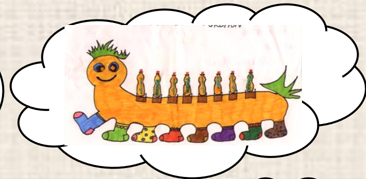


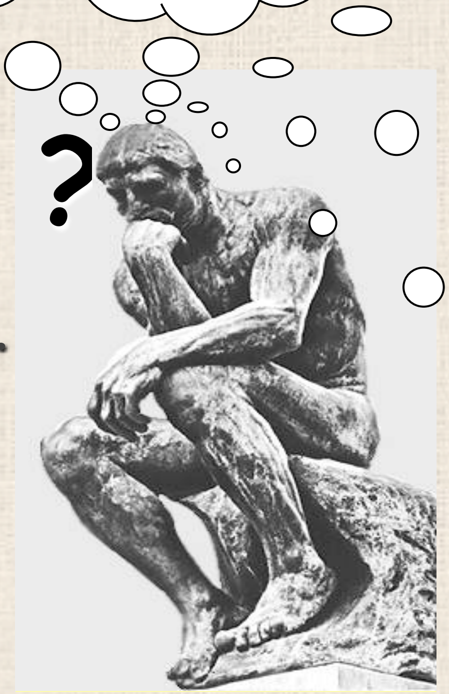
Shrinkage of the diffraction cone (prediction).

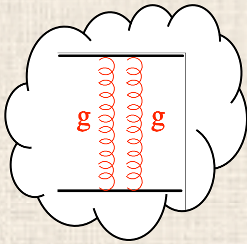
The Pomeron:
¿How does it look like?

At Harvard,
we think of the
Pomeron as a
French wine...
(Sid Coleman)









¿Regge pole? - Probably not.
The intercept is higher for J/Ψ photoproduction and varies with Q^2 in DIS.

¿DGLAP Pomeron?
- Why ordering of transverse momenta of radiated gluons? There are indications on gluon saturation which breaks up the DGLAP.
Besides, perturbative QCD is not legitimate for the soft Pomeron.

¿BFKL Pomeron?
-Has no QCD evolution. Next-to-leading (log) order calculations revealed ~100% large corrections. The intercept is far too high for the soft Pomeron.

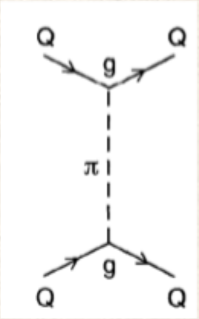
¿More ideas?..

Building the Pomeron

Any material is good: gluons, pions, sigmas, instantons...

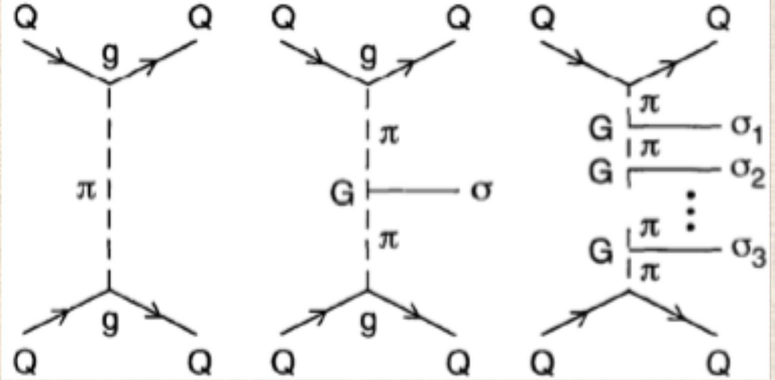
Pion is the lightest hadron, it can reach furthest distances. The π-N coupling is large. **A good candidate!** The first step, however, is quite **discauraging:** the cross section related to spinless **pion exchange steeply falls with energy**.

$$\frac{d\sigma}{dt} \propto \frac{1}{s^2}\left(\frac{g^2}{4\pi}\right)^2 \frac{t^2}{(t-m_\pi^2)^2}$$



Nevertheless, "**Reggeization**" helps, integration over the phase space of the radiated sigmas provides powes of ln(s)

$$\int_0^{\ln s} dy_1 \int_{y_1}^{\ln s} dy_2 \times\times\times \int_{y_{n-1}}^{\ln s} dy_n = \frac{(\ln s)^n}{n!}$$

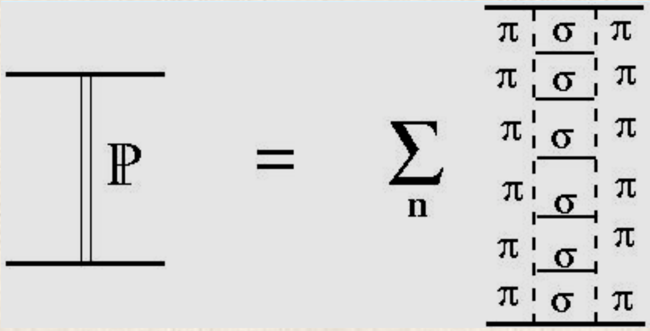


Summing up power of logs we get the cross section

$$\sigma = \frac{g^4}{s^2} \sum_{n=0}^{\infty} \frac{1}{n!} \left[\int \frac{d^2 p_T}{(2\pi)^3} G^2(p_T) \ln s \right]^n \propto s^{2\alpha_P(0)-2}$$

Corresponding Pomeron intercept reads,

$$\alpha_P(0) = \frac{1}{2} \int \frac{d^2 p_T}{(2\pi)^3} G^2(p_T)$$

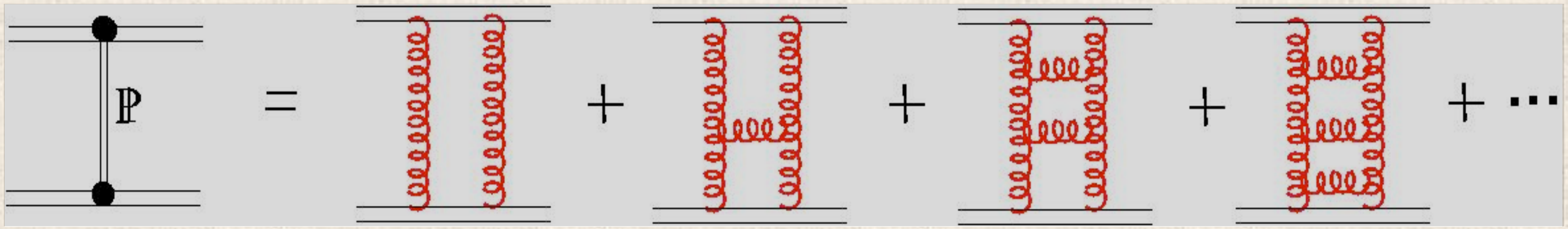


Although pion is spinless, the "pionic" Pomeron may have spin 1.

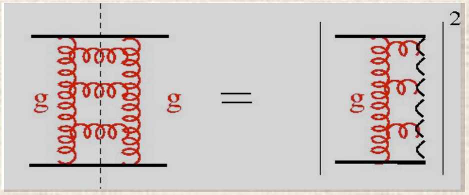
So far it is not clear how realistic is this model. It has enough freedom to reproduce the observed Pomeron intercept.
It also explains well data on different inclusive reactions.
However closeness of the Pomeron intercept to unity looks accidental.

Perturbative Pomeron:

Gluons seem to be the most suitable building material: already the Born graph provides $\alpha_P(0)=1$. The higher order corrections are expected to pull the intercept above one.



The ladder is a shadow of gluon bremsstrahlung according to the unitarity relation



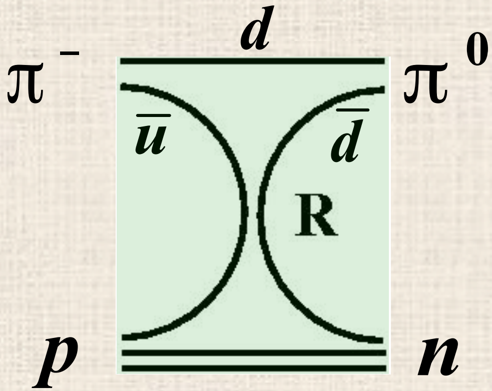
The leading-log approximation (LLA) corresponds to keeping only those terms where each coupling a_s has a big factor **ln(s)** (similar to the pionic Pomeron). For fixed coupling the **BFKL** result is not a Regge pole, but a cut with the intercept

$$\alpha_P(0) - 1 = \frac{12\alpha_s}{\pi}\ln 2$$

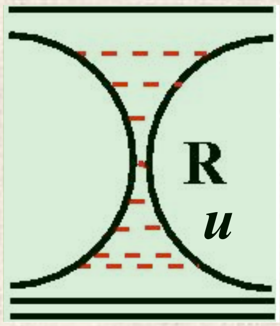
Unfortunately, the next-to-leading-log corrections (extra powers of α_s) to the intercept are of the same order as $a_p - 1$.

Reggeons

Reggeons correspond to exchange of valence quarks.



Born



Reggeized

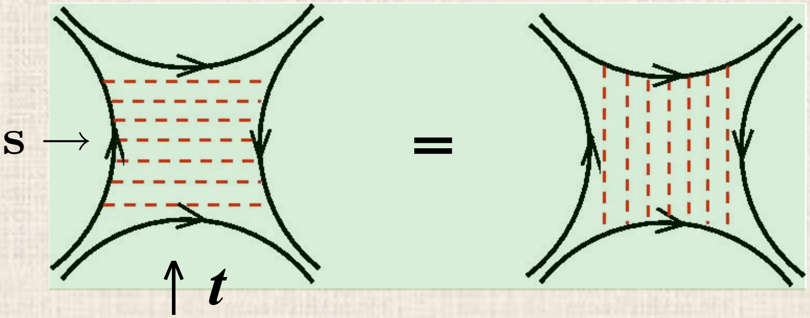
$$\alpha_R(0) = 0.5$$

$$\alpha'_R = \frac{1}{2\pi\kappa} = 0.9 GeV^{-2}$$

$$\kappa = 1 GeV / fm$$

string tension

Duality and exchange degeneracy

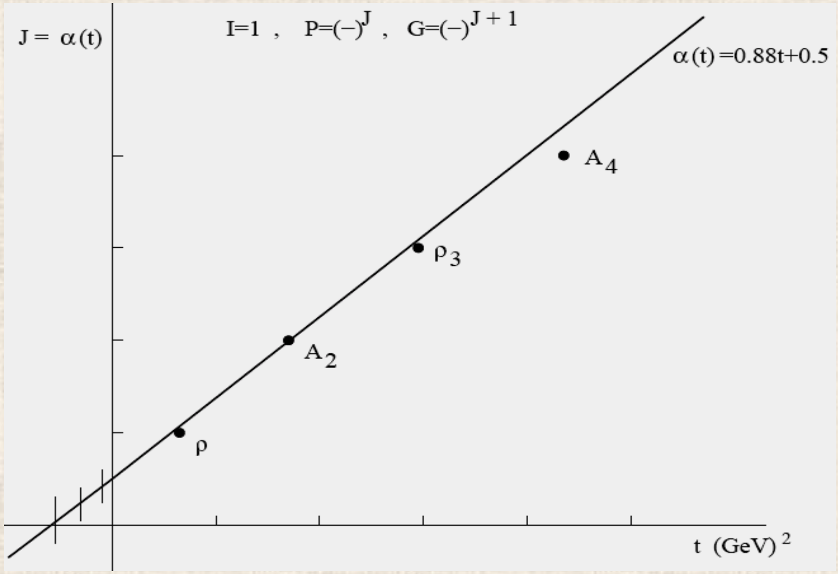


t-channel resonances

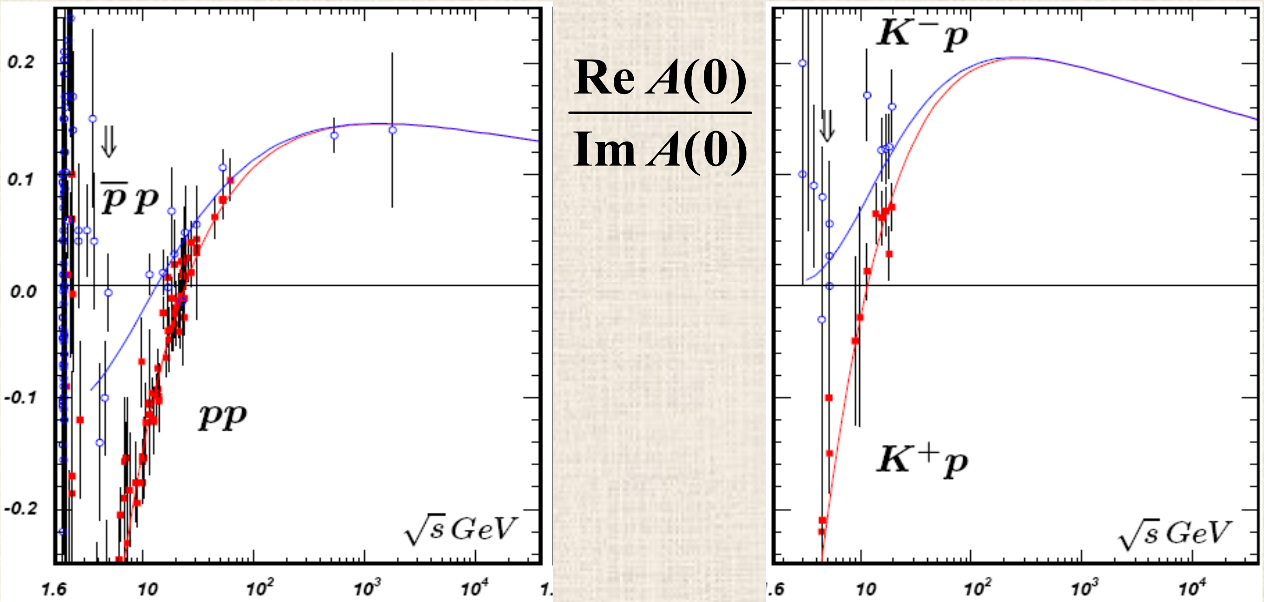
s-channel resonances

No s-channel resonances is possible in pp and K^+p elastic amplitudes. However t-channel Reggeons are present. To comply with duality they must cancel each other in the imaginary part of the amplitude.

