Pulse Shape Simulation for Germanium Detectors

An Overview

MaGe Pulse Shape Simulation Force

Jing Liu @ GERDA Collab. Meeting, Gran Sasso
Why we need it

Multi-site event in unsegmented detector

We need pulse shape analysis AND
Good signal & background samples to train PSA method
And verify discrimination efficiency

Multi-site event in one piece of cake
Why we need it

DEP

Single Compton Scattering events

$^{238}$Th

18-fold Ge detector

REGe

top view

Seg14

SEG17

side view

$^{238}$Th
1. Simulate $E_{\text{dep}}$ using MaGe

2. Group hits according to limits on bandwidth and sampling rate, determine the amount of e-h pairs and their original position

3. Calculate electric field

4. Calculate the drift of charge carriers

5. Calculate induced signals in the electrodes based on the drift trajectory and the weighing potential

6. Fold in effects of electronics, eg. noise, bandwidth, decay time, etc.
Group hits

Example Projected Electron Track

1.02 MeV electron

Stolen from Reyco Henning
Electric field – please refer to Daniel’s talk for greater detail

Poisson’s Equation:

$$\nabla \cdot E = \frac{\rho}{\varepsilon}$$
Calculate the drift :: Mobility

One can define the mobility $\mu_{e,h}$ of the electrons $e$ and holes $h$ as the variable which gives the relation between the electric field $E(r)$ and the drift velocity

$$v(r) = \mu_{e,h} E(r),$$

where $r$ indicates the position. $\mu_{e,h}$ depend on the temperature, the electric field and the structure of the germanium crystal. As long as the electron and hole temperatures do not differ much from the lattice temperature, the drift velocity is proportional to the electrical field and the lattice orientation has no influence. In this case the mobility can be simplified to just a number $\mu_{e,h} = \mu_0$. In germanium detectors cooled at liquid nitrogen temperature the electron/hole pairs are hotter than the lattice. The drift velocity in this condition is influenced by the crystal lattice orientation and not always parallel to the applied electrical field.
Germanium has the same crystalline structure as silicon and diamond, namely, a face-centered cubic (FCC) structure, in which each atom lies at the center of a regular tetrahedron, and is surrounded at its apices by four atoms as shown in Fig. 1.

![Diagram of germanium crystal structure]

- atom
- convalent bond
- crystal axes directions

Figure 1: Structure of germanium crystal.

Due to the crystal lattice symmetry in germanium, in three directions, the crystallographic \(\langle100\rangle\), \(\langle110\rangle\) and \(\langle111\rangle\), the mobility is always aligned with the electrical field.
Experimental data on the longitudinal anisotropy in these specific directions can be found in literature. The mobility data can be well fitted in any principal crystallographic direction with the parameterization:

\[ v = \frac{\mu_0 E}{[1 + (\frac{E}{E_0})^\beta]^{1/\beta}} - \mu_n E, \]  

(10)

where \( E, v \) are the magnitudes of the electric field and drift velocity, respectively, \( \mu_0, \mu_n, E_0 \) and \( \beta \) are the fitting parameters. At low fields, the mobility becomes isotropic and therefore the mobility fit parameter \( \mu_0 \) is expected to become independent of the crystallographic direction. For hot electrons, the departure from a linear \( v \sim E \) relation is modeled through the parameters \( E_0 \) and \( \beta \). At high field, Mihailescu et al. [8] have added the term \( \mu_n E \) to account for the Gunn effect that was observed by Ottaviani et al. [9] for field strengths above 3 kV/cm at 80 K. However, this effect is insignificant in our detector operated with field strengths 10-300 V/mm.
Calculate the drift :: Determine anisotropy in any direction

The anisotropy in any direction is related to the longitudinal anisotropy in the $\langle 100 \rangle$ and $\langle 111 \rangle$ directions. The drift velocity in any direction can be calculated accordingly.

\[
v = \frac{\mu_0 E}{[1 + \left(\frac{E}{E_0}\right)^\beta]^{1/\beta}} - \mu_n E
\]
Calculate the drift :: Drift trajectories

Figure 9: Charge carrier drift trajectories on X-Y plane. The transverse anisotropy causes the bend of the trajectories. Also shown are the cross section of a true coaxial cylindrical germanium detector with inner radius of 5 mm and outer radius of 37.5 mm. The crystal axes are indicated with the signs $\langle 100 \rangle$, $\langle 110 \rangle$ and $\langle 010 \rangle$. 

