

Radiation Dominant Regimes in Electromagnetic Wave Interaction with Electrons

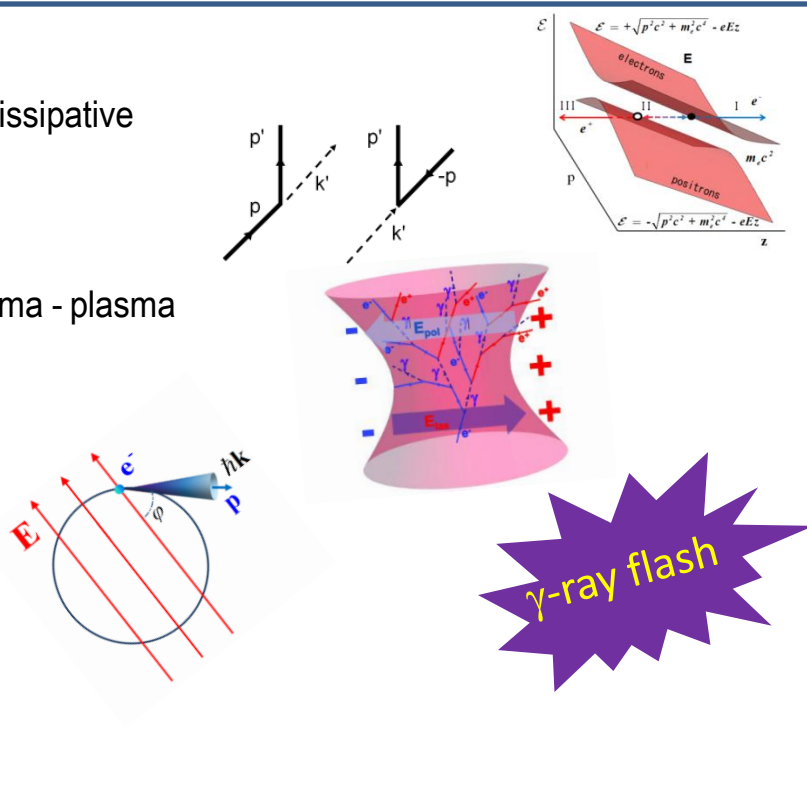
Sergei V. Bulanov

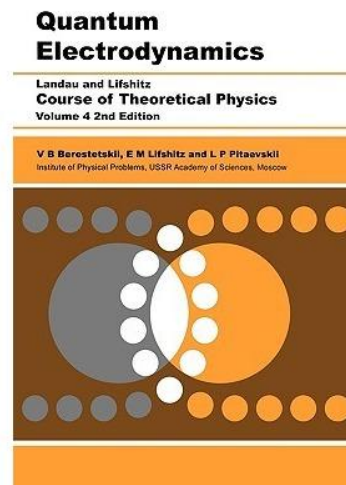
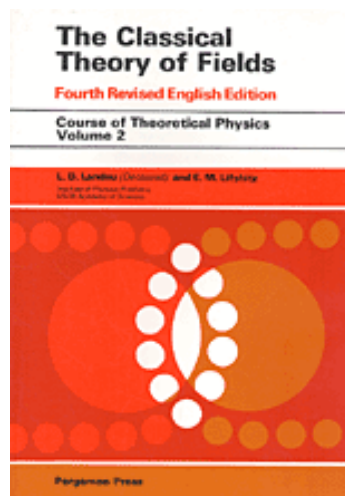
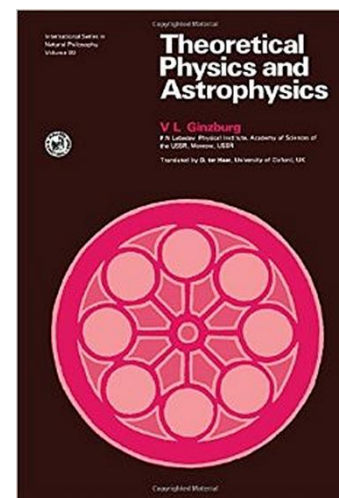
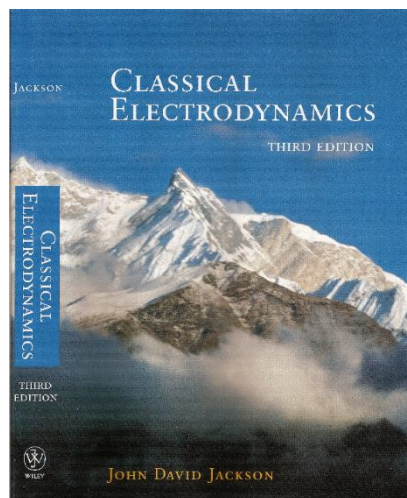
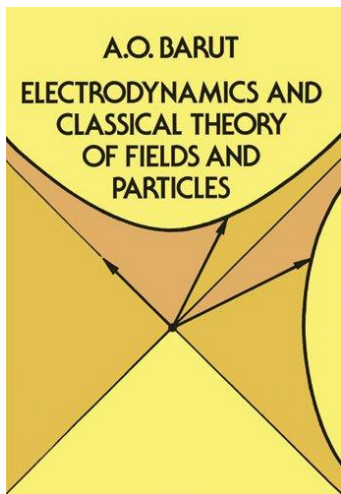
QuBS, Japan Atomic Energy Agency, Kizugawa, Kyoto, Japan

Conference
on Extremely High Intensity Laser Physics
Heidelberg, Germany, 23 July 2015

Extreme Field Limits

in high intensity laser interaction with matter and vacuum

Focused intensity (W/cm ²)	non - perturbative QED $E \leq E_S = m_e^2 c^3 / e \hbar$	10 ²⁹ nonlinear - dissipative vacuum	
	multiphoton QED processes $e + N \hbar \omega_0 \rightarrow \hbar \omega_\gamma + e'$ $\hbar \omega_\gamma + N \hbar \omega_0 \rightarrow e^- + e^+$	10 ²⁷ / γ_e^2 lepton - gamma - plasma	
	radiation reaction $a_0 > (\lambda / r_e)^{1/3} \approx 400$	10 ²⁴ / γ_e^2 X, γ - rays	
	relativistic nonlinear optics $a_0 = eE_0 / m_e \omega_0 c > 1$	10 ²³ / $\gamma_e^{2/3}$ plasma	
	bound electrons $E_0 \leq e^2 / a_B = m_e^2 e^5 / \hbar^4$	10 ¹⁸	



Lorentz-Abraham-Dirac,
Pomeranchuk,
Landau-Lifshitz, ...

Basic Equations: Minkovski & Maxwell Equations

Minkovski equations: $\frac{du^\mu}{ds} = \frac{e}{m_e c} F^\mu{}_\nu u^\nu$ where $u^\mu = \frac{dx^\mu}{ds}$, $ds = \frac{dt}{\gamma}$

EM field tensor: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Maxwell equations: $\varepsilon^{\mu\nu\sigma\rho} \partial_\rho F_{\nu\sigma} = 0$ and $\partial_\nu F^{\mu\nu} = \frac{4\pi}{c} j^\mu$

In 3D notations: $\dot{\mathbf{p}} = e \left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right)$, $\dot{\mathbf{x}} = c \frac{\mathbf{p}}{m_e \gamma}$, $\gamma = \left(1 + \frac{p_\mu p^\mu}{m_e^2 c^2} \right)^{1/2}$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \partial_t \mathbf{E}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \partial_t \mathbf{B}$$

$$\nabla \cdot \mathbf{E} = 4\pi \rho$$

Radiation Losses

Intensity of radiation emitted by electron is given by

$$I = \frac{2e^2}{3m_e^2 c^3} \left(\frac{dp_i}{ds} \frac{dp_i}{ds} \right)$$

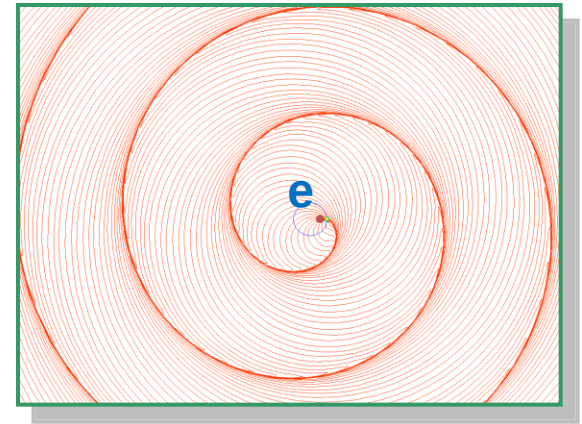
In circularly polarized EM wave (**in plasma**), whose

amplitude is equal to $a_0 = \frac{eE_0}{m_e \omega_0 c}$ electron energy

losses are $\dot{\mathcal{E}}^{(-)} = \frac{2e^4 E_0^2}{3m_e^2 c^3} \left[1 + \left(\frac{eE_0}{m_e \omega_0 c} \right)^2 \right]$

For linearly polarized wave we have

$$\dot{\mathcal{E}}^{(-)} = \frac{e^4 E_0^2}{3m_e^2 c^3} \left[1 + \frac{3}{8} \left(\frac{eE_0}{m_e \omega_0 c} \right)^2 \right]$$



Pattern of field emitted by electron. T.Shintake, 2003

L.D.Landau & E.M.Lifshitz
“The Classical Theory of Fields”

LAD-form of radiation friction force

Equations of electron motion are:

$$m_e c^2 \frac{du^\mu}{ds} = \frac{e}{c} F^{\mu\nu} u_\nu + g^\mu$$

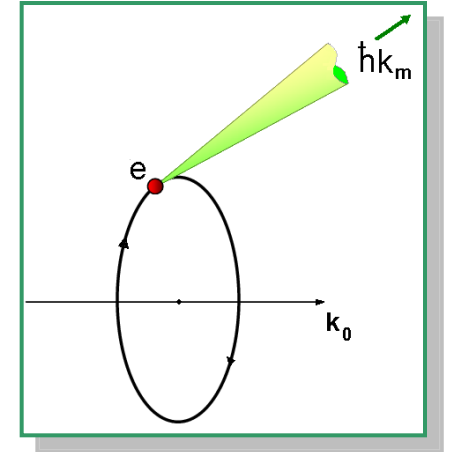
Radiation friction force is given by

$$g^\mu = \frac{2e^2}{3c} \left(\frac{d^2 u^\mu}{ds^2} - u^\mu u^\nu \frac{d^2 u_\nu}{ds^2} \right)$$

Here $\mu=0,1,2,3$, s is proper time: $ds = c dt / \gamma$

4-velocity is $u^i = \frac{dx^i}{ds} = \left(\gamma, \frac{\mathbf{p}}{m_e c} \right)$

and $F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}$ is 4-tensor of EM field



Radiation reaction force

Nonrelativistic case: $m \ddot{\mathbf{x}} = \mathbf{F}_{\text{ex}} + \mathbf{g}_{\text{rad}}$ $\mathbf{g}_{\text{rad}} = \frac{2e^2}{3c^3} \ddot{\mathbf{x}}$ $\mathbf{F}_{\text{ex}} = e\mathbf{E}_{\text{ex}} + \frac{e}{c} \dot{\mathbf{x}} \times \mathbf{B}_{\text{ex}}$