Pairs of leading Reggeons are exchange degenerate, f with ω, and a_2 with ρ, i.e. their Regge trajectories and residue functions must be identical, only the signature factors (phases) are different.



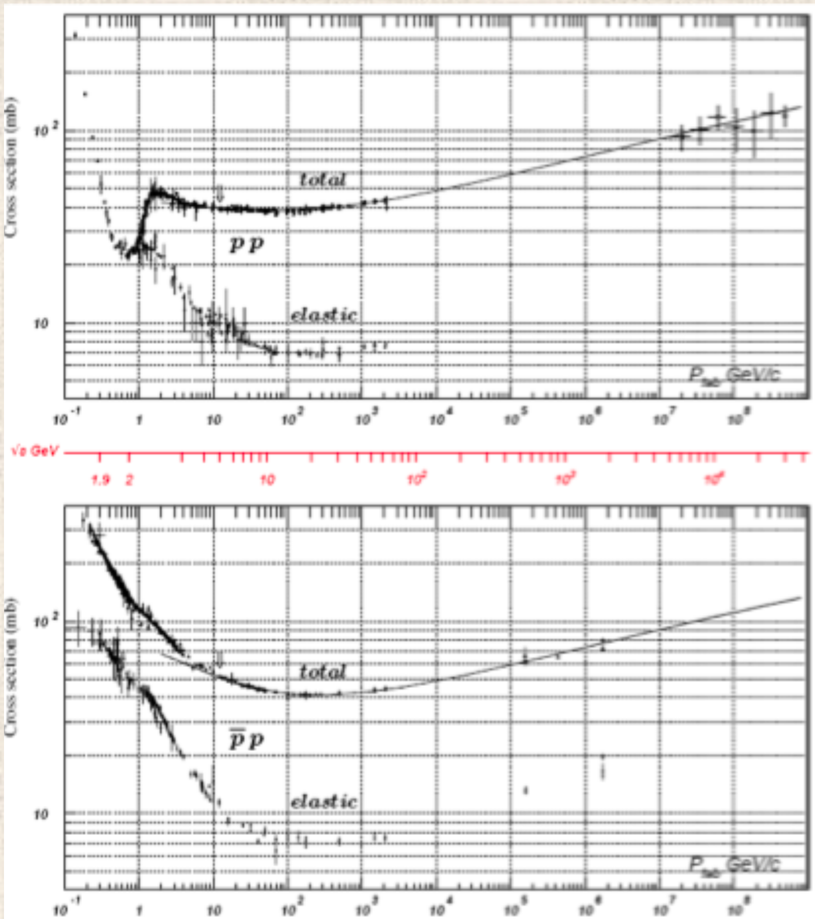
The sums, $f+\omega$ and $a_2+\rho$ must be real for pp and K^+p, but imaginary for $\bar{p}p$ and K^-p.

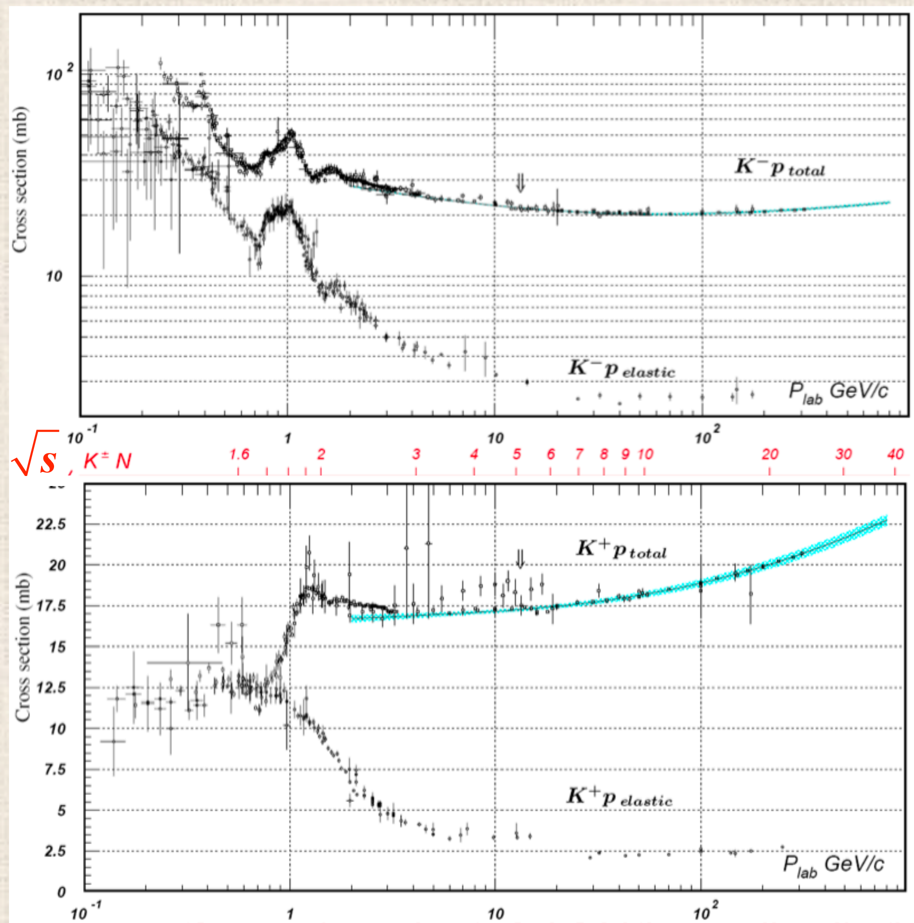


For the same reason spin effects are much stronger in $\bar{p}p$ and K^+p, than in pp and K^-p

As far as the sums, $f+\omega$ and $a_2+\rho$ are imaginary for $\bar{p}p$ and K^-p, the total cross section decreases with energy, until the Reggeon part becomes very small.

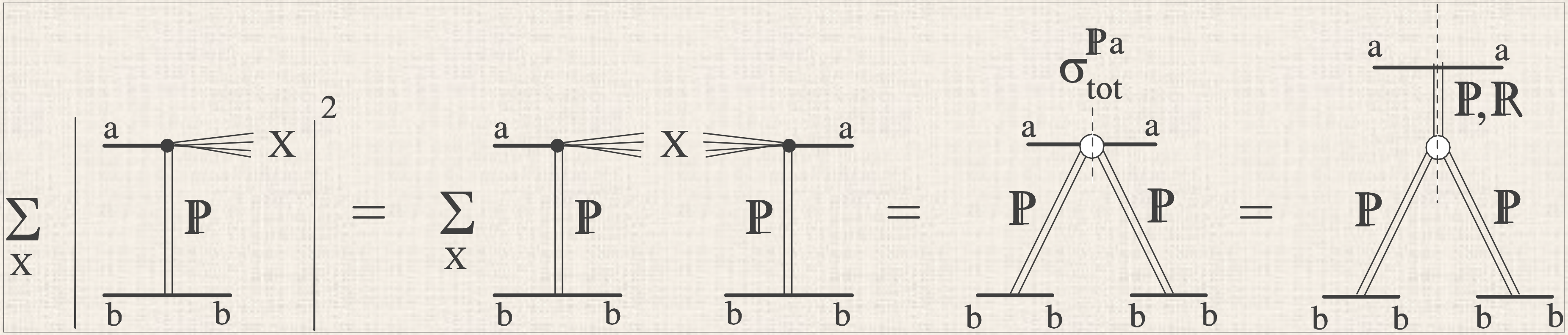
For pp and K^+p the Reggeon part is real and doesn't contribute to the total Xsection. The latter is expected to rise already at low energies





Triple Regge phenomenology

Diffractive excitation of a hadron: a+b->X+b

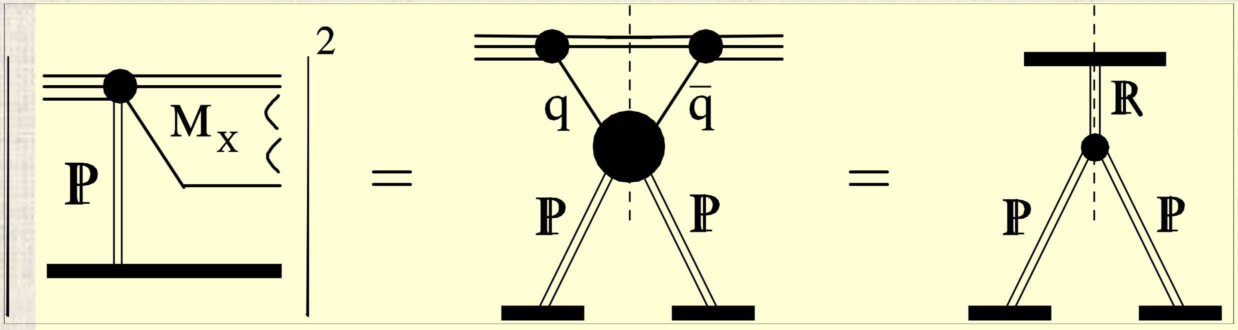


Kinematics: $s_o \ll M_X^2 \ll s;$ $\qquad x_F \equiv \dfrac{2p_b^*}{\sqrt{s}} = 1 - \dfrac{M_X^2}{s}$

$$\frac{d\sigma_{sd}^{ab\to Xb}}{dx_F\,dt} = \sum_{i=\mathbb{P},\mathbb{R}} G_{\mathbb{P}\mathbb{P}i}(t)(1-x_F)^{\alpha_i(0)-2\alpha_{\mathbb{P}}(t)}\left(\frac{s}{s_0}\right)^{\alpha_i(0)-1}$$

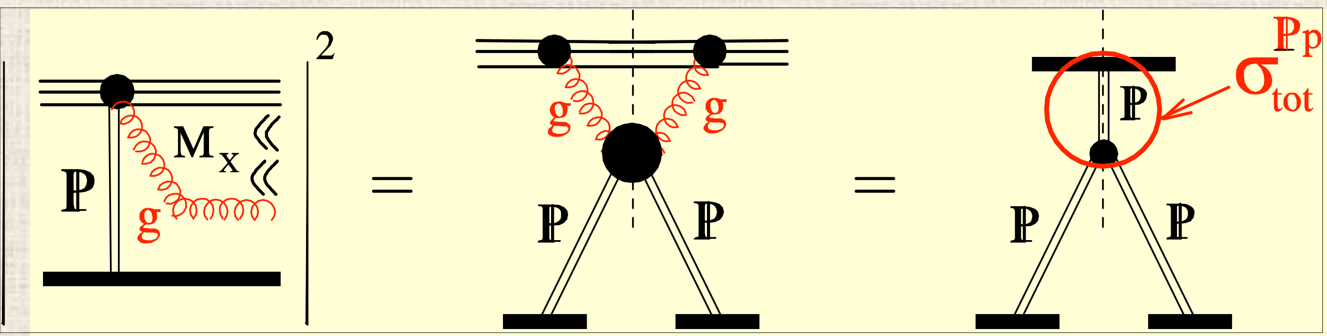
The triple-Regge couplings G_{PPP}, G_{PPR} are fitted to data.

