

Monte Carlo simulations and applications in air-shower simulations for VHE astronomy

by

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Overview

- ◆ The Monte Carlo technique
- ◆ Air-shower development
- ◆ Cherenkov light emission
- ◆ Imaging atmospheric Cherenkov telescopes
- ◆ H.E.S.S.: Simulations and measurements
- ◆ The next step: CTA

The Monte Carlo technique

- ◆ Except for a few early experiments, the start of the MC technique is in 1947 (“Stan Ulam, John von Neumann and the Monte Carlo Method”), applied to the problem of neutron diffusion.
- ◆ The name of the method relates to the randomness in roulette but the historical origin of the name may also be due to Ulam's uncle.
- ◆ In contrast to a Monte Carlo algorithm, a Las Vegas algorithm always leads to the same result (e.g. used in quicksort).

The Monte Carlo technique

“Monte Carlo codes can produce highly precise wrong answers.” (S.A. Dupree and S.K. Fraley, *A Monte Carlo Primer – A practical approach to radiation transport*)

The Monte Carlo technique

- ♦ The source of randomness in MC is a *pseudo-random number generator*.
- ♦ Development of random number generators is a difficult and error-prone task. So better don't try to develop your own generator.
- ♦ Even selecting a generator from collections of existing generators should be done with care.
- ♦ Random number generators in use for the MC technique will always produce the same sequence of numbers, given the same initial *seed*. So you need randomness in your seed.

Historical (and infamous) example of a random number generator

RANDU:

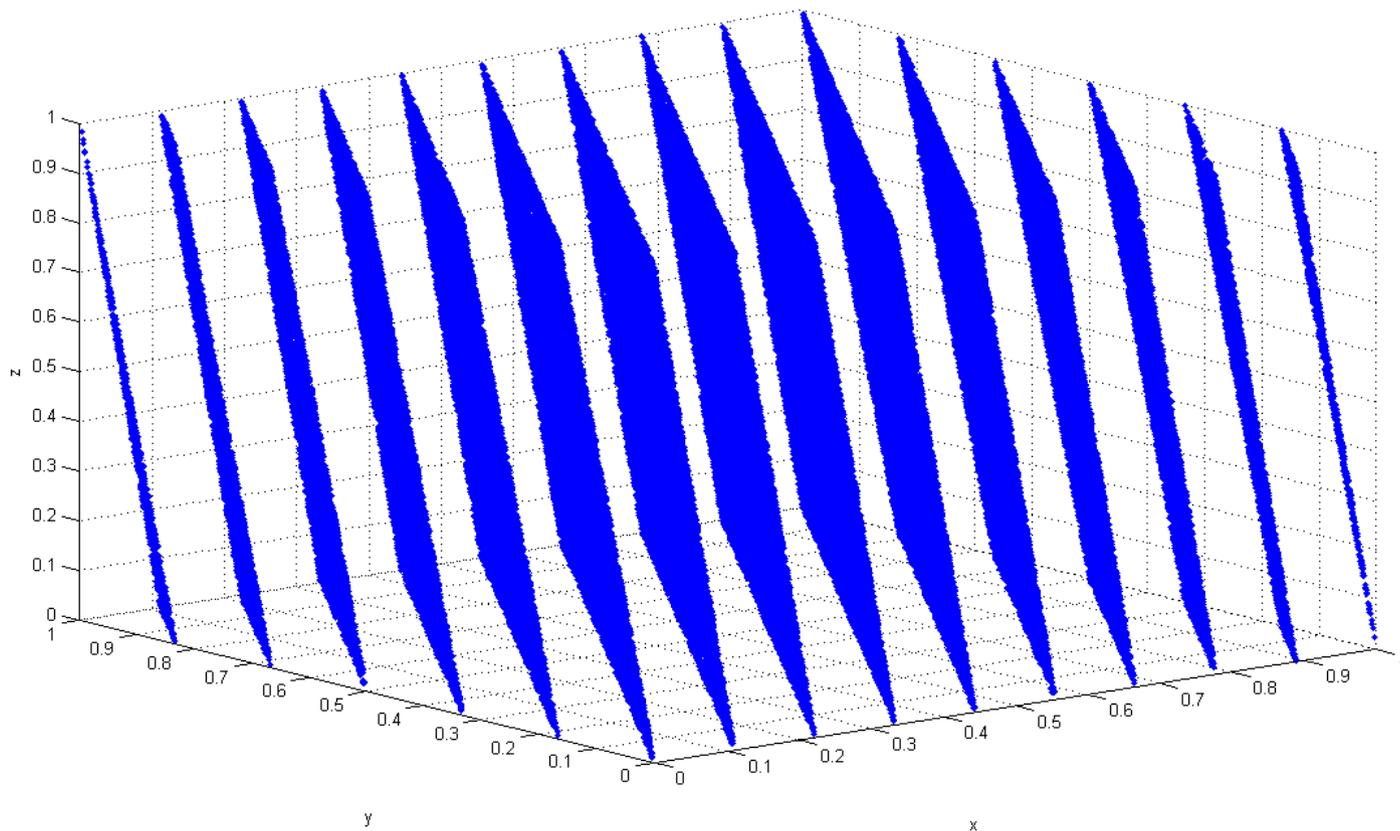
$$V_{j+1} = 65539 * V_j \text{ mod } 2^{31}$$

Was in heavy use in the 1960s and early '70s.

“... its very name RANDU is enough to bring dismay into the eyes and stomachs of many computer scientists!” (Donald Knuth, *The Art of Computer Programming*).

Historical (and infamous) example of a random number generator

RANDU: all (V_j, V_{j+1}, V_{j+2}) in 15 planes in 3-D:



Main deficiencies of bad pseudo-random number generators

- ◆ Correlations among subsequent numbers (often in higher dimensions than with RANDU).
- ◆ Short periods. Many old generators with periods of the order of 2^{31} were fine with slow computers in the old days, but unacceptable these days.
- ◆ Some seeds could result in short periods or other deficiencies while other seeds may be fine.

Take care of generators to use, even today

Example: ROOT provides four generators:

- ◆ TRandom
- ◆ TRandom1
- ◆ TRandom2
- ◆ TRandom3

Which one would *you* pick?

ROOT TRandom

- ◆ TRandom is the default.
- ◆ TRandom based on RN32.
- ◆ F. James (1990):

“Old-fashioned generators like RNDM, RN32, . . . should be archived and made available only upon special request for historical purposes or in situations where the user really wants a bad generator.”
- ◆ Failing 81 out of 94 randomness tests.
- ◆ Even with odd seeds, the sequence length is just $5 \cdot 10^8$, any power of 2 in the seed reduces the length by a factor of 2.
- ◆ TRandom is documented to be bad.

Random number distributions

- ♦ Standard distribution is flat in $[0:1]$ or $]0:1[$.
- ♦ We often need numbers following other distributions, e.g. $f(x) \propto \exp(-x/\lambda)$.
- ♦ The alternative would be very inefficient, playing accept or reject, e.g. in $0 < x < 100\lambda$:

```
while ( !accepted )
{
    x = 100 * Lambda * Random();
    if ( Random() < exp(-x/Lambda) )
        accepted = true;
}
```

Random number distributions

1-D custom probability density:

$f(x) \geq 0$ in the range $x_1 \leq x \leq x_2$.

has
$$\bar{F}(x) = \int_{x_1}^x f(x') dx' / \int_{x_1}^{x_2} f(x') dx'$$

equivalent to our uniform random number $r \in [0:1]$,
i.e. we invert $F(x)=r$. Example: $f(x)=\exp(-x/\lambda)$

$$x = -\lambda \ln(1-r)$$

or, equivalent because $1-r$ is also uniform in $[0:1]$:

$$x = -\lambda \ln(r)$$

Efficient simulations

A major problem in any large-scale simulation is to get the code more efficient, e.g. by

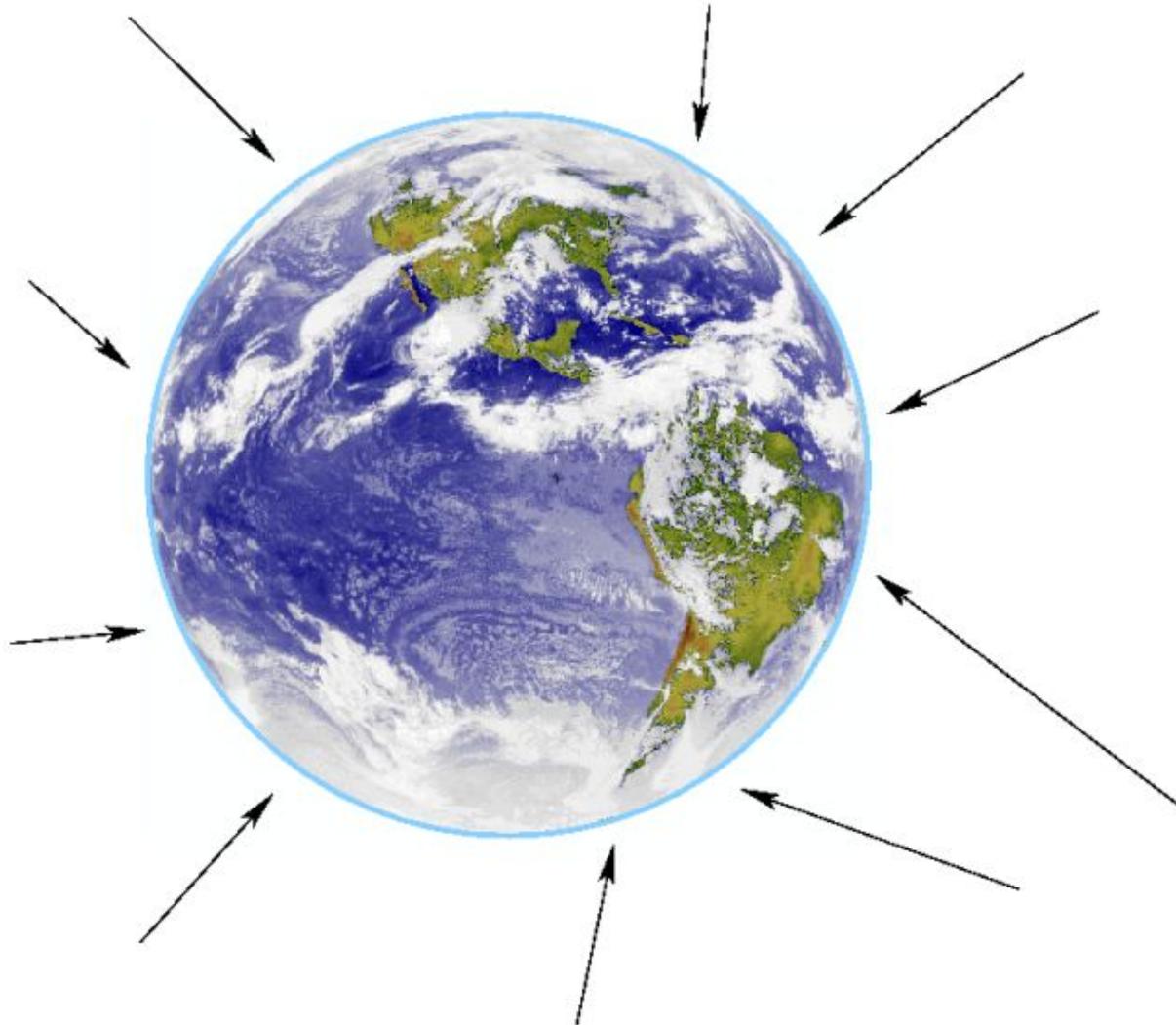
- ♦ checking where approximations can be made,
- ♦ less frequent production of the most common cases, corrected by applying event weights.
Example: instead of $dN/dE \propto E^{-2.7}$ spectrum we could use $dN/dE \propto E^{-2.0}$ simulation and assign weights $w_i \propto E^{-0.7}$ to each event.
- ♦ applying variance-reduction methods, if possible, ...

Correct simulations (?)

To avoid `highly precise wrong answers`:

- ◆ Find test cases with analytical solutions
- ◆ Cross-check with independent codes
- ◆ Check all of your approximations again
- ◆ Check against experimental data
- ◆ Use common sense ...

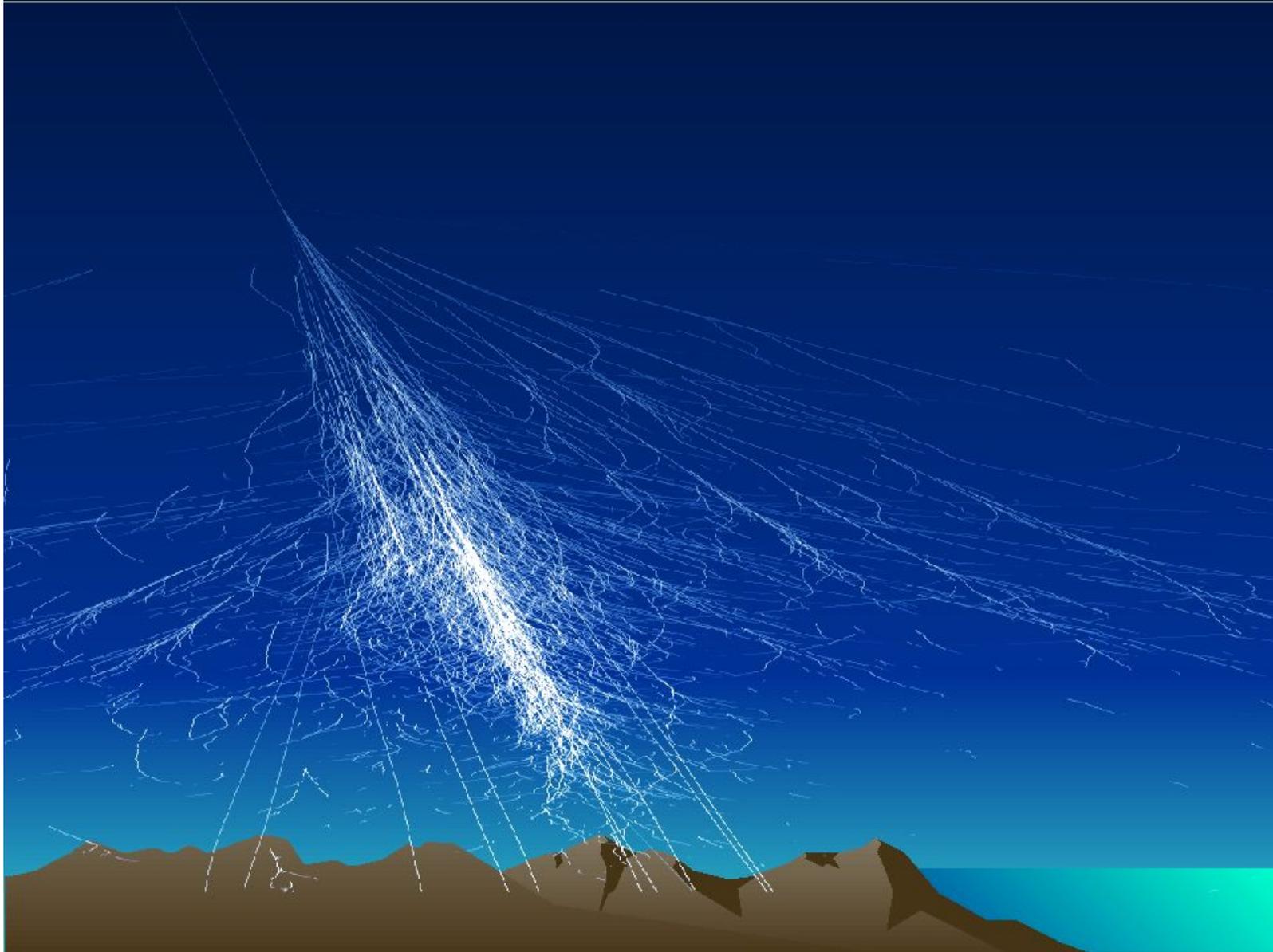
Cosmic rays and γ -rays



Cosmic rays and γ -rays

- Cosmic origin detected in 1912 by Victor Hess
- ◆ Called “Ultragammastrahlung” by Hess.
 - ◆ Only a tiny fraction of it is actually γ -rays.
 - ◆ Most are protons and nuclei, some electrons, positrons, antiprotons, ...
 - ◆ “Cosmic rays” by Millikan.
 - ◆ All charged particles are scattered in the Galactic magnetic field and arrive isotropically from random directions.
 - ◆ Need to find the γ -rays in the haystack of cosmic rays in order to identify the sources ...

