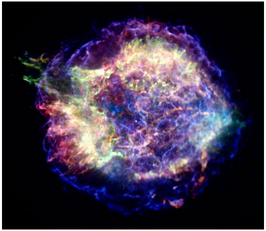
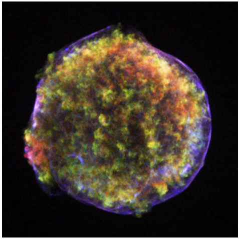
On the amplification of magnetic fields at supernova remnant shocks

Brian Reville
Clarendon Laboratory, University of Oxford
(in collaboration with Tony Bell)

5th International Symposium on High-Energy Gamma-Ray Astronomy
Heidelberg, July 9-13, 2012

Motivation (What we want / what we see)

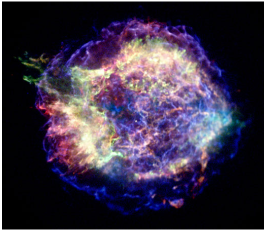


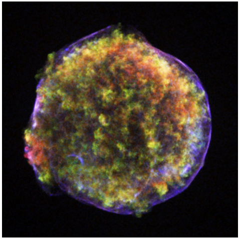


Observations imply $B_{\rm rms} \gg B_{\rm ISM}$ *downstream* of shocks

Advantageous if fields primarily amplified upstream

Motivation (What we want / what we see)





Observations imply $B_{\rm rms} \gg B_{\rm ISM}$ *downstream* of shocks

Advantageous if fields primarily amplified upstream

- Accel. time $t_{\rm acc} \approx \kappa/u_{\rm sh}^2$ where $\kappa \sim \lambda_{\rm MFP} c$

$$E_{\rm max} \approx 10^{13} \left(\frac{r_{\rm g}}{\lambda}\right) \left(\frac{u_{\rm shock}}{10,000{\rm km/s}}\right)^2 \left(\frac{t_{\rm snr}}{100{\rm yr}}\right) \left(\frac{B}{1\mu{\rm G}}\right) {\rm eV}$$

- magnetic field amplification vital to reach “knee”

But must be produced on sufficiently short timescales, and on appropriate lengthscales

Outline

Non-linear amplification – time constraints

Length-scale constraints

3D MHD–Vlasov simulations

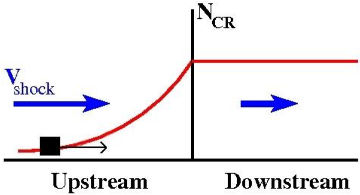
Non-linear amplification time constraints

Maximum rms magnetic field

$$E_{\rm max} \approx 10^{13} \left(\frac{r_{\rm g}}{\lambda}\right) \left(\frac{u_{\rm shock}}{10,000{\rm km/s}}\right)^2 \left(\frac{t_{\rm snr}}{100{\rm yr}}\right) \left(\frac{B}{1\mu{\rm G}}\right) {\rm eV}$$

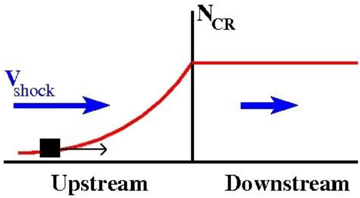
Non-linear magnetic field amplification

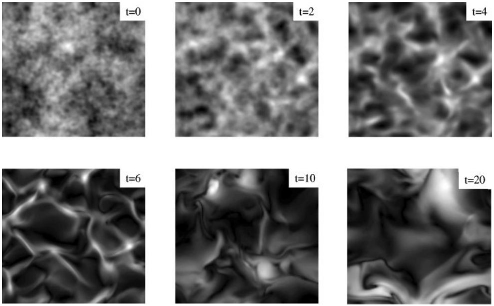
Linear theory (Bell ’04, BR et al. ’06, etc.)
Numerical simulations required to investigate non-linear behaviour



Non-linear magnetic field amplification

Linear theory (Bell ’04, BR et al. ’06, etc.)
Numerical simulations required to investigate non-linear behaviour