The graph PPR corresponds to excitation of the valence quark skeleton



$$\left.\frac{d\sigma_{sd}}{dM_X^2}\right|_{PPR} \propto \frac{1}{M_X^3}$$

The triple-Pomeron graph corresponds to diffractive gluon radiation

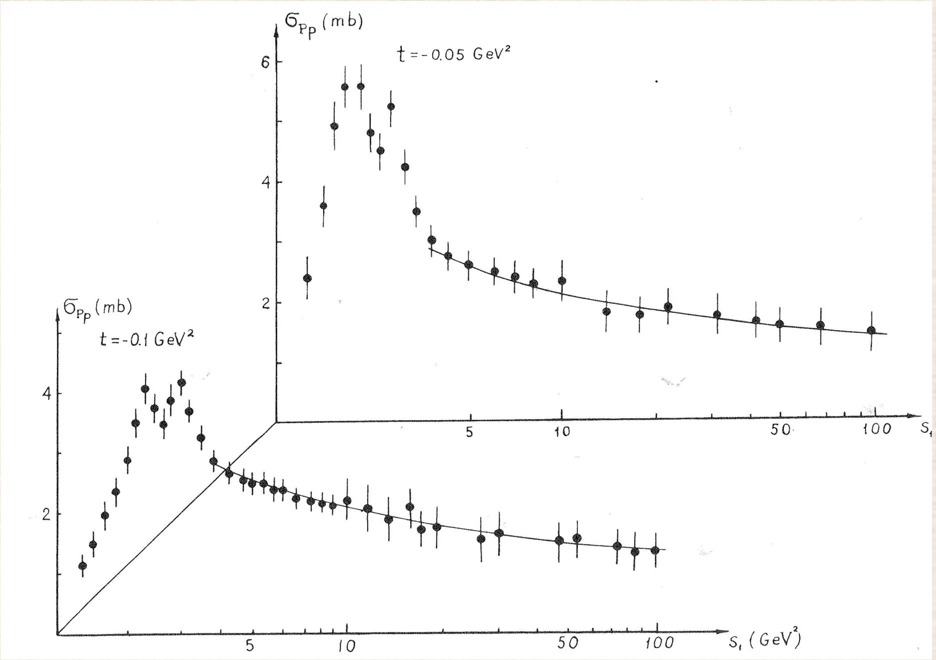


$$\left.\frac{d\sigma_{sd}}{dM_X^2}\right|_{PPP} \propto \frac{1}{M_X^2}$$

One can discriminate the two mechanisms via their M_X-dependences and find the Pomeron-proton cross section from data.

Since the Pomeron is a gluonic object it should interact stronger than a quark-antiquark meson, so one could expect

$$\sigma^{tot}_{Pp} \approx \frac{9}{4}\sigma^{tot}_{\pi p} \approx 50mb$$

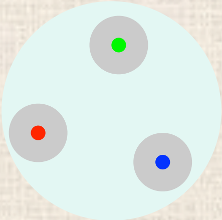


However, diffractive data suggest a much smaller value

$$\sigma^{tot}_{Pp} = \frac{M_x^2/s}{\left(g^P_{pp}(t)\right)^2} M_x^2 \frac{d^2\sigma}{dM_x^2 dt} \approx 2mb$$

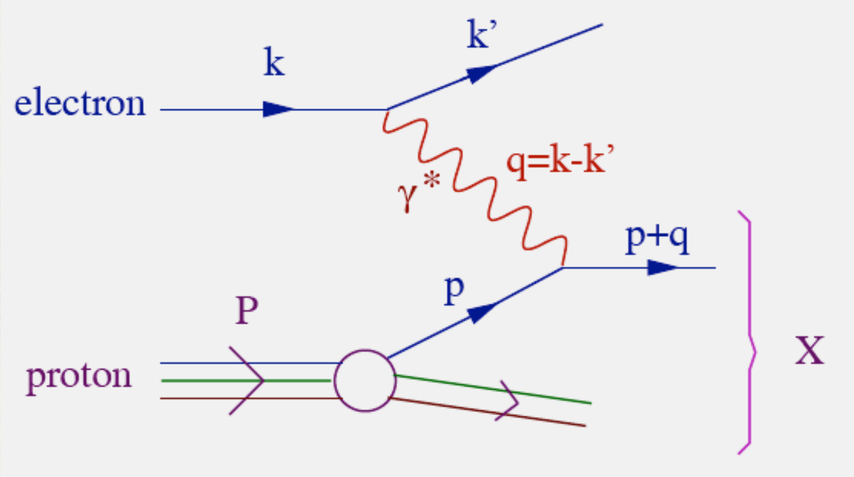
The only solution is to assume that the Pomeron is a small size object, and its cross section is small due to Color Transparency.

This means that gluons in the proton are located within small spots of radius r ~ 0.3 fm.



There are many more evidences for that.

DIS at small x



Inclusive Deep-Inelastic Scattering of electrons (muons) on protons.

Two independent Lorentz invariants :

$$Q^2 \equiv -q^\mu q_\mu = -(k-k')^2 \qquad (Q^2 \geq 0)$$

$$x \equiv \frac{Q^2}{2P\cdot q} = \frac{Q^2}{Q^2+W^2-M^2} \qquad (\text{Bjorken's } x)$$

DIS is characterized by two structure functions F_1 and F_2.

$$\frac{d^2\sigma^{\rm em}}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[y^2 F_1^{\rm em} + \left(\frac{1-y}{x} - \frac{xy^2M^2}{Q^2}\right) F_2^{\rm em}\right]$$

energy loss:	$\nu = (P\cdot q)/M = E - E'$
rel. energy loss:	$y = (P\cdot q)/(P\cdot k) = 1 - E'/E$
recoil mass M_X^2:	$W^2 = (P+q)^2 = M^2 + \frac{1-x}{x}Q^2$

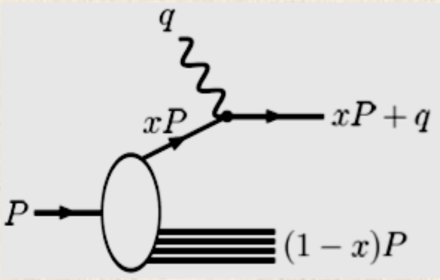
One can split the DIS cross section into the flux of virtual photons,

$$\Gamma = \frac{\alpha}{2\pi^2}\frac{E_e'}{E_e}\frac{1}{Q^2}\frac{1}{1-\epsilon}\frac{W^2-M^2}{2M}$$

and the virtual photo-absorption cross section summed over photon polarization ε,

$$\boxed{\sigma_{\gamma^* p}(x,Q^2) = \sigma_T + \sigma_L = \frac{4\pi^2\alpha_{\rm em}}{Q^2}F_2(x,Q^2)}$$

Thus, F_2 maybe viewed simply as the total X-section. On the other hand, it also can be interpreted as a quark distribution function in the proton. Indeed, assuming partons massless (or m~Λ) and point-like (i.e. unable to get excited) in the Breit frame ($E\gamma$ =0),



$$\begin{aligned}(p')^2 &= P^2 + 2\xi\, P\cdot q + q^2\\ &\simeq 2\,\xi\, P\cdot q - Q^2\\ &= \frac{\xi - x}{x}Q^2 \simeq 0\end{aligned}$$

The scaling variable x turns out to be the fractional momentum ξ of the parton in the infinite momentum frame (p>>M).

Thus, F_2 is also the parton distribution function (PDF)

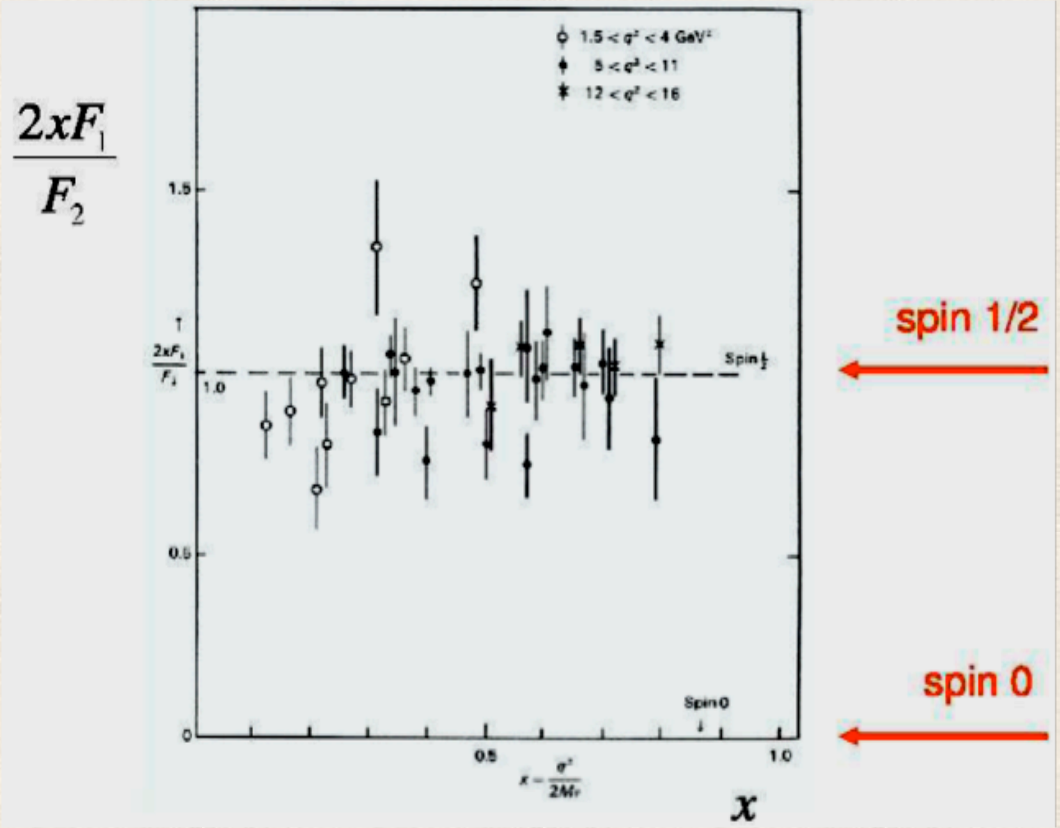
$$F_2(x) = \sum_{q,\bar{q}} e_q^2\, x q(x)$$

The stricture functions depend on the photon polarization,

$$F_2 = F_T + F_L$$

$$F_1 = \frac{1}{2x} F_T$$

A spinless parton cannot absorb a transversly polarized photon (helicity ±1), while a fermion can.



For partons with spin 0

$$F_T = 2xF_1 = 0$$

For partons with spin 1/2

$$F_2 = 2xF_1$$

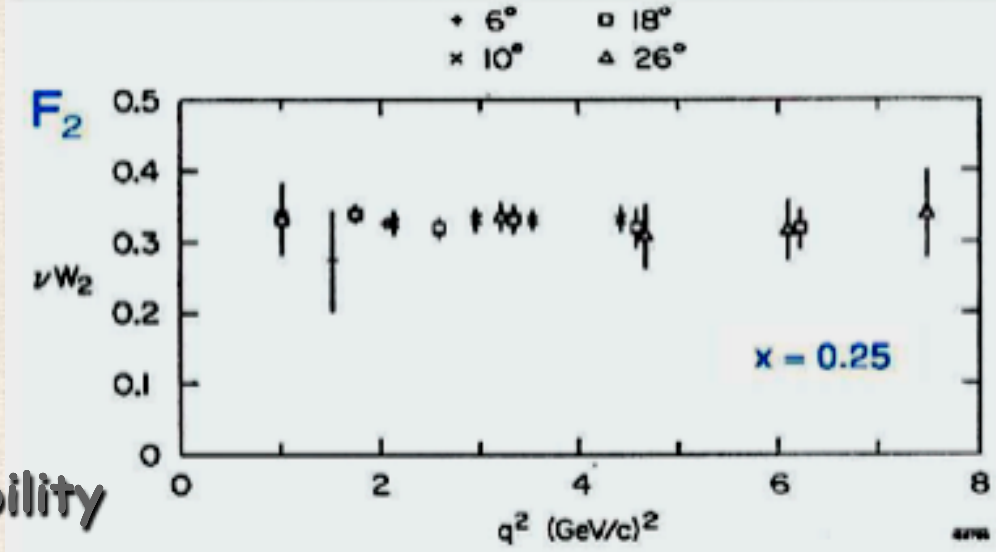
Callan-Gross relation

• "Deep inelastic" or Bjorken limit

Both Q^2 and $P \cdot q \gg M^2$ with $x =$ fixed

Bjorken scaling:
$F_2(x,Q)$ depends only on x

Why the proton formfactor $F(Q)$ steeply falls with Q, while the structure function does not?



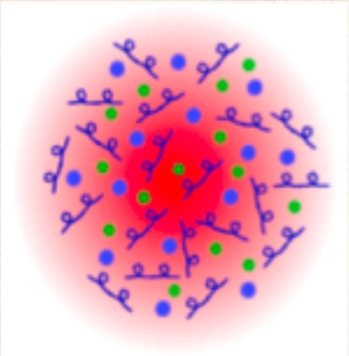
J.T.Friedman + H.W.Kendall, Ann. Rev. Nucl. Sci. **22** (1972) 203

Answer: formfactor is the probability for a hadron to survive a kick of strength Q. The stronger the kick is, the less is the chance to survive. However, in the case of inclusive DIS all final states are allowed, the total probability saturates and is independent of Q.