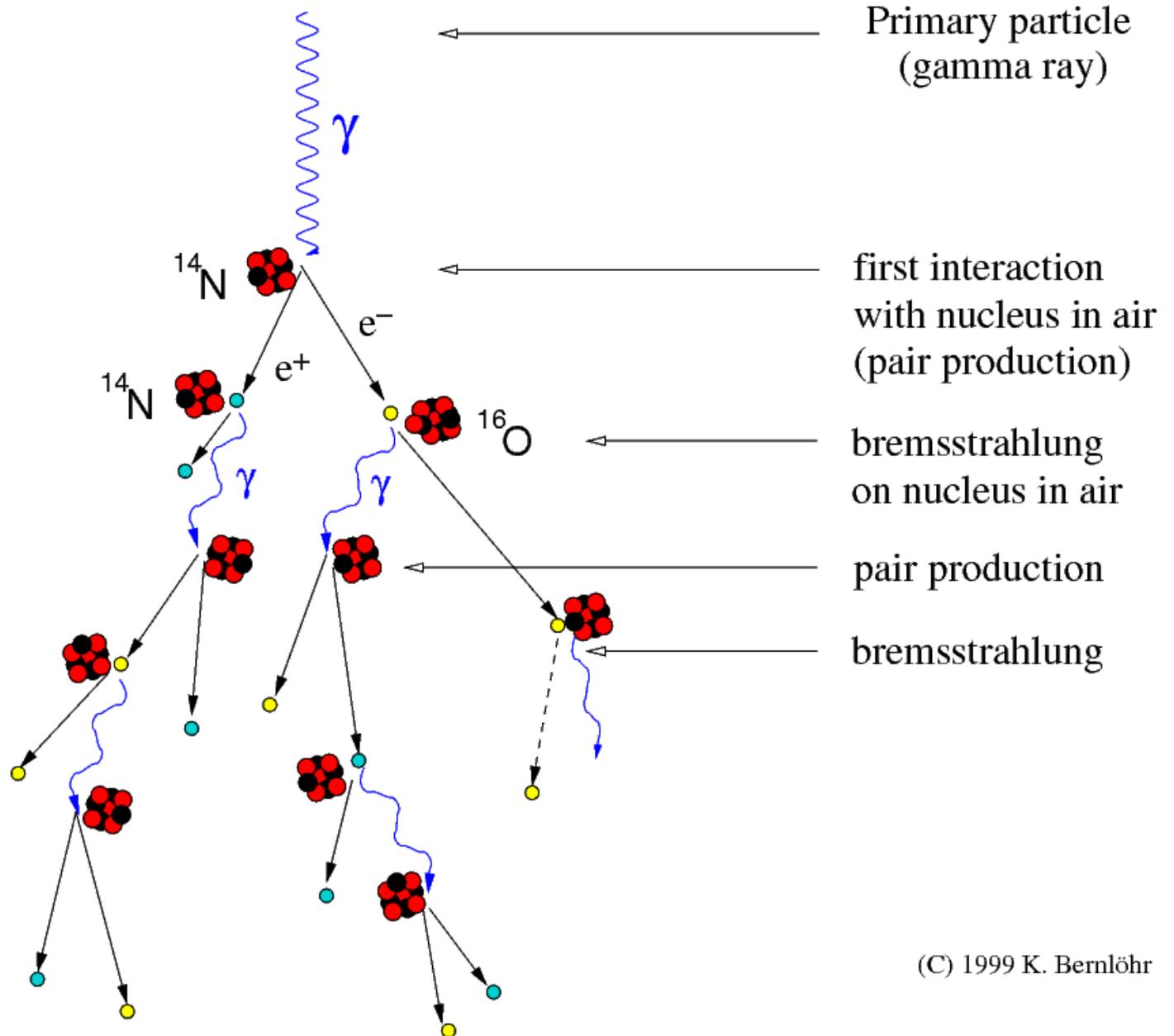
Air shower development



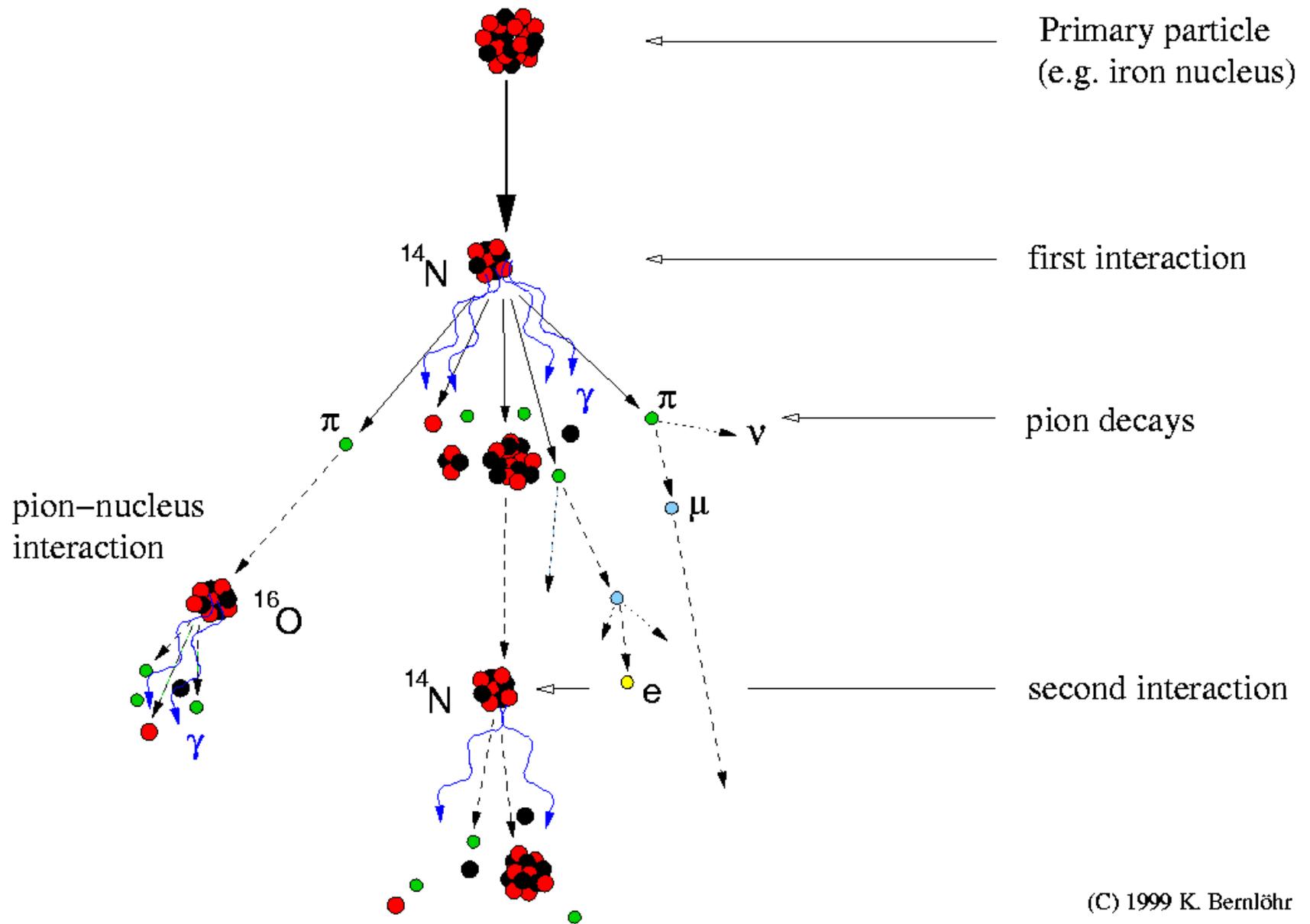
Air shower development

- ◆ Primary particle interacts with air, typically a few ten km high – producing new particles.
- ◆ Secondary particles interact again and so on, but eventually energy loss, e.g. due to ionization, will stop the growth of the particle cascade (“*extensive air shower*”).
- ◆ With high-energy primaries, some secondary particles reach the ground.
- ◆ Even those not reaching ground may radiate and become detectable.
- ◆ The atmosphere becomes part of the detector.

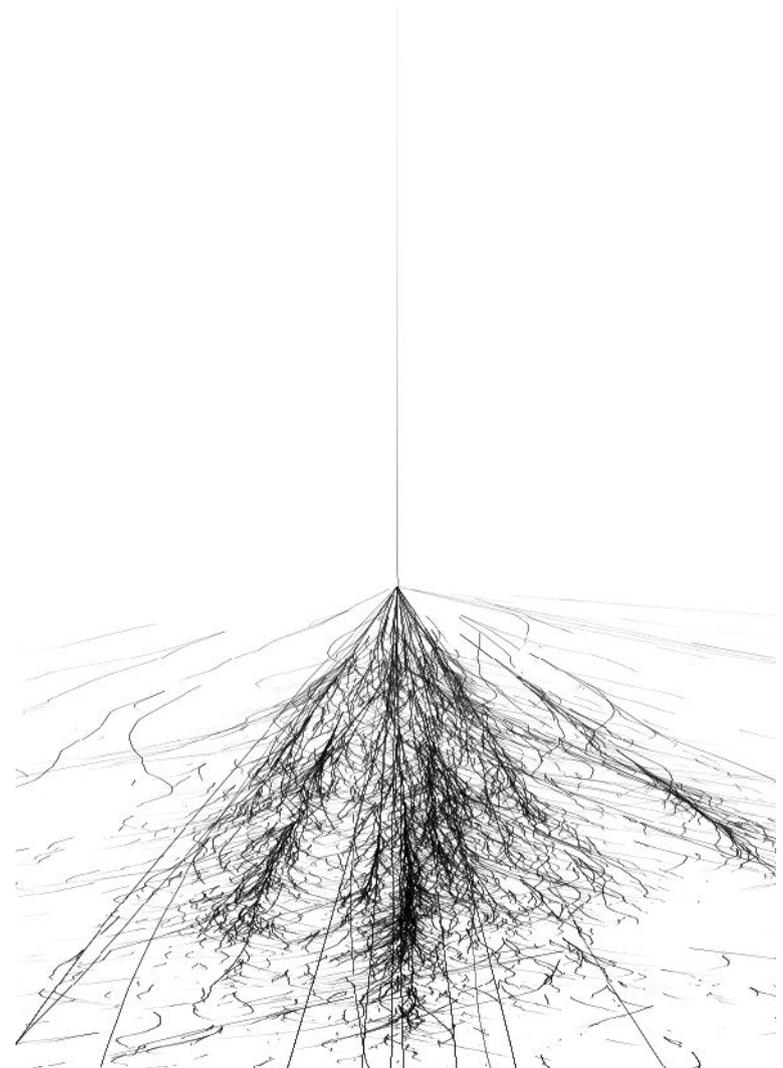
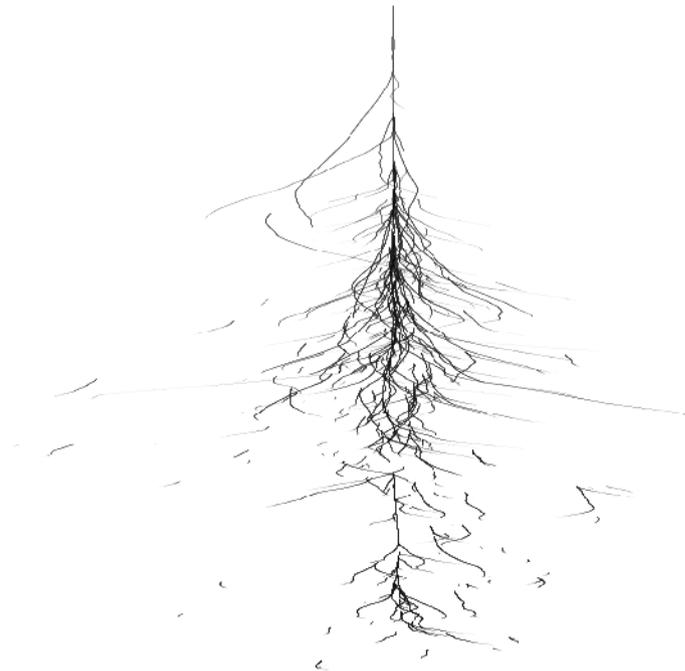
Development of gamma-ray air showers



Development of cosmic-ray air showers

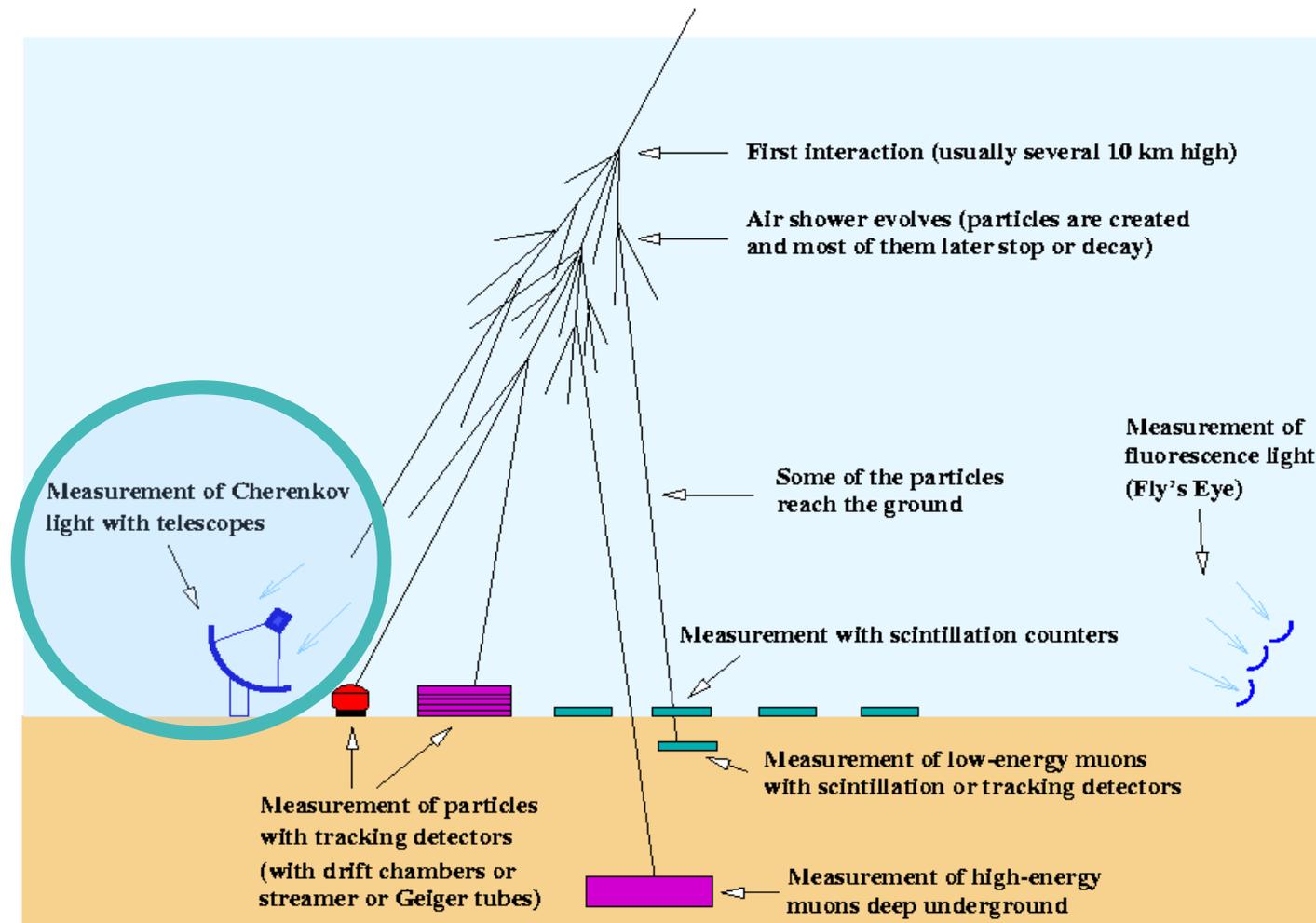


Examples of shower development



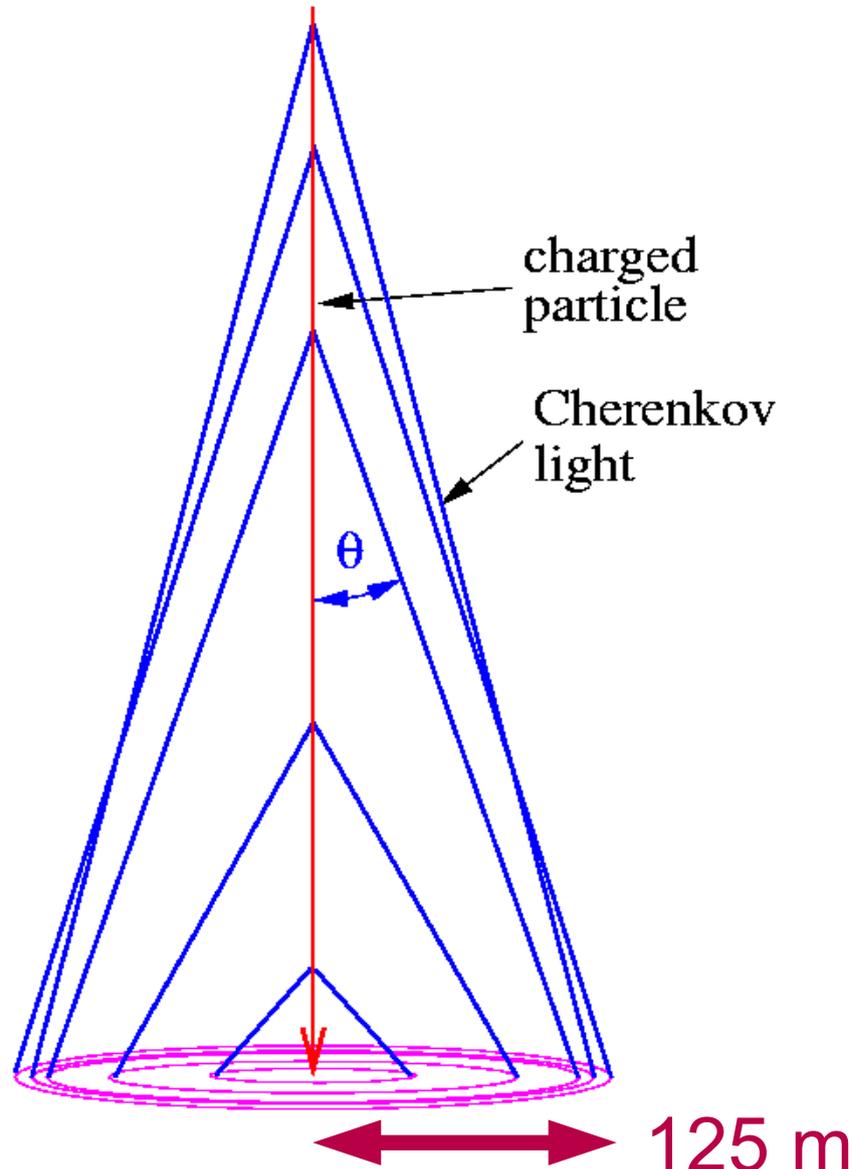
Air shower measurement methods

Measuring cosmic-ray and gamma-ray air showers

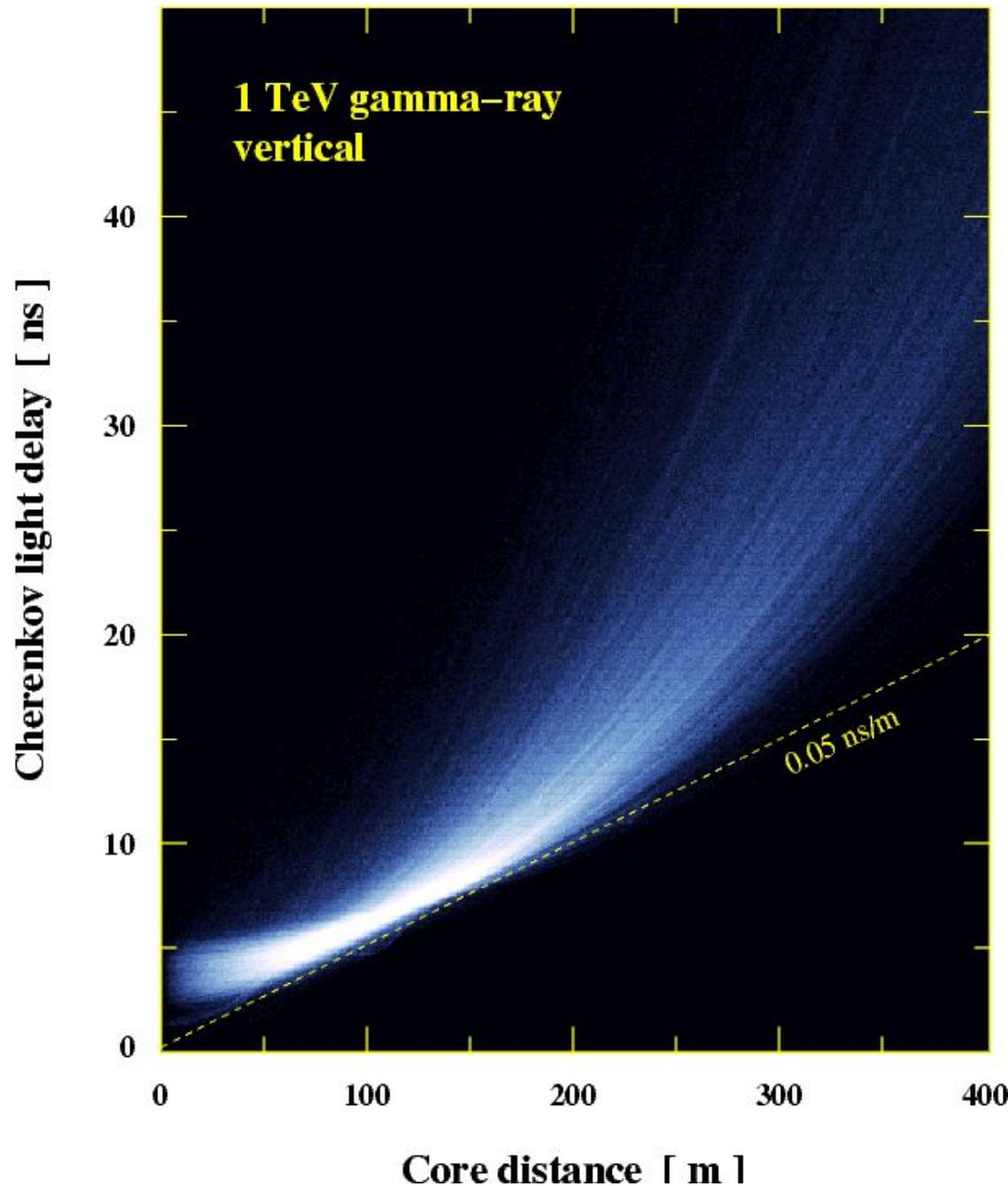


(C) 1999 K. Bernlöhr

Faster than the light



- ◆ Particles can be faster than light in air.
- ◆ Charged particle polarize the medium, leading to emission under angle θ w.r.t. particle direction.
- ◆ θ depends on air density and particle velocity:
$$\cos \theta = 1/(n\beta)$$



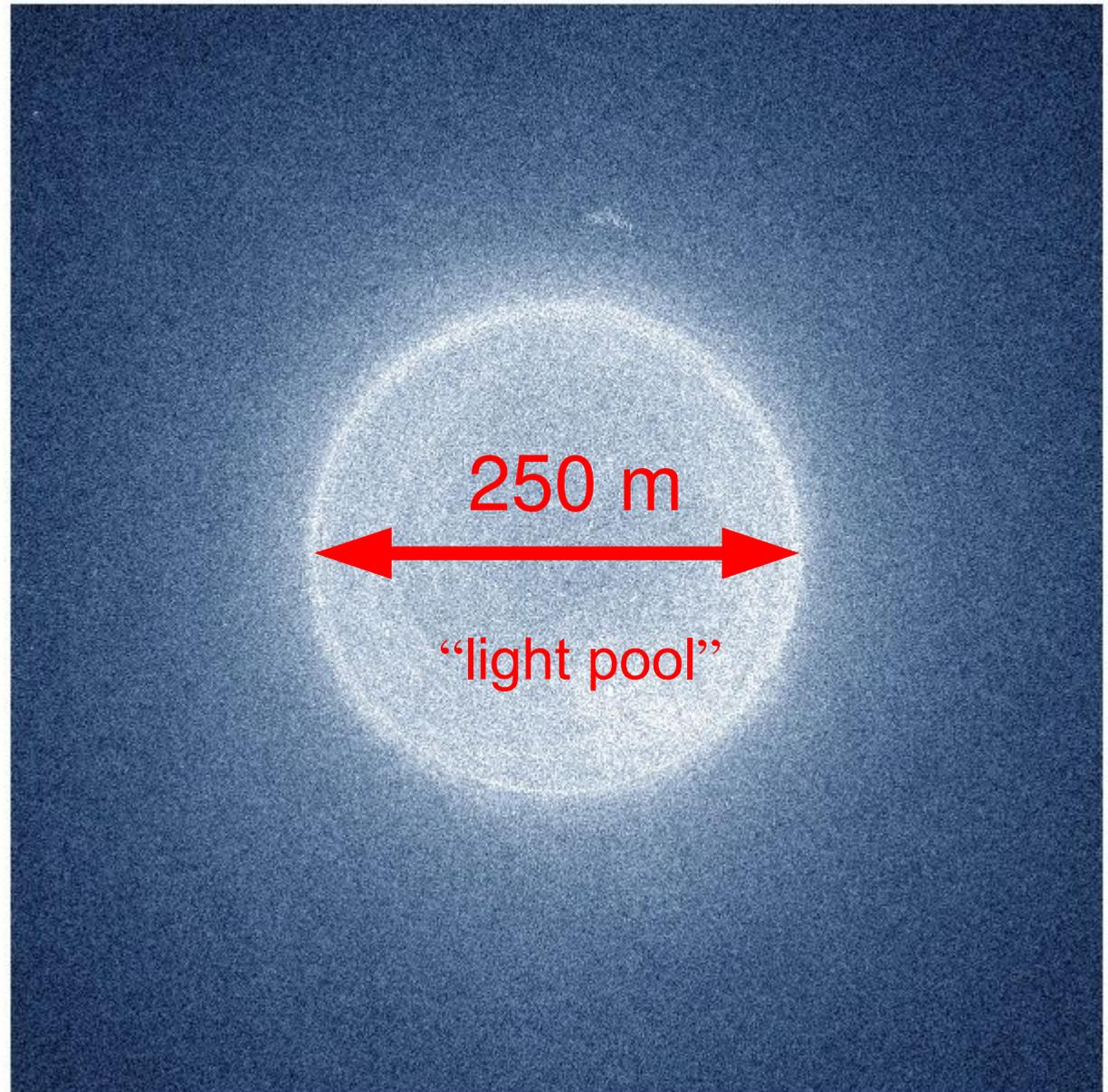
Since the index of refraction is so close to 1, the Cherenkov light almost keeps pace with the particles.