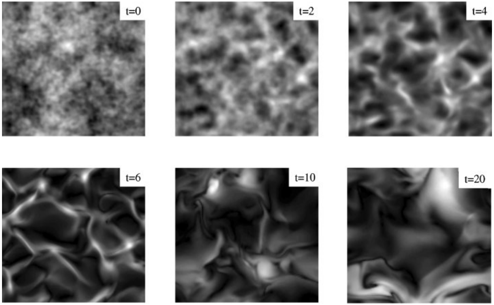
3D MHD simulations

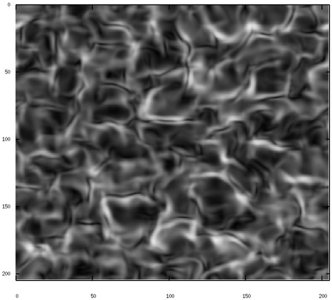
- $L_{\mathrm{box}} \ll r_{\mathrm{g}}$, $j_{\mathrm{cr}} \| B_0$, j_{cr} constant & uniform
- saturated magnetic field $B_{\mathrm{rms}} \approx 30 B_0$

Consistent with observations

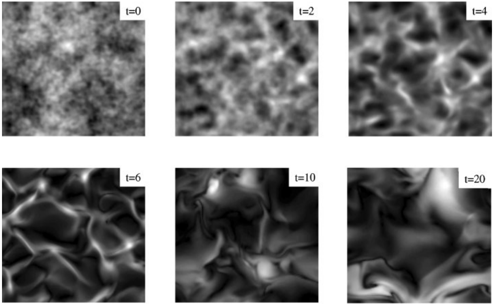
Bell 2004

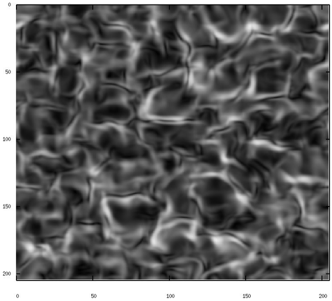
Non-linear evolution of magnetic field

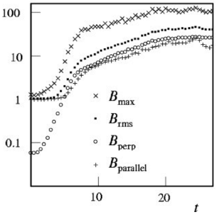


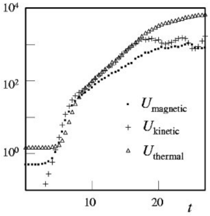


Non-linear evolution of magnetic field



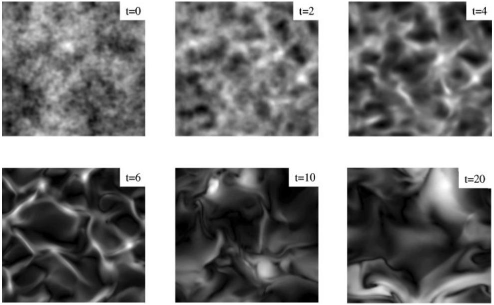


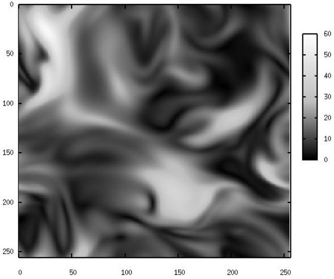


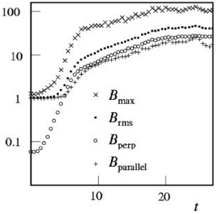


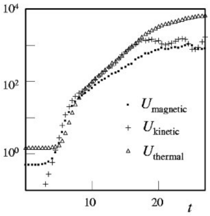
Non-linear growth not an artefact of box-size

Non-linear evolution of magnetic field



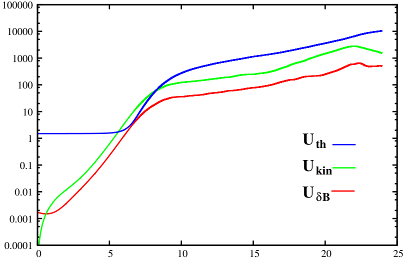






Non-linear growth not an artefact of box-size

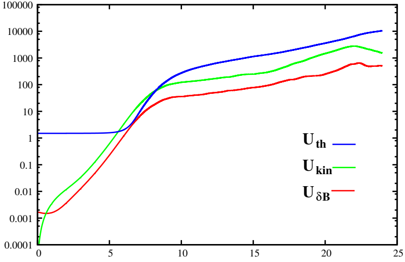
Non-linear evolution of magnetic field



For NR instability, non-linear growth of magnetic energy density determined by continued mixing. Grows approx. in equipartition with kinetic energy $\propto t^2$.