Self-accelerating solution: $\mathbf{v} = \mathbf{v}_0 \exp\left(\frac{3m_e c^3}{2e^2} t\right)$

Characteristic timescale: $t_r = \frac{2e^2}{3m_e c^3} = \frac{2r_e}{3c} \approx 6.25 \times 10^{-24} \text{ s}$

$r_e = \frac{e^2}{m_e c^2} \approx 2.82 \times 10^{-13} \text{ cm}$ - classical electron radius

Typically $x/r_e \gg 1$, $t/t_r \gg 1$, and $E/E_{\text{cr}} \ll 1$,

where $E_{\text{cr}} = \frac{m_e^2 c^4}{e^3} = \left(\frac{\lambda_C}{r_e}\right) \frac{m_e^2 c^3}{e\hbar} = \frac{1}{\alpha} E_S \approx 137.036 E_S$ is the critical field of classical

Electrodynamics with the Compton wavelength $\lambda_C = \frac{\hbar}{m_e c} \approx 3.86 \times 10^{-11} \text{ cm}$

Radiation Force as Perturbation

Self-consistent field:

$$m \ddot{\mathbf{x}} = \int \rho(\mathbf{x} - \mathbf{x}') E(\mathbf{x}') d\mathbf{x}' \quad \mathbf{E} = \mathbf{E}_{\text{ex}} + \mathbf{E}_e$$

Using smallness $r_e \ll |\delta \mathbf{x}|$ we obtain

$$m \ddot{\mathbf{x}} = -m_{\text{em}} \ddot{\mathbf{x}} + \frac{2e^2}{3c^3} \ddot{\mathbf{x}} + \dots$$

i.e. $m_e = m + m_{\text{em}}$, $m_{\text{em}} = \frac{4}{3c^2} \iint \frac{\rho(\mathbf{x})\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d\mathbf{x}d\mathbf{x}'$

“classical mass renormalization”

Weak radiation friction: $|\mathbf{g}_{\text{rad}}| \ll |\mathbf{F}_{\text{ex}}|$

Equations of electron motion
$$m_e \ddot{\mathbf{x}} = \mathbf{F}_{\text{ex}} + \frac{2e^3}{3m_e c^3} \dot{\mathbf{E}}_{\text{ex}} + \frac{2e^4}{3m_e^2 c^4} \mathbf{E}_{\text{ex}} \times \mathbf{B}_{\text{ex}} \quad \text{[L-L]}$$

E.M. wave in vacuum $\omega^2 = k^2 c^2$ in plasma $\omega^2 = k^2 c^2 + \omega_{pe}^2$ $\omega_{pe}^2 = 4\pi n e^2 / m_e$
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Pomeranchuk (1939)

Covariant and 3D forms of L-L expression

Radiation reaction force in the Landau-Lifshitz approximation (L-L II, §76):

$$m_e c^2 \frac{du^\mu}{ds} = \frac{e}{c} F^{\mu\nu} u_\nu + \frac{2e^3}{3m_e c^3} \left\{ \partial_\lambda F^{\mu\nu} u_\nu u_\lambda - \frac{e}{m_e c^2} \left[F^{\mu\lambda} F_{\nu\lambda} u^\nu - (F_{\nu\lambda} u^\lambda) (F^{\nu\kappa} u_\kappa) u^\mu \right] \right\}$$

$$\begin{aligned} m_e c^2 \frac{d\mathbf{p}}{dt} = & e \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right) + \\ & + \frac{2e^3}{3m_e c^3} \gamma \left\{ (\partial_t + \mathbf{v} \cdot \nabla) \mathbf{E} + \frac{1}{c} \mathbf{v} \times ((\partial_t + \mathbf{v} \cdot \nabla) \mathbf{B}) \right\} + \\ & + \frac{2e^4}{3m_e^2 c^4} \left\{ \mathbf{E} \times \mathbf{B} + \frac{1}{c} \mathbf{B} \times (\mathbf{B} \times \mathbf{v}) + \frac{1}{c} \mathbf{E} (\mathbf{v} \cdot \mathbf{E}) \right\} - \\ & - \frac{2e^4}{3m_e^2 c^5} \gamma^2 \mathbf{v} \left\{ \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right)^2 - \frac{1}{c^2} (\mathbf{v} \cdot \mathbf{E})^2 \right\} \end{aligned}$$

$\propto \varepsilon_{rad} \gamma \omega a_0$

$\propto \varepsilon_{rad} a_0^2$

$\propto \varepsilon_{rad} \gamma^2 a_0^2$

Dimensionless parameter:

$$\varepsilon_{rad} = \frac{2e^2 \omega}{3m_e c^3} = \frac{4\pi r_e}{3\lambda} \approx 10^{-8} \left(\frac{1\mu m}{\lambda} \right)$$

"Pomeranchuk theorem"

We consider the electron colliding with the EM wave given by

$$a_0(t - x/c) \approx a_0(2t) \quad x = -ct$$

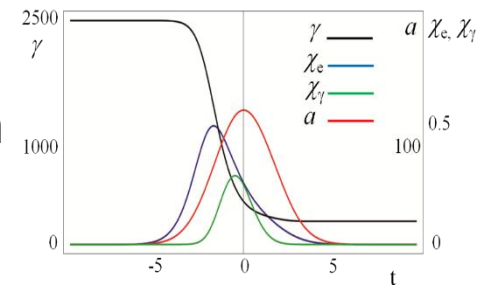
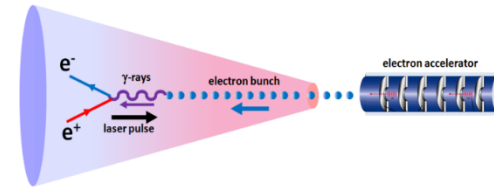
Retaining the main order terms in the L-L radiation friction force, we obtain equation for the x-component of the electron momentum

$$\frac{d\gamma}{dt} = -\varepsilon_{rad} \omega a_0^2(2t) \gamma^2, \quad \varepsilon_{rad} = \frac{2r_e}{3\lambda}$$

Its solution is

$$\gamma(t) = \frac{\gamma_0}{1 + \varepsilon_{rad} \omega \gamma_0 \int_0^t a_0^2(2t') dt'}, \quad \gamma \xrightarrow[t \rightarrow \infty]{} \frac{1}{\varepsilon_{rad} \omega \tau_{las} a_0^2}$$

For $\varepsilon_{rad} = 10^{-8}$ $\omega \tau_{las} = 100$ $a_0 = 300$ the electron gamma - factor becomes equal to

$$\gamma = 10$$


Landau, L. D. and Lifshitz, E. M., *The Classical Theory of Fields*, Pergamon, 1975

Pomeranchuk, I.: 'Maximum Energy that Primary Cosmic-ray Electrons Can Acquire on the Surface of the Earth as a Result of Radiation in the Earth's Magnetic Field', J. Phys. USSR, 2, 65 – 69, 1940

Radiation Force in 1+1 D Electrodynamics

We consider the case of normal incidence of a plane electromagnetic wave on an infinitely thin foil. The foil is located in the plane $x = 0$. The interaction of the wave with the foil is described by Maxwell's equations for the vector potential $A(x, t)$ which yield

$$\partial_{tt}A - c^2 \partial_{xx}A = 4\pi c \delta(x) J(A) + \dot{\delta}(t) A(x, 0) + \delta(t) \dot{A}(x, 0)$$

Convolution of the Green function for the one-dimensional wave equation

$$G(x, t; s, \tau) = \theta[(t - \tau) - |x - s|/c]/2$$

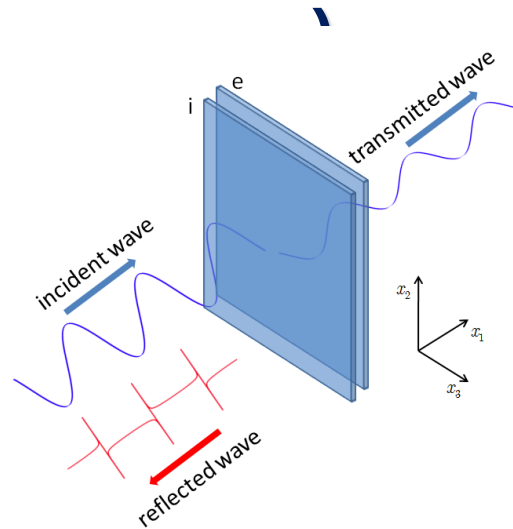
with the terms in the right-hand side of the wave equation yields

$$A(x, t) = A_{in}(x, t) + 2\pi \int_0^{t-|x|/c} J(A(0, s)) ds$$

Assuming $x = 0$ and taking the derivative with respect to time, we obtain for $p = -eA/c$

$$\dot{p} + \frac{2\pi n e^2 l}{c} v = e E_{in}(t)$$

Nonlinear Electrodynamics (1+1 D) of a Thin Foil



V. I. Bratman, S. V. Samsonov,
Phys. Lett. A 206, 377 (1995)

S. V. Bulanov, T. Zh. Esirkepov, M. Kando,
S. S. Bulanov, S. G. Rykovanov, F. Pegoraro,
Phys. Plasmas 20, 123114 (2013)

.....

Four-vector el.
current

$$j^\nu = \sum_\alpha j_\alpha^\nu = \sum_\alpha Z_\alpha(c, v_\alpha) e n_0 l_0 \delta(x - x_\alpha(t)) \quad \nu = 0, 1, 2, 3$$

Solution to the
Maxwell
equations
for emitted
EM wave

$$\mathbf{E}_\alpha = 2\pi n_0 l_0 Z_\alpha e \left[s_\alpha(x, \bar{t}_\alpha) \mathbf{e}_1 + \frac{v_{2,\alpha} \mathbf{e}_2 + v_{3,\alpha} \mathbf{e}_3}{c - s_\alpha(x, \bar{t}_\alpha) v_{1,\alpha}(\bar{t}_\alpha)} \right]$$

$$\mathbf{B}_\alpha = -2\pi n_0 l_0 Z_\alpha e s_\alpha(x, \bar{t}_\alpha) \frac{v_{3,\alpha} \mathbf{e}_2 - v_{2,\alpha} \mathbf{e}_3}{c - s_\alpha(x, \bar{t}_\alpha) v_{1,\alpha}(\bar{t}_\alpha)}$$

Retarded time, $\bar{t}_\alpha = t - |x - x_\alpha(\bar{t}_\alpha)| / c$, and $s_\alpha(x, \bar{t}_\alpha) = \text{sgn}(x - x_\alpha(\bar{t}_\alpha))$