Near the edge of the 'light pool' most light from a gamma-ray shower arrives within 2 ns.

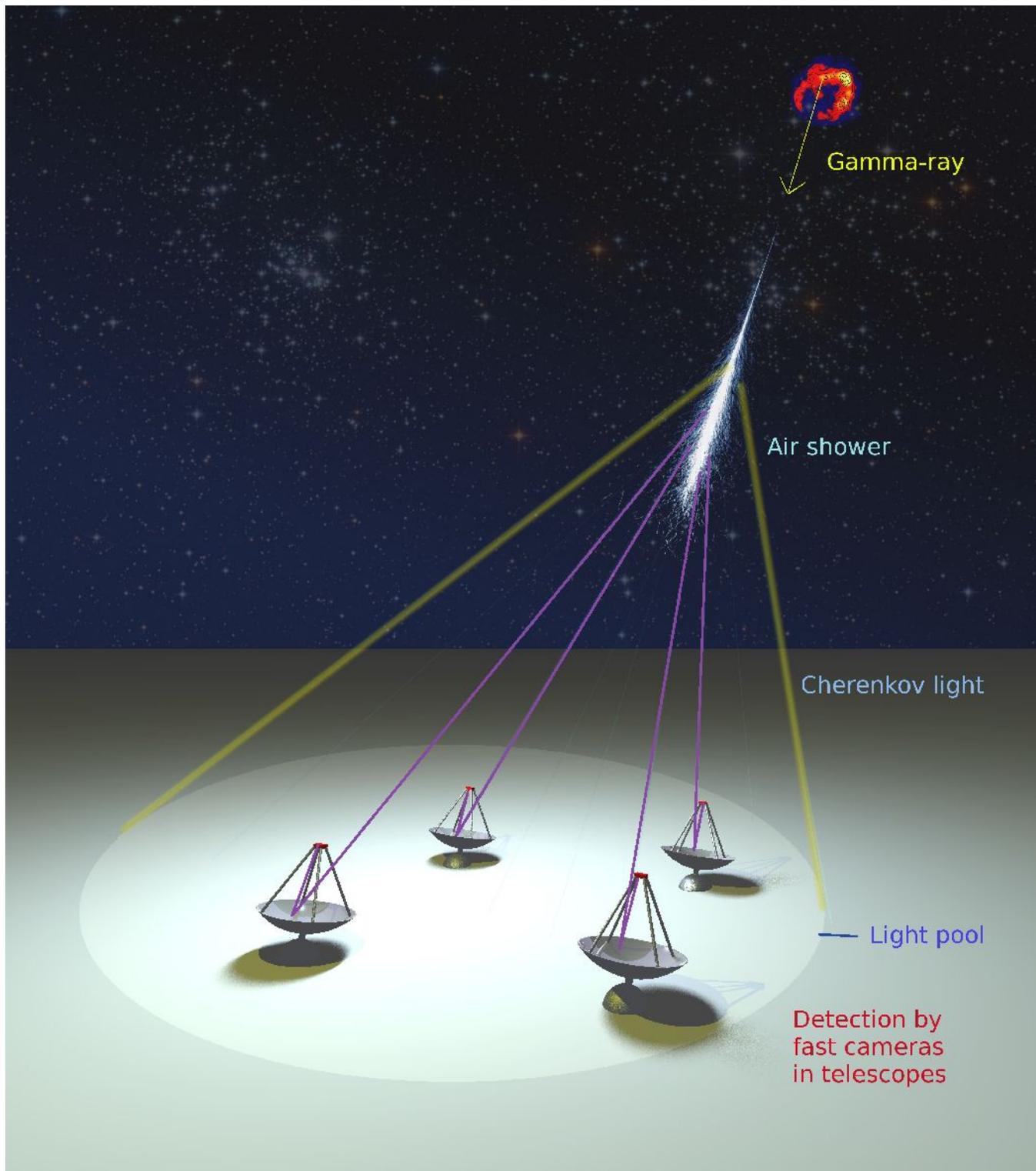
Cherenkov
light as
arriving on
the ground.

Observation
level here at
1800 m a.s.l.

Size of area as shown: 600 m * 600 m



Now put some
telescopes into
the picture ...



Imaging atmospheric Cherenkov telescopes (IACT)

CAT

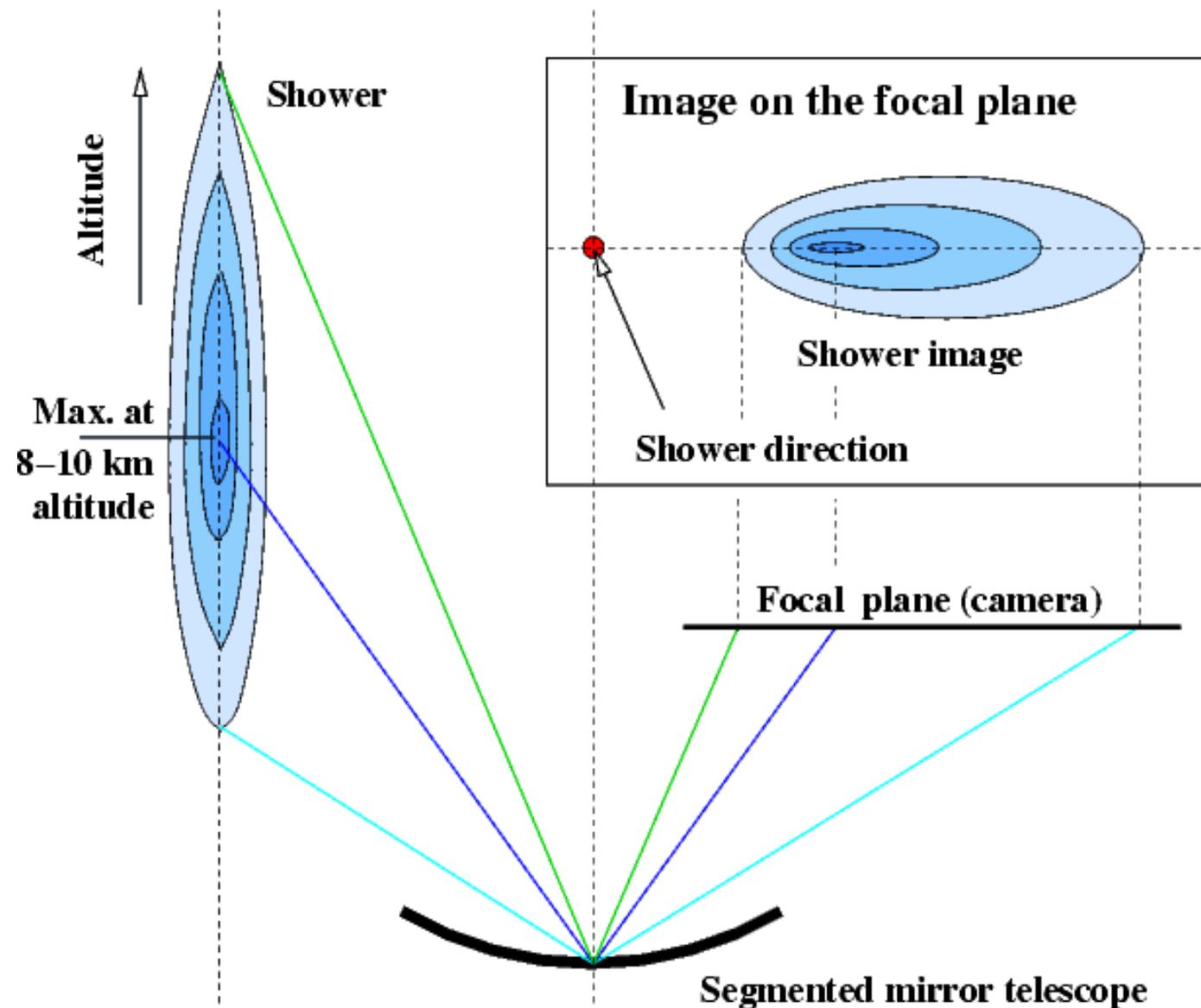
H.E.S.S.

HEGRA

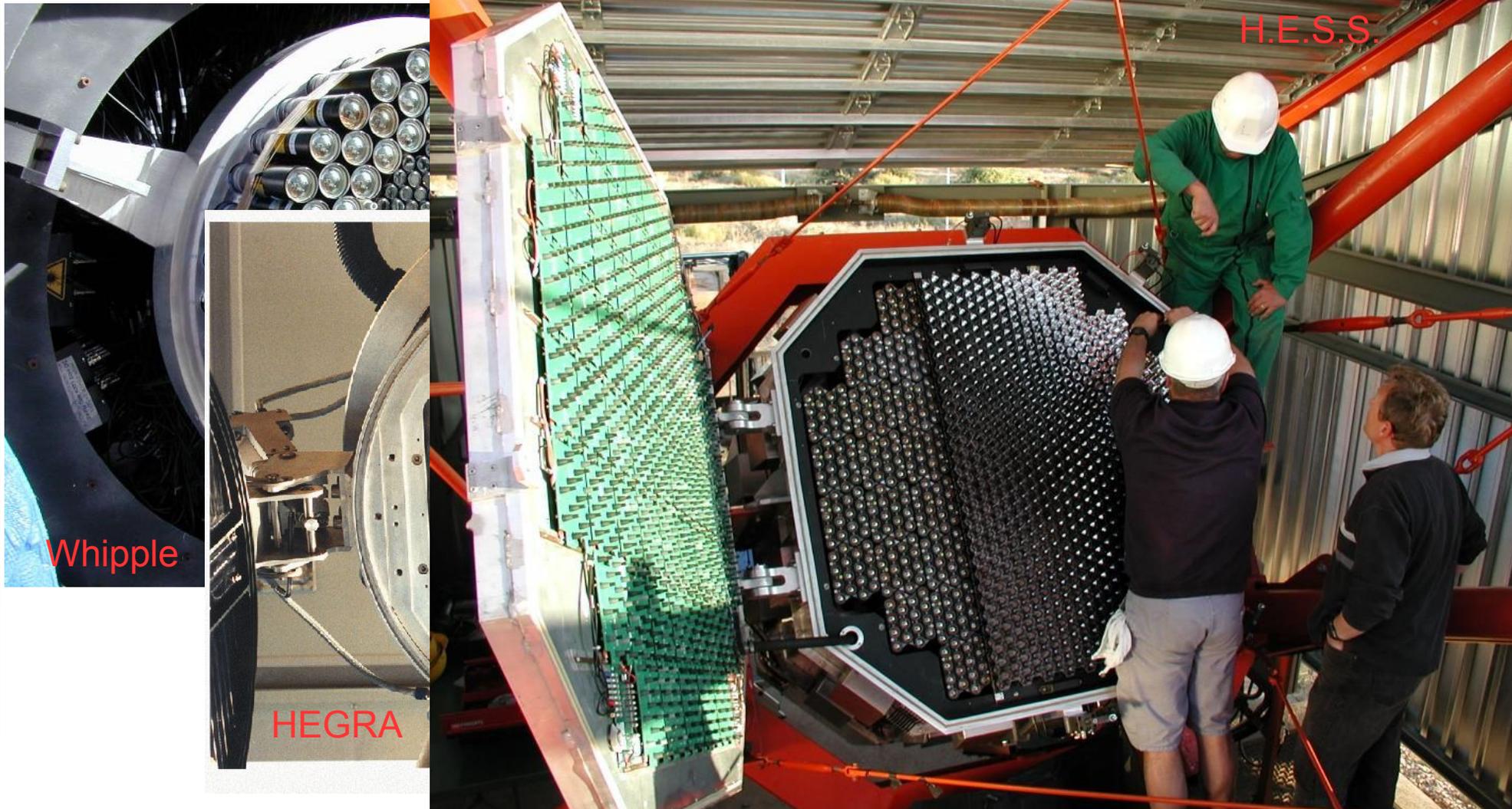
Whipple



Shower imaging by telescopes

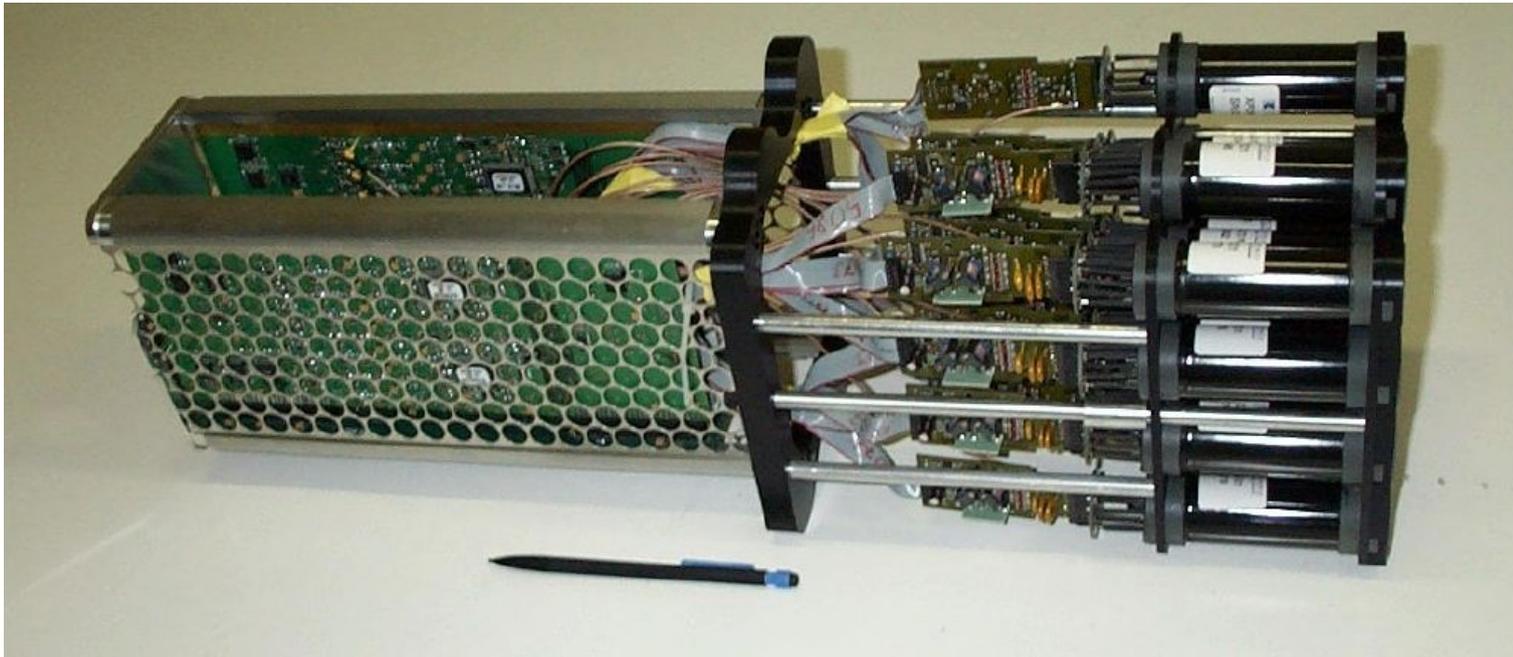


Cameras of Cherenkov telescopes



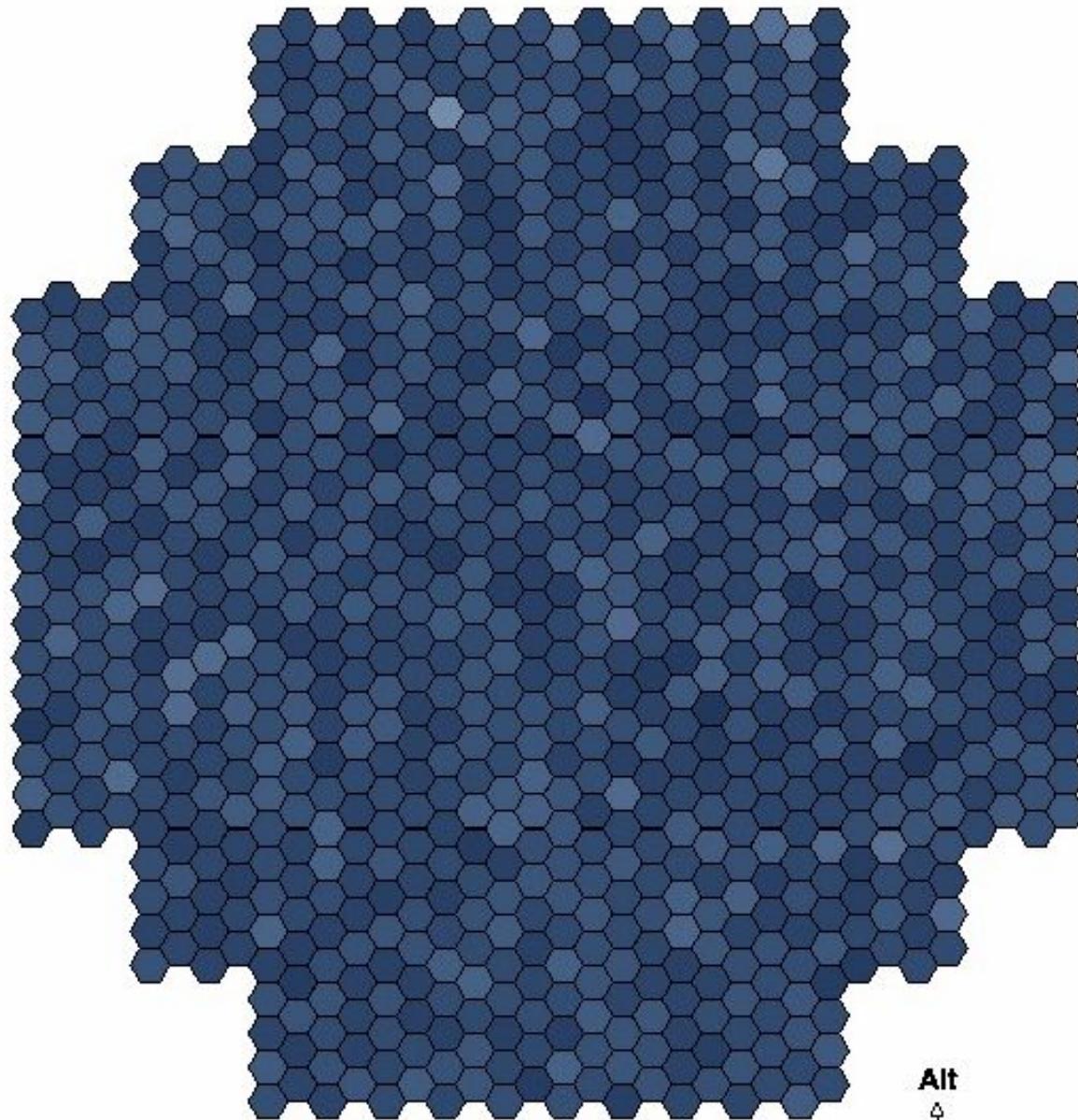
H.E.S.S. camera components

Camera organized with 60 drawers of 16 PMTs each:

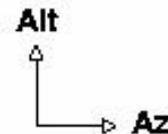


H.E.S.S. cameras contain complete electronics and readout system. Connected to outside world by 4 optical fibres.

