Filamentary diffusion of cosmic rays

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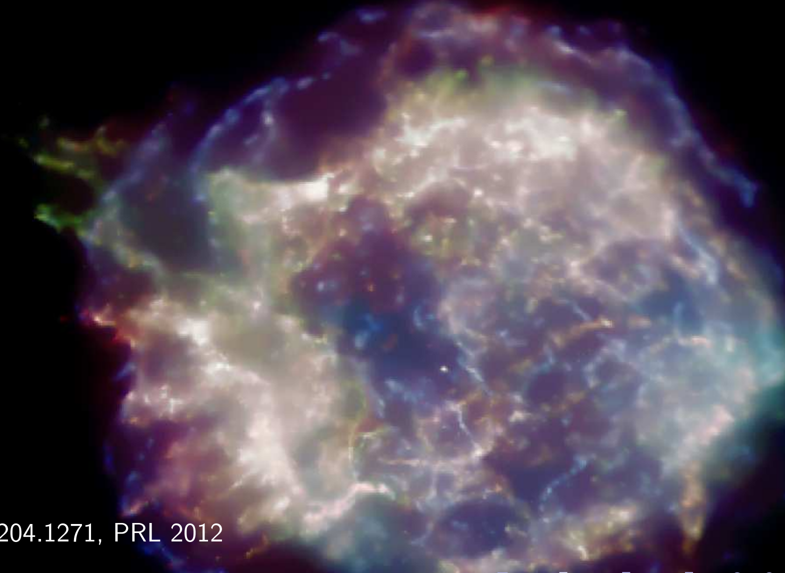

arxiv:1204.1271, PRL 2012

Diffusion in magnetic fields

- Galactic magnetic field: regular + turbulent component
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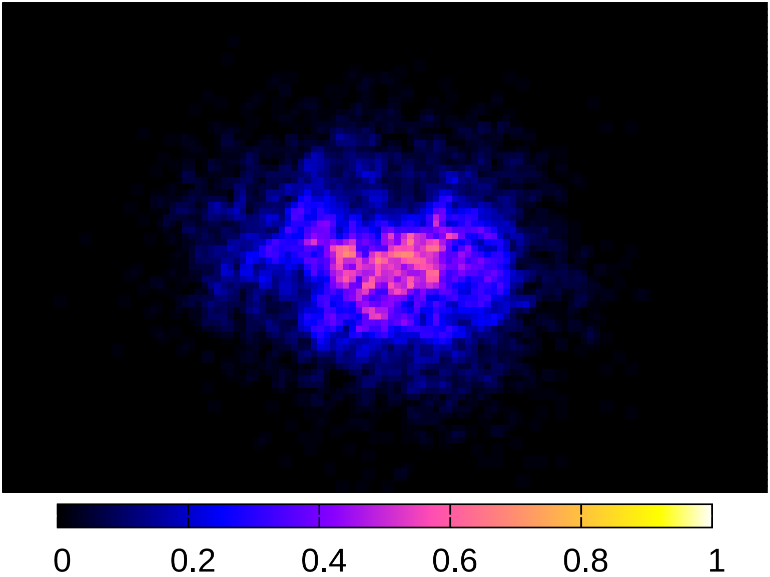
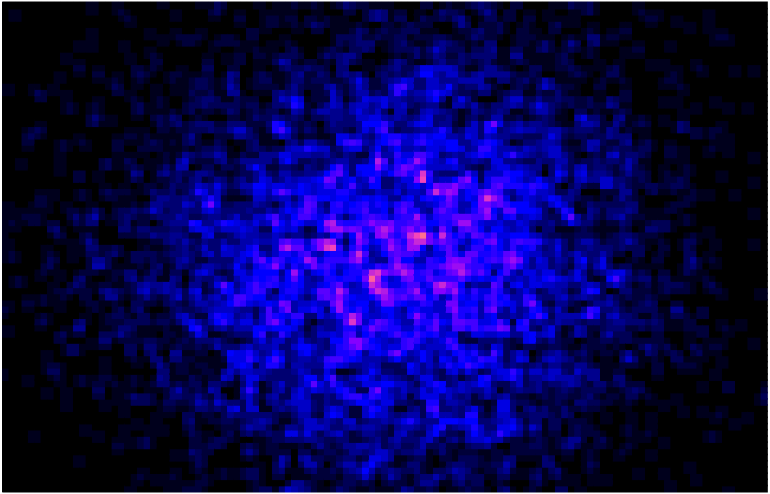
- For a pure random field, $\langle r^2 \rangle = \sqrt{6Dt}$ and