Self-Action: Radiation Friction in Cooperative Mode

At the α - th layer we have for electric and magnetic field

$$\mathbf{E}_{\alpha,l} = 2\pi n_0 l_0 c Z_\alpha e \frac{v_{2,\alpha} \mathbf{e}_2 + v_{3,\alpha} \mathbf{e}_3}{c^2 - v_{1,\alpha}^2}, \quad \mathbf{B}_{\alpha,l} = -2\pi n_0 l_0 Z_\alpha e v_{1,\alpha} \frac{v_{3,\alpha} \mathbf{e}_2 - v_{2,\alpha} \mathbf{e}_3}{c^2 - v_{1,\alpha}^2}$$

Equations of the α - th layer motion in components

$$\dot{p}_{1,\alpha} = Z_\alpha \mu_\alpha \left(E_1 + \frac{p_{2,\alpha} B_3 - p_{3,\alpha} B_2}{\gamma_\alpha} \right) - \epsilon_\alpha \frac{p_{1,\alpha} (p_{2,\alpha}^2 + p_{3,\alpha}^2)}{\gamma_\alpha (\gamma_\alpha^2 - p_{1,\alpha}^2)},$$

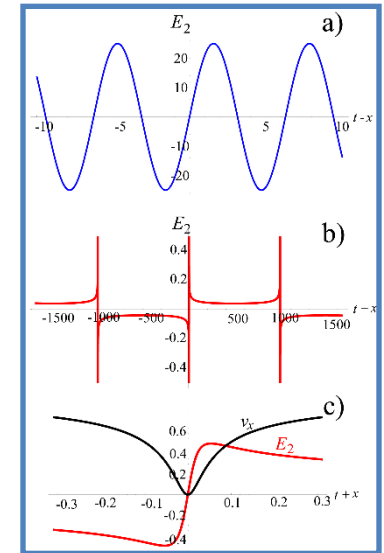
$$\dot{p}_{2,\alpha} = Z_\alpha \mu_\alpha \left(E_2 - \frac{p_{1,\alpha} B_3}{\gamma_\alpha} \right) - \epsilon_\alpha \frac{p_{2,\alpha}}{\gamma_\alpha},$$

$$\dot{p}_{3,\alpha} = Z_\alpha \mu_\alpha \left(E_3 + \frac{p_{1,\alpha} B_2}{\gamma_\alpha} \right) - \epsilon_\alpha \frac{p_{3,\alpha}}{\gamma_\alpha},$$

$$\dot{x}_\alpha = \frac{p_{1,\alpha}}{\gamma_\alpha}$$

with dimensionless parameter

$$\epsilon_\alpha = \frac{2\pi n_0 e^2 l_0}{m_\alpha \omega_0 c}$$



High Order Harmonics

Relativistic Oscillating Mirror

Relativistic Flying Mirror with Thin Foil Target

Theoretical language to describe the process in the configuration considered by

V. V. Kulagin, V. A. Cherepenin, M. S. Hur, H. Suk,
Phys. Plasmas 14, 113101 (2007)

D. Kiefer et al., *Nat. Comm.* (2013)

Two counter-propagating waves interact with thin foil target

Reflected wave phase and frequency are

$$\psi_r(u) = \omega_s \left(u + \frac{a_0^2}{2} u - \frac{a_0^2}{4\omega} \sin 2\omega u \right)$$

and

$$\omega_r(u) = \omega_s \left(1 + a_0^2 \sin^2 \omega u \right)$$

Frequency upshifting factor $g = \frac{\gamma + p_1}{\gamma - p_1}$

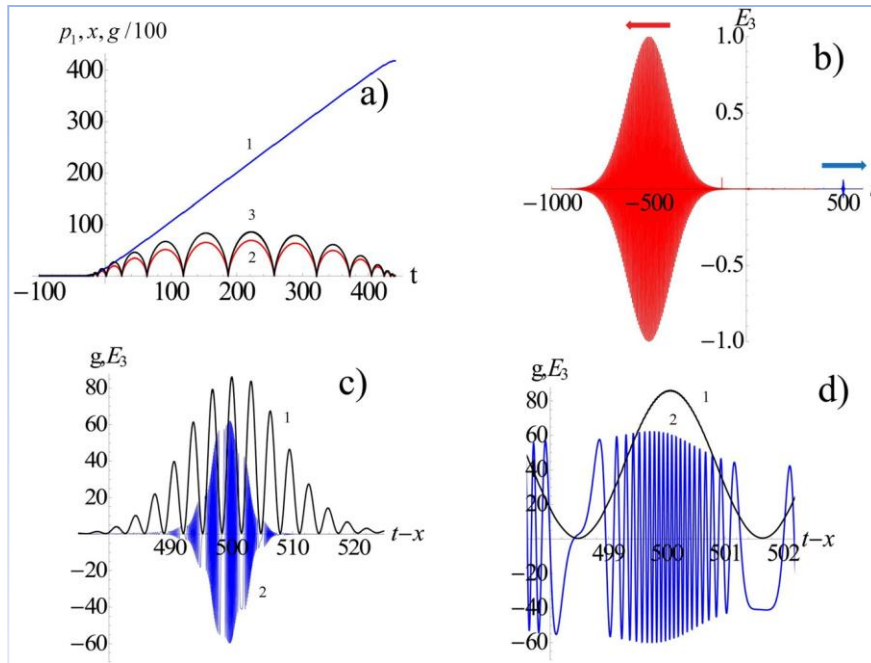
Reflected pulse frequency changes from ω_s to $\omega_s (1 + a_0^2)$

S. V. Bulanov, T. Zh. Esirkepov, M. Kando, A. S. Pirozhkov, and N. N. Rosanov

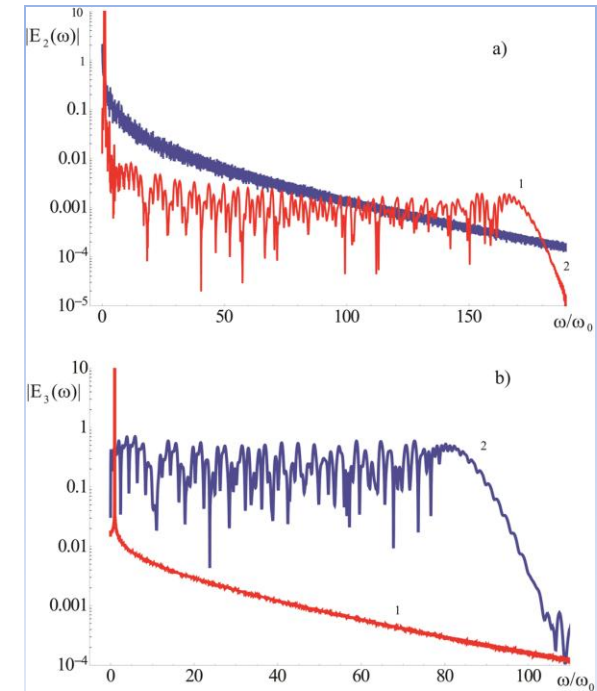
"Relativistic Mirrors in Plasmas – Novel Results and Perspectives"

Physics Uspekhi 183, 429 - 464 (2013)

Reflected at DS-FM EM Wave

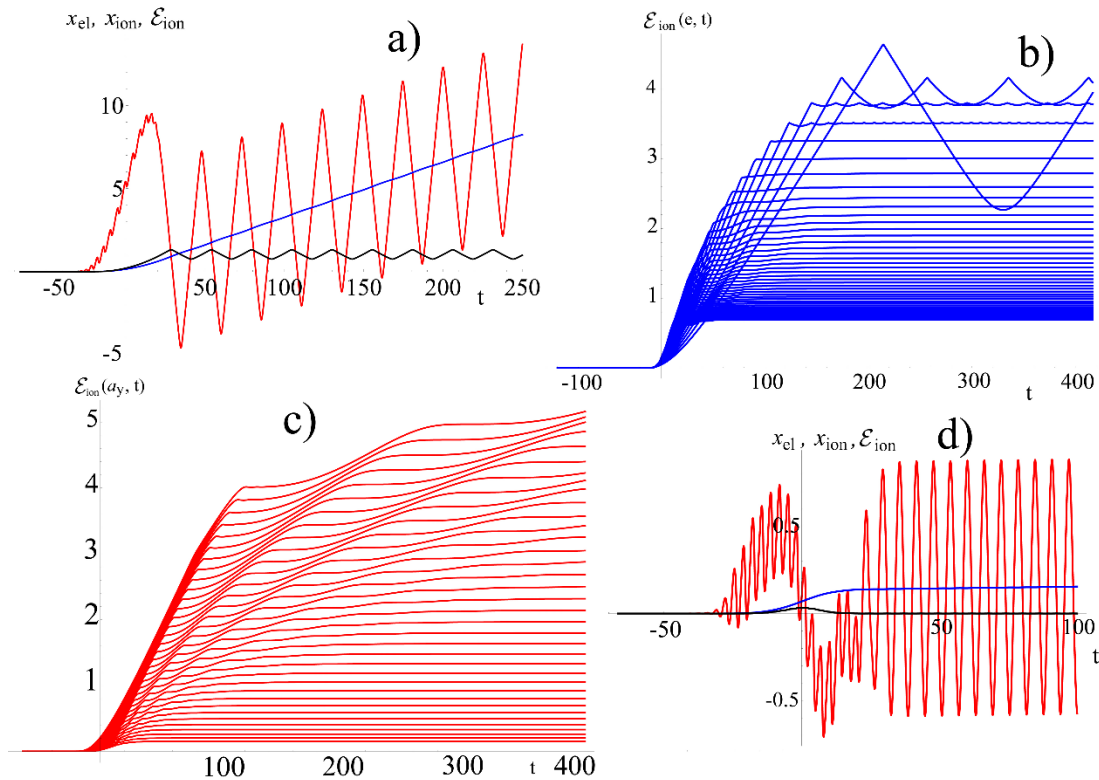


a) Time dependence of the longitudinal electron layer momentum (red curve), of the layer coordinate (blue curve) and of the factor g (black curve). Counterpropagating source pulse:
 b) Reflected (blue curve) and transmitted, (red curve) waves. c) Reflected pulse (blue curve) and frequency upshifting factor (black curve). d) Close up of the reflected pulse (blue curve) and frequency upshifting factor (black curve).



The frequency spectrum of the driver and source pulses. a) The dependence of the absolute value of the Fourier transform of the component of the electric field, corresponding to the incident and transmitted electromagnetic of the driver pulse. b) The dependence of the absolute value of the Fourier transform of the component of the electric field, which corresponds to the incident and reflected electromagnetic of the source pulse.

Ion Acceleration in RPDA Regime



Ion acceleration by the radiation pressure. (a) Time dependence of the electron (red curve) and ion (blue curve) layer co-ordinates and of the ion energy (black) (b) Normalized ion energy vs time for laser amplitude and the parameter ϵ varying from 45 to 250 from bottom to top with the step equal to 5. (c) Normalized ion energy vs time for the EM pulse amplitude varying from bottom to top from 100 to 450 with the step equal to 10. (d) Time dependence of the electron (red curve) and ion (blue curve) layer co-ordinates and of the ion energy (black) for the case without radiation friction.