0.0 Nanosekunden (Milliardstel Sekunden)



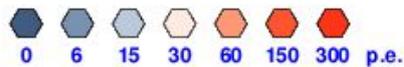
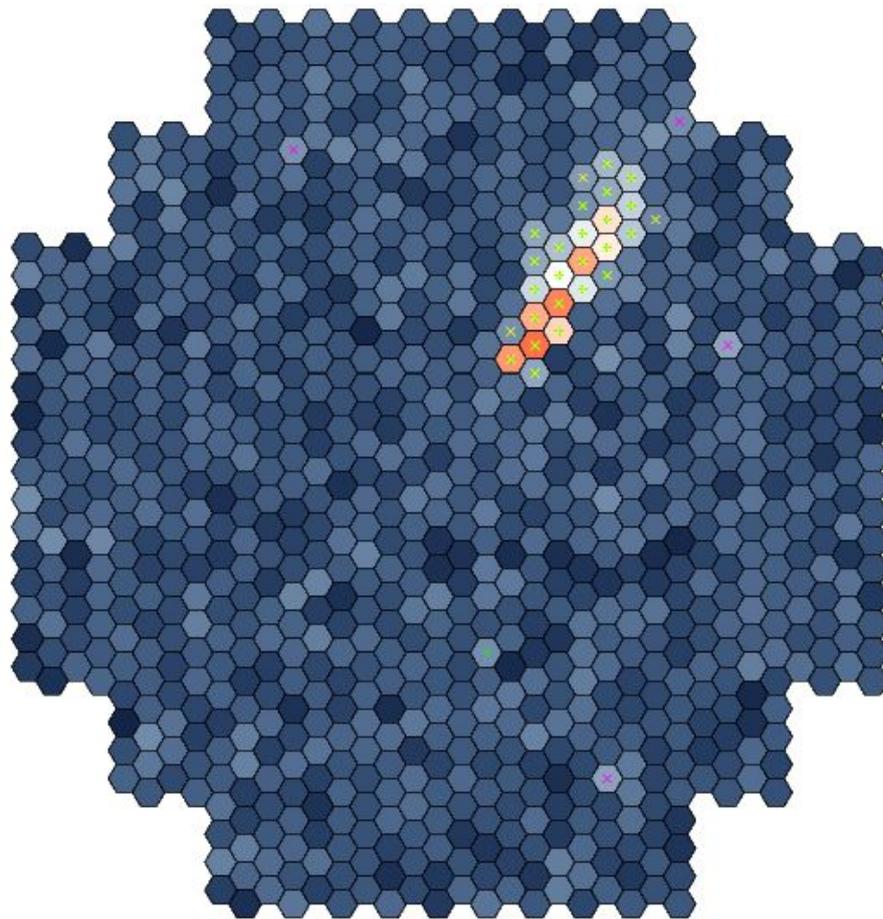
Intensität: 0 6 15 30 60 150 300 p.e.



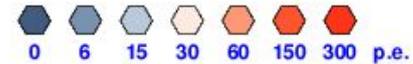
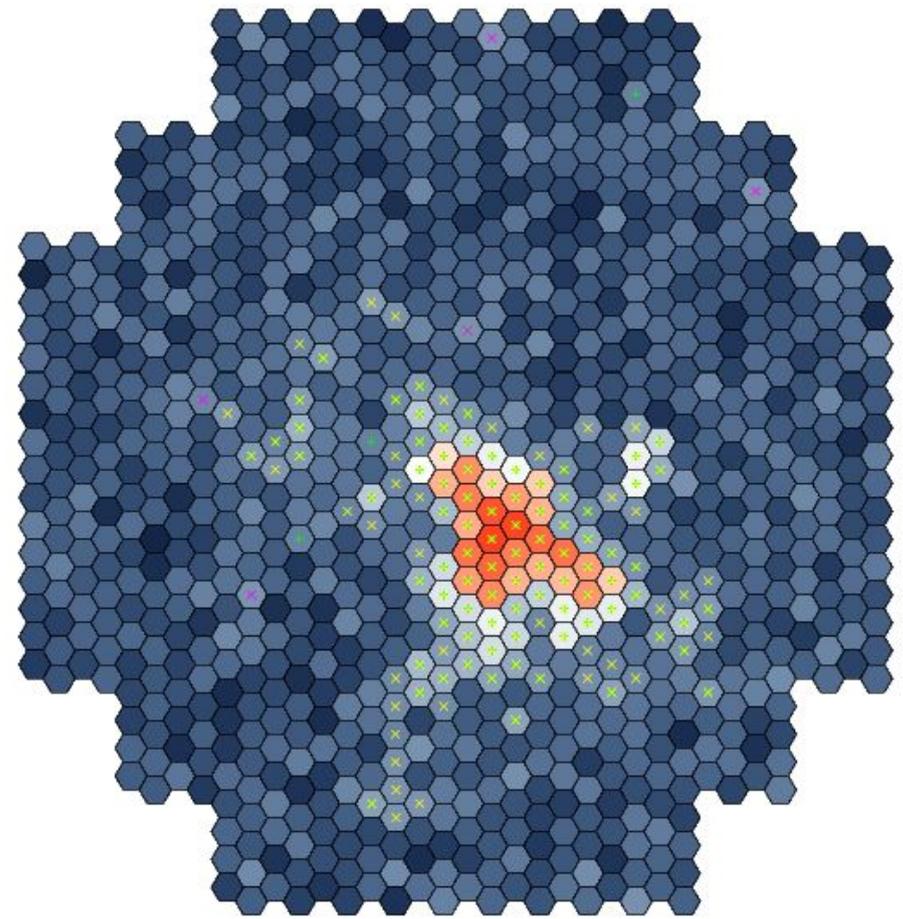
Signal is only nanoseconds long.
Nightsky background of 10^8 p.e./s/pixel.

Fast cameras are needed!

Gamma and hadron showers

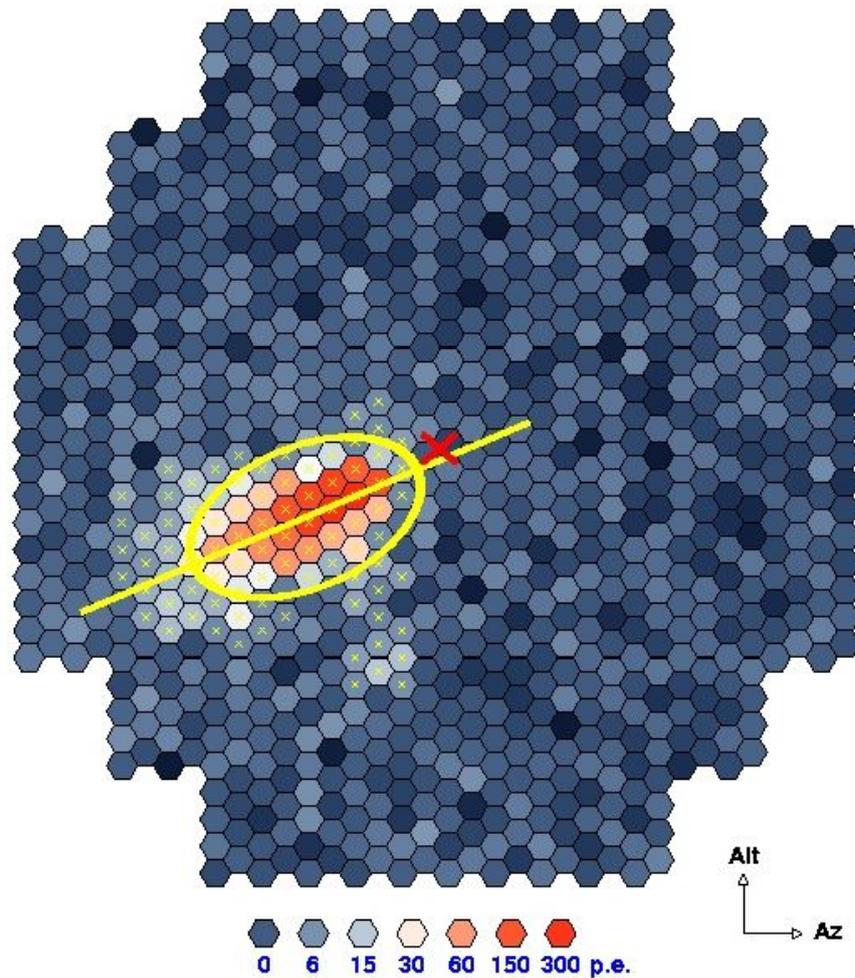


Gamma shower

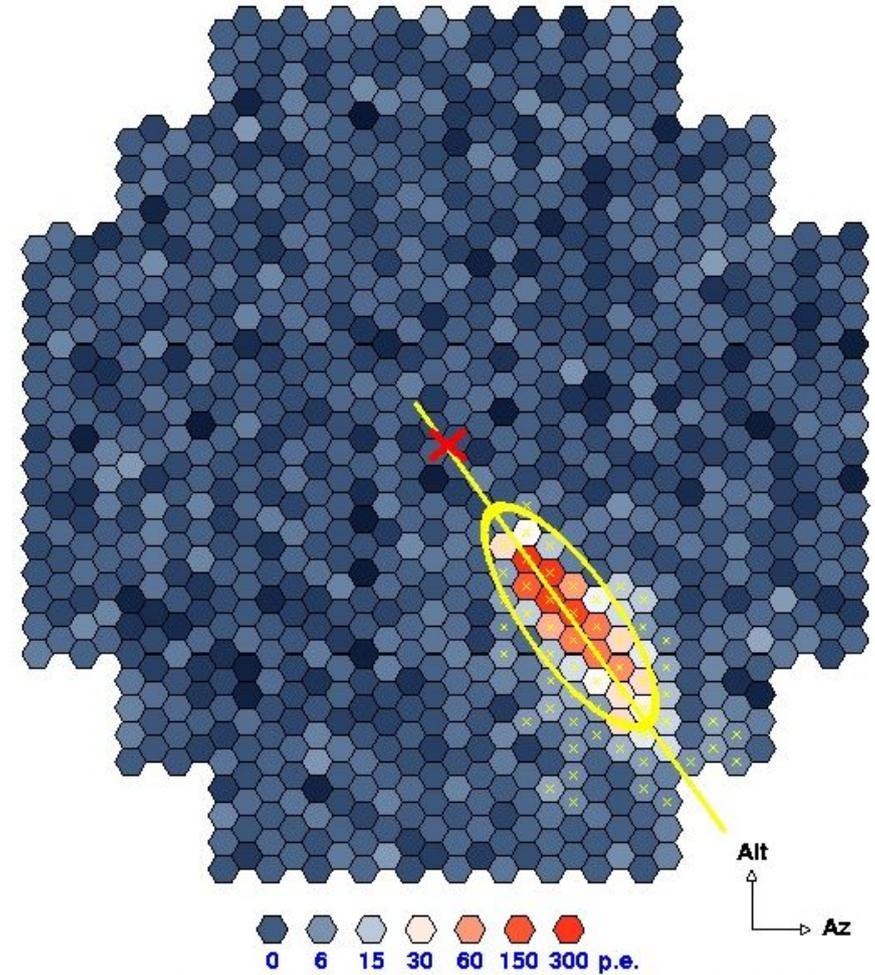


Hadron shower

Stereoscopic observations

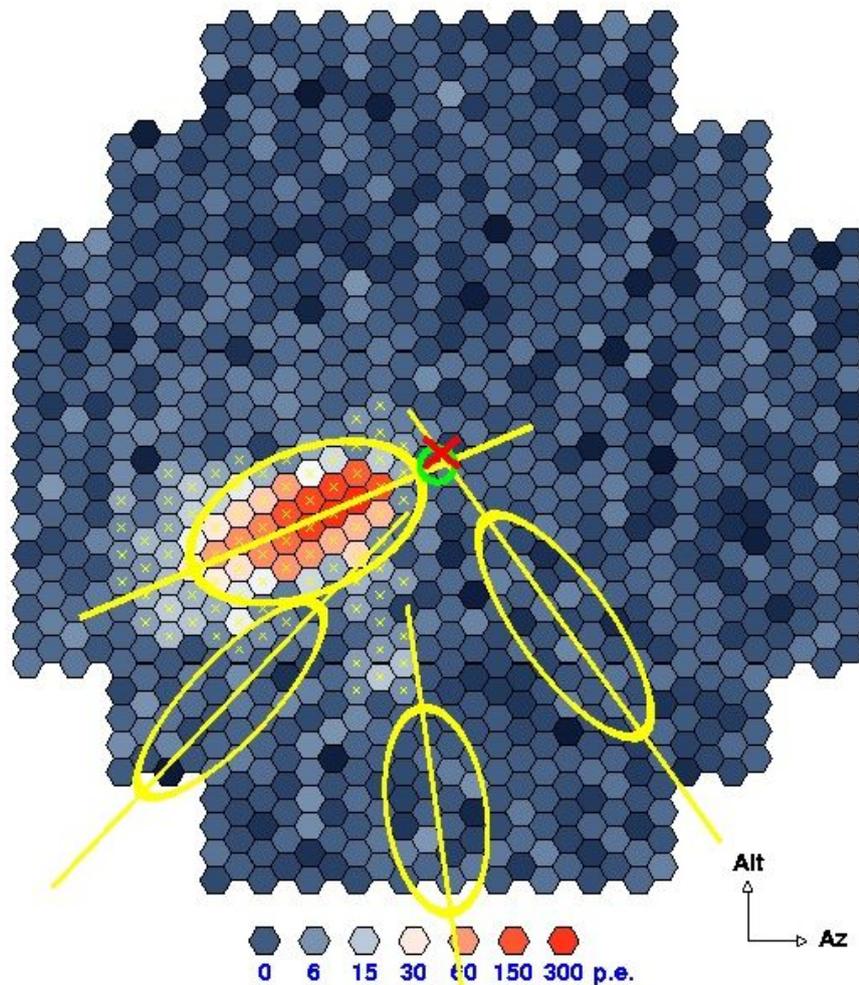


Primärteilchen: Gamma von 1.000 TeV Energie in 49 m Abstand



Primärteilchen: Gamma von 1.000 TeV Energie in 101 m Abstand

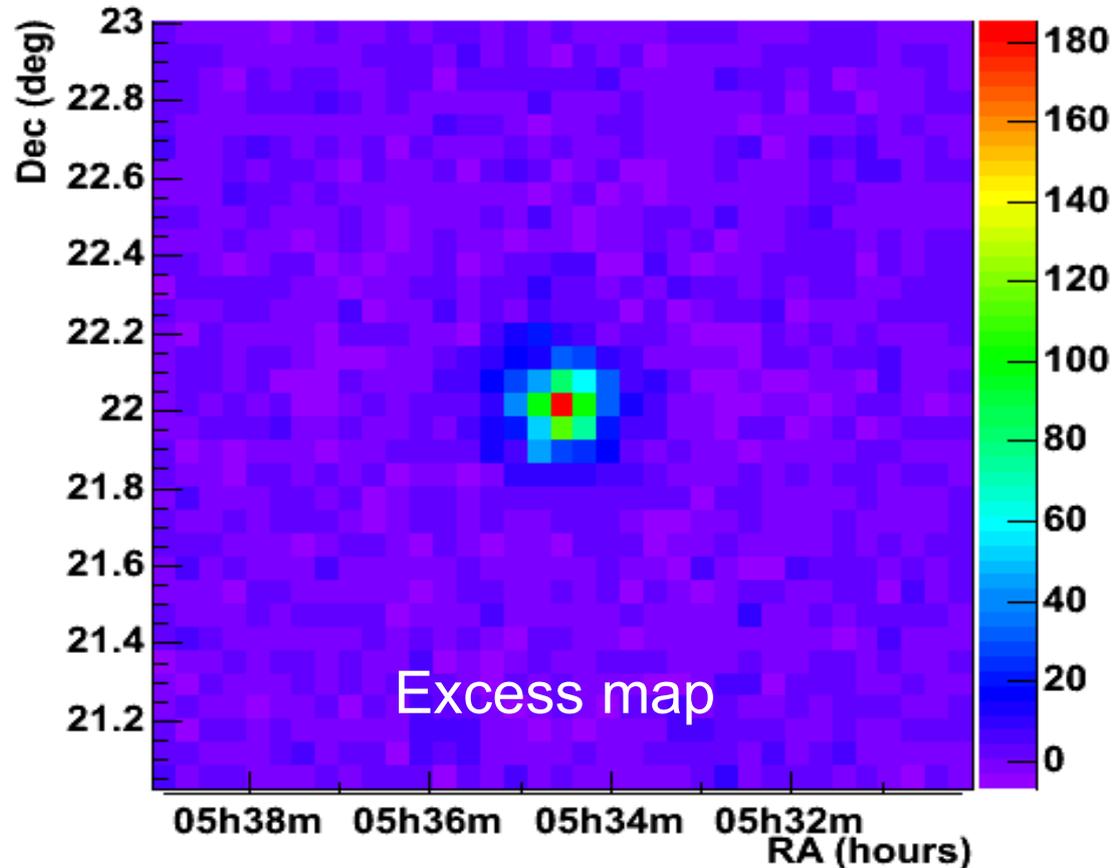
Stereoscopic observations



Primärteilchen: Gamma von 1.000 TeV Energie in 49 m Abstand

- Advantages:
- ◆ No local muon triggers.
 - ◆ Shower reconstruction in 3-D including height of shower maximum.
 - ◆ Better gamma-hadron separation.

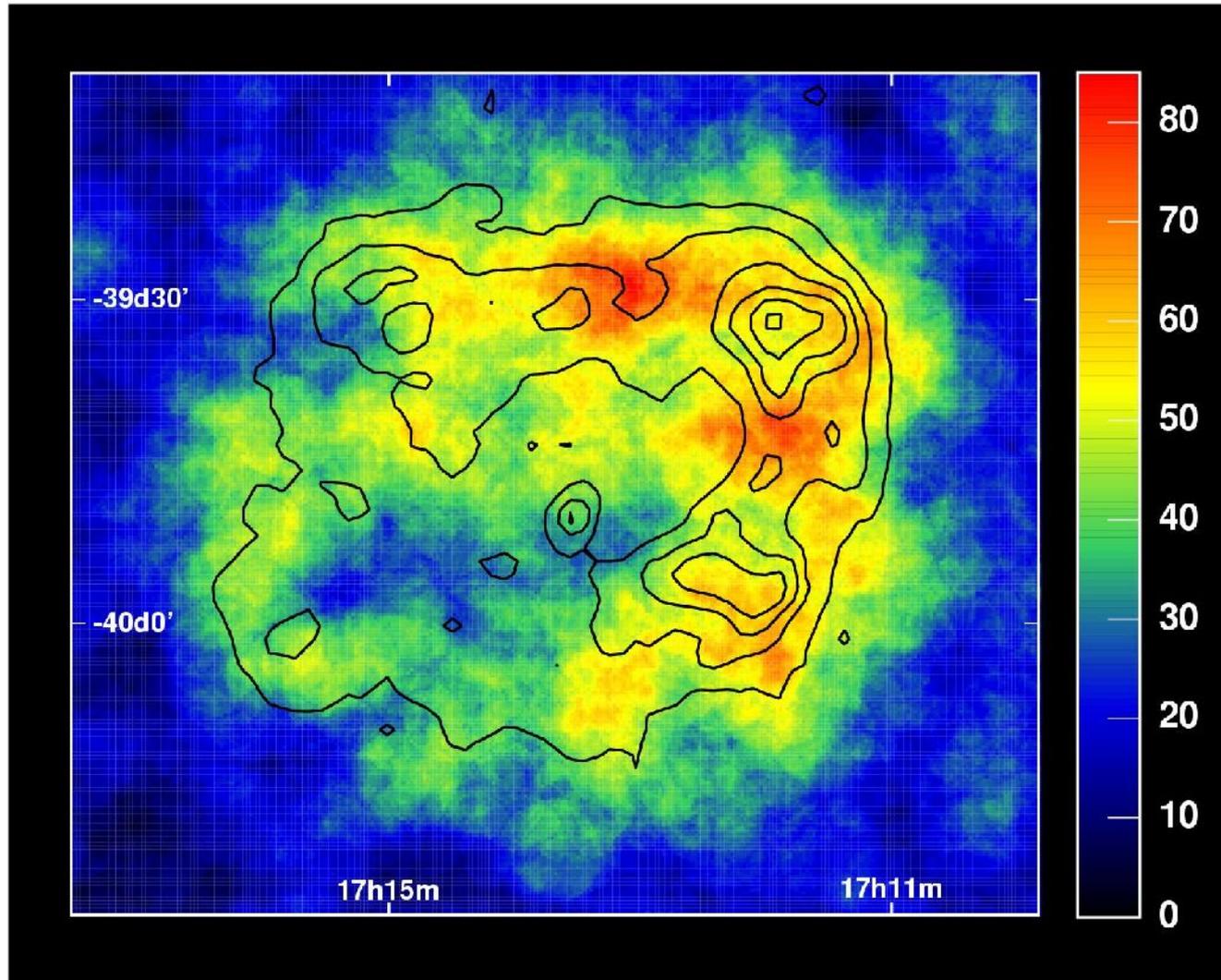
Example source: Crab Nebula



Crab Nebula is an unresolved source.