$$n(r) \propto \exp\left(-\frac{r^2}{4Dt}\right)$$

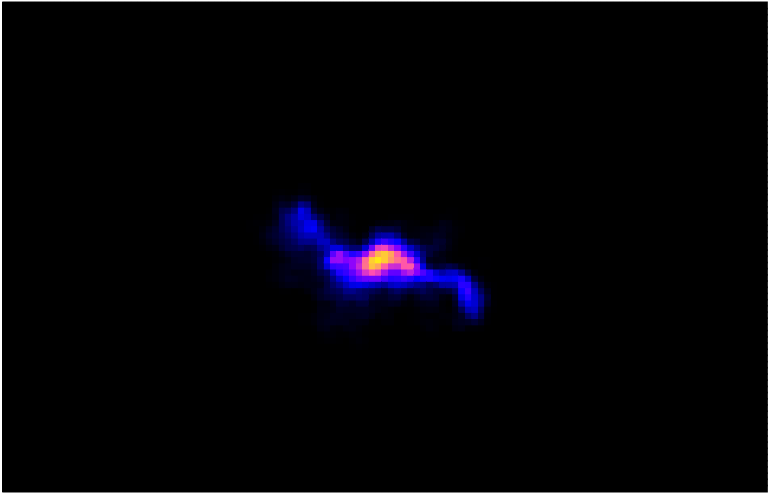
CR diffusion close to source, $E = 10\,\mathrm{PeV}$, $t = 2000\,\mathrm{yr}$



CR diffusion close to source, $E = 10\,\mathrm{PeV}$, $t = 7000\,\mathrm{yr}$



CR diffusion close to source, $E = 10\,\text{PeV}$, $t = 500\,\text{yr}$



Filamentary CR diffusion close to source:

Explanation:

- CRs scatter on modes with $kR_L \sim 1$
- fast modes with $kR_L \gg 1$: irrelevant
- slow modes with $kR_L \ll 1$: act as regular, uniform field B_0
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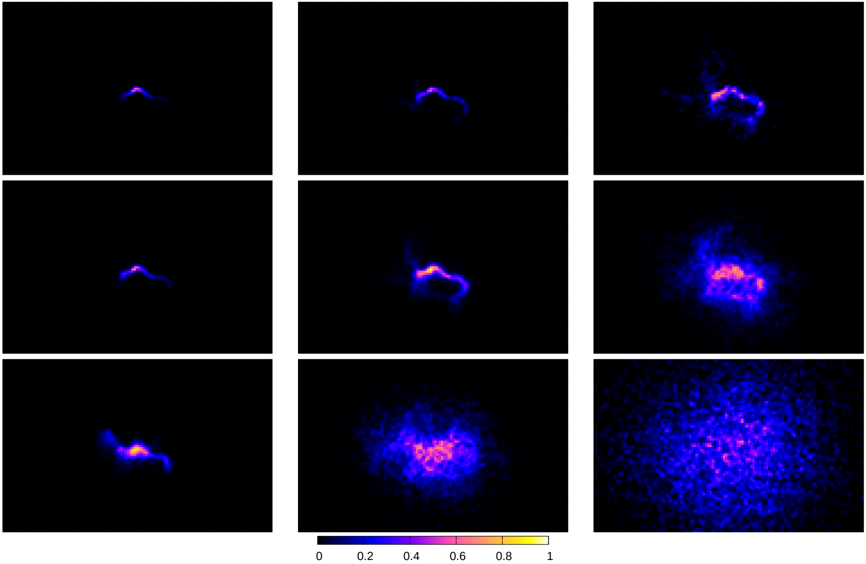
Why not seen earlier in simulations?

- too large scales, $l \gg l_{\max}$, considered
- anisotropy vanishes averaging over field realizations
- anisotropy vanishes for random start positions

$E = 100\,\text{TeV} \to 1\,\text{PeV} \to 10\,\text{PeV}$ $\qquad t = 500\,\text{yr} \downarrow 2000\,\text{yr} \downarrow 7000\,\text{yr}$



