GRB modelling with NAIMA toolbox

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Contents

1 Introduction

This document describes in more details the algorithms and the physical constrains that are used in the GRB modelling code used in the H.E.S.S. paper on GRB190729A [H.E.S.S. Collaboration(2020)]. The details about the physics of GRB afterglows can be checked there and in the references contained.

The code was developed by CR together with the team while working on GRB190729A, however we made it a general purpose code that could be easily used to analyse multiwavelength data of any other GRB afterglows.

The modelling of the multiwavelength emission of GRBs is done using the NAIMA software (using version 0.8.3) [Zabalza(2015), and references therein] and uses this framework to fit multiwavelength GRB data using a Markov-Chain Monte Carlo (MCMC) method.

What the present module does is to create an interface between the radiation processes already coded in NAIMA and the physical parameters that would be needed to explain the multiwavelength emission of a GRB in a simple 1-zone Synchrotron-Self-Compton (SSC) scenario.

2 The physical parameters

Here we highlight basic expressions for GRB afterglows and the physical parameters used in the code to explain their emission. The focus of this section is simply to describe and explain the assumptions that are made.

2.1 Theoretical framework

2.1.1 The Lorentz factor of the shock

(I thank D. Khangulyan for explaining this part so clearly)

We start with some relations coming from the physics of the environment. The most important step is to compute the Lorentz factor of the forward shock. For this purpose we use the Blandford-McKee relation [Blandford, & McKee(1976)]:

$$\Gamma^2 = \frac{E_{iso}}{Mc^2} \tag{1}$$

where M is the mass of the material swept by the shock and E_{iso} is the isotropic energy of the GRB.

We have 2 possible scenarios: a constant density of the interstellar medium around the progenitor or a density profile resulted by a stellar wind and hence depends as r^{-2} with r being the distance from the central source.

Relativistic calculations give then that in the radius of the shock front is

$$R \simeq \Gamma^2(R) c\Delta t \begin{cases} 8 & \text{for ISM} \\ 4 & \text{for wind}\,, \end{cases} \tag{2}$$

with Δt being the time since the explosion in the observer's frame. Then:

$$\Gamma^2 = \frac{3E_{iso}}{4\pi R^3 n c^2} \tag{3}$$

At this point we have 2 possibilities depending if we have ISM or Wind scenario.

ISM scenario

In the ISM scenario the density is constant, so with a volume $V = \frac{4}{3}\pi R^3$, the swept mass is $M = nm_pV$ with n the number density of the particles and m_p is the proton mass.

If now we substitute 2 in 3, then we obtain:

$$\Gamma = \left(\frac{1}{8}\right)^{3/8} \left(\frac{3E_{iso}}{4\pi n m_p c^2 (c\Delta t)^3}\right)^{1/8} \tag{4}$$

Wind scenario

In this case, we have to first assume that the progenitor star has a mass loss rate $\dot{m}$ and a wind speed v. With these quantities, the number density of the material at distance r is

$$n(r) = \frac{\dot{m}}{4\pi v r^2 m_p} \tag{5}$$

then when we substitute in equation 3, we have

$$\Gamma^2 = \frac{3E_{iso}}{4\pi c^2 (4\Gamma^2 c\Delta t)^3 n} \tag{6}$$

$$= \frac{3E_{iso}}{4\pi c^2 (4\Gamma^2 c\Delta t)^3} \frac{4\pi v R^2}{\dot{m}} \tag{7}$$

where we used equation 5. After simplifications we end up with

$$\Gamma = \left(\frac{3E_{iso} v}{4c^3 \Delta t \dot{m}}\right)^{1/4} \tag{8}$$

2.1.2 The number density of the target photon field

For the way NAIMA is constructed, to compute the SSC component of the spectrum, we need to calculate the number of synchrotron photons produced in the emission zone.

This is done using the simple approximation:

$$n_{ph} \simeq \frac{(d^2N/dEdt)_{syn}}{4\pi R^2 c} \tag{9}$$

where $d^2N/dEdt$ is the number of photons emitted per second per energy computed in NAIMA (it has units of [ph/s/eV]). With respect to the expressions in the NAIMA documentation[1], we drop the numerical factor 2.24 in the computation of the number density because we are assuming that the emission comes from a thin shell and not from a full spherical region[2].

