1. The Neutrinoless Double Beta Decay

The two neutrinoless double beta decay ($2\nu\beta\beta$) is a second order process described by the Standard Model (SM), which takes place in case the usual beta decay is energetically forbidden. In the neutrinoless decay mode ($0\nu\beta\beta$), one neutrino is exchanged between two decaying neutrons: it is not known by the SM but could happen if:
- the neutrino is a Majorana particle ($\nu = \bar{\nu}$) of non-zero mass;
- the lepton number conservation is violated (by two units).

The $0\nu\beta\beta$ decay half-life is given by:

$$T_{1/2} = \frac{\tau_{0\nu\beta\beta}}{\sqrt{M^0 \cdot \varepsilon}}$$

Where $F^{0\nu\beta\beta}$ is the phase space, $M^0$ the nuclear matrix-element, $\varepsilon = \sum_i C_i^2 m_i^2$ and $Q$ is the $Q$-value.

The experimental signature is a continuum energy spectrum that is different with respect to the $2\nu\beta\beta$ mode and a peak at the $Q$-value for the $0\nu\beta\beta$.

2. The Experimental Sensitivity to $0\nu\beta\beta$ Decay

The energy resolution is a crucial parameter for obtaining physics results. It is usually given as the Full Width at Half Maximum (FWHM) of the spectral peaks, once fitted by a Gaussian and in GERDA is quoted at $Q_{Q^2}$.

An improved resolution involves:
- higher sensitivity to the presence of background-induced-gamma peaks;
- more precision in the construction of the background model;
- in case a $0\nu\beta\beta$ decay signal is present, more precision in the computation of the $0\nu\beta\beta$ decay events;
- in case no $0\nu\beta\beta$ decay signal is present, a stronger limit for the value of the half-life, $T^{0\nu\beta\beta}_{1/2}$.

The achievable limit on $T^{0\nu\beta\beta}_{1/2}$ is:

$$T^{0\nu\beta\beta}_{1/2} \approx \frac{\tau_{0\nu\beta\beta}}{\sqrt{M^0 \cdot \varepsilon}}$$

where $\alpha$, $\beta$, $M$, $\varepsilon$, $\tau$, $Q$, $H$ and $\Delta E$ are parameters that depends on the filter type, and $\alpha$ is the root mean square of the series and parallel noise and $A_P$ is the coefficient of $1/f$ noise. The optimization of $\tau$ allows to filter out peculiar noise frequencies.

3. Digital Signal Processing in the GERDA Experiment

The GERManium Detector Array (GERDA) experiment at Gran Sasso National Laboratory is designed to study the $0\nu\beta\beta$ decay by using enriched germanium diodes deployed in Liquid Argon (LAr). The detectors are protected from the external radiation by a multi-layer shielding made of:
- a water Cherenkov active moon veto;
- copper to absorb gammas;
- LAr to cool down the detectors and further absorb gammas and neutrons.

The energy reconstruction is performed via on-line processing (32 parallel parabolas). The procedure consists of 2 steps:
- a differentiation of the sampled signal $a(t)$ with time delay $\Delta t = 5 \text{ ps}$:
  $$a(t) \rightarrow a(t) = a(t) - a(t - \Delta t)$$
- a Moving Average (MA) with the same width, repeated 25 times:
  $$\bar{a}(t) = \frac{1}{\Delta t} \sum_{i=1}^{25} a(t - i \Delta t)$$

The energy is then given by the height of the possible parabolas. This algorithm is very useful and relates to the differentials of the signals in some single detector.

4. Digital Signal Processing with Germanium Detectors

The energy resolution in a germanium detector depends on two factors:

FWHM$_{\text{signal}}$ = FWHM$_{\text{inst}}$ + FWHM$_{\text{bias}}$

where:

$$\text{FWHM}_{\text{inst}} = 2.35 \sigma_{\text{inst}} / \sqrt{m}$$
$$\text{FWHM}_{\text{bias}} = 2.35 \sigma_{\text{bias}} / \sqrt{m}$$

The energy resolution is a crucial parameter for obtaining physics results. It is usually given as the Full Width at Half Maximum (FWHM) of the spectral peaks, once fitted by a Gaussian and in GERDA is quoted at $Q_{Q^2}$.

An improved resolution involves:
- higher sensitivity to the presence of background-induced-gamma peaks;
- more precision in the construction of the background model;
- in case a $0\nu\beta\beta$ decay signal is present, more precision in the computation of the $0\nu\beta\beta$ decay events;
- in case no $0\nu\beta\beta$ decay signal is present, a stronger limit for the value of the half-life, $T^{0\nu\beta\beta}_{1/2}$.

The achievable limit on $T^{0\nu\beta\beta}_{1/2}$ is:

$$T^{0\nu\beta\beta}_{1/2} \approx \frac{\tau_{0\nu\beta\beta}}{\sqrt{M^0 \cdot \varepsilon}}$$

where $\alpha$, $\beta$, $M$, $\varepsilon$, $\tau$, $Q$, $H$ and $\Delta E$ are parameters that depends on the filter type, and $\alpha$ is the root mean square of the series and parallel noise and $A_P$ is the coefficient of $1/f$ noise. The optimization of $\tau$ allows to filter out peculiar noise frequencies.