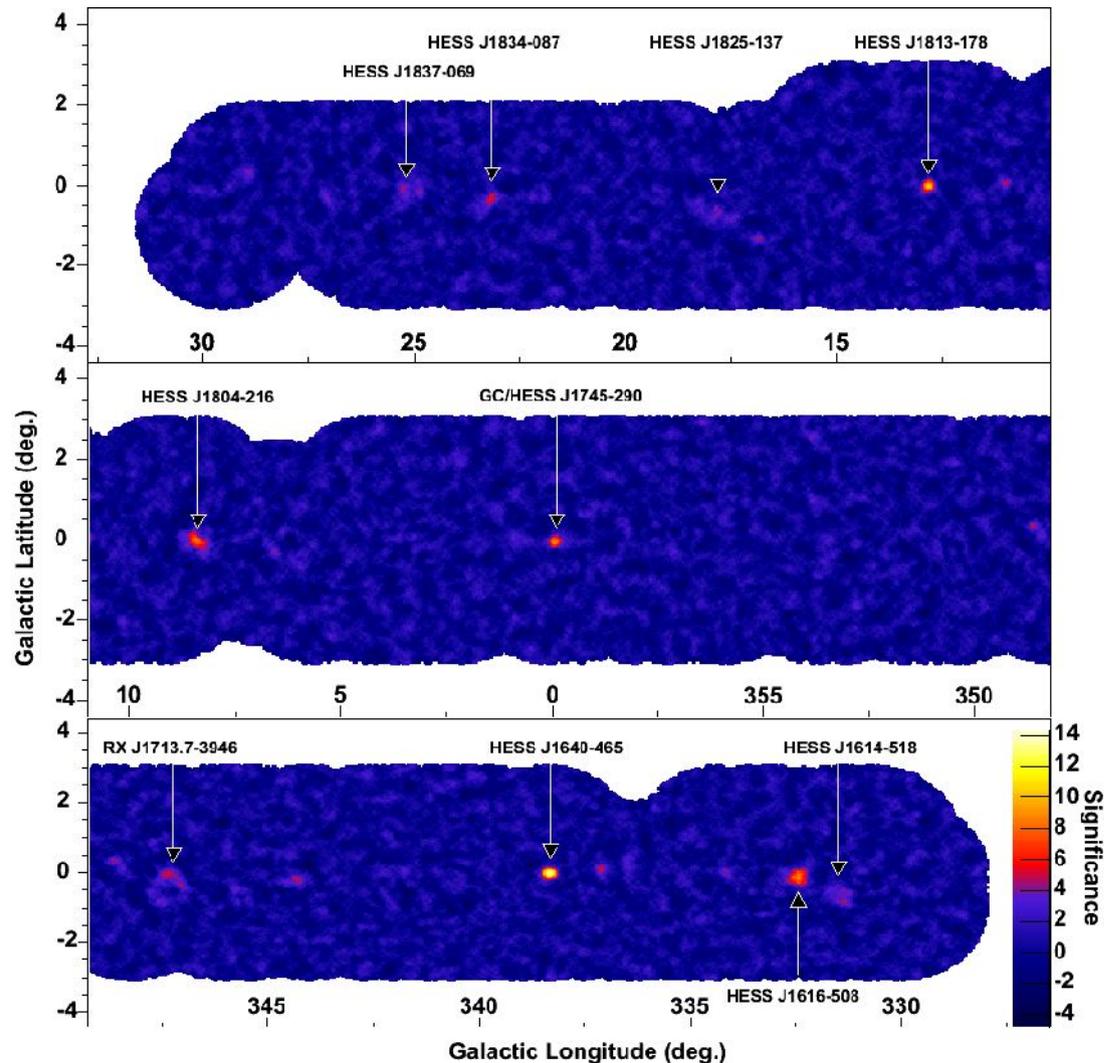
Distribution shows gamma-ray point-spread function (PSF).
Resolution: 0.1° .

Another one: RX J1713.7-3946



Nature 432, 75 (2004)

Many sources in the Galactic plane

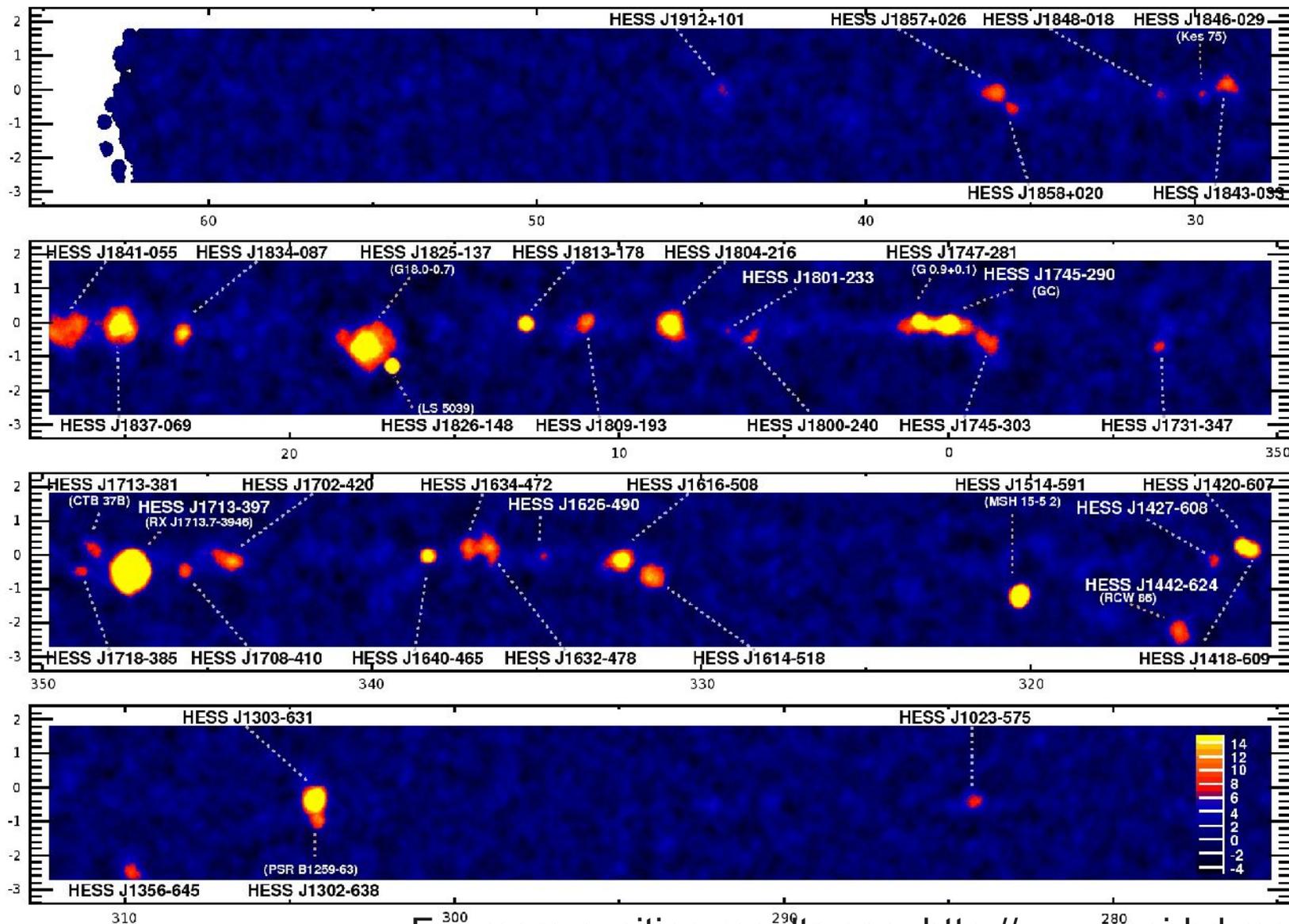


Galactic plane survey revealed many more Galactic gamma-ray sources than we knew before.

Science 307, 1938 (2005)

For more exciting results see: <http://www.mpi-hd.mpg.de/HESS/>

Many sources in the Galactic plane



For more exciting results see: <http://www.mpi-hd.mpg.de/HESS/>

Simulation outline

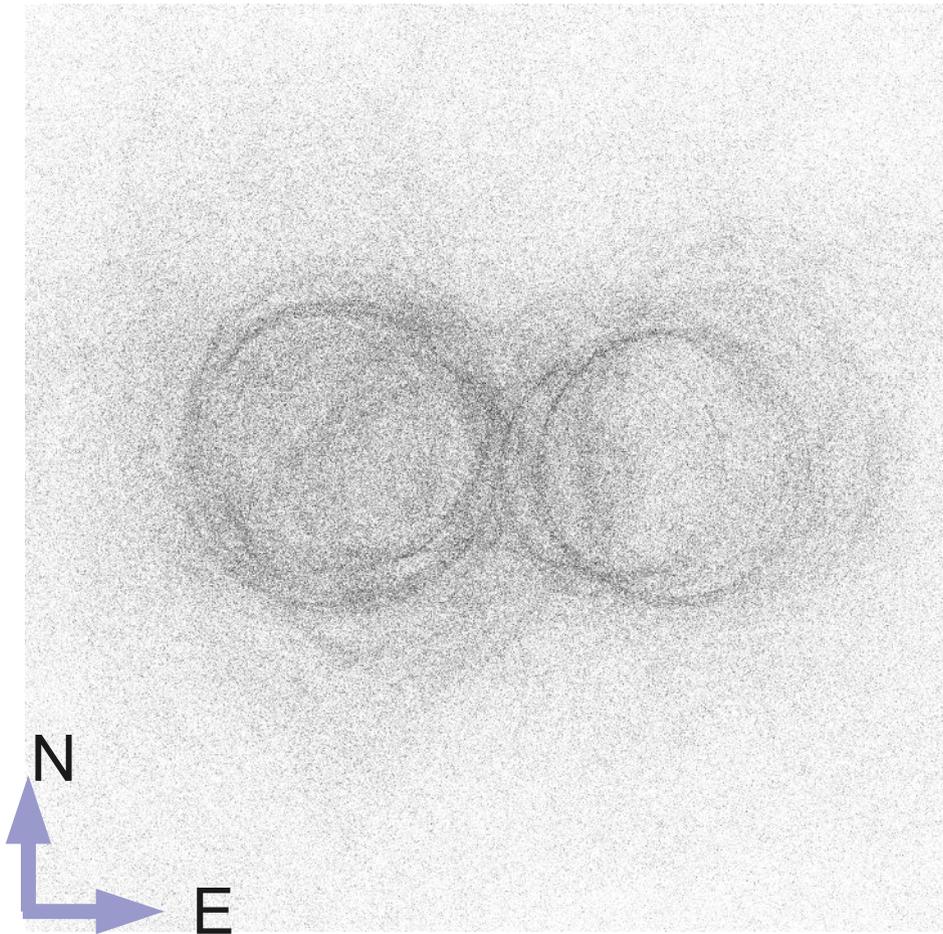
Two stages:

- ◆ Shower simulation (CORSIKA)
 - ◆ Particle interactions, decays, and propagation in the atmosphere.
 - ◆ Emission of Cherenkov light.
 - ◆ Recording of the light arriving at a telescope.
- ◆ Telescope simulation (sim_telarray)
 - ◆ Optical ray tracing.
 - ◆ Response of PMTs as light sensors.
 - ◆ Electronics response and shower detection.
 - ◆ Digitization and conversion to “raw data” like in a real experiment.

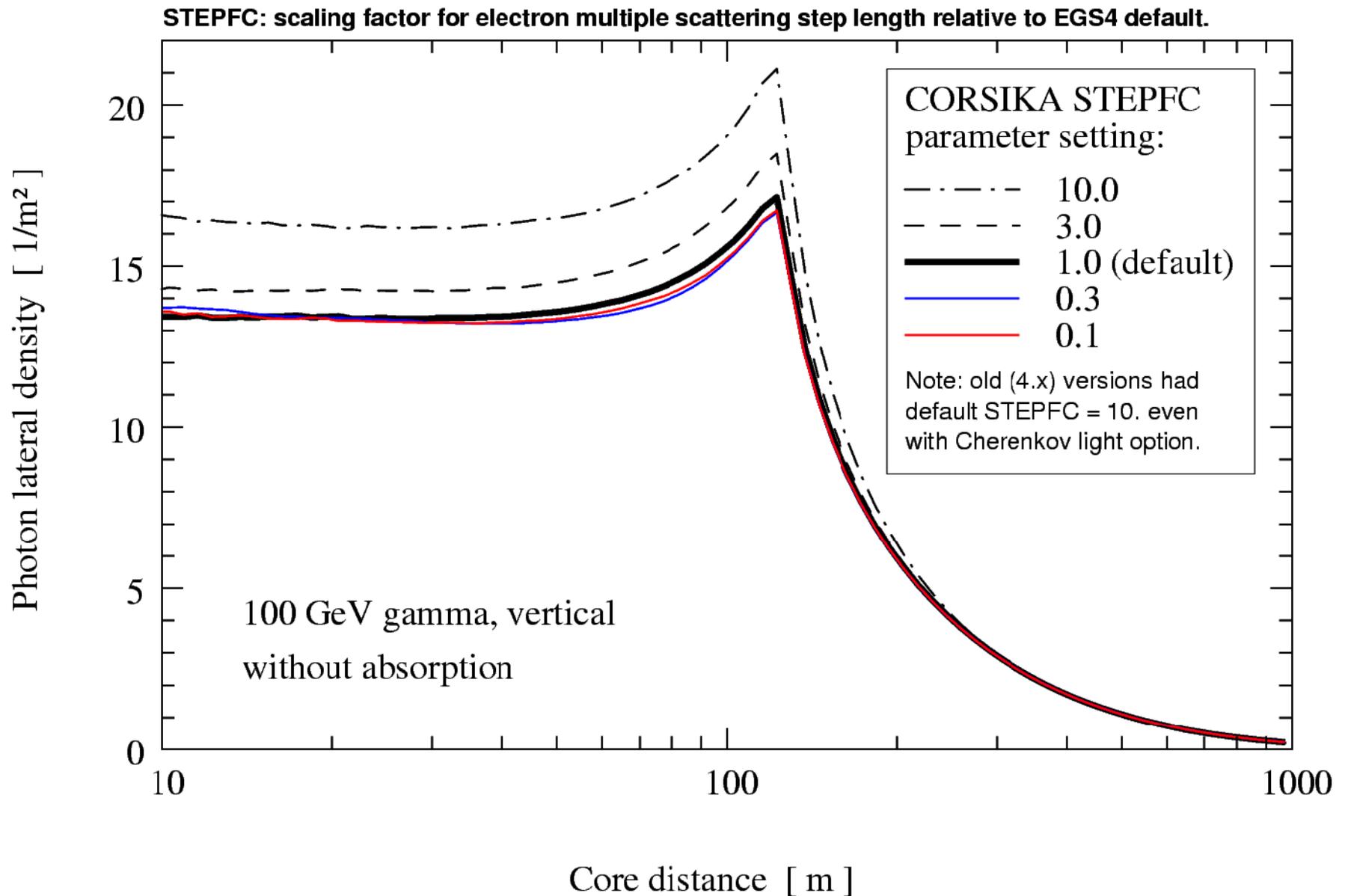
Shower simulation

- ◆ CORSIKA provides the framework, including
 - ◆ interface to interaction models,
 - ◆ particle propagation in the atmosphere and geomagnetic field,
 - ◆ decays of unstable particles,
 - ◆ emission of Cherenkov light,
 - ◆ different storage formats.
- ◆ Particle interactions are handled by plug-in interaction model codes
 - ◆ EGS4 for interactions of gammas and electrons,
 - ◆ QGSJET/SIBYLL/... for high energy particles,
 - ◆ GHEISHA/URQMD/FLUKA for low energy particles

Geomagnetic field is important



Simulation: step size is important



Simulating H.E.S.S. and other IACT instruments

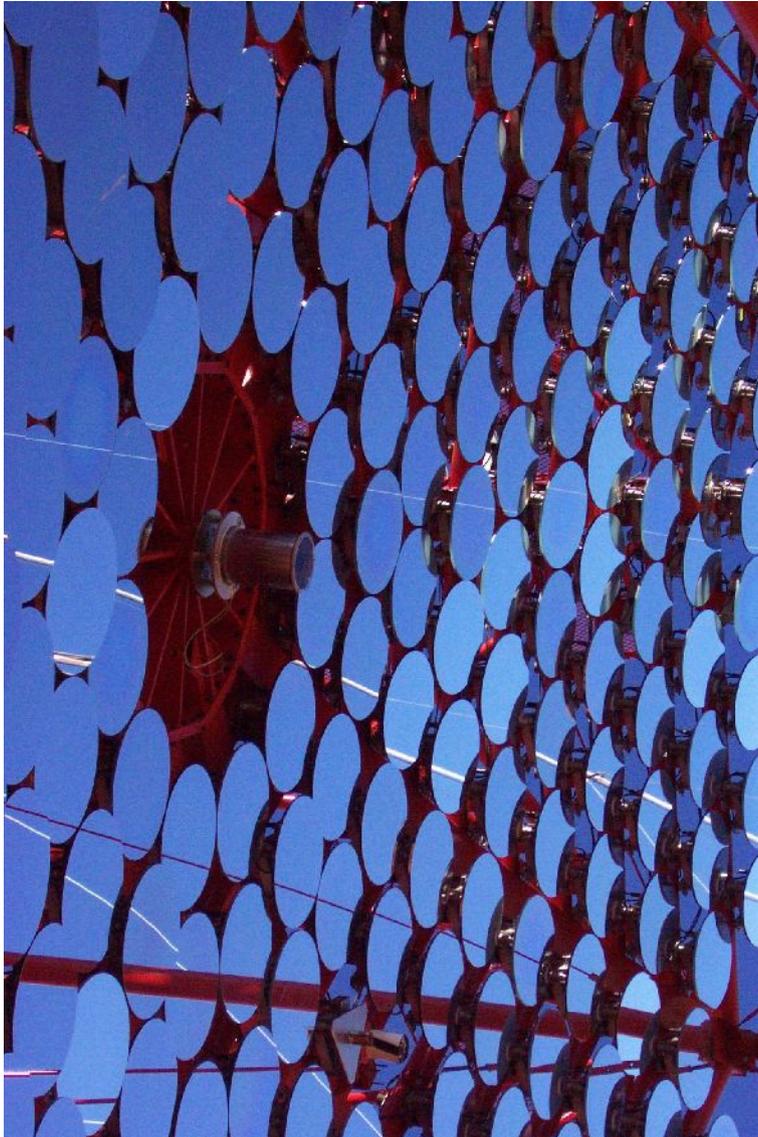
- ♦ All results of the H.E.S.S. experiment are cross-checked by at least two independent processing schemes, with separate calibration and analysis schemes, with gamma-ray selection based on completely different Monte Carlo simulations.
- ♦ I will describe here only the CORSIKA / sim_telarray (sim_hessarray) line of simulations.

Optics simulation

Optics simulation includes

- ◆ Atmospheric transmission.
- ◆ Ray-tracing and reflection on segmented mirrors.
- ◆ Shadowing by camera support structure.
- ◆ Angle-of-incidence dependent acceptance in pixels (PMTs with Winston-cone funnels).
- ◆ Wavelength dependence of mirror reflectivity, funnel efficiency, PMT quantum+collection efficiency.

Optics simulation: ray-tracing



- ◆ Photons are fully ray-traced on non-perfect mirror facets with random misalignments.
- ◆ Acceptance in camera depending on angle of incidence.
- ◆ Shadowing by camera support optional.