High Efficiency Gamma-Ray Generation during Interaction of Extremely Intense Laser Radiation with Overdense Plasma Targets

C. P. Ridgers, C. S. Brady, R. Duclous, J. G. Kirk, K. Bennett, T. D. Arber, A. P. L. Robinson, A. R. Bell,
Phys. Rev. Lett. 108 (2012) 165006

T. Nakamura, J. K. Koga, T. Z. Esirkepov, M. Kando, G. Korn, S. V. Bulanov,
Phys. Rev. Lett. 108 (2012) 195001

High Power Gamma-Ray Source

Applications

- Photo-nuclear reactions
- Electron-positron pair creation
- Gamma laser pumping
- Medicine
-
- **Radiation safety**

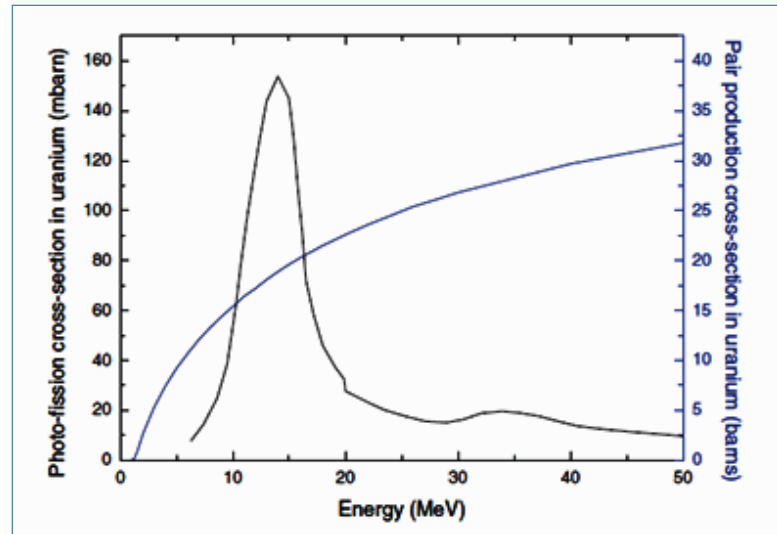
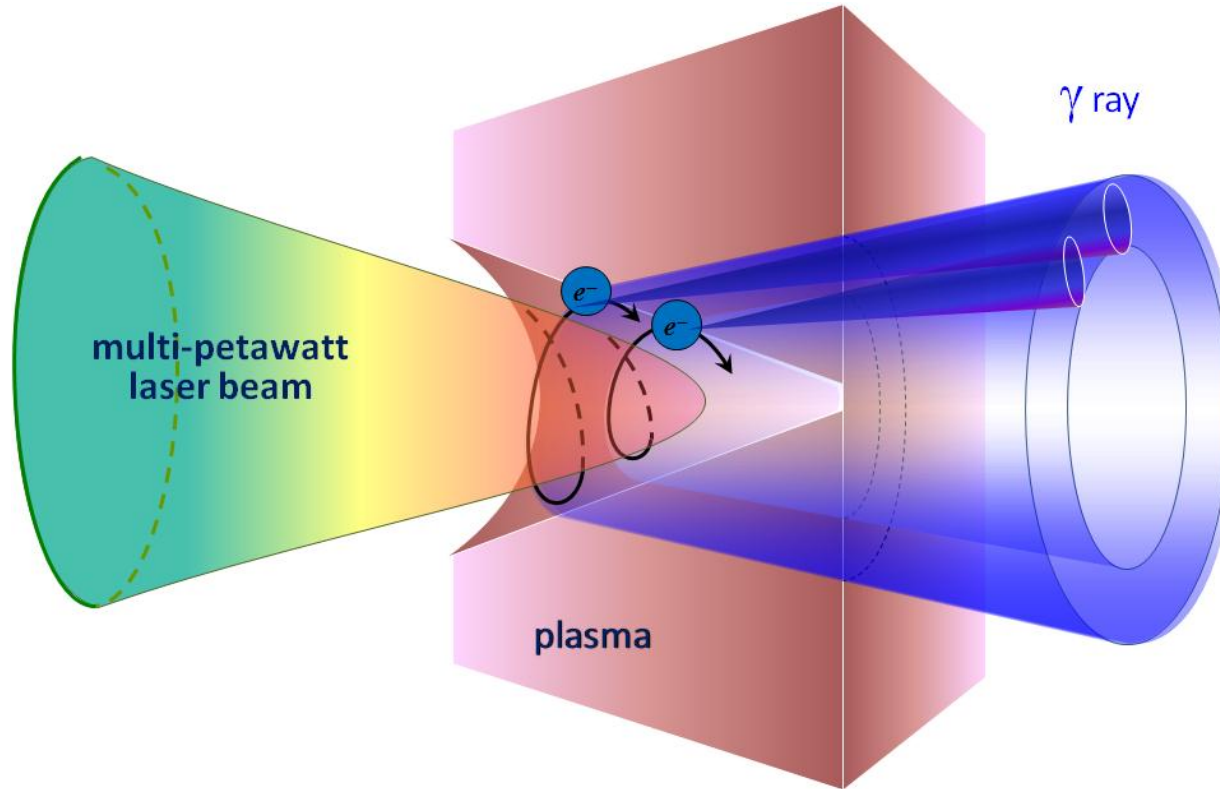


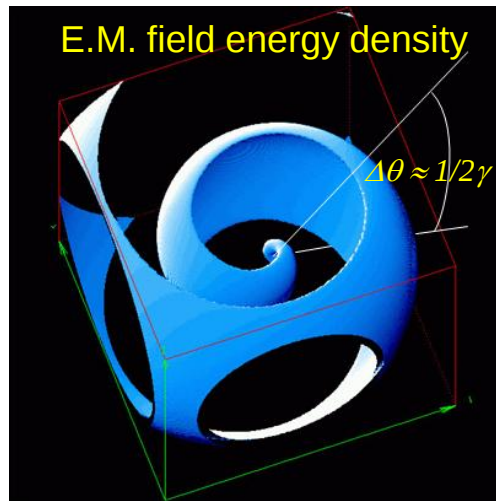
Photo-fission cross-section and pair production cross-section in uranium
[J. Galy, et al., New J. Phys. 9 (2007) 23]

Concept of high power gamma-flash generation



Nonlinear Thomson Scattering

High energy photons are also emitted in high-intensity laser light interaction with plasmas where electrons quivering with ultrarelativistic energy produce nonlinear Thomson scattering, which has much in common with synchrotron radiation



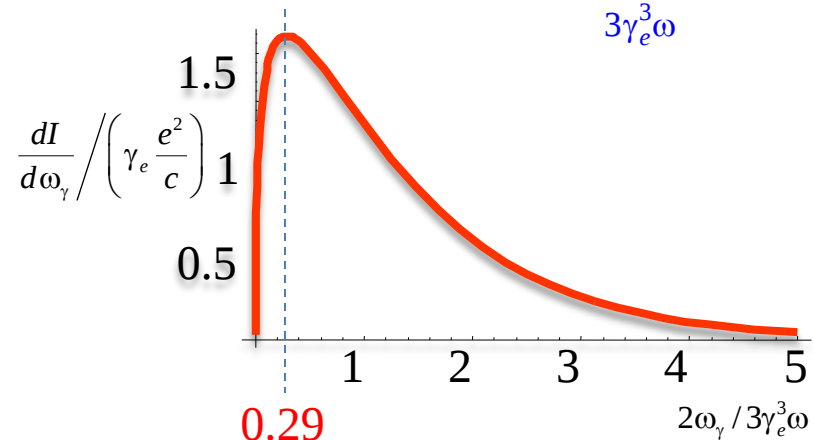
Energy loss by radiation

$$\frac{d\gamma_e}{dt} = -\frac{2e^2}{3m_e c^3} \omega^2 \gamma_e^2 (\gamma_e^2 - 1)$$

SYNCHROTRON RADIATION

Frequency distribution of the total energy emitted by rotating electron

$$\frac{dI}{d\omega_\gamma} = \frac{\sqrt{3}}{2\pi} \gamma_e \frac{e^2}{c} \frac{2\omega_\gamma}{3\gamma_e^3 \omega} \int_{\frac{2\omega_\gamma}{3\gamma_e^3 \omega}}^{\infty} K_{5/3}(\xi) d\xi$$



Photon Energy

The characteristic energy of the photon emitted via nonlinear Thomson scattering scales with the electron quiver energy, $\gamma_e m_e c^2$, in the limit $\gamma_e \gg 1$ as

$$\mathcal{E}_\gamma = \hbar \omega_\gamma \approx \hbar \omega \gamma_e^3$$

where ω is the laser frequency.

The energy of the electron quivering in plasma under the action of an electromagnetic wave with an amplitude of $a = eE / m_e \omega c \gg 1$ is of the order of $m_e c^2 a$.

For a laser frequency of the order of 10^{15} s^{-1} the emitted photon energy is in the γ - ray range if $a > 10^2$ which corresponds to a laser intensity higher than 10^{22} W/cm^2 and to $\lambda_\gamma \ll n^{-1/3}$.

The radiation generated by present - day lasers approaches this limit.

At this limit radiation friction effects change the electromagnetic wave interaction with matter rendering the electron dynamics dissipative, with efficient transformation of the laser energy into γ - ray photons.

Electron Dynamics in Rotating Electric Field

The electron dynamics in the boosted frame of reference (c-pol EM wave):

electron equations of motion are

$$\begin{cases} \dot{\mathbf{q}} = -\mathbf{a} - \frac{\varepsilon_{\text{rad}}}{\gamma} \left\{ \gamma^2 \dot{\mathbf{a}} - \mathbf{a} (\mathbf{q} \cdot \mathbf{a}) + \mathbf{q} \left[(\gamma \mathbf{a})^2 - (\mathbf{q} \cdot \mathbf{a})^2 \right] \right\} \\ \mathbf{q} = (q_2, q_3) \end{cases}$$

were

$$n = \frac{n}{n_{cr}}, \quad \tau = \Omega t, \quad \mathbf{q} = \frac{\mathbf{p}}{m_e c}, \quad \mathbf{a} = \frac{e\mathbf{E}}{m_e \Omega c}, \quad \gamma = \left(1 + q_1^2 + q_2^2 + q_3^2 \right)^{1/2}$$

The dimensionless parameter $\varepsilon_{\text{rad}} = 2e^2 \Omega / 3m_e c^3$

determines the role of the radiation friction. The radiation friction effects become dominant when the laser pulse amplitude is equal to or greater than

$$a_{\text{rad}} = \varepsilon_{\text{rad}}^{-1/3}$$

which corresponds to $\approx 10^{23} \text{ W/cm}^2$ with $a_{\text{rad}} \approx 400$.

For $\varepsilon_{\text{rad}} = 0$ the wave frequency is $\Omega = \omega_{pe} \left(1 + q_1^2 + a^2 \right)^{-1/4}$ (Akhiezer and Polovin, 1956)

Ya. B. Zel'dovich (1975); A. Zhidkov (2002); SVB (2004); A. Di Piazza (2008), ...