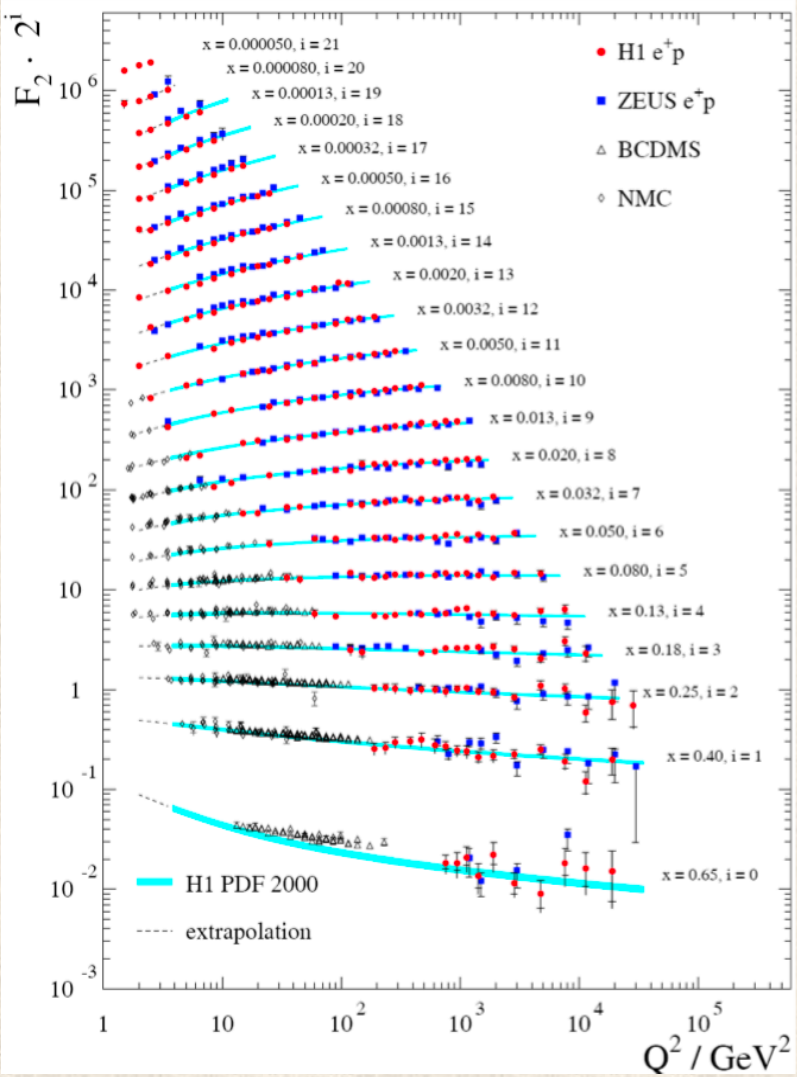
Similar situation is in hadronic collisions: the t-slope of single diffraction is half of that for elastic pp, because of disappearance of one proton vertex.

All that could be correct, if the number of parton were constant. However, they are not classical particles, but quantum fluctuations which number depends on reference frame and resolution.

- A photon of virtuality Q can resolve partons with transverse momenta $k_T < Q$,



but is blind to harder fluctuations. Increasing Q, one can see more partons in the proton.



Correspondingly, the parton distribution slowly changes with Q: it is getting shifted to small x, due to momentum conservation, i.e. it is expected to rise with Q at small x, but fall at large x.

This evolution with the scale is controlled by DGLAP evolution equations (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi)

$$\mu\frac{\partial}{\partial\mu}\begin{pmatrix} q_i(x,\mu) \\ g(x,\mu) \end{pmatrix} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\begin{pmatrix} P^{q_iq_j}(x/\xi) & P^{q_ig}(x/\xi) \\ P^{gq_j}(x/\xi) & P^{gg}(x/\xi) \end{pmatrix}\begin{pmatrix} q_j(\xi,\mu) \\ g(\xi,\mu) \end{pmatrix}$$

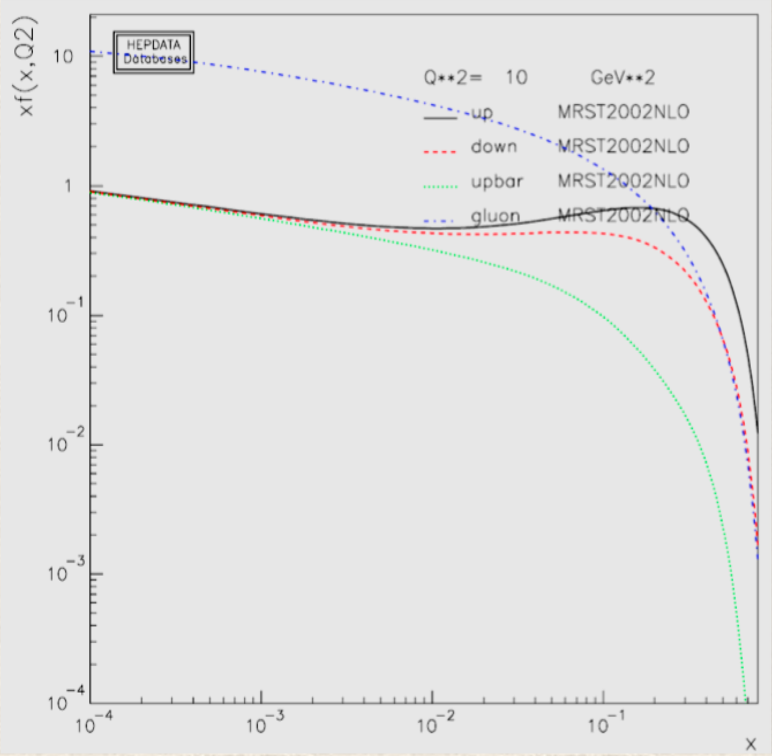
The splitting functions P are calculated perturbatively

pQCD is unable to calculate the PDFs, since that involves essential nonperturbative effects. However, one can calculate how PDFs vary when the hard scale changes.

The typical strategy for extracting PDFs from DIS data:

1. Introduce an ad hoc PDF at some scale
2. Evolve them with DGLAP to other scales
3. Compare with data and adjust the starting PDFs

Having good data with high statistics one can single out PDFs for different parton species.

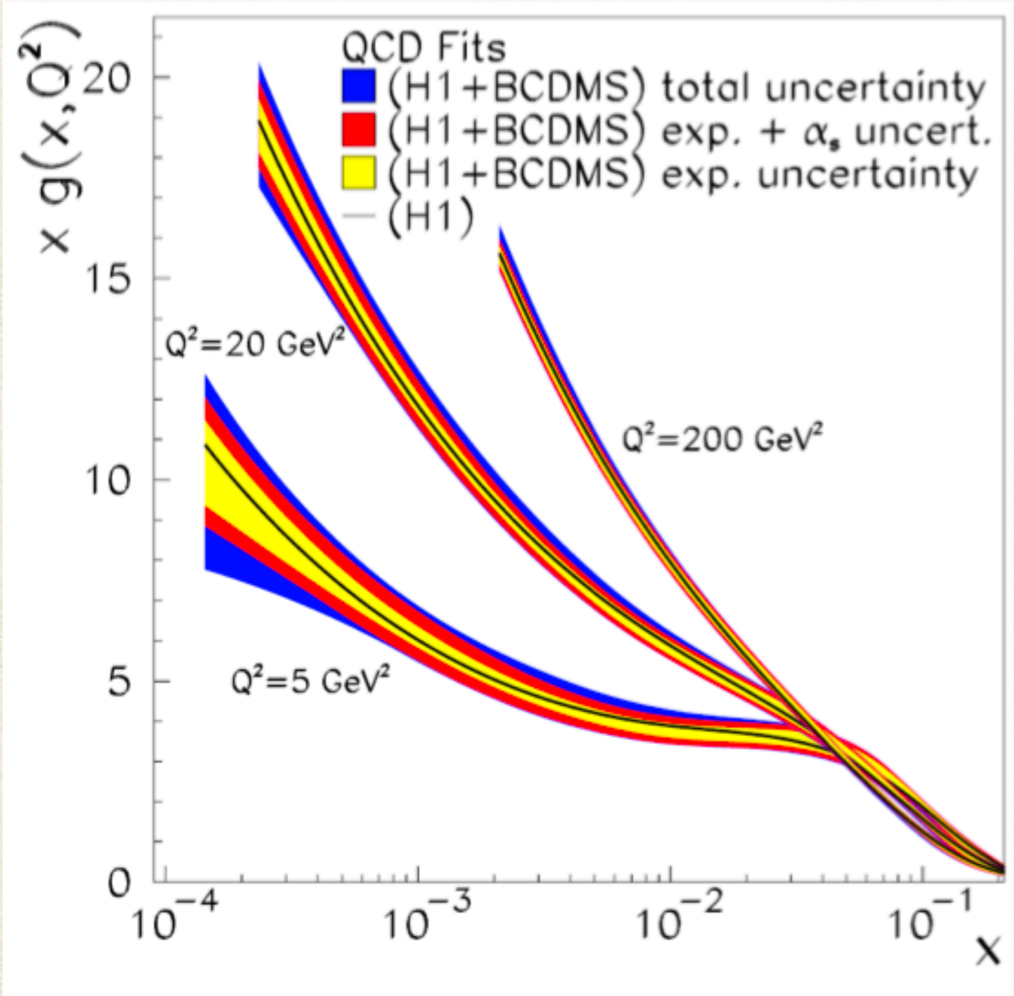


Gluons dominate PDFs at small $x<0.1$ and steeply rise.

Actually, importance of gluons has been known since the early days of the parton model.

The momentum conservation sum rule:

$$\int_0^1 F_2(x) = \int_0^1 \sum_{q,\bar{q}} e_q^2 xq(x)$$

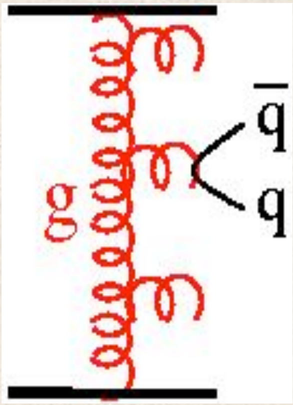


is the fraction of the total momentum carried by all quarks and antiquarks in the proton.

It turns out to be only about half. Another half of the proton momentum is carried by partons which don't interact with the photon, apparently gluons.

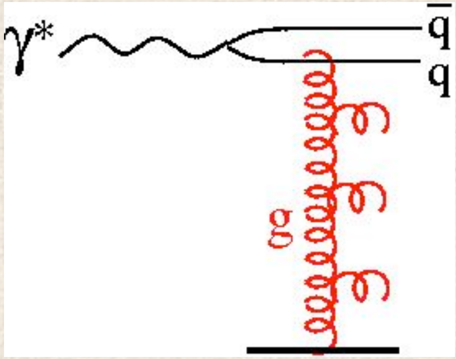
• "Small-x" or high-energy limit

$$W^2 \gg Q^2 > M^2 \quad \Longrightarrow \quad x \simeq \frac{Q^2}{W^2} \ll 1$$

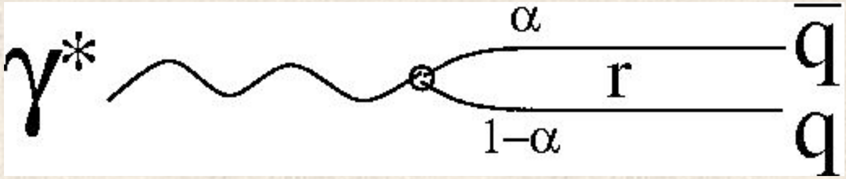


The parton model description is not Lorentz invariant, only observables are.
One cannot even say where a sea parton has originated from, who is the owner, the beam or the target.

In the domain of small x sea quarks and gluons dominate, and another reference frame, the rest frame of the proton is more convenient.
In this reference frame the proton has no partons, which all belong to the incoming photon. The photon converts into a quark-antiquark pair which then develops a parton cloud.



The wave function of a virtual photon



$$\varepsilon^2 = \alpha(1-\alpha)Q^2 + m_f^2$$

$$\left|\Psi^T_{q\bar{q}}(\alpha,\rho)\right|^2 = \frac{2N_c\alpha_{em}}{(2\pi)^2}\sum_{f=1}^{N_f} Z_f^2\left\{\left[1-2\alpha(1-\alpha)\right]\varepsilon^2 \mathrm{K}_1^2(\varepsilon\rho) + m_f^2\mathrm{K}_0^2(\varepsilon\rho)\right\}$$

$$\left|\Psi^L_{q\bar{q}}(\alpha,\rho)\right|^2 = \frac{8N_c\alpha_{em}}{(2\pi)^2}\sum_{f=1}^{N_f} Z_f^2 Q^2\alpha^2(1-\alpha)^2\mathrm{K}_0^2(\varepsilon\rho),$$

$$\sigma^{\gamma^* p}_{T,L} = \int_0^1 d\alpha \int d^2\rho \left|\Psi^{T,L}_{q\bar{q}}(\alpha,\rho)\right|^2 \sigma_{q\bar{q}}(\rho)$$

The dipole X-section has the property of Color Transparency. If the mean dipole size is r~1/Q, the cross section is $1/Q^2$ which corresponds to the Bjorken scaling.

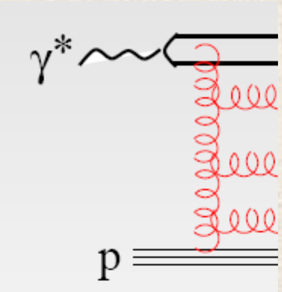
Q. Is inclusive DIS hard or soft reaction?