[1] `https://naima.readthedocs.io/en/latest/radiative.html`

[2] The proper calculation on a shell might lead to a factor few difference in the photon density.

2.1.3 The electron distribution

NAIMA does not perform time evolution, so the electrons injected have a broken power law distribution with an exponential cut-off at high energy. The expression, as taken from the NAIMA documentation, is the following:

$$f(E) = \exp\left(-\frac{E}{E_c}\right) \begin{cases} A(E/E_0)^{-\alpha_1} & : E < E_b \\ A(E_b/E_0)^{\alpha_2-\alpha_1}(E/E_0)^{-\alpha_2} & : E > E_b \end{cases} \tag{10}$$

where E_c is the cutoff energy, E_0 is the reference energy (set to 1 TeV), E_b the break energy and α_1 and α_2 the indices for the power law above and below the break energy and A is the normalization. All these quantities are in the shock frame.

Before just plugging in the function in the model, there are some other physical constraints that need to be considered.

2.2 Calculation of the minimum injection energy

The minimum injection energy is a parameter that cannot be fixed *a-priori*, but depends on the fraction of energy that goes into the particles with respect to the total available energy at the shock. This parameter is usually called η_e (the equivalent parameter that describes the amount of energy going into magnetic field is called η_B).

Using this parameter, the minimum injection energy can be computed following the relations written below:

$$\int_{E_{min}}^{\infty} AE^{-p_{inj}+1}dE = AE_{min}^{2-p_{inj}}/(p_{inj}-2) = \eta_e \dot{E} \tag{11}$$

$$\int_{E_{min}}^{\infty} AE^{-p_{inj}}dE = AE_{min}^{1-p_{inj}}/(p_{inj}-1) = \dot{N} \tag{12}$$

If the above equations are solved analytically (under the obvious assumptions that $p_{inj} > 2$ and $E_{min} \ll E_{max}$) the ratio gives:

$$E_{min}\frac{p_{inj}-1}{p_{inj}-2} = \eta_e E_{tot}/N_{tot} = \eta_e \Gamma m_p c^2 \tag{13}$$

To avoid problems in the MCMC and make the calculations stable, the ratios are computed numerically and the minimum energy is computed with an iterative procedure. From the equations above, it follows that:

$$\frac{\int_{E_{min}}^{\infty} AE^{-p_{inj}+1}dE}{\int_{E_{min}}^{\infty} AE^{-p_{inj}}dE} = \eta_e \Gamma m_p c^2 \tag{14}$$

Using the full electron distribution in the integrals, we first set the minimum energy to 1 GeV (as it is close to the rest mass energy of a proton) and then we compute how much the ratio between the first and the second term of the equation (calling it K) is away from 1. After this, we shift down the value of E_{min} by K. One iteration is enough to obtain a value $K \sim 1$ for a reasonable range of indices (most of the times within a factor 2 or less). Introducing additional iterations would make the value more precise, but slow down too much the fitting (this operation has too be done for every step of the MCMC).

2.3 Calculation of the normalization of the electron distribution

As we use η_e as free parameter, we need to associate it with the actual normalization of the electron distribution because this is the parameter that in the end NAIMA uses to build the electron distribution.

It starts with the internal energy density of the shocked plasma, which is

$$w = 2\Gamma^2 n_0 m_p c^2 \tag{15}$$

At this point we compute the E_{min} of the distribution from the values of α_1 and η_e, and we construct an electron distribution with unitary normalization.

We then compute the integral to get the total energy density for this electron distribution:

$$T = \frac{1}{V}\int_{E_{min}}^{E_{max}} E\frac{dN}{dE}dE \tag{16}$$

where the assumed volume is that of a spherical shell $V = 4\pi R^2(R/(9\Gamma))$. In this case the width of the shell is $D = R/(9\Gamma)$. The numerical factor comes from assuming that we are in the ISM environment, while in the wind case the width would be $R/(3\Gamma)$.

From this follows that the normalization parameter of the electron distribution A is

$$A = \frac{\eta_e w}{T}. \tag{17}$$

Once found A, we can proceed with the rest of the modelling because we now have all the ingredients that NAIMA needs to compute the emitted radiation.