Radiation Friction Effects

In order to describe the electron motion we write the electron momentum as

$$\begin{pmatrix} q_{\perp} \\ q_{\parallel} \\ q_{\perp} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\tau) & \sin(\tau) \\ 0 & -\sin(\tau) & \cos(\tau) \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

Here $q_{\parallel}$ and $q_{\perp}$ are the components of the electron momentum parallel and perpendicular to the electric field.

We assume here that the wave is given.

Neglecting the change of the $q_{\perp}$ -component (near-critical plasma density),

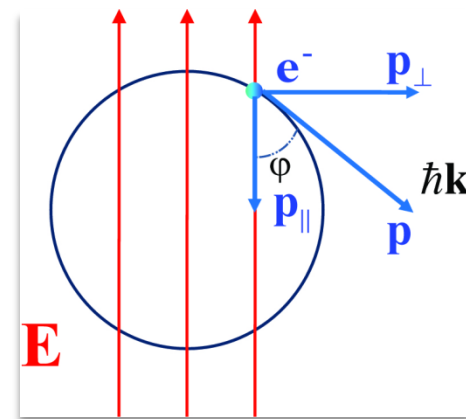
we obtain

$$\dot{q}_{\perp} - q_{\parallel} = -\varepsilon_{\text{rad}} \left[\gamma a + a^2 \frac{q_{\perp}}{\gamma} (1 + q_{\perp}^2) \right]$$

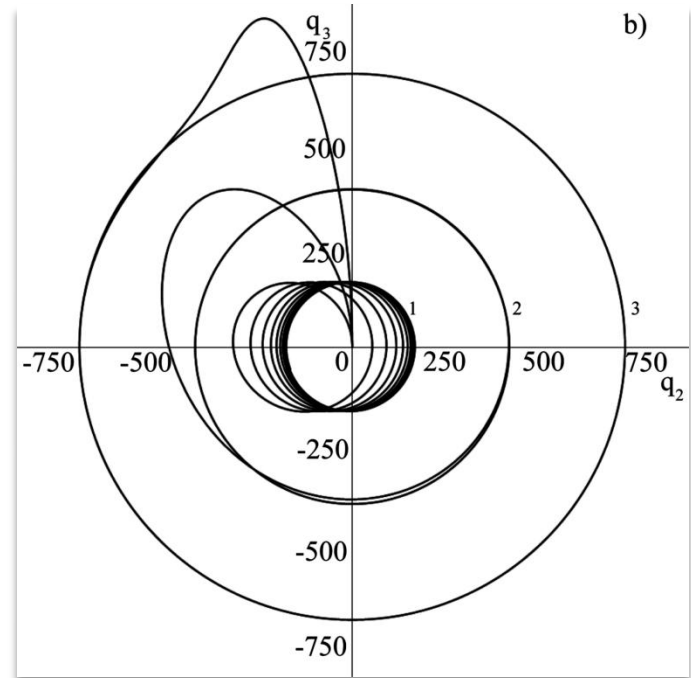
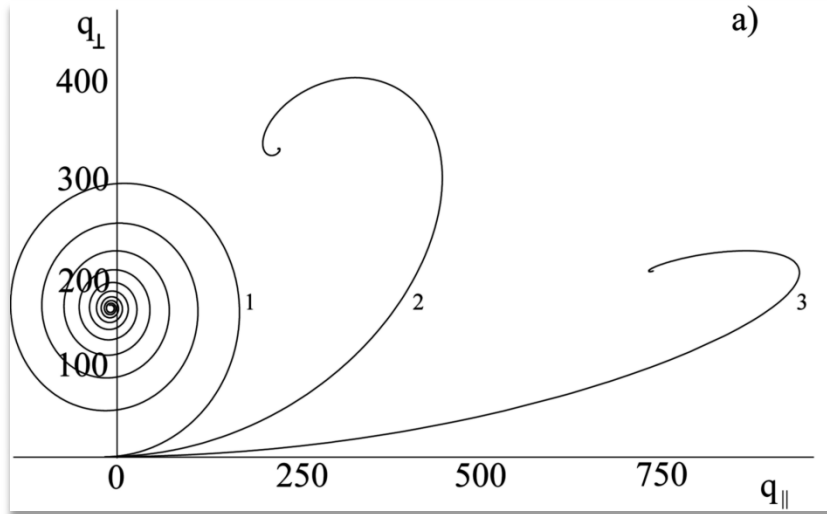
$$\dot{q}_{\parallel} + q_{\perp} = a - \varepsilon_{\text{rad}} a^2 q_{\parallel} \frac{q_{\perp}^2}{\gamma}$$

with the energy balance equation

$$\dot{\gamma} = a u_{\parallel} - \varepsilon_{\text{rad}} (a q_{\perp} + a^2 q_{\perp}^2)$$



Trajectories



Electron orbits in a) the $(q_{\parallel}, q_{\perp})$ plane, and b) the (q_2, q_3) plane for $\epsilon_{\text{rad}} = 10^{-8}$.

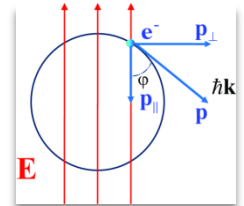
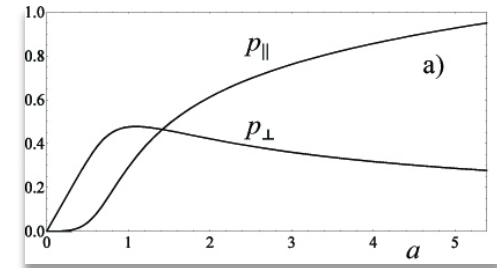
Curves for (1) $a = 0.35 a_{\text{rad}}$, (2) $a = a_{\text{rad}}$, and (3) - $a = 5 a_{\text{rad}}$.

Integral Scattering Cross Section

Stationary solutions

If $1 \ll a \ll a_{\text{rad}} = \varepsilon_{\text{rad}}^{-1/3}$, $q_{\perp} \approx a - \varepsilon_{\text{rad}}^2 a^7$, $q_{\parallel} \approx \varepsilon_{\text{rad}} a^4$

For $a \gg a_{\text{rad}} = \varepsilon_{\text{rad}}^{-1/3}$ we have $q_{\perp} \approx (\varepsilon_{\text{rad}} a)^{-1/2}$, $q_{\parallel} \approx (a/\varepsilon_{\text{rad}})^{1/4}$



The energy flux reemitted by the electron is equal to $e(\mathbf{v} \cdot \mathbf{E})$,

which is $\approx \varepsilon_{\text{rad}} m_e c^2 \Omega \gamma (a q_{\perp} + a^2 q_{\perp}^2)$.

The integral scattering cross section by definition equals the ratio of the reemitted energy flux to the Poynting vector magnitude:

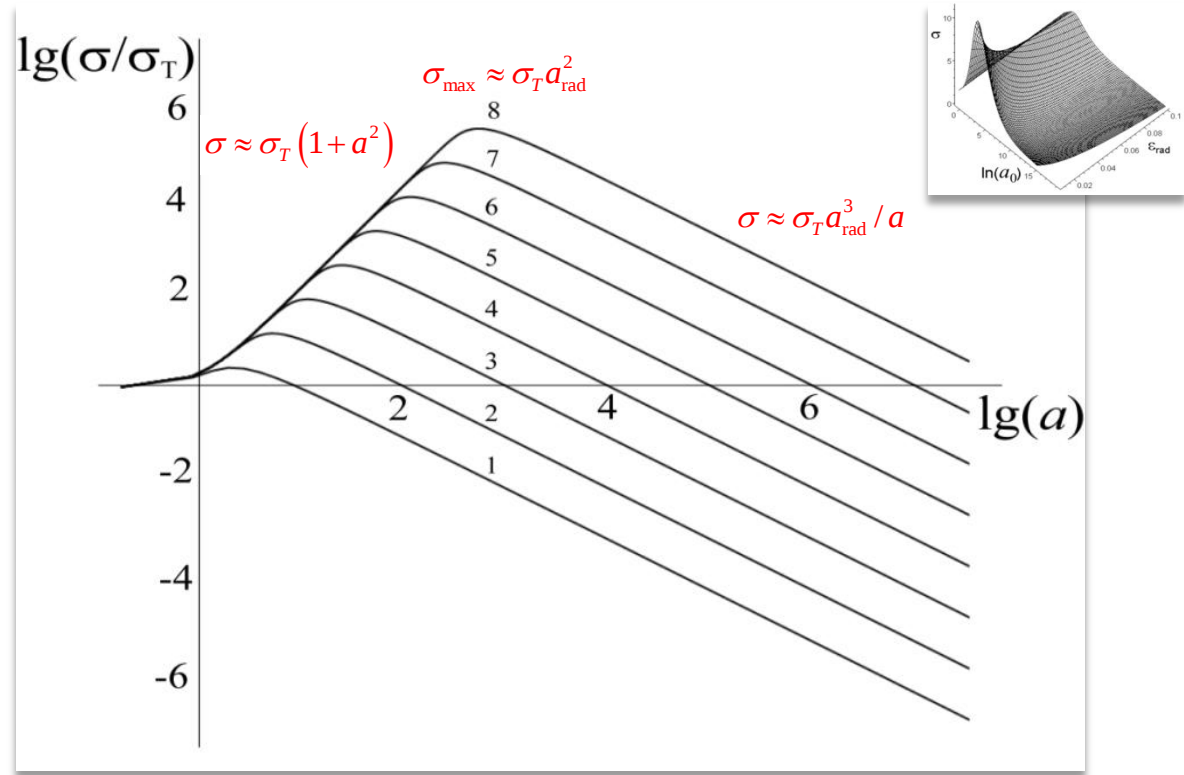
$$\sigma = \sigma_T \left(\frac{q_{\perp}}{a} + q_{\perp}^2 \right)$$

Here σ_T is the Thomson scattering cross section $\sigma_T = 8\pi r_e^2 / 3 = 6.65 \times 10^{-25} \text{ cm}^2$.

S. V. Bulanov, T. Zh. Esirkepov, M. Kando, J. K. Koga, S. S. Bulanov,

“Lorentz-Abraham-Dirac vs Landau-Lifshitz radiation friction force in the ultrarelativistic electron interaction with electromagnetic wave (exact solutions)”, *Phys. Rev. E*, 84, 056605 (2011)

Scattering Cross Section vs Laser Amplitude



Dependence of $\lg \frac{\sigma}{\sigma_T}$ on $\lg a$.

For each curve the integer label n corresponds to $\epsilon_{\text{rad}} = 10^{-n}$.

Depletion Length

The EM waves decay in underdense plasma in the limit $1 \ll a \ll a_{\text{rad}} = \epsilon_{\text{rad}}^{-1/3}$

with the damping time

$$\tau_d \propto \frac{1}{\epsilon_{\text{rad}} a^3}$$

The laser pulse depletion length is of the order of $l_{\text{dep}} \approx \frac{1}{\sigma n_e}$.