in non-linear phase, growth rate significantly lower than linear rate

Non-linear evolution of magnetic field



For NR instability, non-linear growth of magnetic energy density determined by continued mixing. Grows approx. in equipartition with kinetic energy $\propto t^2$.

in non-linear phase, growth rate significantly lower than linear rate

But thermal energy also continues to grow!
For growth $\delta B \gg B_0$, we might expect $U_{\rm th} \gtrsim U_{\rm B}$

If 100 μG fields are produced **upstream**, $U_{\rm th} > 0.5$ keV cm^{-3}
Unlikely for neutral Hydrogen to survive crossing the precursor.
Interesting for remnants such as Tycho & RCW86, which have Hα, non-thermal x-ray filaments and TeV gamma-ray detections.

Reville & Bell in prep.

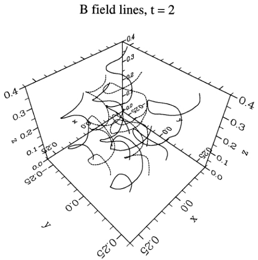
Length-scale constraints

Including the back-reaction on cosmic rays

$$E_{\rm max} \approx 10^{13} \left(\frac{r_{\rm g}}{\lambda}\right) \left(\frac{u_{\rm shock}}{10,000{\rm km/s}}\right)^2 \left(\frac{t_{\rm snr}}{100{\rm yr}}\right) \left(\frac{B}{1\mu{\rm G}}\right) {\rm eV}$$

Feed-back on the particles

- non-resonant instability amplifies magnetic field (at least initially) on short lengthscales (less than gyroradius of CRs driving the growth)
- need to generate field structure on larger lengthscales to reduce λ (or κ) significantly
- Numerical investigation of magnetic field on these scales requires kinetic treatment of cosmic rays



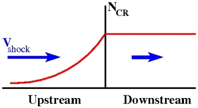
Lucek & Bell '00

Following Zachary 89, Lucek & Bell 00 use PIC treatment for cosmic rays coupled to MHD code

Cosmic ray filamentation

Vlasov Eqn: $$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla f + e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \cdot \frac{\partial f}{\partial \boldsymbol{p}} = 0$$

distribution isotropic (on average) in the shock rest frame



$$\boldsymbol{E} = -\boldsymbol{u} \times \boldsymbol{B} = -\boldsymbol{u} \times (\nabla \times \boldsymbol{A}) \approx u_{\rm sh} \nabla(\boldsymbol{A} \cdot \boldsymbol{x})$$

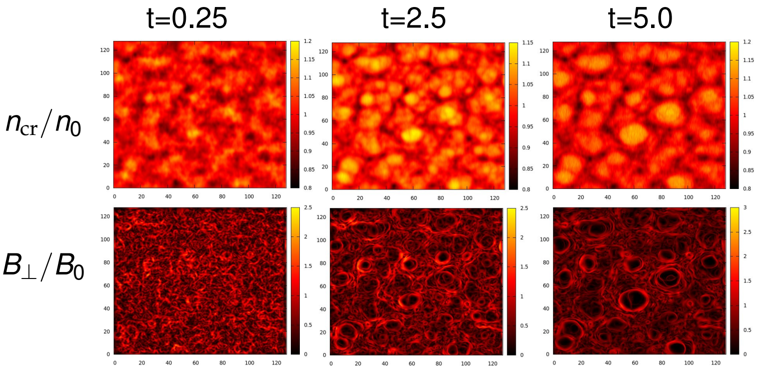
$$\frac{\partial f}{\partial t} + c\frac{\boldsymbol{p}}{p} \cdot \nabla f + e\nabla(u_{\rm sh} A_x) \cdot \frac{\partial f}{\partial \boldsymbol{p}} = 0$$