2.4 Constrains from the age of the system

The age of the system is given by the Lorentz-corrected time from trigger:

$$\Delta t' = (t - t_0)\Gamma \tag{18}$$

where the time has been corrected for the relativistic boost (primed quantities are in the shock frame). This means that the cooling break should be in a position in the spectrum for which the energy of the particles have the same cooling time as the age of the system. For a generic GRB, using this relations

can give some constrains on the intensity of the magnetic field imposing that the cooling time of the electrons at the break is at the same level of the age of the system.

This constrain is implemented in the prior function and can be switched off in the initialization (setting the `cooling_constrain` option to `False`).

2.5 Calculation of the internal absorption

The code takes into account the gamma-gamma absorption that affects the emitted high energy photons when they interact with the synchrotron photons inside the source (internal absorption).

To take this effect into account, we need the number density of the target radiation field and the cross section of the absorption process.

The number density of the target radiation field n_{ph} is actually already computed by NAIMA when we ask for the SSC component. The cross section for the process is the analytical approximation coded in [Aharonian(2004)]. In this way we can compute the optical depth parameter τ. Assuming that the size of the region in which absorption and emission happens is $R/(9\Gamma)$ for the ISM case and $R/(3\Gamma)$ for the Wind case.

In the code there are 2 different implementations for the absorption. The default one takes into account that in the same region we have both emission and absorption. In this case we have that:

$$F = \frac{F'}{\tau}(1 - \exp(-\tau)) \tag{19}$$

where with F' we indicate the intrinsic flux and with F the observed one [Rybicki & Lightman(1979)]. In the code there is also the simple implementation $F = F' \exp(-\tau)$.

2.6 Physical parameters fitted to the data

The final implementation puts together the inputs from the previous subsections to build the model using NAIMA.

In the basic implementation, there are 5 parameters that are set free to vary:

- η_e: the fraction of energy available as non-thermal electrons
- E_{break}: energy of the break in the electron distribution
- α_2: the power law index above the break in the electron distribution
- E_{cut}: cut-off energy of the electron distribution
- B: the intensity of the magnetic field

With the exception of α_2, all parameters are fitted in logarithmic space (base 10). The low energy index α_1 is not a free parameter because we are using the

assumption that the break in the electron distribution is a synchrotron cooling break so that $\alpha_1 = \alpha_2 - 1$. This is a plausible assumption.

The limits in which these parameters can vary are defined in the prior function. Thanks to the module structure, it is possible to add different model functions and different priors that are called in the implementation. As an example, the released code includes a different prior function in which the cut-off of the electron distribution is not limited by the equilibrium between acceleration and synchrotron cooling. The use of this prior can be initialized through the flag `synch_nolimit` set to `True`.

3 Easy extensions

The module is written in such a way that it is possible to simply add more models and more priors and ensure they are called correctly by modifying the `load_model_and_prior` function.

4 Full code documentation

For practical use, the code has been documented using the `numpy` style and it can be found here:

`https://bitbucket.org/hess/grbmodelling/src/master/`

This link is accessible by H.E.S.S. members using the instruction at this internal page. This repository contains all the models explored when writing the article on GRB190829A [H.E.S.S. Collaboration(2020)]. Every attribute and method of the code has been nicely commented in the source code and the example on how to run the code can be seen in the `README` file in the repository.

A public version of the software is available on my personal github account: `https://github.com/Carlor87`

This version includes only a subset of the available models used for HESS.

5 Proof of concept: GRB190114C

Here we report the results that can be obtained using the data on the GRB GRB190114C observed by the MAGIC telescope [MAGIC Coll. (2019)]. The data are extracted from Figure 2 of the paper [MAGIC Coll. (2019)] for the interval 1 (68 to 110 seconds after trigger) and we ran our code on those points. The outcome is shown below in Figure 1.

The value of the magnetic field corresponds to $\eta_B = 0.007$.

These values were obtained using 64 parallel walkers, 50 steps of burn-in and 100 steps for the MCMC.