It reaches its minimum for given electron density, when the integral scattering cross section is maximal:

$$\min\{l_{\text{dep}}\} \approx \frac{1}{\sigma_{\text{max}} n_e} = \frac{1}{\sigma_T n_e} \left(\frac{2r_e \Omega}{3c} \right)^{2/3}$$

Gamma-beam Divergence

Performing the Lorentz transform to the laboratory frame of reference we find that

for $1 \ll a \ll a_{\text{rad}} = \epsilon_{\text{rad}}^{-1/3}$

$$p_1 = m_e c \frac{\left[\beta_{\text{ph}}^2 + a^2 (\beta_{\text{ph}}^2 - 1) \right]^{1/2} - \beta_{\text{ph}}}{\beta_{\text{ph}}^2 - 1}, \quad p_{\parallel} \approx 0, \quad p_{\perp} \approx m_e c a$$

when $1 \ll a_{\text{rad}} = \epsilon_{\text{rad}}^{-1/3} \ll a$, we have

$$p_1 \approx \frac{m_e c}{\beta_{\text{ph}}^2 - 1} \left(\frac{a}{\epsilon_{\text{rad}}} \right)^{1/4}, \quad p_{\parallel} \approx m_e c \left(\frac{a}{\epsilon_{\text{rad}}} \right)^{1/4}, \quad p_{\perp} \approx \frac{m_e c}{(\epsilon_{\text{rad}} a)^{1/2}}$$

The radiating electrons move in the direction of the laser pulse propagation.

This results in the gamma-photon energy upshifting by a factor $2(\beta_{\text{ph}}^2 - 1)^{-1/2}$

and to the gamma-beam collimation within the angle

$$\theta \approx (\beta_{\text{ph}}^2 - 1) \ll 1$$

Laser Power Required for Gamma-Flash Emission

For $\Omega \approx \omega$ corresponding to a one-micron wavelength laser, the maximal value of the integral scattering cross section is of the order of 10^{-19} cm^2 at $I \approx 10^{23} \text{ W/cm}^2$

If the laser with this intensity irradiates a solid density target $n \approx 10^{23} \text{ cm}^{-3}$, then

$$\min\{l_{\text{dep}}\} \approx 1 \mu\text{m}$$

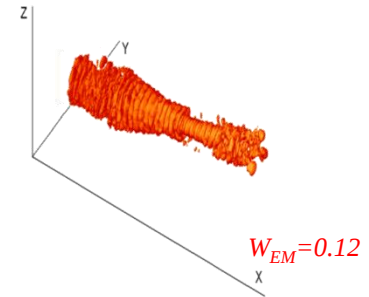
This results in a gamma-ray flash with the duration and power comparable, within an order of magnitude, to the incident laser pulse duration and power.

Laser power required for realization of the optimal conditions for the gamma-flash emission. Inside the self-focusing channel $a^3 = 8\pi (\mathcal{P}_{\text{las}} / \mathcal{P}_c) (\omega_{\text{pe}} / \omega)^2$

with $\mathcal{P}_c = 2m_e^2 c^5 / e^2 \approx 17 \text{ GW}$

The optimal condition, $a^3 = \epsilon_{\text{rad}}^{-1}$, yields $\mathcal{P}_{\text{las}} \approx 10^2 (\omega / \omega_{\text{pe}})^2 \text{ PW}$

i.e., the required laser power is about **10 PW**.



2D PIC Simulations with Radiation Friction

Laser:

Power: 10 PW, $\lambda=1 \mu\text{m}$, p-pol

Pulse duration: 30 fs

Spot size: $5.2 \mu\text{m}$

Intensity: $4.8 \times 10^{22} \text{ W/cm}^2$ ($a_0=150$)

Tailored plasma target:

Maximum density : $350 n_c$

Preplasma scale length L : from $0.1 \mu\text{m}$ to $20 \mu\text{m}$

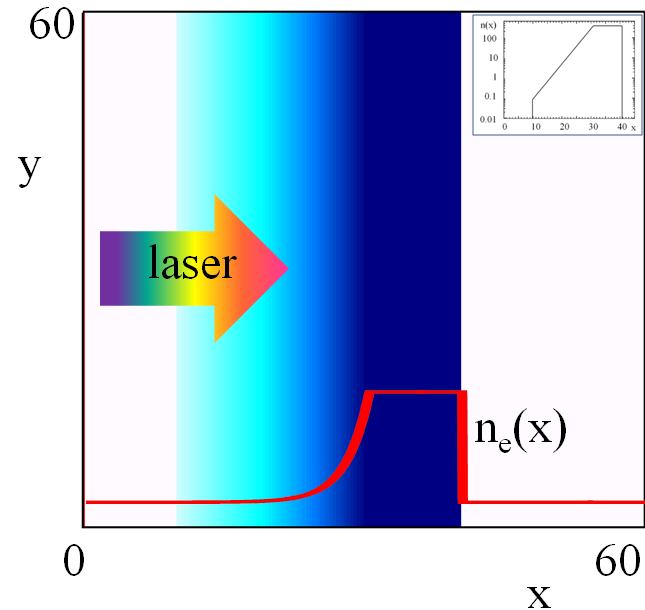
Ions: $A/Z=2$

Simulation parameters:

$\Delta x = \Delta y = 1/40 \sim 1/200 \mu\text{m}$

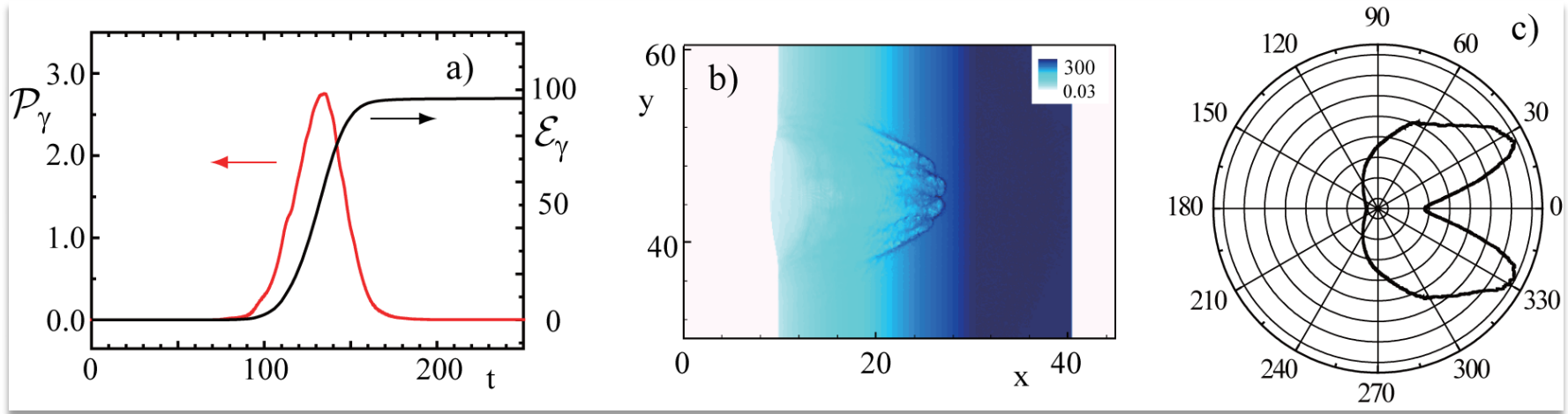
$\Delta t = 0.0025 \text{ fs}$

Simulation box: $l_x = 50 \sim 210 \mu\text{m}$, $l_y = 80 \mu\text{m}$



A. G. Zhidkov, et al., Phys. Rev. Lett. 88, 185002 (2002)
N. M. Naumova et al., Phys. Rev. Lett. 102, 025002 (2009)
T. Schlegel et al., Phys. Plasmas 16, 083103 (2009)
M. Tamburini et al., New J. Phys. 12, 123005 (2010)

Simulation Results for the Parameters of Interest



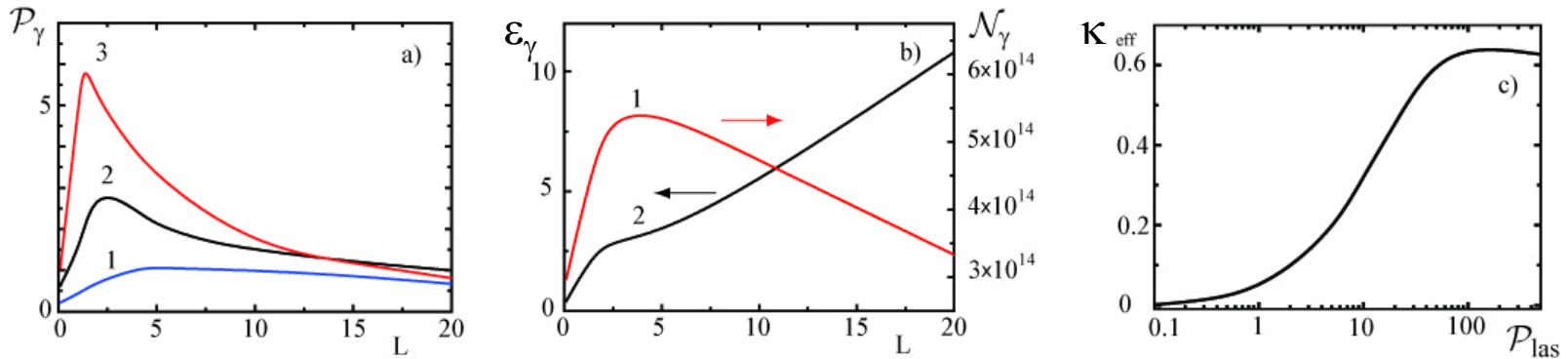
- a) The radiation power, $\mathcal{P}_\gamma$ (PW), and energy, $\mathcal{E}_\gamma$ (J), vs time t (fs).
- b) The ion density distribution in the (x, y) plane for $t = 260$ fs.
- c) The gamma ray intensity angular distribution. For $L = 10 \mu\text{m}$.

The angular distribution of the emitted radiation has been calculated according to the formula

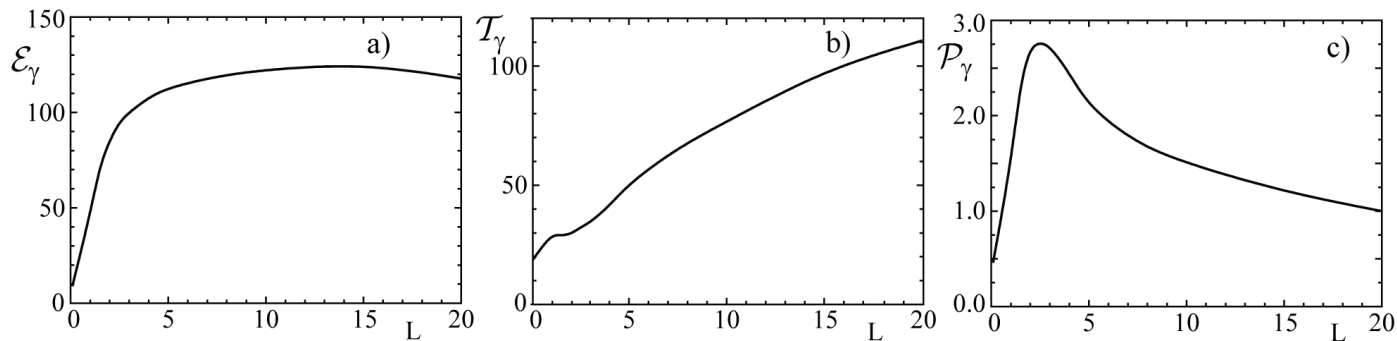
$$dI = \frac{e^2}{4\pi c^3} \left\{ \frac{2(\mathbf{n} \cdot \mathbf{w})(\mathbf{v} \cdot \mathbf{w})}{c \left(1 - \frac{(\mathbf{v} \cdot \mathbf{n})}{c} \right)^5} + \frac{\mathbf{w}^2}{\left(1 - \frac{(\mathbf{v} \cdot \mathbf{n})}{c} \right)^4} - \frac{\left(1 - \frac{v^2}{c^2} \right) (\mathbf{n} \cdot \mathbf{w})^2}{\left(1 - \frac{(\mathbf{v} \cdot \mathbf{n})}{c} \right)^6} \right\} d\Omega$$

The summation was performed over all radiating electrons.