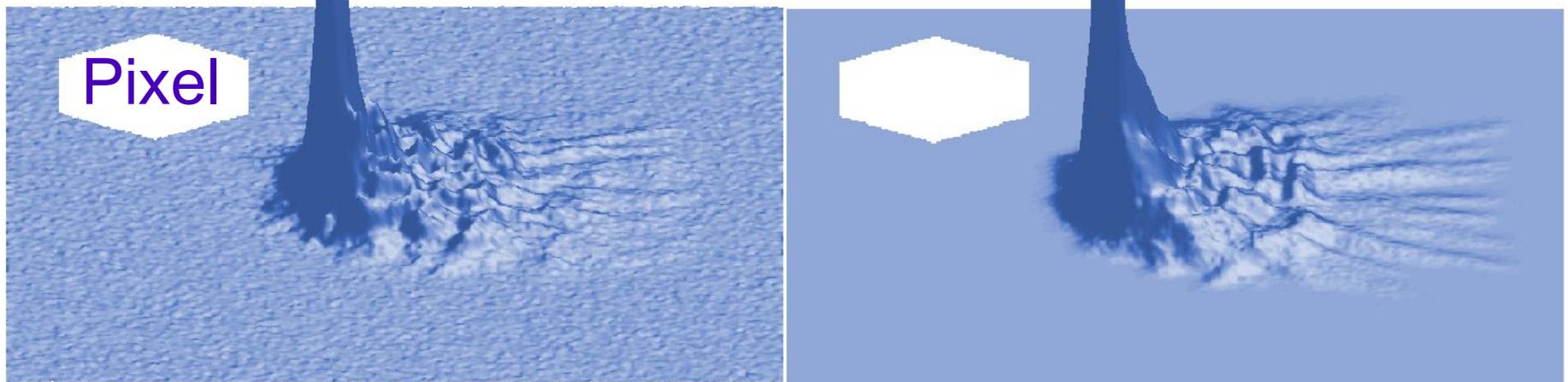
Star light imaging

Measurement of star light imaged on the closed camera lid by a dedicated CCD camera mounted at the dish centre.

CT 4, measured

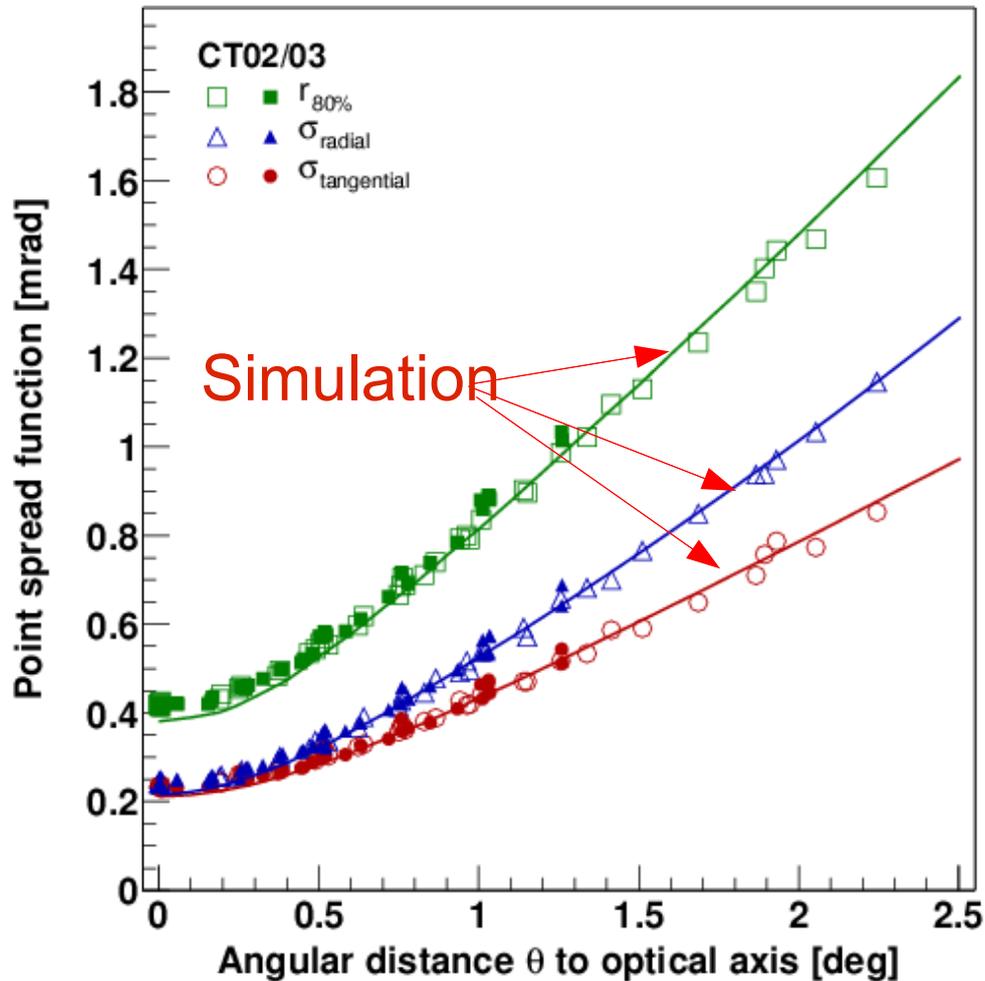
sim_telarray ray-tracing

2.3° off-axis



R. Cornils

Star light imaging



Star-light point spread function depending on off-axis angle is fully reproduced with just the measured mirror quality as well as the mounting and alignment accuracy as inputs.

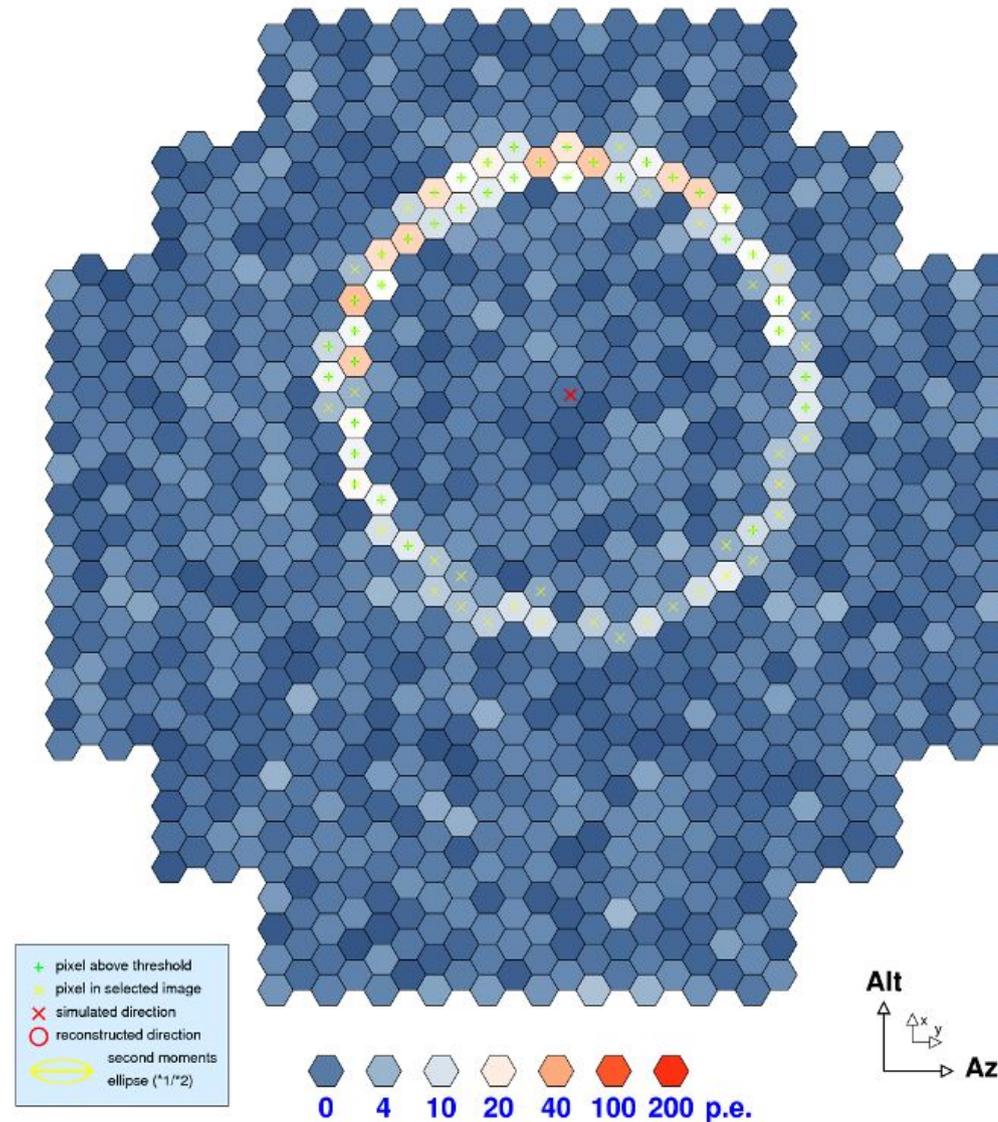
Pulse shapes and electronics

- ♦ PMT amplitude distribution, including afterpulses for night sky photons
⇒ calculate NSB (night sky backgr.) triggers.
- ♦ Pulse shapes at input of comparator or discriminator.
- ♦ Pulse shapes of comp./discr. outputs to tel. trigger.
- ♦ Pulse shapes at analog ring sampler / FADC inputs (multiple gains).
- ♦ All configurable.

Electronics simulated in detail

- ◆ Pixel trigger by fast comparator, then summing up comparator outputs and forming a sector trigger (=telescope trigger).
- ◆ System trigger by overlapping of at least two telescope triggers.
- ◆ Read-out with two different pre-amplifier gains, full simulation of raw data with night-sky background (may include stars).

Cross-checks with muons



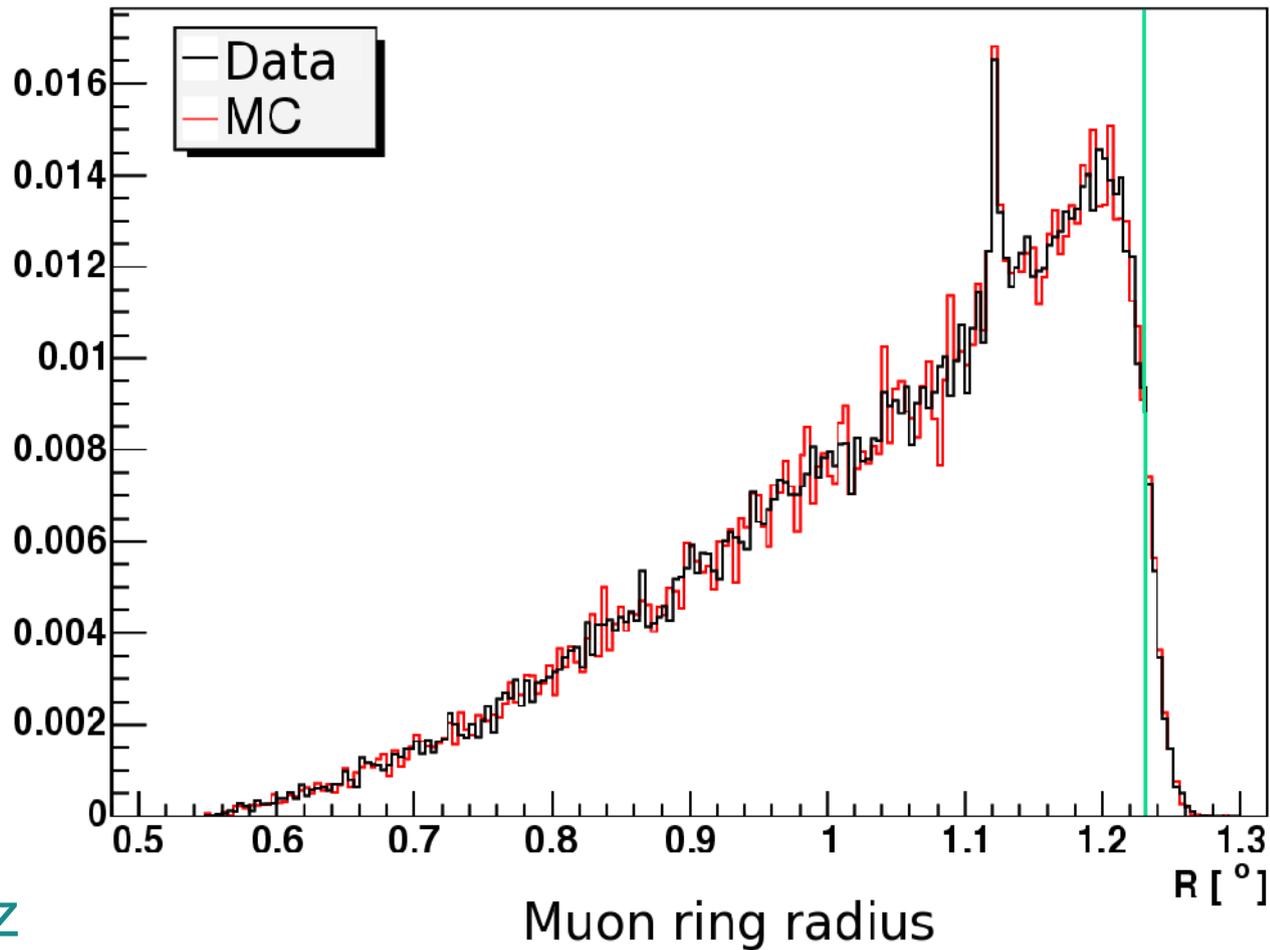
Primary: muon of 0.023 TeV energy at 4 m distance

Cross-checks with muons

- ◆ Muons directly hitting the telescope mirror are imaged onto complete rings. Light of such rings is emitted over (at least) the last 600 m.
- ◆ Maximum physical ring radius is 1.23° (HESS).
- ◆ Slower muons result in rings of smaller opening angle (radius) and experience more multiple scattering (rings more diffuse).