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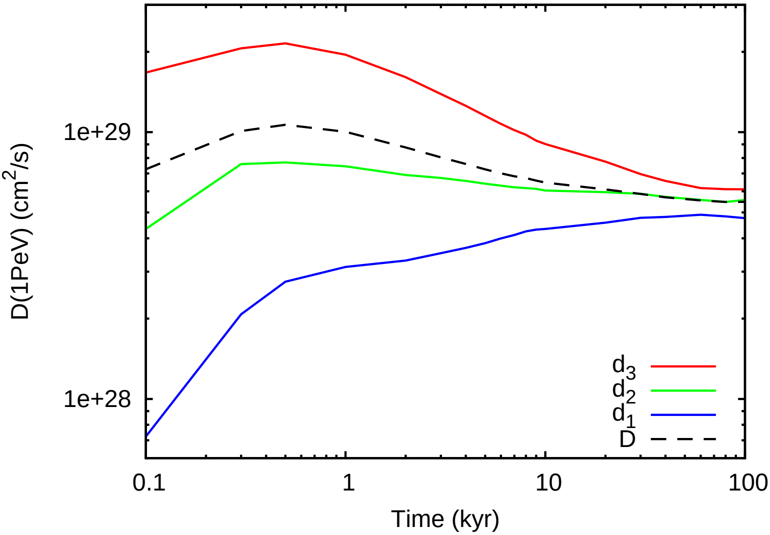
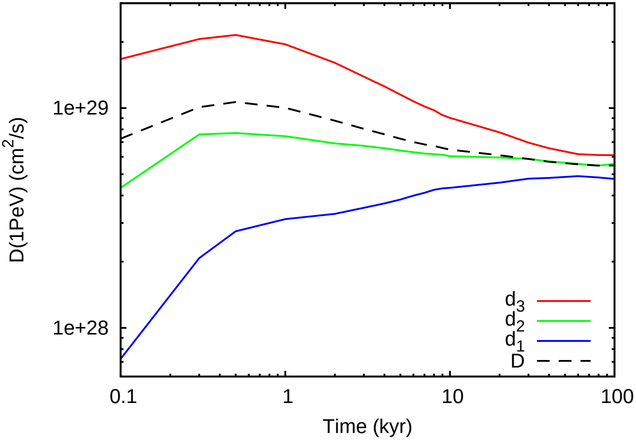
- diagonalizes $D_{ij}^{(b)}$, determine eigenvalues $d_i^{(b)}$
- average the ordered eigenvalues, $d_1^{(b)} < d_2^{(b)} < d_3^{(b)}$, over the M realizations,

$$d_i = \frac{1}{M} \sum_{b=1}^{M} d_i^{(b)}$$

Eigenvalues of $D_{ij} = \langle x_i x_j \rangle/(2t)$ for $E = 10^{15}$ eV

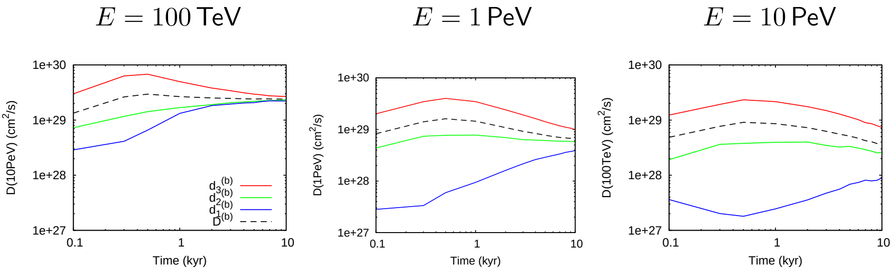


Eigenvalues of $D_{ij} = \langle x_i x_j \rangle / (2t)$ for $E = 10^{15}$ eV



- asymptotic value is ~ 4 smaller than "Galprop value"

Transition time to standard diffusion:

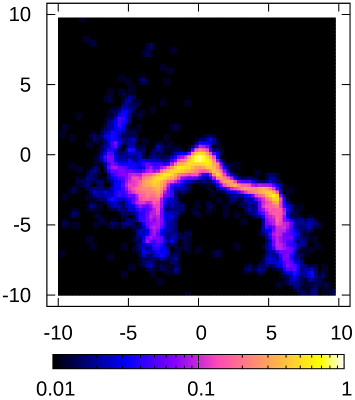


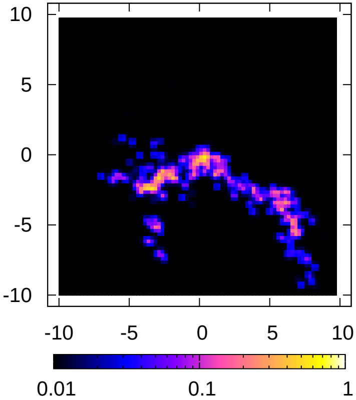
$$t_* \sim 10^4\,\mathrm{yrs}\ \ (l_{\max}/150\,\mathrm{pc})^{\beta}\,(E/1\,\mathrm{PeV})^{-\gamma}$$

with $\beta \simeq 2$ and $\gamma = 0.25$–0.5

for Kolmogorov turbulence and $B_{\mathrm{rms}} = 4\,\mu$G.

Comparison CR density vs. photon flux





$\Rightarrow$ irregular gamma-ray halos as tracker of CR density

Tycho–irregular halo?