Parameters of γ -rays vs Plasma Scalelength & P_{las}

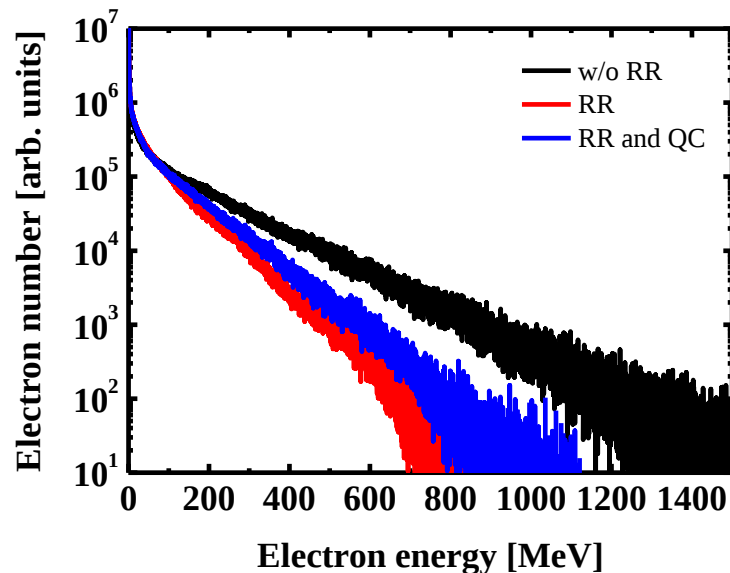


- a) Dependence of the gamma-ray power $\mathcal{P}_\gamma$ (PW) on the plasma scale length, L (m) for the laser pulse energy of 300 J and the laser power, P_{las} , varying from 5 to 20 PW: 1 - 5 PW, 60 fs, 2 - 10 PW, 30 fs, 3 - 20 PW, 15 fs.
- b) The photon number $\mathcal{N}_\gamma$ (curve 1) and gamma-ray photon energy ε_γ (MeV) (curve 2) vs the plasma scale length, L (μm) for 10 PW, 30 fs laser pulse.
- c) The efficiency of the laser energy conversion to gamma rays, κ_{eff} , vs the laser pulse power.

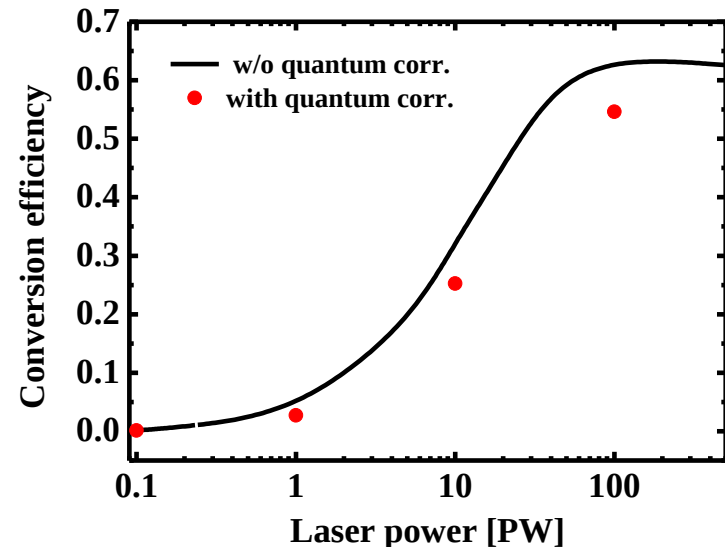


- a) Emitted gamma-ray pulse energy $\mathcal{E}_\gamma$ (J), b) duration (fs), and c) power $\mathcal{P}_\gamma$ (PW) vs the plasma scale length, L (μm).

Electron energy spectra & Conversion efficiency

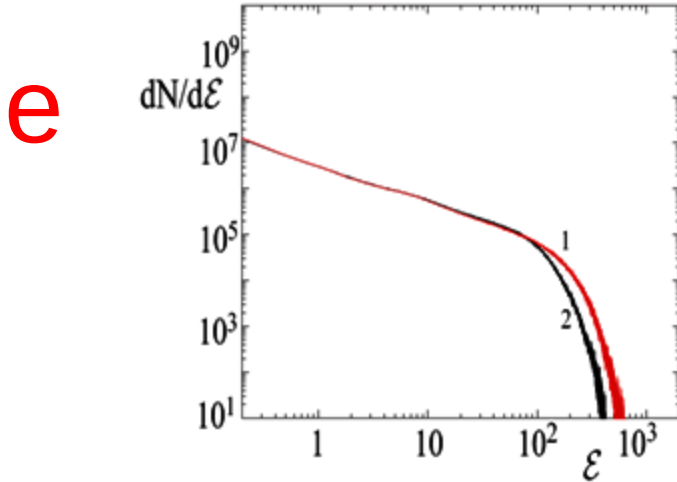


Electron energy spectra
without radiation friction and QED
effects,
with radiation friction and without
QED effects,
with radiation friction and with
QED effects taken into account



Comparison of the conversion
efficiency of the laser energy to
the energy of the γ -rays when the
quantum correction of the radiation
friction is taken into account (dots)
and when it is neglected (solid
curve)

ENERGY SPECTRUM OF EMITTED GAMMA-RAYS



Electron energy spectrum without (1) and with (2) radiation friction effects taken into account

Electron energy spectrum can be approximated by

$$\frac{dN}{d\mathcal{E}} = K_e \mathcal{E}^{-\kappa} \text{Exp}\left(-\frac{\mathcal{E}}{\mathcal{E}_m}\right)$$

In simulations: $\kappa = 0.8$ and $\mathcal{E}_m = 38$ MeV

The spectral distribution of the photons emitted by rotating with the frequency ω electron of the energy $m_e c^2 \gamma_e$ is given by γ

$$\frac{dI(\omega_\gamma, \mathcal{E})}{d\omega_\gamma} = \frac{\sqrt{3}}{2\pi} \frac{e^2}{c} \left(\frac{\mathcal{E}}{m_e c^2} \right) u(\mathcal{E}) \int_{u(\mathcal{E})}^{\infty} K_{5/3}(x) dx$$

with $u(\mathcal{E}) = \frac{2\omega_\gamma}{3\omega} \left(\frac{m_e c^2}{\mathcal{E}} \right)^3$

The averaged spectrum of the gamma photons is given by the integral

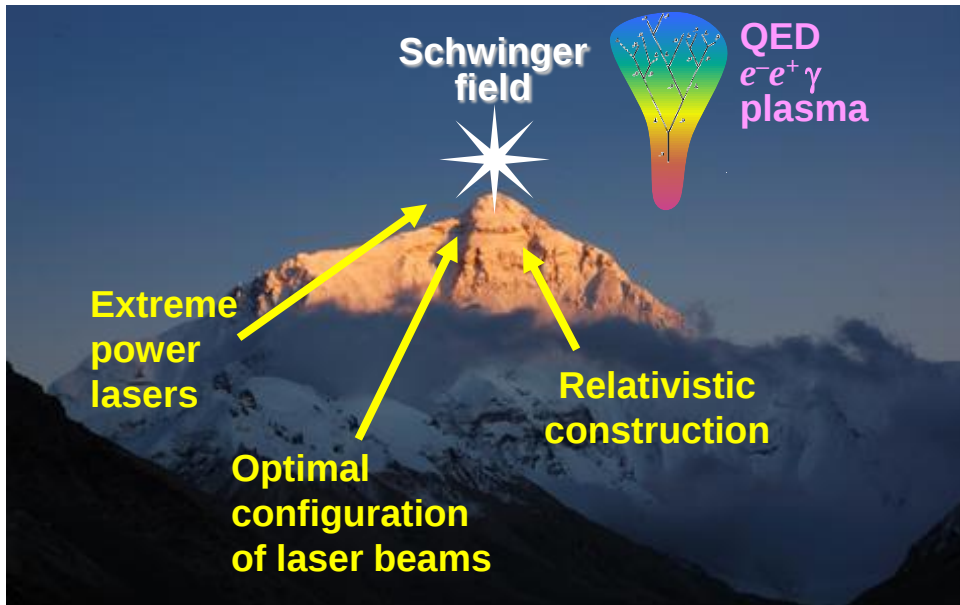
$$J(\omega_\gamma) = \int_0^\infty \frac{dI(\omega_\gamma, \mathcal{E})}{d\omega_\gamma} \frac{dN}{d\mathcal{E}} d\mathcal{E}$$

Calculating it we obtain

$$J(\omega_\gamma) = \frac{\sqrt{3} e^2 K_e}{2\pi c} (m_e c^2)^{(1-\kappa)} \left(\frac{2\omega_\gamma}{3\omega} \right)^{\frac{2-\kappa}{3}} F(\mathcal{E}_m, \kappa)$$

where $\mathcal{E}_m = \frac{\mathcal{E}_m}{m_e c^2} \left(\frac{3\omega}{2\omega_\gamma} \right)^{1/3}$

Laser Driven $e^-e^+\gamma$ Plasma

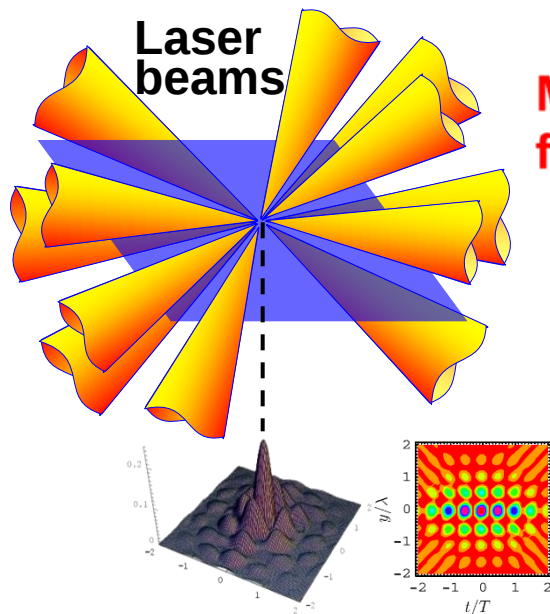


Electron-positron pairs can be created before the laser field reaches the Schwinger limit, due to a large phase volume occupied by a high-intensity EM field.