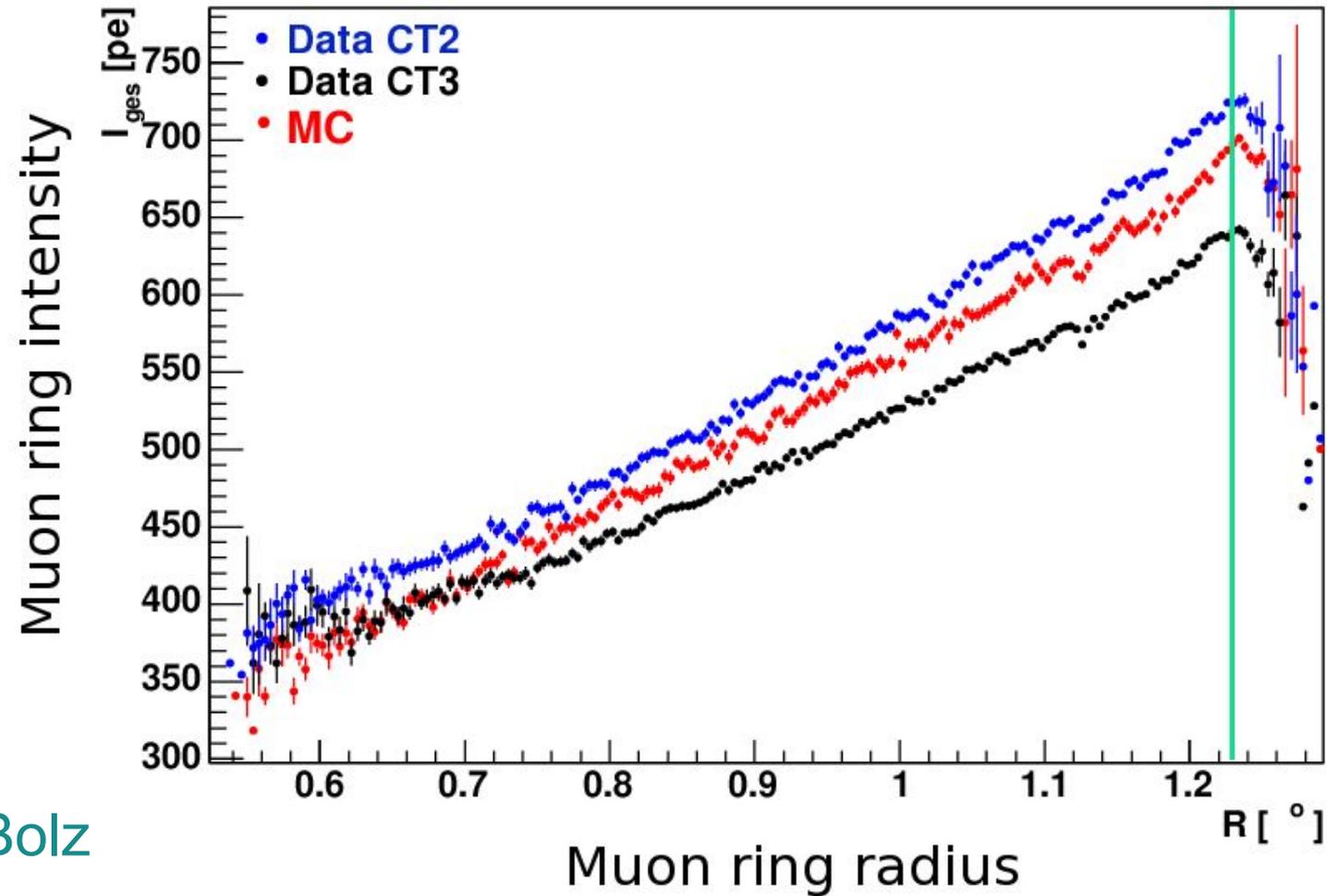
Muon ring radius

Note: requires use of proper muon energy spectrum



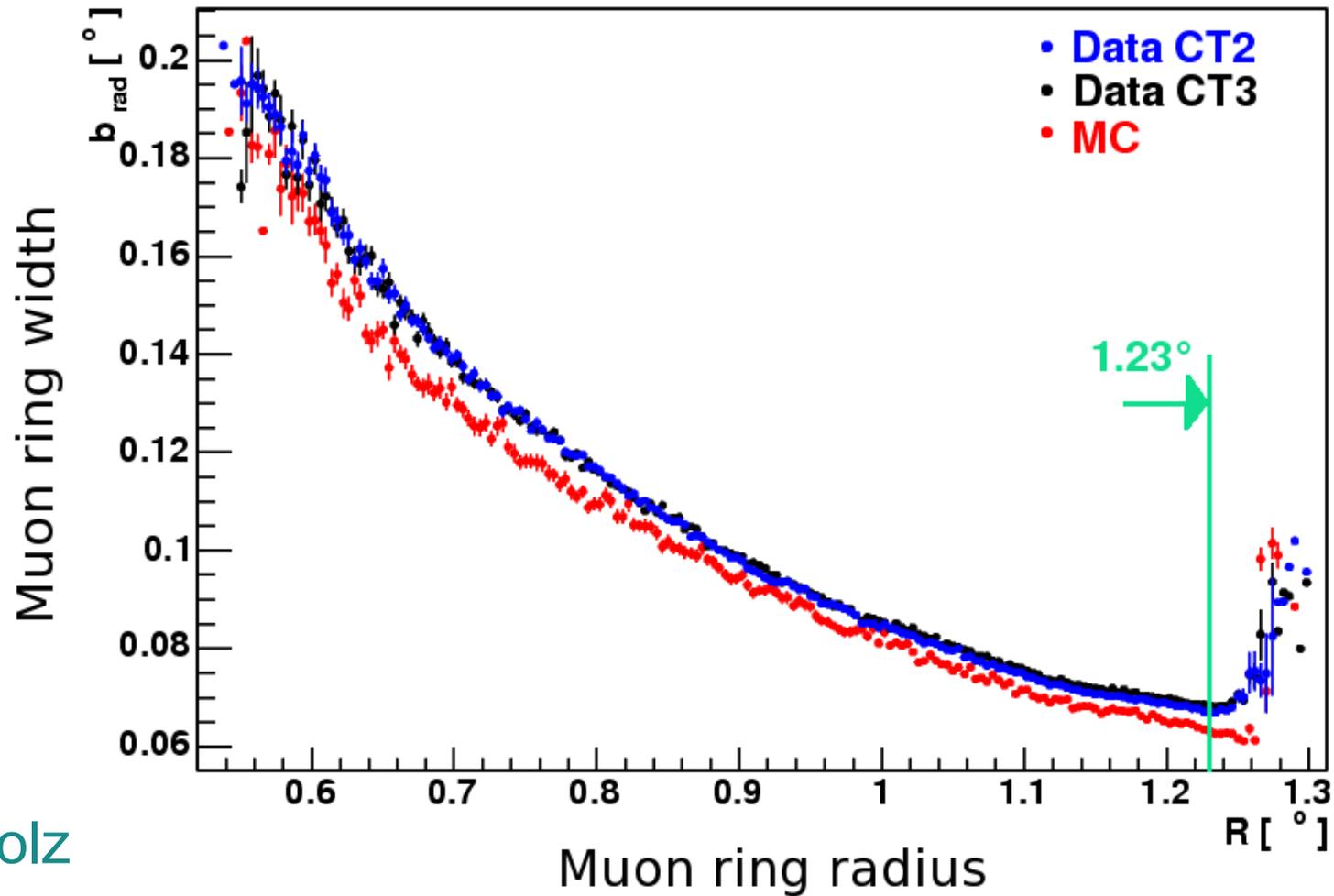
O. Bolz

Muon ring intensity



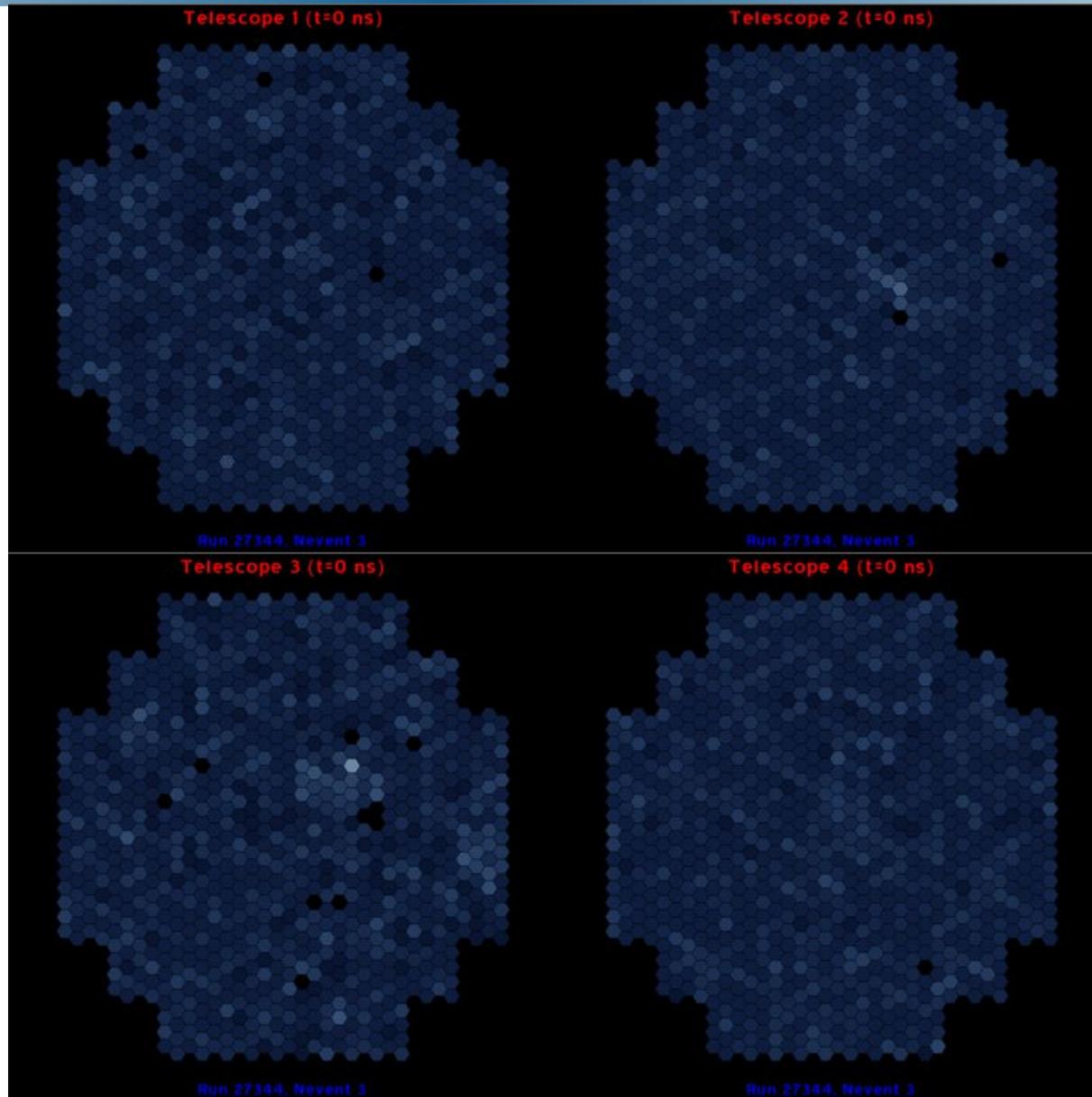
O. Bolz

Muon ring width



O. Bolz

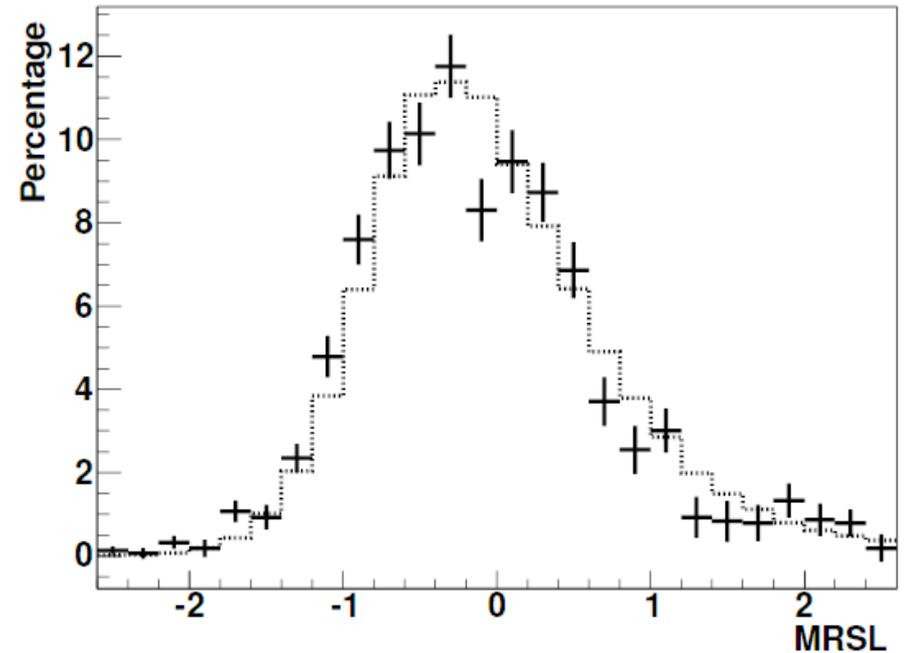
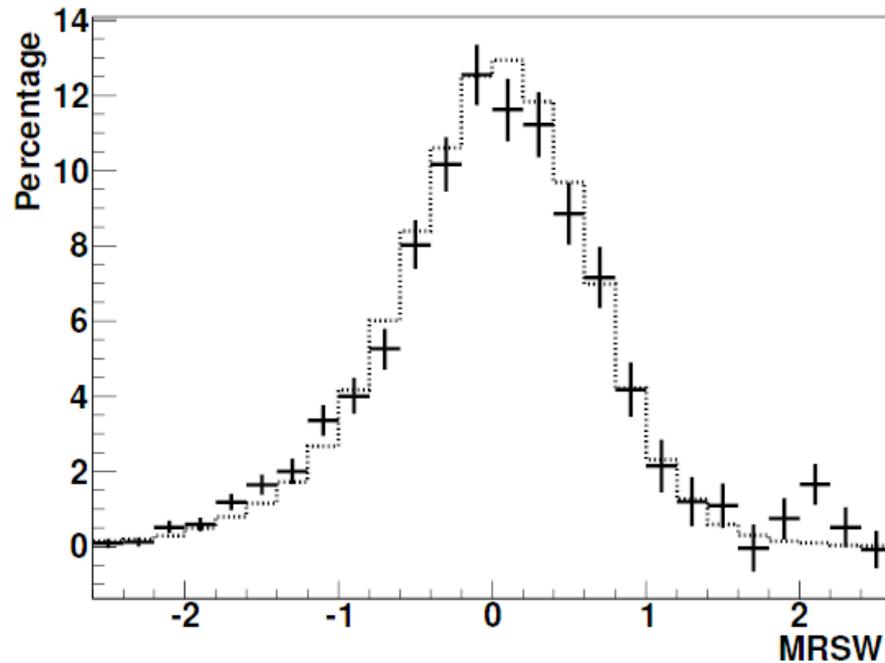
Cross-checks with showers



Cross-checks with showers

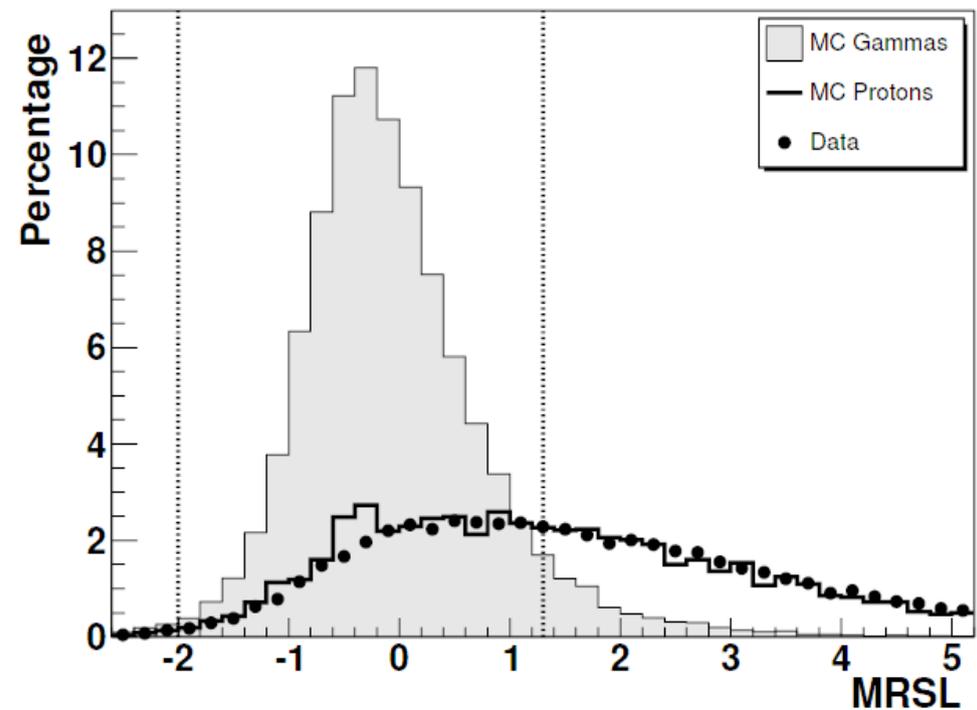
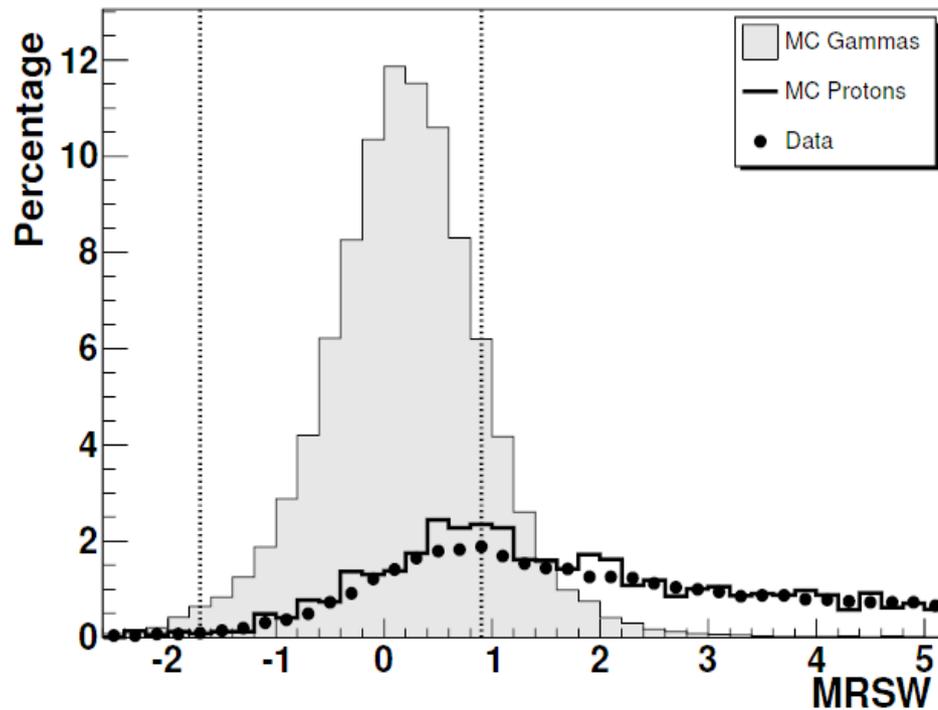
- ◆ Shapes of air shower images can be compared to simulations.
- ◆ This can be done separately with gamma-ray showers from a source and with background (hadron) showers without a gamma-ray source.
- ◆ For gamma-ray showers, a correction for remaining background can be determined from an off-source region in the sky.

Scaled width and length



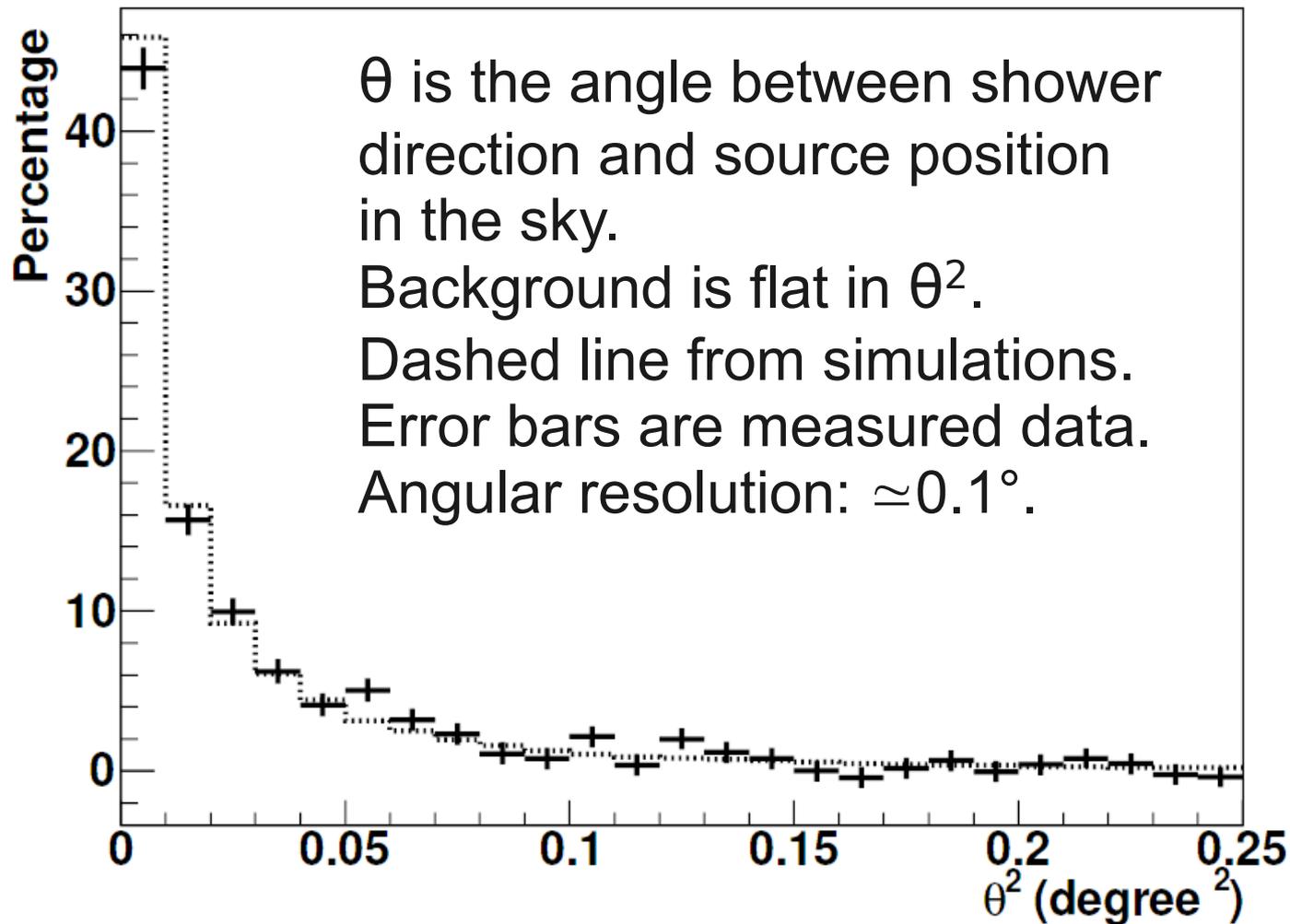
Mean reduced scaled width (MRSW) and length (MRSL) defined such that simulated gamma-rays have mean 0. and variance 1. Here: comparing simulations and measured data from the Crab Nebula.

Scaled width and length



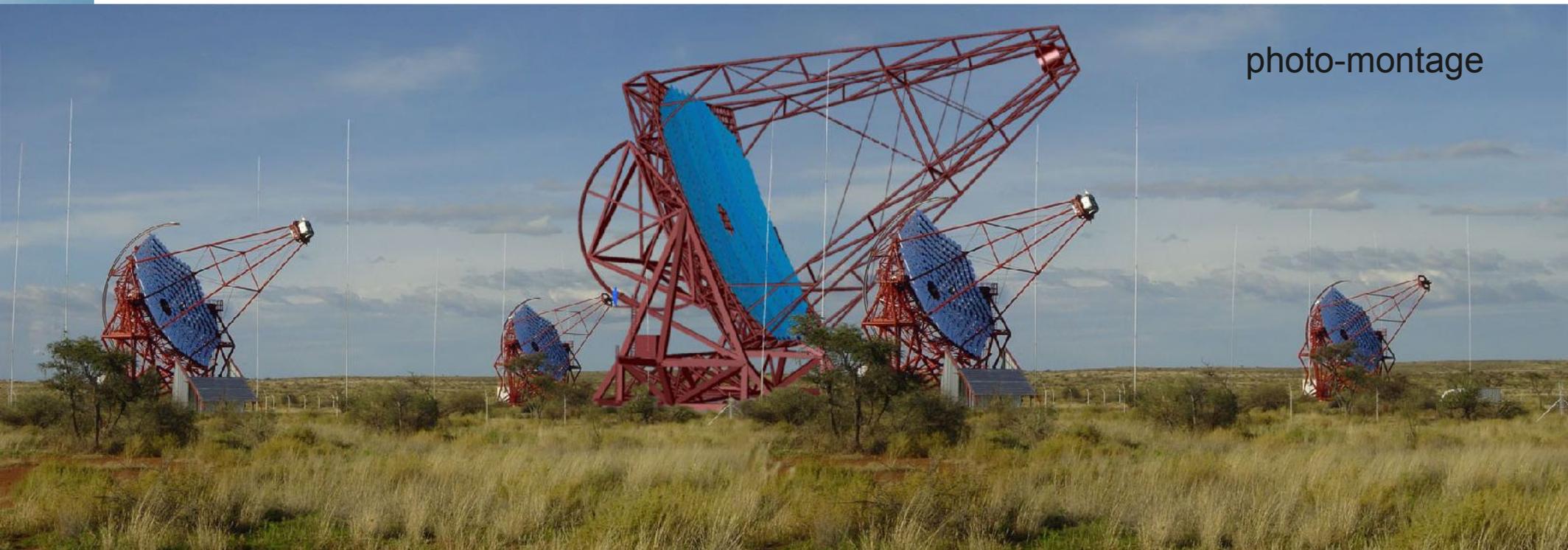
For hadrons both MRSW and MRSL are larger. Vertical bars indicate typical cut parameters for rejecting most hadrons and losing only few gamma rays.