Equilibrium solution :

$$f = f(p - e u_{\rm sh} A_x / c) \Rightarrow n_{\rm cr} = n_0 + \eta A_x, \quad j_{\rm cr} = e n_{\rm cr} u_{\rm sh}$$

Cosmic rays – A_x correlation: Magnetic fields act to focus CRs

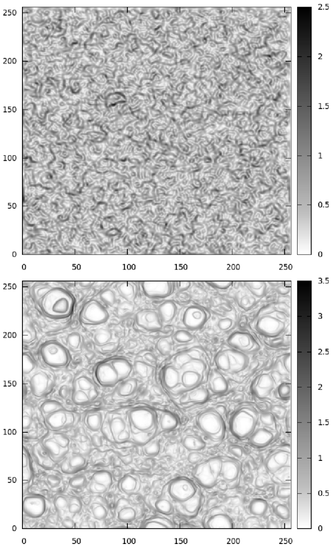
Hybrid MHD–CR simulations

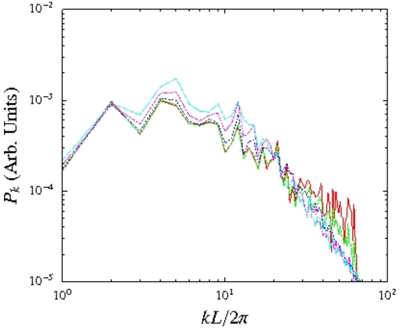


BR & Bell ’12

Cosmic ray structure is non-uniform on large scales
Expanding magnetic loops focus result in cosmic-rays filaments
Filamentation developmet can be used to calculate growth of largescale fields

Growth of large scale field





Growth observed on long wavelengths $\lambda > L_{\rm box}/10$

$$t_{\rm fil} \approx 50 \left(\frac{\eta}{0.2}\right)^{-1} \left(\frac{U_{\rm cr}/\rho u_{\rm sh}^2}{0.1}\right)^{-\frac{1}{2}} \left(\frac{u_{\rm sh}}{10^7 {\rm m/s}}\right)^{-2} \left(\frac{B_\perp^{\rm rms}}{100\mu{\rm G}}\right)^{-1} \left(\frac{E_{\rm min}}{10^{14}{\rm eV}}\right) {\rm yrs}$$

3D MHD–Vlasov simulations

Putting it all together....

$$E_{\mathrm{max}} \approx 10^{13} \left(\frac{r_{\mathrm{g}}}{\lambda}\right) \left(\frac{u_{\mathrm{shock}}}{10,000\mathrm{km/s}}\right)^2 \left(\frac{t_{\mathrm{snr}}}{100\mathrm{yr}}\right) \left(\frac{B}{1\mu\mathrm{G}}\right) \mathrm{eV}$$

MHD–Vlasov-Fokker-Planck

Diffusive shock acceleration theory assumes particle distribution close to isotropy

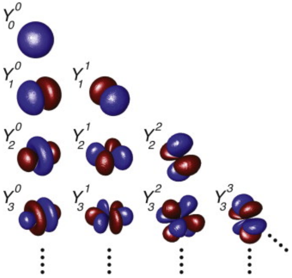
$$F(\boldsymbol{p},\boldsymbol{x},t) \approx f_0(p,\boldsymbol{x},t) + \delta f(\boldsymbol{p},\boldsymbol{x},t) \quad , \quad |\delta f| \ll f_0$$

typically an good approximation provided $u/c \ll 1$.