A. - Both

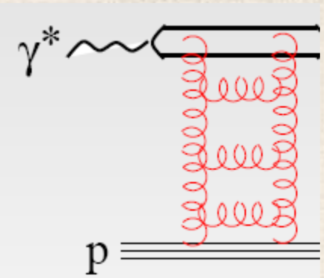
Q. Is diffractive DIS hard or soft?

A. - Soft !

$$\langle r^2 \rangle \sim \frac{1}{\varepsilon^2} \sim \frac{1}{Q^2\alpha\left(1-\alpha\right)+m_q^2}$$



inclusive DIS



diffractive DIS

In very asymmetric fluctuations $\alpha \sim 1/Q^2$, or $1-\alpha \sim 1/Q^2$ the fluctuations are soft

	$\lvert C_\alpha\rvert^2$	σ_α	$\sigma_{tot}=\sum_{\alpha=soft}^{hard}\lvert C_\alpha\rvert^2\sigma_\alpha$	$\sigma_{sd}=\sum_{\alpha=soft}^{hard}\lvert C_\alpha\rvert^2$
Hard	~ 1	$\sim \frac{1}{Q^2}$	$\sim \frac{1}{Q^2}$	$\sim \frac{1}{Q^4}$
Soft	$\sim \frac{m_q^2}{Q^2}$	$\sim \frac{1}{m_q^2}$	$\sim \frac{1}{Q^2}$	$\sim \frac{1}{m_q^2 Q^2}$

$$\sigma_{tot}^{\gamma^* p}(x,Q^2) \propto F_2(x,Q^2) \propto \left(\frac{1}{x}\right)^{\lambda_{eff}(Q^2)}$$

The effective Pomeron intercept is related to the effective exponent:

$$\alpha_P(0) - 1 = \lambda_{eff}(Q^2)$$

Data show that the Pomeron intercept is moving with Q to higher values. This clearly demonstrates that the Pomeron is not a pole, as QCD predicts.

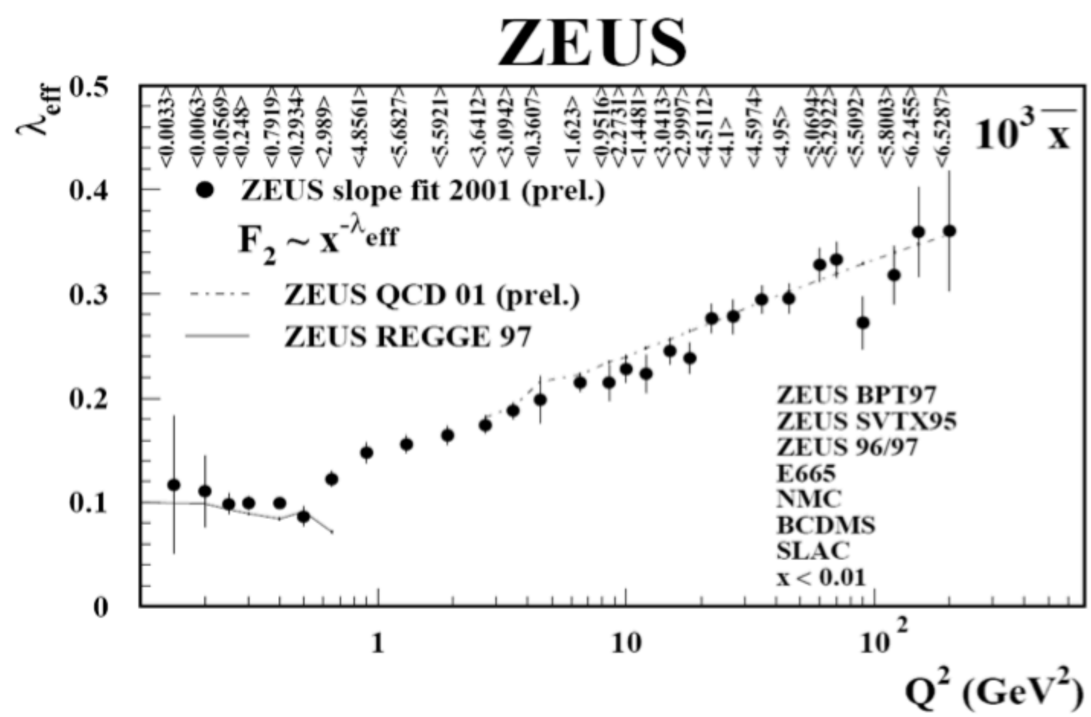
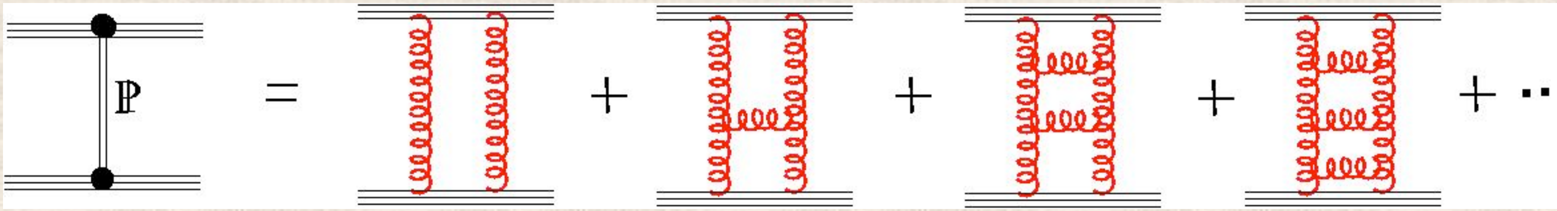


Figure 1: $\lambda_{\rm eff}$ *as a function of* Q^2. *The estimate for the ZEUS REGGE97 and ZEUS QCD 01 parametrizations are also shown.*

DGLAP Pomeron



Double-Leading-Log approximation.

Each radiated gluon is integrated over its phase space:

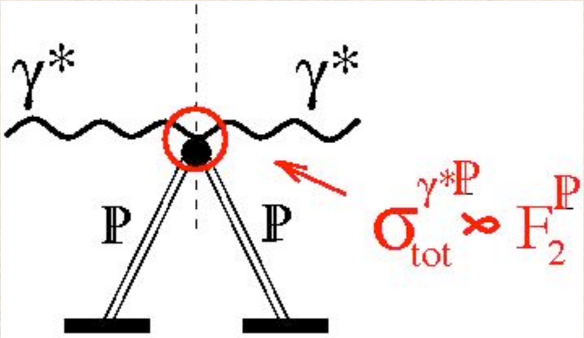
$$\alpha_s(k_i^2)\frac{dk_i^2}{k_i^2}\frac{dx_i}{x_i}$$

The cross section is some of powers of double logs:

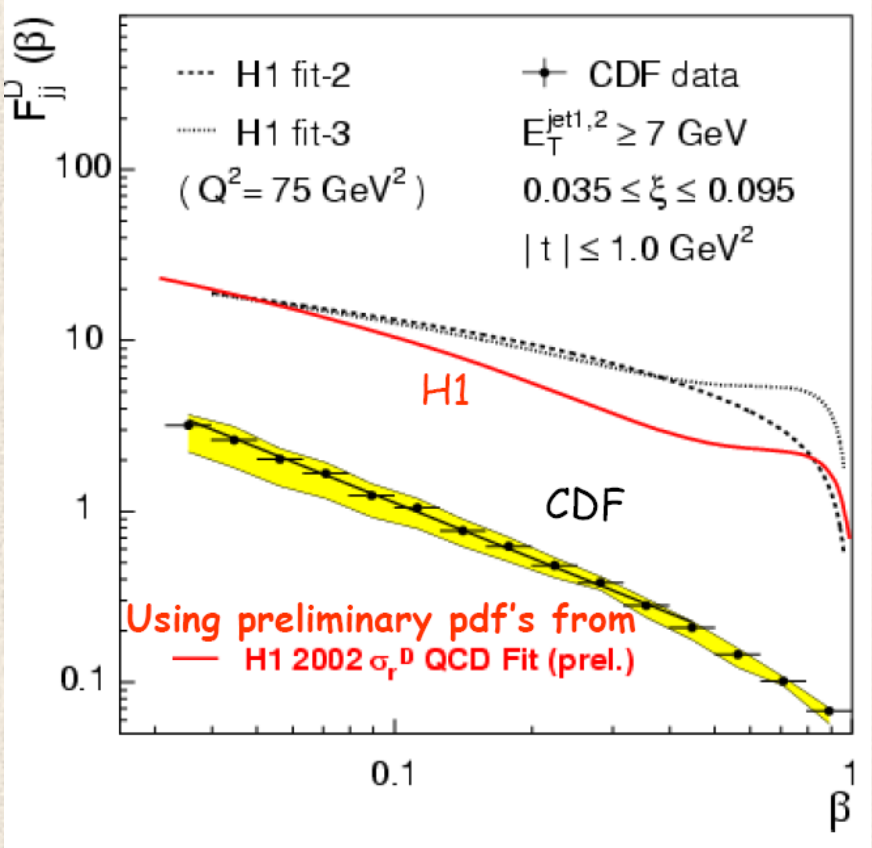
$$F_2(x,Q^2) \propto \sum_{n=0}^{\infty}\frac{[\ln\ln(Q^2/\Lambda^2)]^n[\ln(1/x)]^n}{(n!)^2} \propto \exp\left[2\sqrt{\ln\ln(Q^2/\Lambda^2)\ln(1/x)}\right]$$

The higher the scale Q is, the steeper rises the cross section with 1/x.
The effective Pomeron intercept is a rising function of Q.

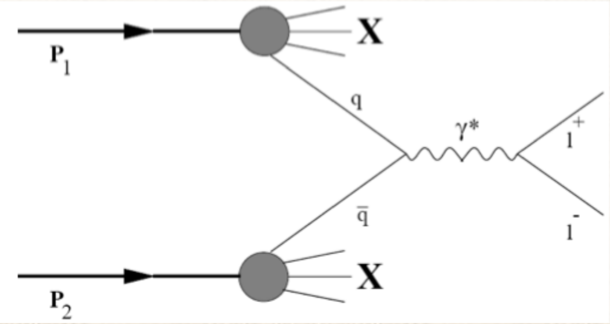
The same triple-Regge graph, but for DIS can be interpreted as a way to measure the structure function (PDFs) of the Pomeron. It was found that the Pomeron is mostly a gluonic object, but has a sizeable quark contribution.



Unfortunately attempts to use this diffractive PDFs of the Pomeron for hadronic hard diffraction badly failed: data from the Tevatron contradict such predictions by an order of magnitude. And for a good reason well undrstood theoretically.



Drell-Yan reaction



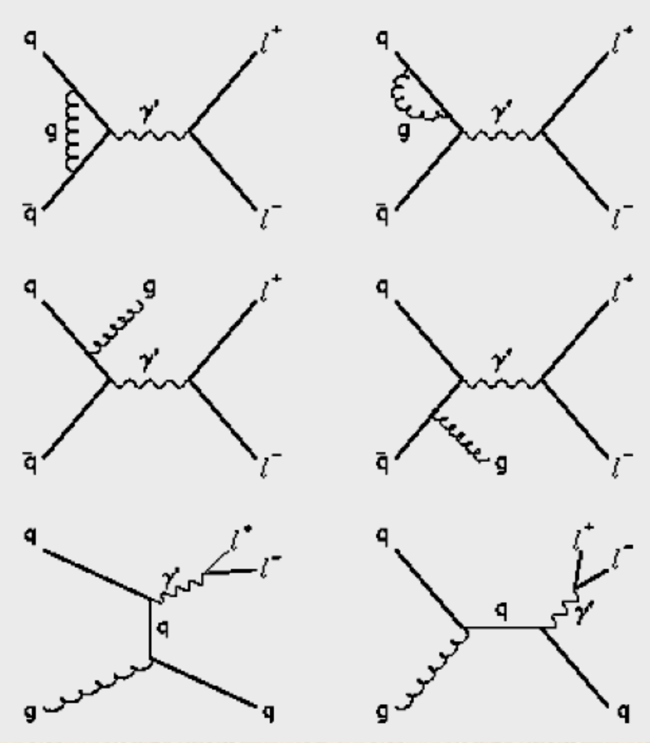
$$x_F = \frac{2p_L^{cm}}{\sqrt{S}} \approx x_1 - x_2$$

$$x_1 x_2 = \frac{M^2}{S}$$

$$M^2 \frac{d\sigma}{d\tau} = \frac{4\pi\alpha_{em}^2}{3N_c} \int_0^1 dx_1 \sum_f Z_f^2 \{q_f(x_1)\bar{q}_f(\tau/x_1) + (1 \leftrightarrow 2)\}$$

The cross section was found to be less than twice as small as data suggest.