The plots below show also the behaviour of the MCMC chains for every parameter (Figure 2) and the correlation plot between them (Figure 3). Of course a greater number of walkers and more steps for the burn-in phase and

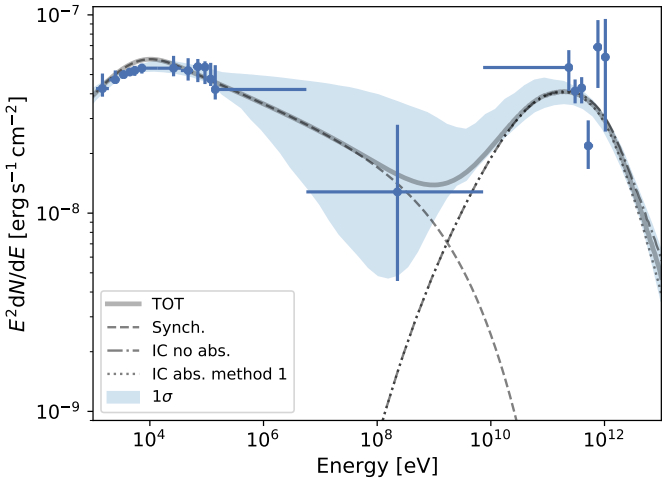

Figure 1: MCMC fit on the data from the GRB 190114C in the time interval [68-110] seconds after trigger as reported in [MAGIC Coll. (2019)]. The shaded blue area indicates teh 1 σ confidence interval derived from the posterior distribution of the model parameters. the "method 1" in the IC absorption line refers to the computation of internal absorption using the simple model that does not account for emission and absorption in the same region.

Parameter	Night 1
$\log_{10}(\eta_e)$	-1.44 ± 0.03
$\log_{10}(E_b)$ [TeV]	$-1.65^{+0.3}_{-0.19}$
α_2	$3.14^{+0.4}_{-0.11}$
$\log_{10}(E_c)$ [TeV]	1.3 ± 0.2
$\log_{10}(B)$ [G]	$0.27^{+0.05}_{-0.04}$
η_e	0.036
η_B	0.007

Table 1: Final parameters values for the interval 1 of GRB190114C as in Figure 1.

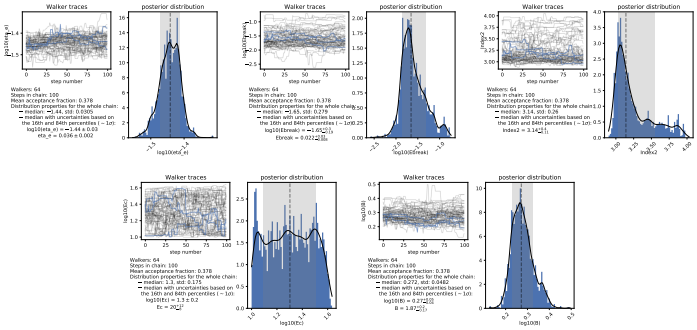

Figure 2: Posterior distributions of the parameters from the fit reported in Figure 1 and Table 1

for the actual chain could converge to a better result. This is however a good proof of concept.

The `README` file shows how to run this example.

6 Acknowledgments

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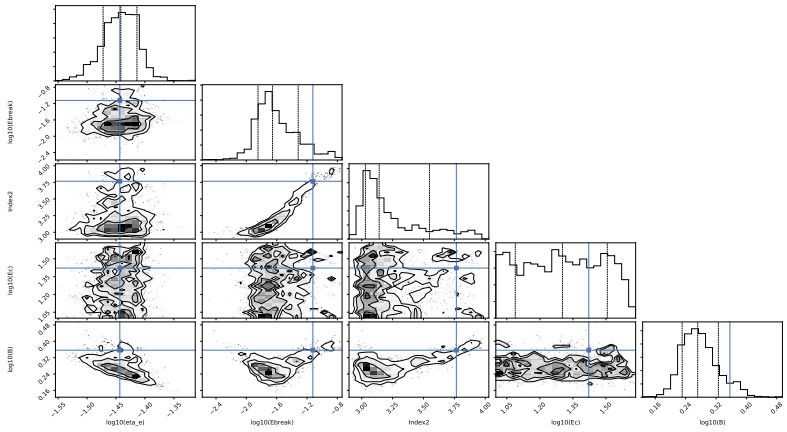

Figure 3: Corner plot between the parameters showing distributions and correlations between the values. The light blue lines indicate the best likelihood parameters.

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