Make a spherical harmonic expansion of the Vlasov-Fokker-Planck equation. Ideally suited to studying cosmic-ray behaviour (as featured in Tony Bell's talk on Monday)

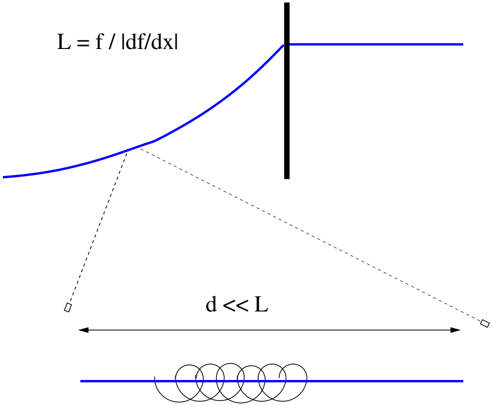
$$f(p,\theta,\varphi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} f_\ell^m(p) P_\ell^{|m|}(\cos\theta) e^{im\varphi}$$

collisions on magnetic fluctuations allow us to truncate series after a few terms



iso-surfaces from Tzoufras et al. 2011

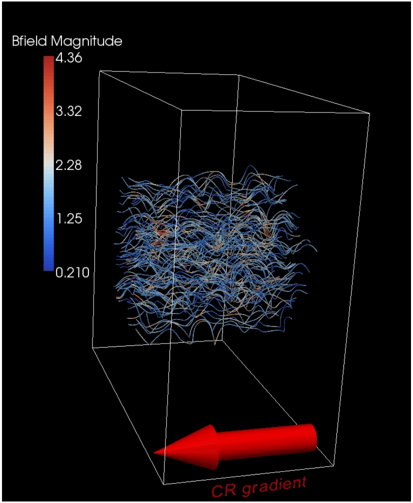
MHD–Vlasov-Fokker-Planck

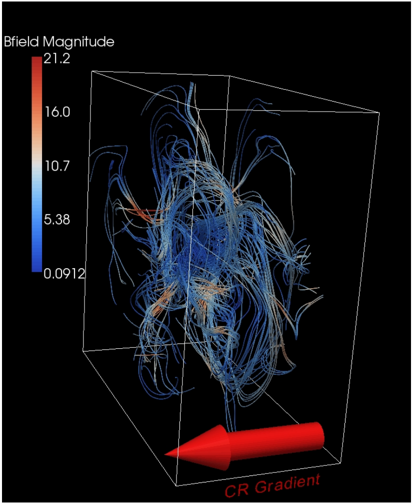


On scales $\ll$ precursor scaleheight L, cosmic-ray density is approximately uniform, but has net current due to large scale gradient.

With a spherical harmonic expansion, we can keep the large scale gradients of f_0^0 and f_1^0, thus keeping connection with the shock.

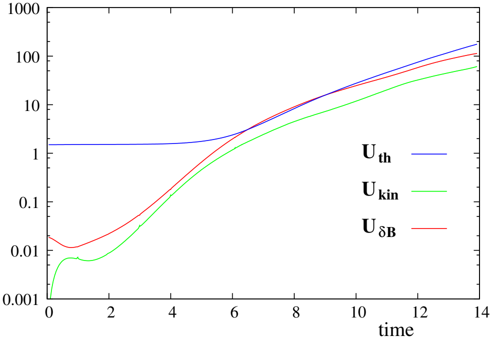
Magnetisation of cosmic-rays





Cosmic rays become magnetised ($r_g \lesssim L_{box}$).
See an effective perpendicular magnetic field structure.

Application to supernovae



Growth of magnetic field fundamentally different than case of non-resonant instability.

Time units normalised to 2.5 Larmor periods
e.g. For 100TeV protons, $B_0 = 3\mu$G, $P \approx 1$yr
$\approx 20 - 30$ yrs to magnetise cosmic rays.
Precursor crossing time for fluid element $\approx 20 - 30$ yrs!!

These runs assumed quite favourable conditions

Summary

- Saturated strength of magnetic field still an open question – may generally be time limited
- Supernovae with Hα emission and non-thermal x-ray filaments may provide information on in-situ acceleration/ magnetic field amplification
- Including the cosmic-ray dynamics vital to explain large scale magnetic field evolution – scattering, acceleration etc.
- Cosmic ray filamentation introduces multi-scale aspect of the problem (removes need for inverse-cascade?)
- 3D Hybrid Vlasov–MHD simulations suggest that filamentary structures are disrupted (at least on long time-scales)
- large-scale ($\sim$ gyroradius of highest energy particles) perpendicular magnetic fields can be produced.