Next-to-leading (NLO) corrections:



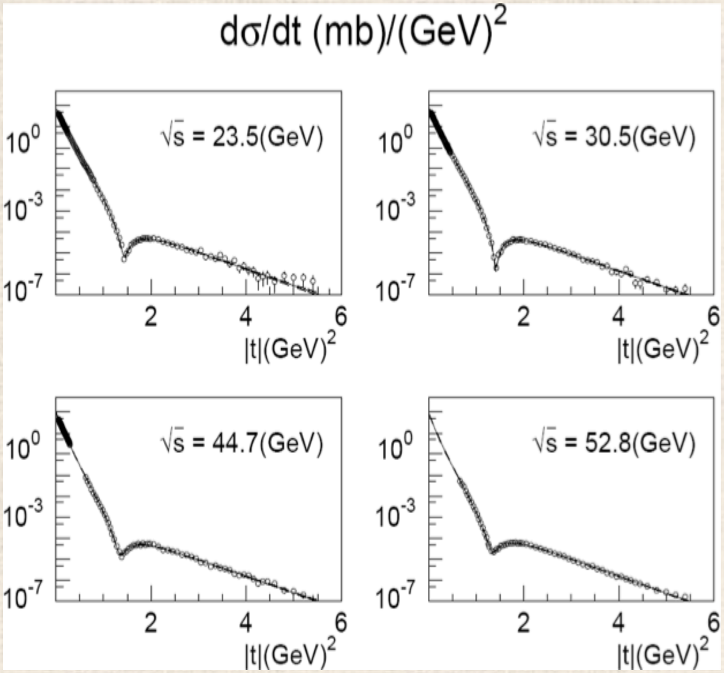
The correction K-factor is big, K≈2.3

Soft elastic diffraction

Diffractive elastic scattering of hadrons is a shadow of inelastic collisions.

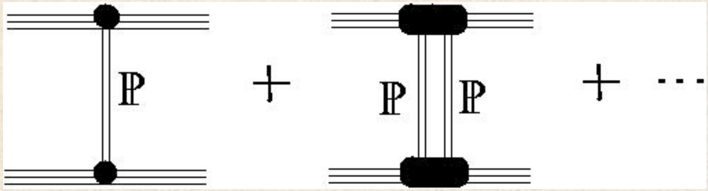
$$2\,\mathrm{Im}\, f_{el}(0) = \sigma_{tot}$$

This quantum mechanical effect has been known in classical optics.
The angular distribution of elastic diffraction has characteristic minima and maxima, for hadrons as well.



This diffraction is soft, since the main bulk of inelastic collisions is soft.

In the Regge approach this dip results from the interference of single and double Pomeron exchanges

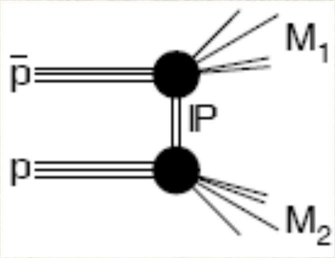


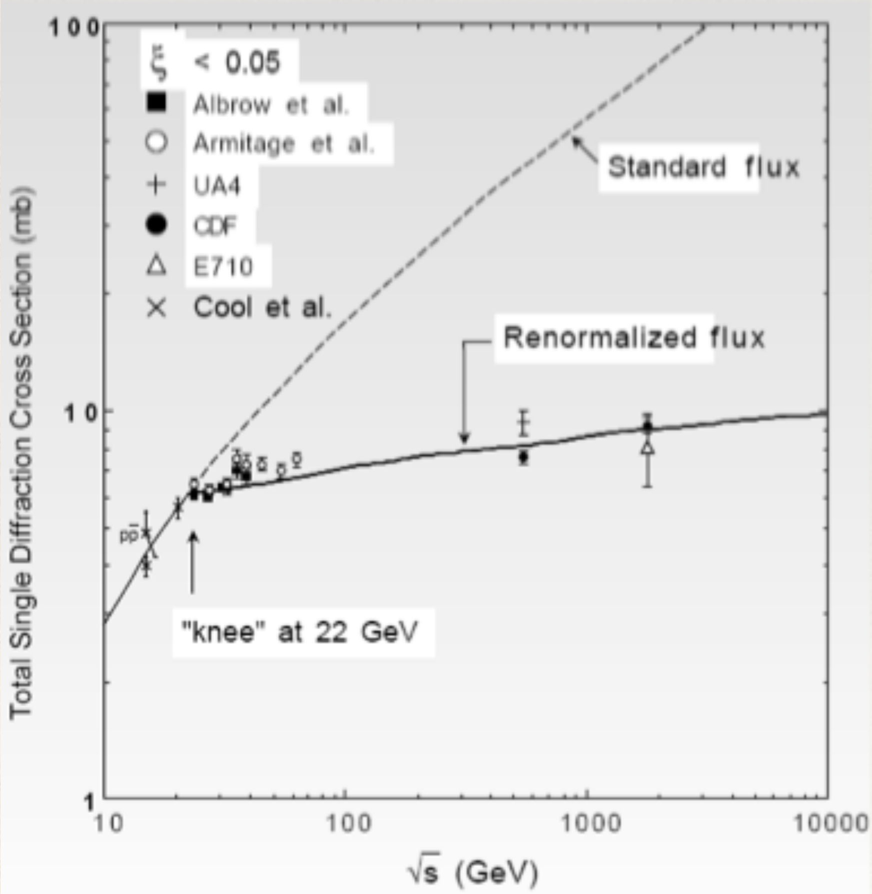
Soft inelastic diffraction

The multi-component structure of hadrons leads to diffractive excitations. Since different components interact differently (i.e. make different shadows), the final state wave packet is modified and can be projected to a new hadronic state.

The cross sections of diffractive excitation channels and elastics scattering are of the same order.

Experimentally diffraction looks like a large rapidity gap event. Particles are produces only at small angles relative the beam or/and target directions. Nothing is produceced in between.

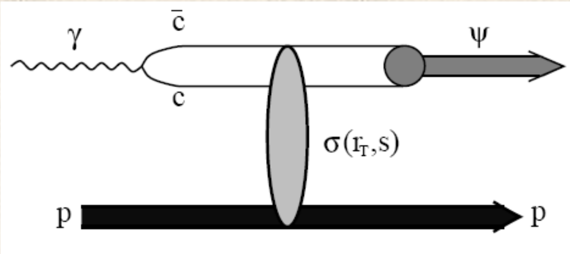


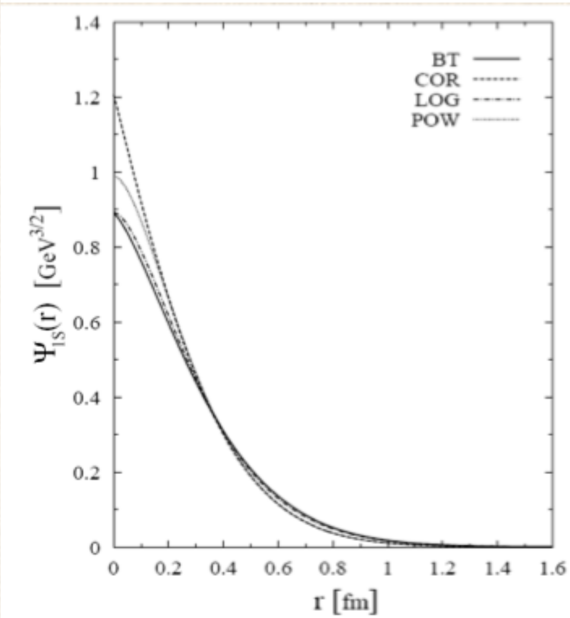


Rapidity Gaps in Fireworks

Hard diffraction: heavy flavor production in DIS

pQCD @ work





$$\mathcal{M}_{\gamma^* p}(s,Q^2) \;=\; \sum_{\mu,\bar{\mu}} \int_0^1 d\alpha \int d^2\vec{r}_T\, \Phi_\psi^{*(\mu,\bar{\mu})}(\alpha,\vec{r}_T)\, \sigma_{q\bar{q}}(r_T,s)\, \Phi_{\gamma^*}^{(\mu,\bar{\mu})}(\alpha,\vec{r}_T,Q^2)$$

Charmonium wave function

$$\left(-\frac{\Delta}{m_c}+V(r)\right)\Psi_{nlm}(\vec{r}) = E_{nl}\,\Psi_{nlm}(\vec{r})$$

$$\Psi(\vec{r}) = \Psi_{nl}(r)\cdot Y_{lm}(\theta,\varphi)$$

Cornell potential: $V(r) = -\frac{k}{r} + \frac{r}{a^2}$

The mean c-cbar separation is

$$\left\langle r_T^2 \right\rangle \sim \frac{1}{m_c^2}$$

in the photon fluctuation

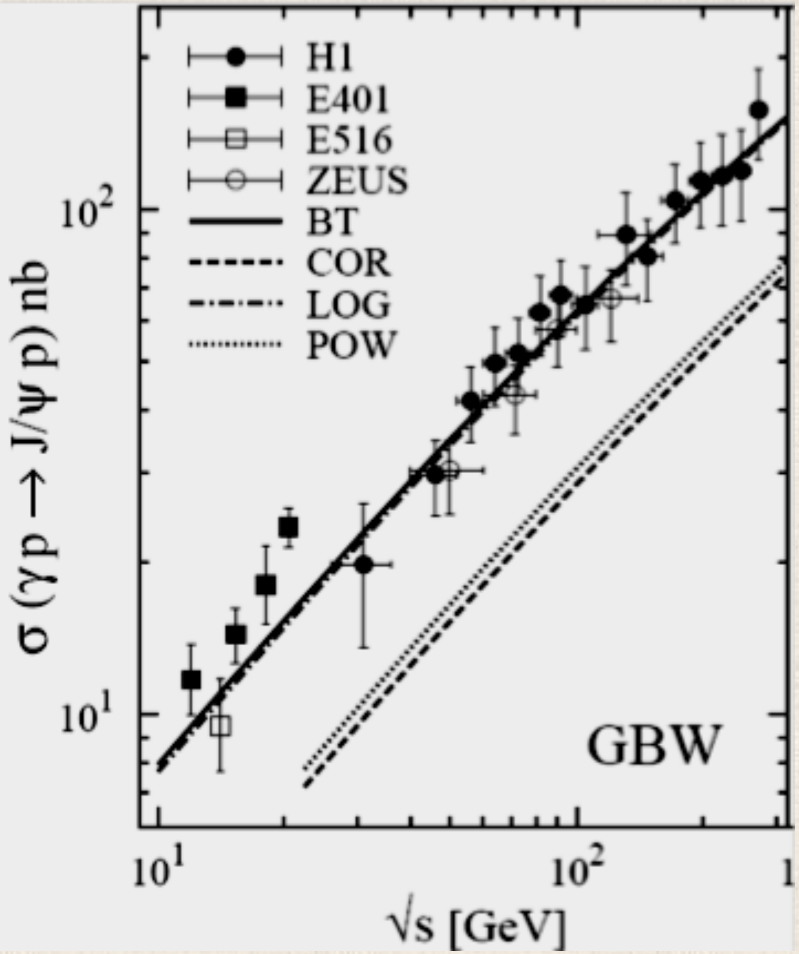
$$\left\langle r_T^2 \right\rangle \sim \frac{1}{m_c\omega}$$