The γ -ray point spread function



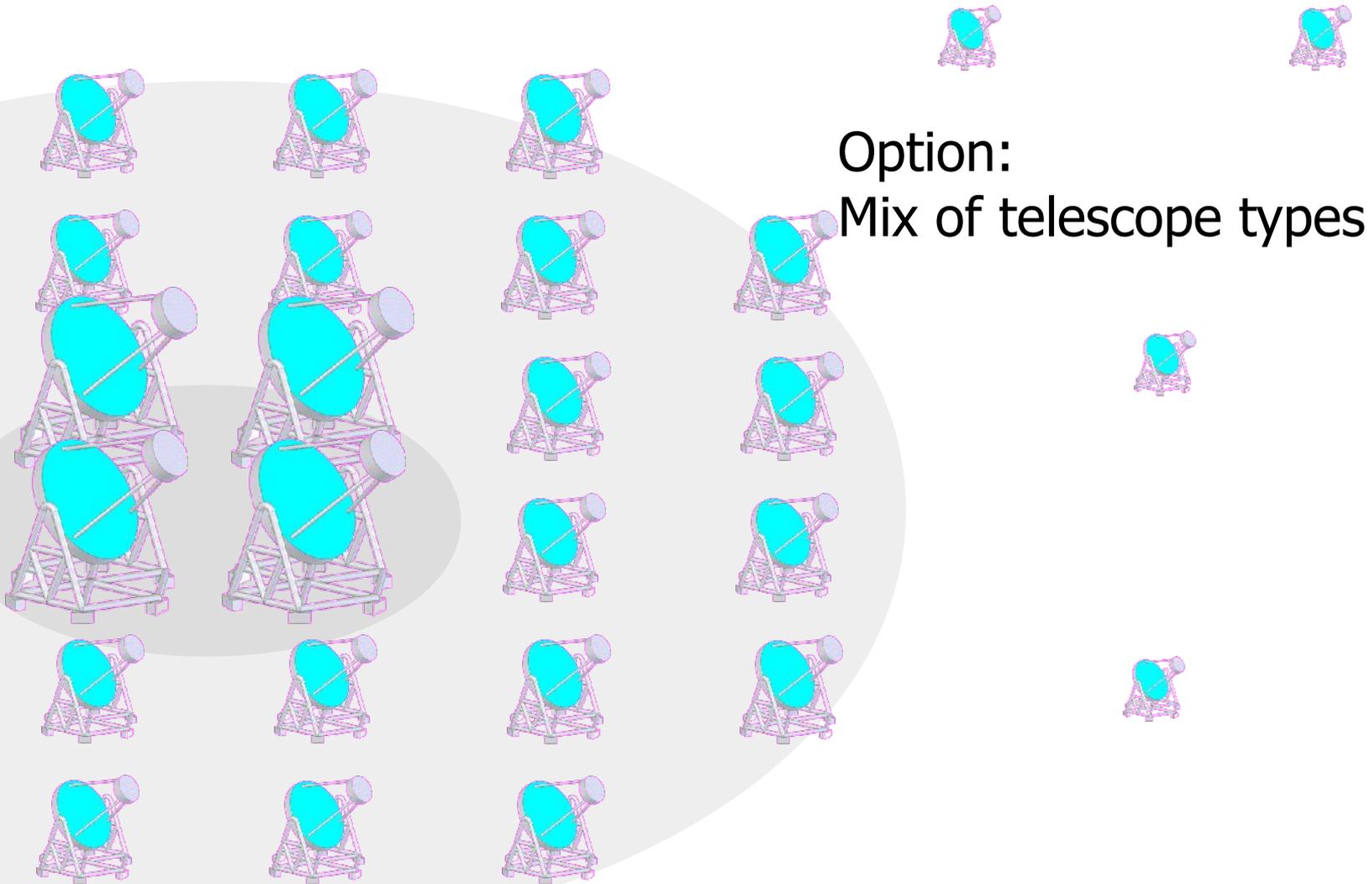
Crab Nebula is consistent with a point source (actual size is much smaller than the PSF).

Future instruments: H.E.S.S. II



H.E.S.S. II is a 28 m diameter (600 m² mirror area) telescope, currently under construction in the centre of the four H.E.S.S. I telescopes.

Future instruments: CTA (Cherenkov Telescope Array)



Not to scale !

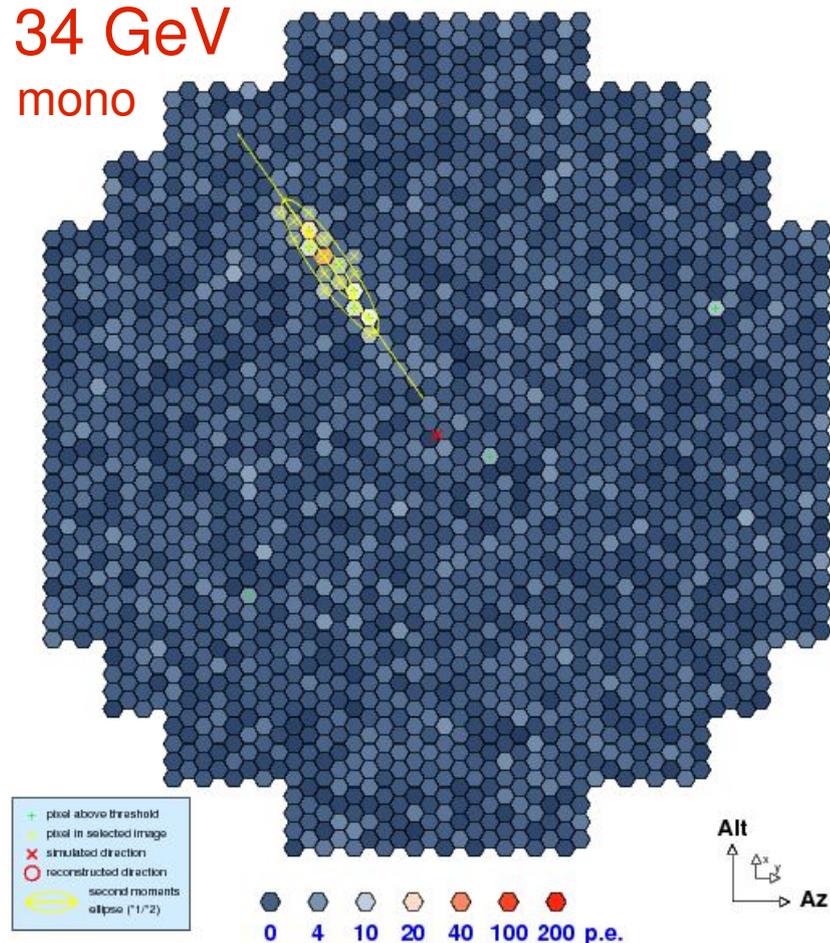
Simulations for future instruments

- ◆ Completed and ongoing simulations for H.E.S.S. II (28 m diameter telescope, under construction in Namibia).
- ◆ Simulations for high-altitude low-threshold instruments (à la 5@5).
- ◆ Simulations for possible designs of extended telescope arrays (more telescopes, wider energy range than current instruments): CTA.

Simulated H.E.S.S.-II γ -events

Sum of signals in selected pixels: 176.7 p.e.

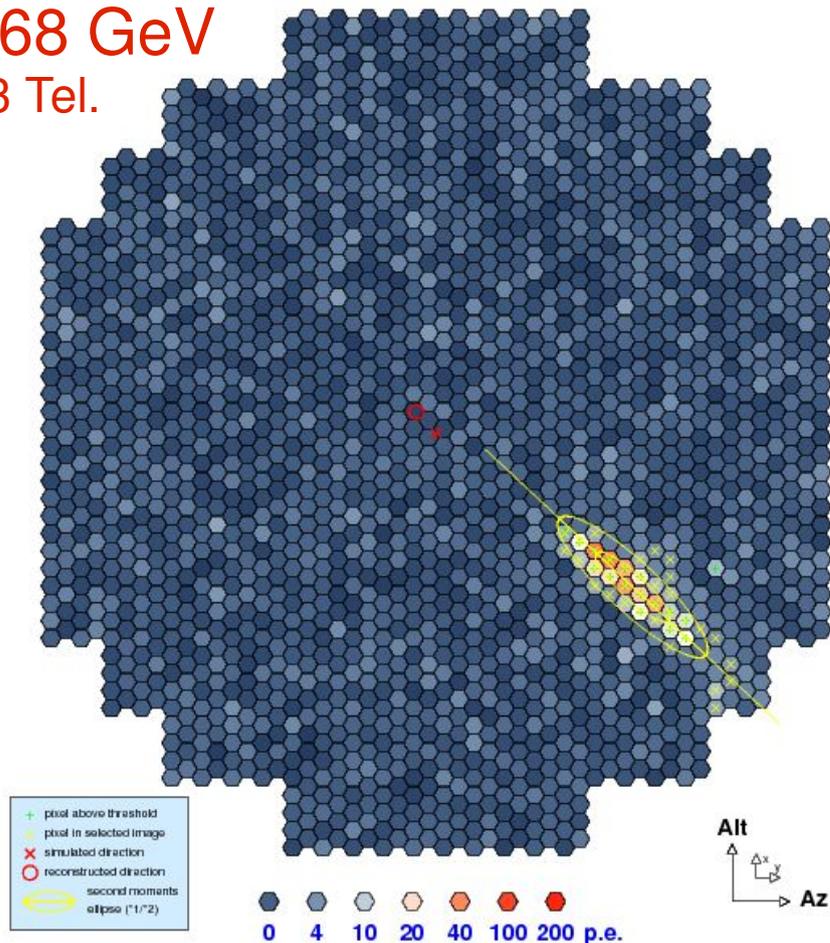
34 GeV
mono



Primary: gamma of 0.034 TeV energy at 156 m distance

Sum of signals in selected pixels: 463.2 p.e.

168 GeV
3 Tel.

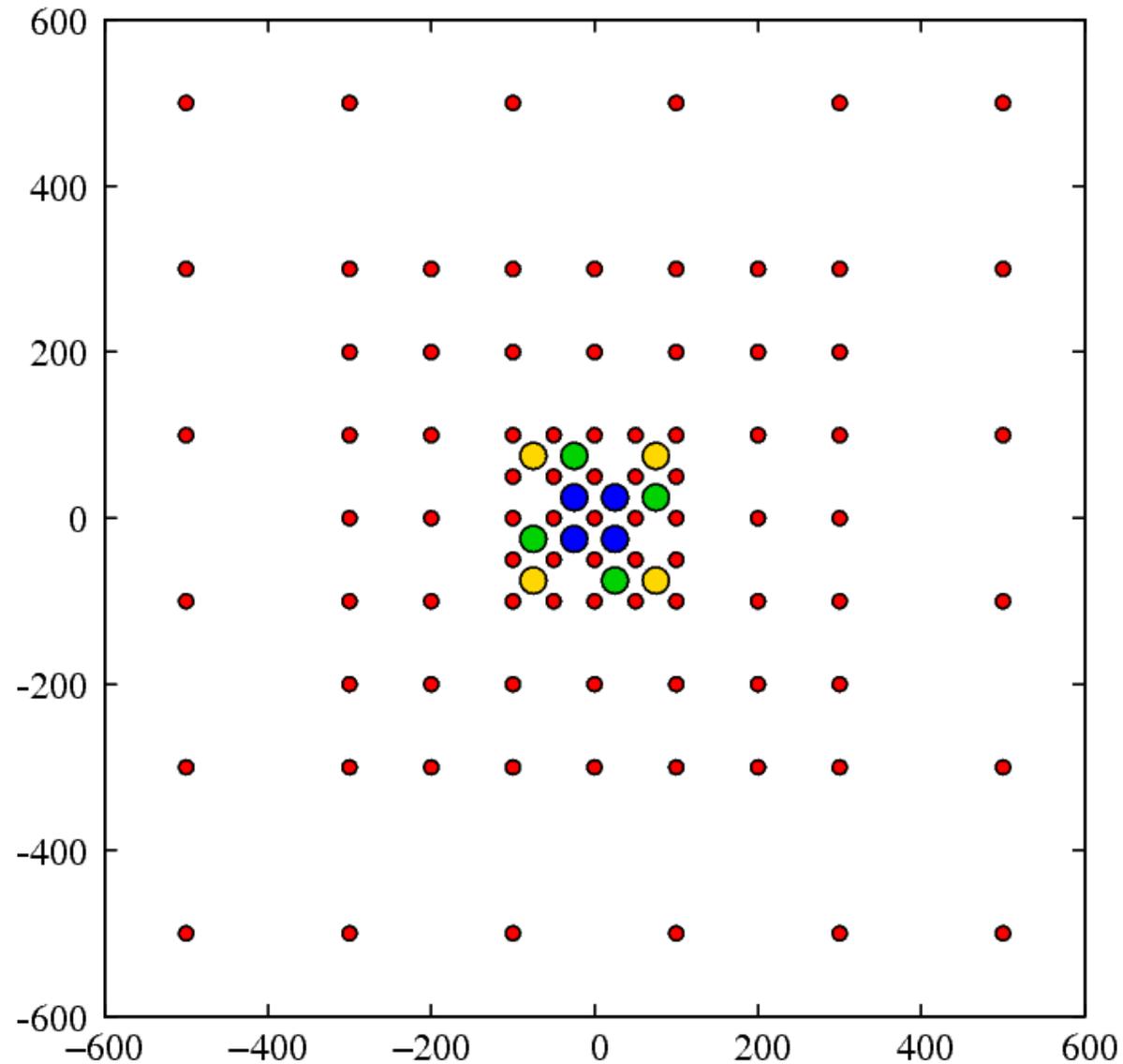


Primary: gamma of 0.168 TeV energy at 178 m distance

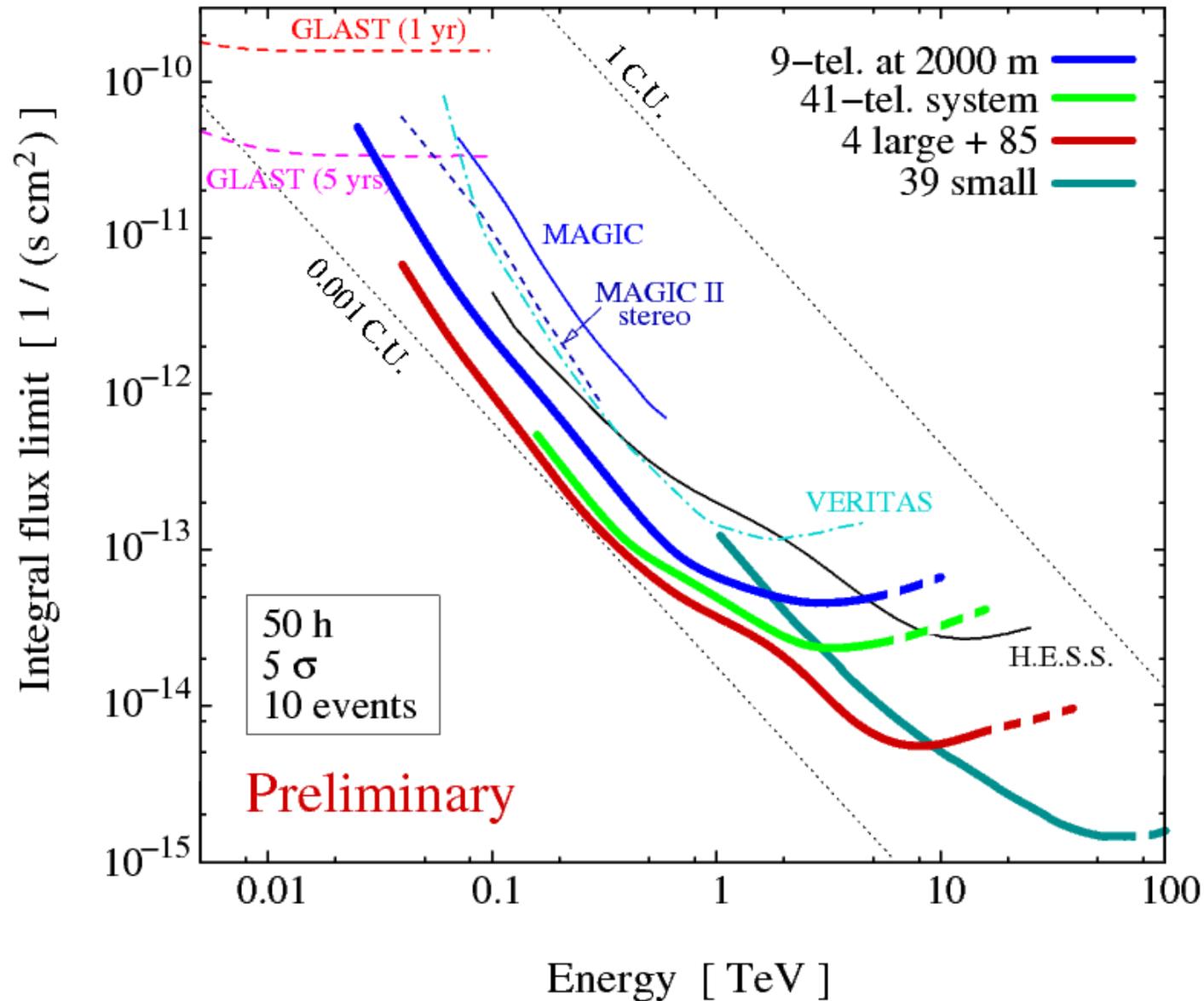
Large array example configuration

- ◆ An example configuration with more telescopes than we could ever hope to build.
- ◆ Turning subsets of telescopes on and off in the analysis.
- ◆ Two basic telescope types, with dish parameters as used for H.E.S.S. I and II, but wider f.o.v. and increased quantum efficiency.

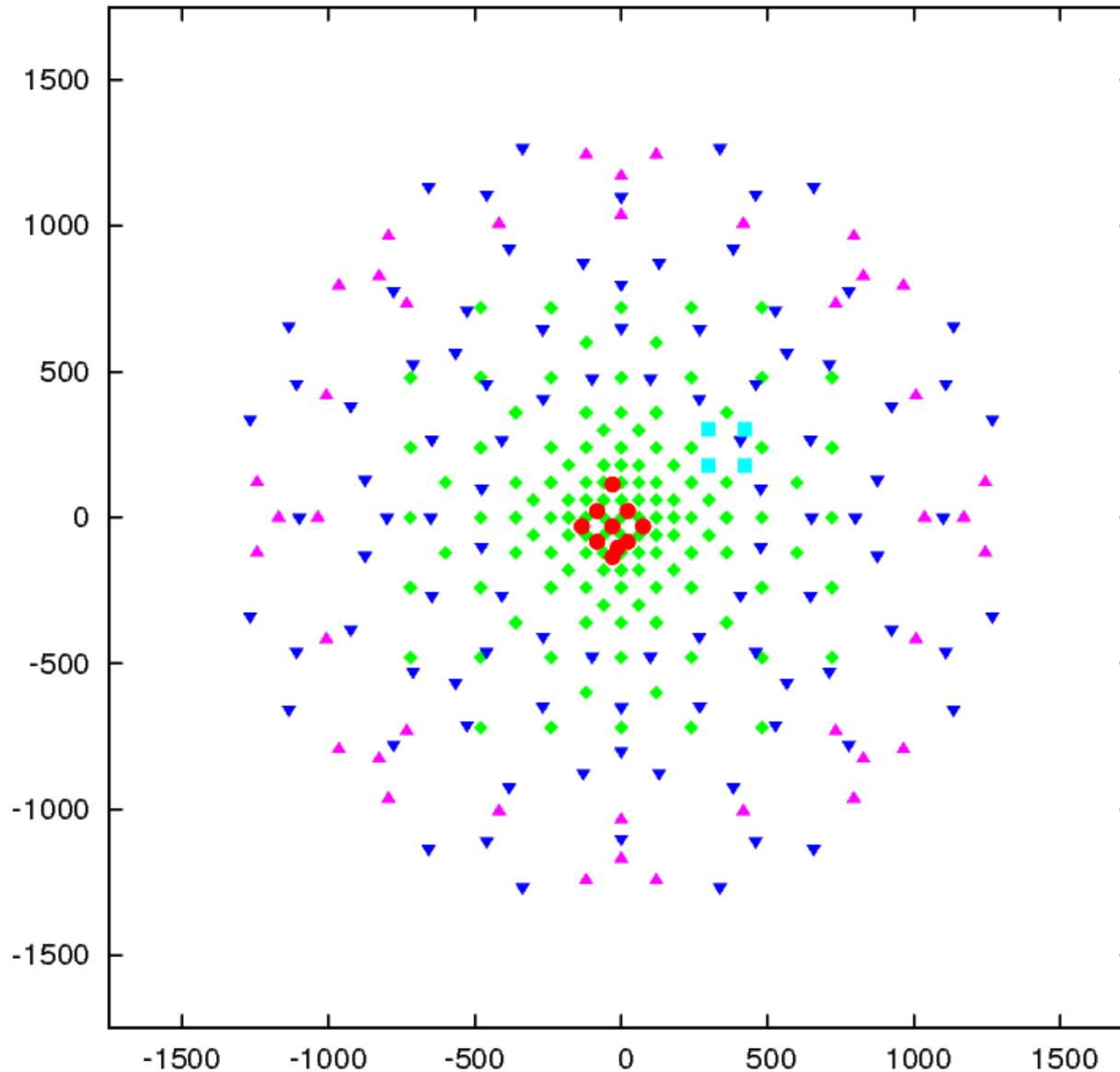
Large array example configuration



CTA: Sensitivity ~ 1 milliCrab



Latest simulations for CTA design



275 telescopes
5 different types

Selecting subsets
in the analysis,
each of ~80 M€,
typically 30-50
telescopes of
two or three types.

The goal is to find
the (near) optimum
design before
asking for a lot
of money.

Summary

- ◆ Simulations of H.E.S.S. (I) can reproduce the experimental behaviour in great detail, without adapting any parameters. All parameters are determined from pre-installation test measurements + muon calibration.
- ◆ Evaluation of large array configurations is easy without having to change any code.
- ◆ The simulations are in use for the design of future instruments like CTA, which will improve the sensitivity by an order of magnitude and enlarge the energy range to 20 GeV-300 TeV.