Numbers of signals recorded by different segments
Determine the crystal axis orientation using the occupancy plot.
Signals induced in the electrodes – please refer to Daniel’s talk for detail

Ramo’s Theorem:

\[ Q(t) = -N_{e/h} (E_{dep}) \cdot \varphi_w (r(t)) \]
\[ I(t) = N_{e/h} (E_{dep}) \cdot \vec{E}_w (r(t)) \cdot v(t) \]

- \( N_{e/h} (E_{dep}) \): Number of e/h created by energy deposition \( E_{dep} \)
- \( \varphi_w, \vec{E}_w \): Weighting potential, field
- \( r(t) \): Drift trajectory
- \( v(t) \): Drift velocity
Ideal pulse shape

Crystal 0, $T_0 = -50$ ns

- Central Contact
- Segment 1
- Segment 2
- Segment 3
- Segment 4
- Segment 5
- Segment 6

Charge vs. time (ns)

- Y-axis: $10^3$ x-axis: 0 to 1000 ns

Diagram showing pulse shapes for different segments of a crystal.
Simulation of decay time

Crystal 0, $T_0 = -50$ ns

Charge vs. time (t [ns])

- Central Contact
- Segment 1
- Segment 2
- Segment 3
- Segment 4
- Segment 5
- Segment 6
Fold in bandwidth

Crystal 0, $T_0 = -50$ ns

Charge vs. Time (ns)
Let's add some noise

**Crystal 0, $T_0 = -50$ ns**
Summary, Acknowledgement & Outlook

- Now we can simulate the whole signal formation process in germanium detector system!
- Many thanks to Majorana MC group!
- Validation of the simulation on the way
- Pulse shape analysis follows
Crystal 0, $T_0 = -50$ ns
eps[0] + hps[0]: \text{time} \times 10^9
Find back $0\nu\beta\beta$ events occur between two segments
Calculate the drift :: Parameters fitted with experimental data

Parameterization values fitted with the experimental data are summarized in Table 1. The values from two different references are quite different from each other. We’d better measure it by ourselves again.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Carrier</th>
<th>Direction</th>
<th>$\mu_0$ [cm$^2$/V·s]</th>
<th>$E_0$ [V/cm]</th>
<th>$\beta$</th>
<th>$\mu_n$ [cm$^2$/V·s]</th>
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<td>$\langle 111 \rangle$</td>
<td>40180</td>
<td>493</td>
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<td>589</td>
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<td>$\langle 100 \rangle$</td>
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Table 1: Fit parameters for the experimental drift velocities in the $\langle 111 \rangle$ and $\langle 100 \rangle$ directions.
Calculate the drift :: Electron drift velocity

Effective electron mass tensor:
\[ \gamma_j = R_j^{-1} \gamma_0 R_j = R_j^T \gamma_0 R_j \]

Electron drift velocity equation:
\[ v = \frac{\mu_0 E}{[1 + (\frac{E}{E_0})^\beta]^{1/\beta}} - \mu_n E \text{ in the } \langle 111 \rangle \text{ and } \langle 100 \rangle \text{ directions} \]

Fraction of electrons in certain directions:
\[ n_i = \frac{n_j}{n} \left( \frac{\sqrt{E_0 \gamma_j E_0}}{\sum_i \sqrt{E_0 \gamma_i E_0}} - \frac{n_e}{n} \right) + \frac{n_e}{n} \]

Effective electron mass tensor:
\[ \gamma_0 = \begin{pmatrix} m_t^{-1} & 0 & 0 \\ 0 & m_t^{-1} & 0 \\ 0 & 0 & m_t^{-1} \end{pmatrix} \]

Mass values:
- \[ m_t = 1.64 m_e \]
- \[ m_l = 0.0819 m_e \]
Calculate the drift :: Hole drift velocity

\[
\begin{pmatrix}
    v_x' \\
    v_y' \\
    v_z'
\end{pmatrix} = R_z(\phi + \frac{\pi}{4} + \phi_{110}) R_y'(\theta) \begin{pmatrix}
    v_x' \\
    v_y' \\
    v_z'
\end{pmatrix}
\]

\[
v_x' = v_r = v_{h100}^1(E)[1 - \Lambda(k_0)(\sin(\theta)^4 \sin(2\phi)^2 + \sin(2\theta)^2)]
\]

\[
v_y' = v_\theta = v_{h100}^1(E)\Omega(k_0)[2 \sin(\theta)^3 \cos(\theta) \sin(2\phi)^2 + \sin(4\theta)]
\]

\[
v_z' = v_\phi = v_{h100}^1(E)\Omega(k_0) \sin(\theta)^3 \sin(4\phi)
\]

\[
\Lambda(k_0) = -0.01322k_0 + 0.41145k_0^2 - 0.23657k_0^3 + 0.04077k_0^4
\]

\[
\Omega(k_0) = 0.006550k_0 - 0.19946k_0^2 + 0.09859k_0^3 - 0.01559k_0^4
\]

\[
k_0(v_{rel}) = 9.2652 - 26.3467v_{rel} + 29.6137v_{rel}^2 - 12.3689v_{rel}^3, \quad v_{rel} = \frac{v_{h111}(E)}{v_{h100}(E)}
\]

\[
v = \frac{\mu_0 E}{[1 + (E/E_0)^{\beta}]^{1/\beta}} - \mu_n E \text{ in the } \langle 111 \rangle \text{ and } \langle 100 \rangle \text{ directions}
\]
Pulse rise time