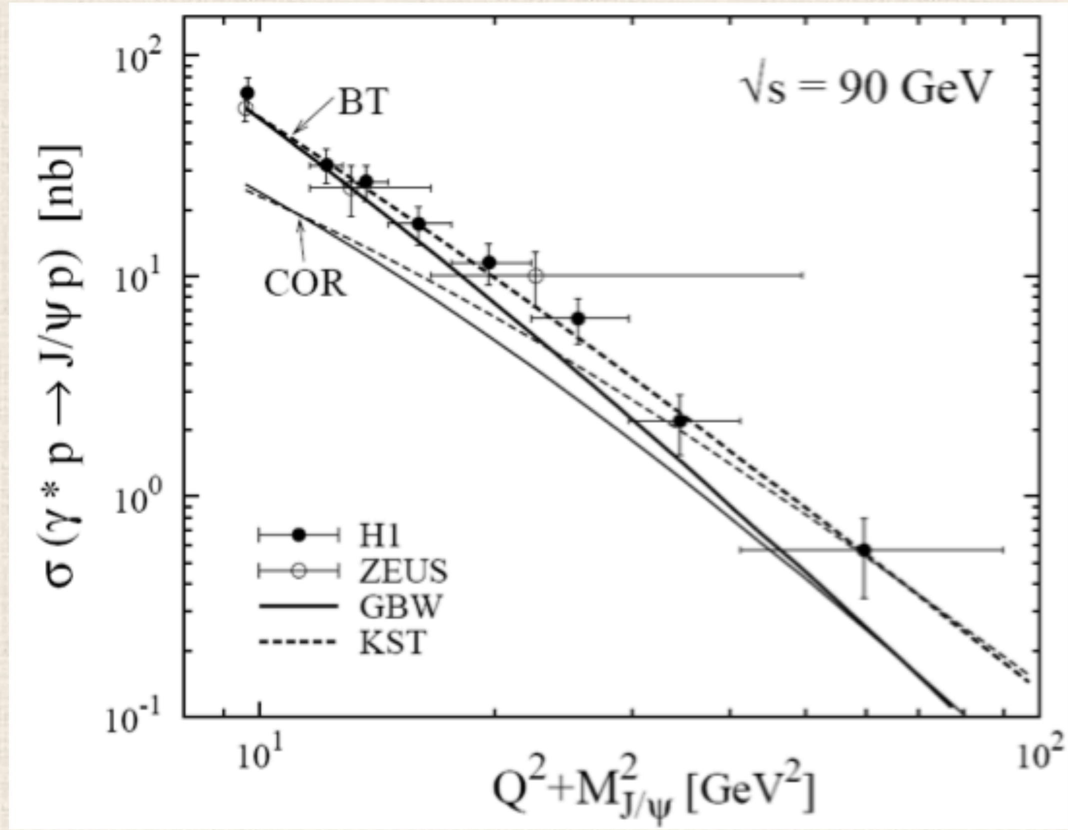
in the J/Ψ

$\omega = 300\,MeV$

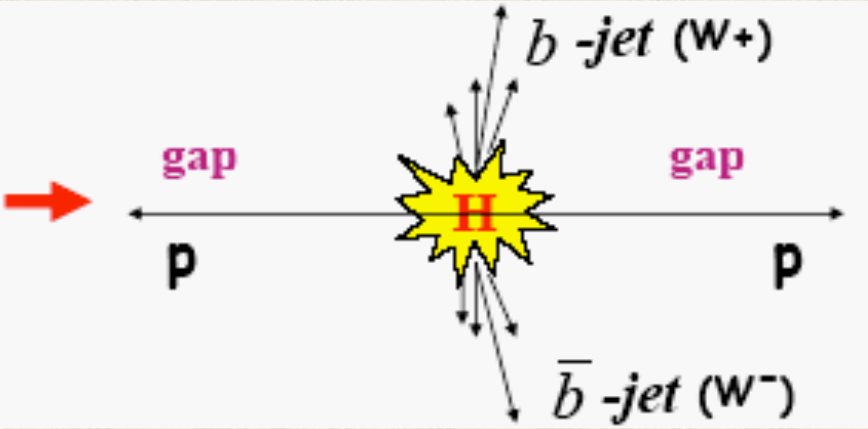
In the convolution of two wave functions small distances dominate. Thus, the diffractive crosssection is $\sigma(\gamma^* p \to J/\Psi p) \propto 1/m_c^4$

Diffractive electroproduction of J/Ψ

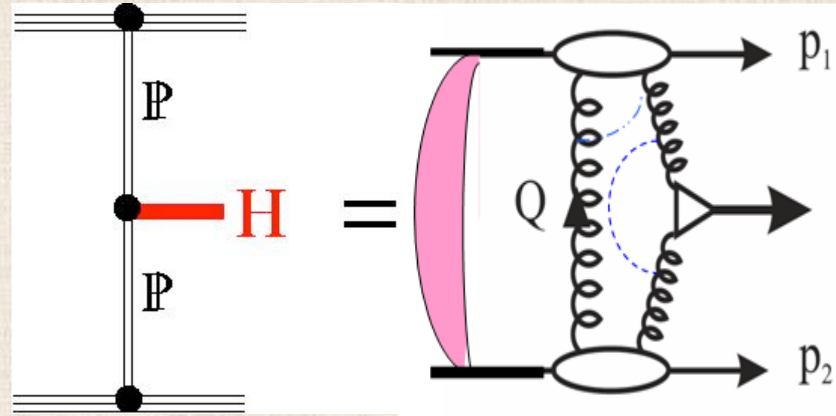


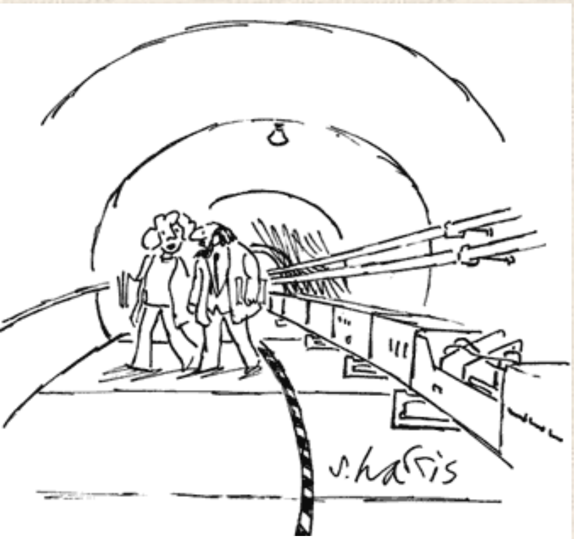


Higgs hunting: double-Pomeron reaction



$$\mathbf{p} + \mathbf{p} \rightarrow \mathbf{p} + \mathbf{H} + \mathbf{p}$$





"What if we spend all these Billions, and there just AREN'T more particles to find?"

Advantages:

- ✓ Missing mass measurement
- ✓ Relatively low background

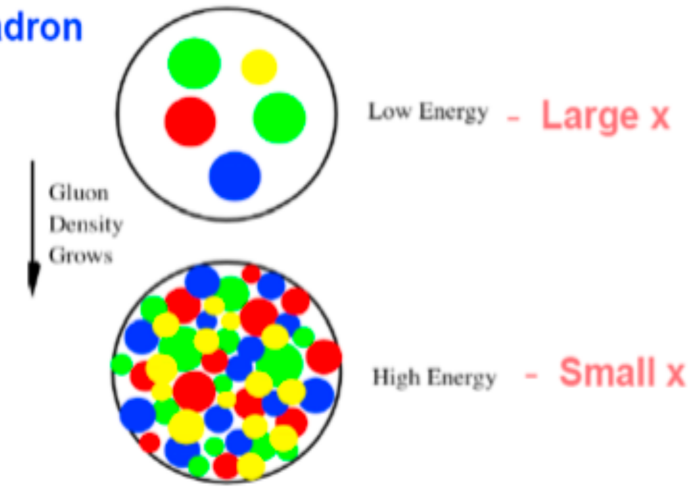
$$L_{eff} \sim \frac{\hat{S}^2}{b^2} \left| N \int \frac{dQ_t^2}{Q_t^4} f_g(x_1, x_1', Q_t^2, \mu^2) f_g(x_2, x_2', Q_t^2, \mu^2) \right|^2$$

Sudakov formfactor

The expected cross section is quite low, **few fm**, but still looks doable.

As we saw, HERA data demonstrate that the gluon density steeply rises down to small x.

Resolving the hadron -BFKL evolution



Gluon density saturates at f= $\frac{1}{\alpha_S}$

$$\frac{1}{2(N_c^2-1)}\,\frac{x\,G(x,Q^2)}{\pi R^2\,Q^2}=\frac{1}{\alpha_S(Q^2)}$$

Saturated for

$$\boxed{Q=Q_s(x)>>\Lambda_{\rm QCD}\approx 0.2\ {\rm GeV}}$$

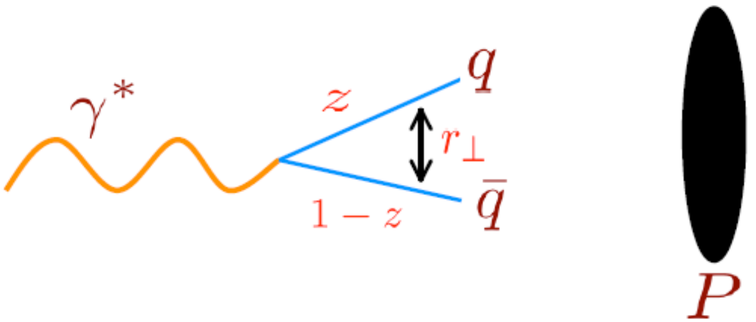
Mechanism for parton saturation:

at small x gluons undergo branching with the rate proportional to the gluon density.
As the density becomes high, an inverse process – fusion becomes important. Its rate is quadratic in the gluon density, so rises faster than branching and eventually stops.

The reached equilibrium density is higher for smaller coupling.

Being optimistic, one can see traces of saturation even in the proton.

Golec-Biernat & Wusthoff's model

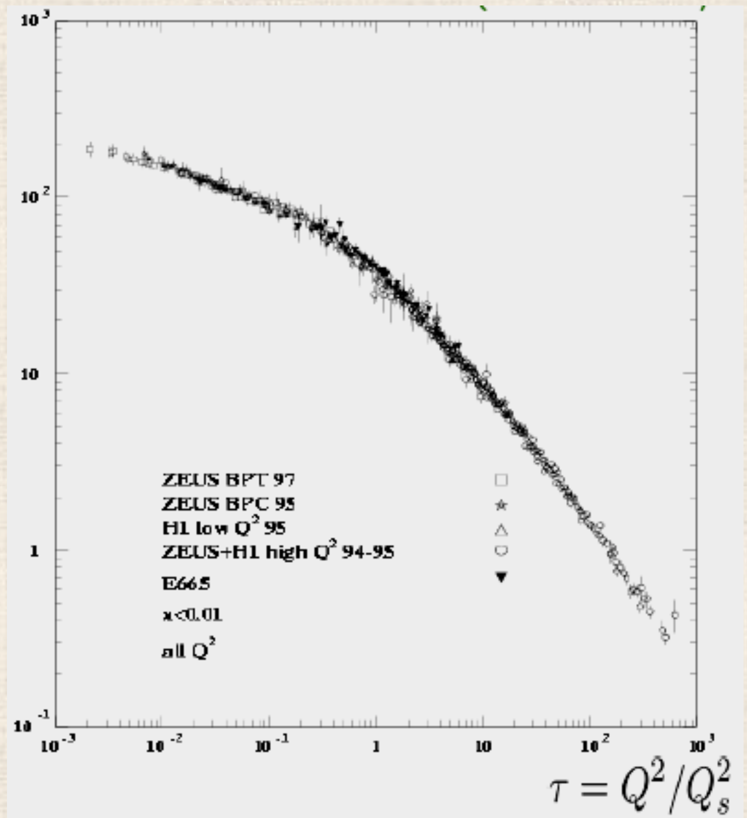


$$\sigma^{\gamma^*,P}_{\mathrm{T,L}} = \int d^2 r_\perp \int dz\, |\psi_{\mathrm{T,L}}(r_\perp, z, Q^2)|^2\, \sigma_{q,\bar{q},P}(r_\perp, x)$$

where $\sigma_{q\bar{q}P}(r_\perp, x) = \sigma_0 \left[1 - \exp\left(-r_\perp^2\, Q_s^2(x)\right)\right]$

& $$\boxed{Q_s^2(x) = Q_0^2 \left(\frac{x_0}{x}\right)^\lambda}$$

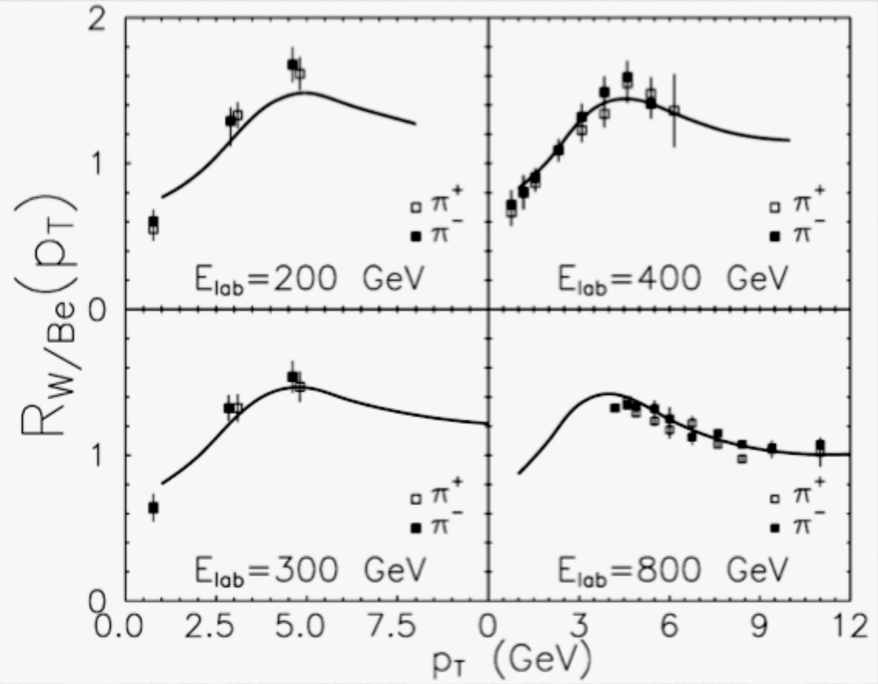
Parameters: $Q_0 = 1\,\mathrm{GeV}\,;\, \lambda = 0.3\,;\, x_0 = 3\cdot 10^{-4}$



However, DGLAP evolution describes the same data as well. So far it is not clear how much saturation is relevant to available DIS data.

Cronin effect

Back in 1973 the Cronin's group discovered that nuclei may not only suppress reactions, but also enhance. A considerable enhancement was found for production of hadrons with large transverse momentum.



This was understood soon as a result of multiple interactions in the nucleus.

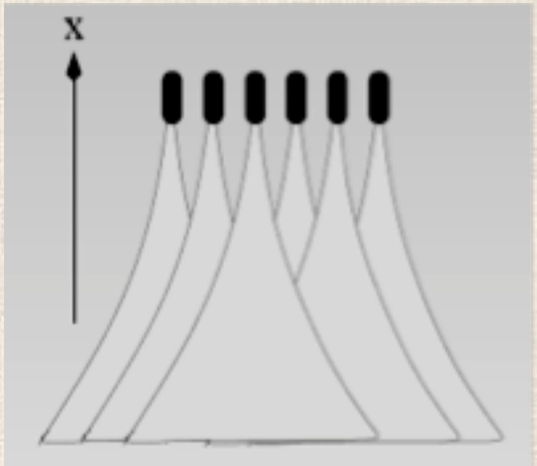
However, in the parton model based on QCD factorization describes this process as a hard parton-parton scattering. No multiple interactions. This should be interpreted as a modification of PDF in the nucleus.

Such a modification is a result of parton saturation in nuclei and the phenomenon is named Color Glass Condensate.

Nuclear shadowing

A particle thrown on a nuclear target has many possibilities of interaction with different bound nucleons. However, the total probability of interactions should not exceed 1. Therefore, a probability of each interaction must be reduced, what can be viewed as a result of shadows produced by the preceding collisions.

Shadowing looks quite differently in the infinite momentum frame of the nucleus. If the bound nucleons are well separated in the nuclear rest frame, both the nucleon size and internucleon spacing are Lorentz contracted and the nucleons still don't "talk" to each other".



The Lorentz contraction factor is m/E for internucleon spacing, but is m/xE for partins. Then, the logitudinal propagation of small-x partons is large. They overlap and do "talk" to each other, i.e. they fuse and reduce parton density at small x. The cross section decreases and this is shadowing.

Back of the envelope estimates

- Quark shadowing

At high energies dipoles are "frozen" by Lorentz time dilation during propagation through the nucleus. Then,

$$\left.\frac{q_A(x)}{A q_N(x)}\right|_{x\ll 1} = \frac{2}{\langle\sigma_{\bar{q}q}(r)\rangle}\int d^2b\left[1-\left\langle e^{-\frac{1}{2}\sigma_{\bar{q}q}(r)T_A(b)}\right\rangle\right]$$

For lead $q_A/Aq_N = 0.35$

- Gluon shadowing is much weaker, since $\sigma_{I\!Pp} \ll \sigma_{\pi p}$

$$\left.\frac{G_A(x)}{A G_N(x)}\right|_{x\ll 1} = \frac{2}{\langle\sigma_{GG}(r)\rangle}\int d^2b\left[1-\left\langle e^{-\frac{1}{2}\sigma_{GG}(r)T_A(b)}\right\rangle\right]$$

$$= 1-\frac{3C}{8}\,r_0^2\,\rho_A R_A+\frac{C^2}{10}\,r_0^4\,\rho_A^2\,R_A^2-\ldots\approx 0.74$$

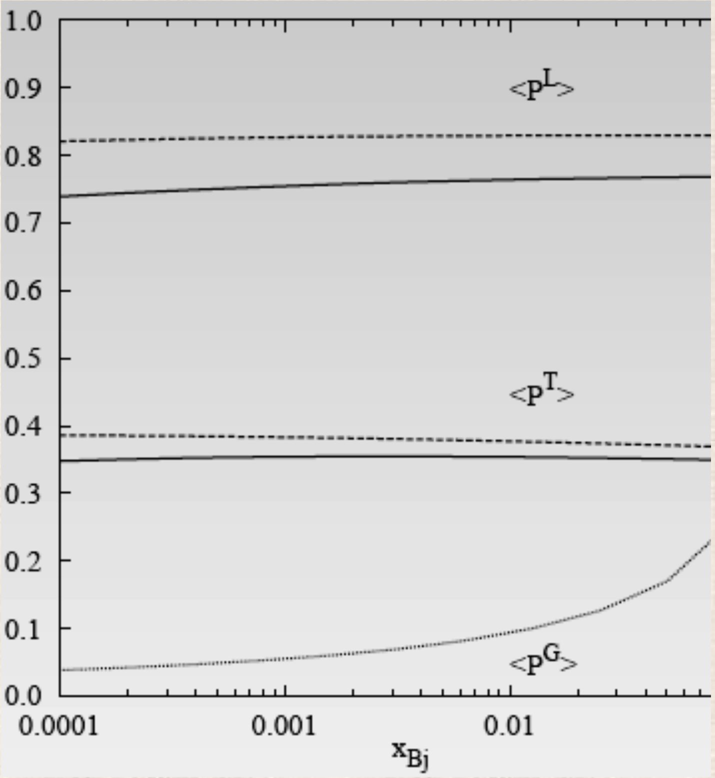
Since gluons in the proton are located within small spots, they have a little chance to overlap in transverse plane, even in heavy nuclei. The mean number of gluonic spots overlapping with this one is

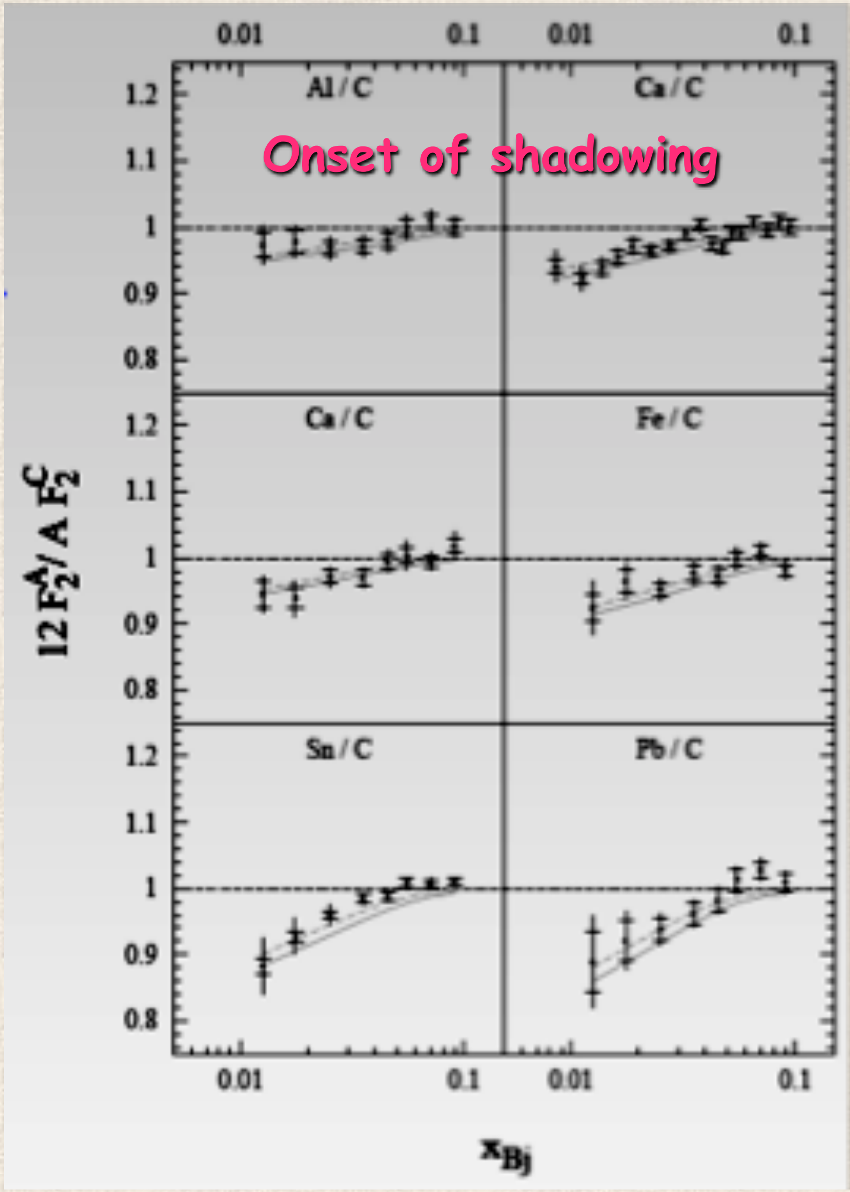
$$\langle n\rangle = \frac{3\pi}{4}\,r_0^2\,\langle T_A\rangle = \pi\,r_0^2\,\rho_A\,R_A = 0.3$$

In real data the photon fluctuations in DIS are never "frozen", but keep breathing during propagation through the nucleus

The fluctuation lifetime varies for longitudinal (L) and transverse (T) photons and for gluon (G)

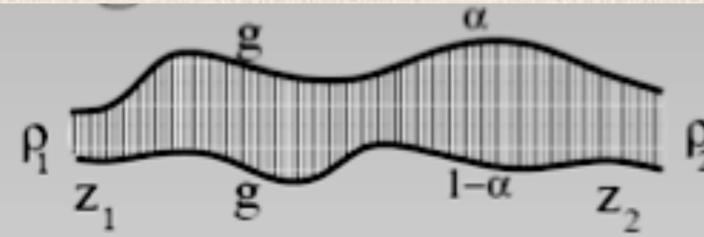
$$t_c = P\, t_{max} = \frac{P}{x\, m_N}$$





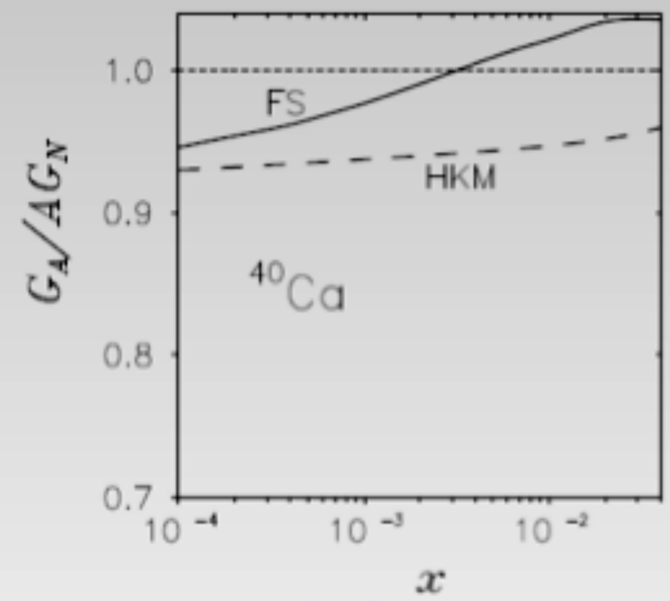
Schrödinger 2-dimensional equation for the Green function of a glue-glue dipole propagating through a medium

$$
i\frac{d}{dz_2}G_{gg}(z_1,\vec{\rho}_1;z_2,\vec{\rho}_2) = \left[-\frac{\Delta_\rho}{2\,p\,\alpha\,(1-\alpha)} + V_{gg}(z_2,\vec{\rho},\alpha)\right]G_{gg}(z_1,\vec{\rho}_1;z_2,\vec{\rho}_2)
$$



DGLAP analysis is able to single out from data nuclear PDFs for different species of partons. A leading order analysis failed to extract the gluon distribution, but the NLO fit turned out to be quite sensitive to gluons.

The results confirm a very weak gluon shadowing



NLO analysis

D. de Florian & R. Sassot(2004)

Optimistic conclusions

- Due to self-interaction of gluons QCD has the desired properties of asymptotic freedom and infra-red confinement

- Perturbative QCD is a legitimate tool for study the scale-evolution of observables, even though the absolute values are difficult to calculate. High precision data from HERA well confirmed the theory.

- QCD factorization provides a possibility to predict cross sections of other hard reactions, Drell-Yan process, heavy flavor production, high transverse momenta, etc.

- QCD gave a new life to a more general approach to soft hadronic collisions, Regge theory. A tremendous progress has been made attempting to calculate the main Regge term, Pomeron

Optimistic conclusions

The theory of QCD was granted the Nobel prize.
This seems to approve QCD as a correct and well developed theory.

2004 Nobel prize
in physics

Shall we get satisfied and retire?

Pessimistic conclusions

Well… This might be true only for perturbative methods which are relevant to the regime of asymptotic freedom.

Unfortunately, we have a rather poor understanding of soft nonperturbative physics which we are never free of.

The QCD based phenomenology is indeed pretty well developed. Nowadays we are able to calculate almost any reaction without fitting to the data to be explained.

However, the theory looks far more complicated and messy than the first principles, the QCD Langrangian.

Amount of models and theoretical tools keep growing.

OUTLOOK

Data from LHC should help us to settle many of controversies in our expectations for small-x physics. LHC is a laboratory for glue-dynamics.

✓The steep rise of gluon density is expected to be stopped by saturation. This might not happen, since even in pp at the Tevatron saturation is reached only for central collisions.

✓The saturation scale in nuclei is expected to reach values of few GeV, and the Cronin enhancement should be replaced by a strong suppression. Alternatively, very weak gluonic effects are expected.

✓LHC data should resolve the large scale controversy about the magnitude of gluon shadowing. This will also bring forth important information on the gluonic structures in the proton.

✓And many more…

Thank you for your attention and patience!