S. S. Bulanov, N. B. Narozhny, V. D. Mur, V.S. Popov, "Electron-positron pair production by electromagnetic pulses". JETP, 102, 9 (2006).

A. R. Bell & J. G. Kirk, "Possibility of Prolific Pair Production with High-Power Lasers". Phys. Rev. Lett. 101, 200403 (2008).

A. M. Fedotov, N. B. Narozhny, G. Mourou, G. Korn, "Limitations on the Attainable Intensity of High Power Lasers". Phys. Rev. Lett. 105, 080402 (2010).

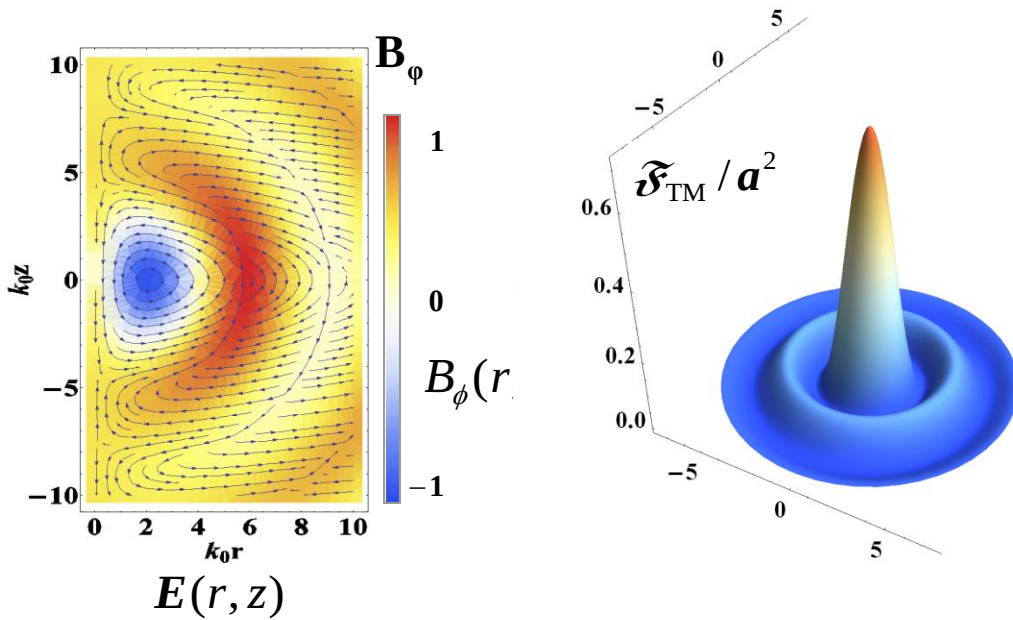


Multiple 10kJ beam system provides necessary conditions for e^-e^+ pairs creation.

Number of pulses	Number of e^-e^+ with 10kJ pulses	Required power (kJ) to create one pair
2	9×10^{-19}	40
4	3×10^{-9}	20
8	4	10
16	1.8×10^3	8
24	4.2×10^6	5.1

S.S.Bulanov, V.D.Mur, N.B.Narozhny, J.Nees, V.S.Popov, Phys. Rev. Lett. 104, 220404 (2010).

3D EM configuration TM - mode



TM configuration :

Magnetic field

$$\begin{aligned} \mathbf{B}(R, \theta) &= \\ &= \mathbf{e}_\phi \frac{B_0 \sin(\omega_0 t)}{(8\pi k_0 R)^{1/2}} J_{n+1/2}(k_0 R) L_n^l(\cos \theta) \end{aligned}$$

Electric field

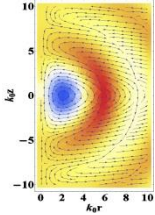
$$\mathbf{E} = i k_0^{-1} (\nabla \times \mathbf{B})$$

The vector field shows r - and z -components of the poloidal electric field in the plane (r, z) :

The color density show toroidal magnetic field distribution.
The first Poincare invariant

$$\mathcal{F}_{\text{TM}} / a_0^2 = (\mathbf{E}^2 - \mathbf{B}^2) / 2a_0^2$$

Instability



$e^- (e^+)$ trajectory in r, z - plane $\omega_0 t \approx 1, a_0 \gg 1$

$$p_z(t) = m_e c a_0 \omega_0 t,$$

$$p_r(t) = m_e c \frac{a_0 k_0 r_0 \omega_0 t}{2^{3/2}} I_1 \left(\frac{\omega_0 t}{2^{3/2}} \right),$$

$$r(t) = \frac{a_0 r_0}{2^{3/2}} I_1 \left(\frac{\omega_0 t}{2^{3/2}} \right) + \frac{a_0 r_0 \omega_0 t}{16} \left[I_0 \left(\frac{\omega_0 t}{2^{3/2}} \right) + I_2 \left(\frac{\omega_0 t}{2^{3/2}} \right) \right]$$

with $k_0 r_0 = (2.5 a_0 / \pi a_s)^{1/2}$; growth rate : $\Gamma = \omega_0 / 2^{3/2}$

**Motion of a Charge
in a Superstrong Electromagnetic
Standing Wave**

Motivation for studying on how electrons move in superstrong standing wave

D. Bauer, P. Mulser, W.-H. Steeb, Phys. Rev. Lett. 75, 4622 (1995)

G. Lehmann, K.H. Spatschek, Phys. Rev. E 85, 056412 (2012)

A. Ts. Amatuni and I. V. Pogorelsky, Phys. Rev. STAB 1, 034001 (1998)

A.M.Fedotov, et al., “Limitations on the Attainable Intensity of High Power Lasers”. PRL 105, 080402 (2010)

Model: antinodes of circularly polarized standing wave.

S.S.Bulanov, et al., “Schwinger Limit Attainability with Extreme Power Lasers”. PRL 105, 220407 (2010)

Model: 3D TM configuration.

L. L. Ji, A. Pukhov, I. Y. Kostyukov, B. F. Shen, K. Akli, Phys. Rev. Lett. 112 (2014) 145003

A.Gonoskov, et al., “Anomalous Radiative Trapping in Laser Fields of Extreme Intensity”. PRL 113, 014801 (2014)

Electrons in a sufficiently intense standing wave are compressed toward, and oscillate synchronously at, the antinodes of the electric field. This opens new possibilities for the generation of high energy, directed, and collimated radiation and particle beams.

A Zhidkov, S Masuda, SS Bulanov, J Koga, T Hosokai, R Kodama, “Radiation reaction effects in cascade scattering of intense, tightly focused laser pulses by relativistic electrons: Classical approach”
Physical Review Special Topics-Accelerators and Beams 17 (5), 05400 (2014)

T. Zh. Esirkepov, et al., “Attractors and chaos of electron dynamics in electromagnetic standing waves”,
Phys. Lett. A 379, 25 (2015)

Quantum Form-factor

Equations of electron motion

$$m_e c^2 \frac{du^\mu}{ds} = \frac{e}{c} F^{\mu\nu} u_\nu + g^\mu$$

with radiation friction force g^μ

EM wave is modelled by rotating E field

$$\dot{\mathbf{q}} = -\mathbf{a} - \frac{\varepsilon_{\text{rad}} G_e(\chi_e)}{\gamma_e} \left\{ \gamma^2 \dot{\mathbf{a}} - \mathbf{a}(\mathbf{q} \cdot \mathbf{a}) + \mathbf{q} \left[(\gamma \mathbf{a})^2 - (\mathbf{q} \cdot \mathbf{a})^2 \right] \right\}$$

$$n = \frac{n}{n_{cr}}, \quad \tau = \Omega t, \quad \mathbf{q} = \frac{\mathbf{p}}{m_e c}, \quad \mathbf{a} = \frac{e\mathbf{E}}{m_e \Omega c}, \quad \gamma_e = \left(1 + q_1^2 + q_2^2 + q_3^2 \right)^{1/2}$$

QED effects incorporated with the form-factor, $G_e(\chi_e)$, equal to the ratio of the full radiation intensity to the intensity emitted by a classical electron

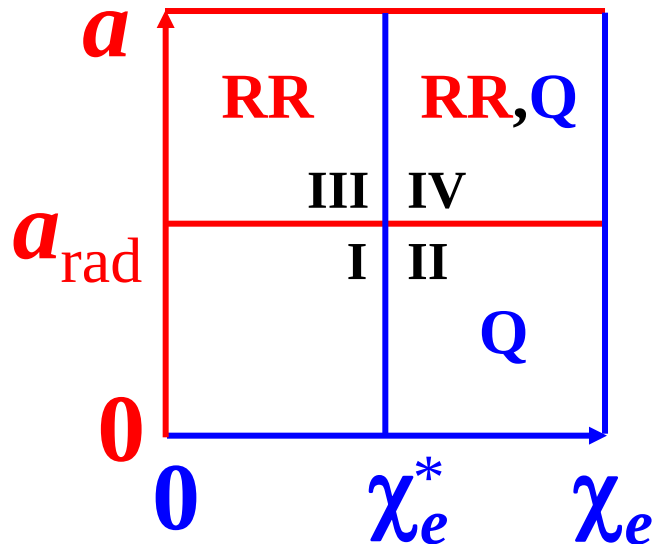
$$G_e(\chi_e) = -\frac{3}{4} \int_0^\infty \left[\frac{4 + 5\chi_e x^{3/2} + 4\chi_e^2 x^3}{(1 + \chi_e x^{3/2})^4} \right] \Phi'(x) x dx$$

where $\Phi(x)$ is the Airy function

J. Schwinger, Proc. Natl. Acad. Sci. U.S.A. 40, 132 (1954)
A.A. Sokolov, et al., Sov. Phys. JETP 24, 249 (1954)
A.R. Bell and J. G. Kirk, Phys. Rev. Lett. 101, 200403 (2008)
C. P. Ridgers, et al., Phys. Rev. Lett. 108, 165006 (2012)
I.V. Sokolov, et al, Phys. Rev. E 81, 036412 (2010)
J. G. Kirk, et al., Plasma Phys. Contr. Fusion 51, 085008 (2009)
R. Duclous, et al., Plasma Phys. Contr. Fusion 53, 015009 (2011)
A. Di Piazza, et al., Rev. Mod. Phys. 84, 1177 (2012)

Electron Motion in Circularly Polarized EM Wave

4 regimes



Dimensionless amplitude

$$a = eE/m_e\omega c$$

At $a=a_{\text{rad}}$ emitted energy becomes equal to the energy received from EM wave.

$$a_{\text{rad}} = \left(\frac{3\lambda}{4\pi r_e} \right)^{1/3} \quad r_e = \frac{e^2}{m_e c^2}$$

When the recoil of the emitted photon is significant, the emission probability is characterized by

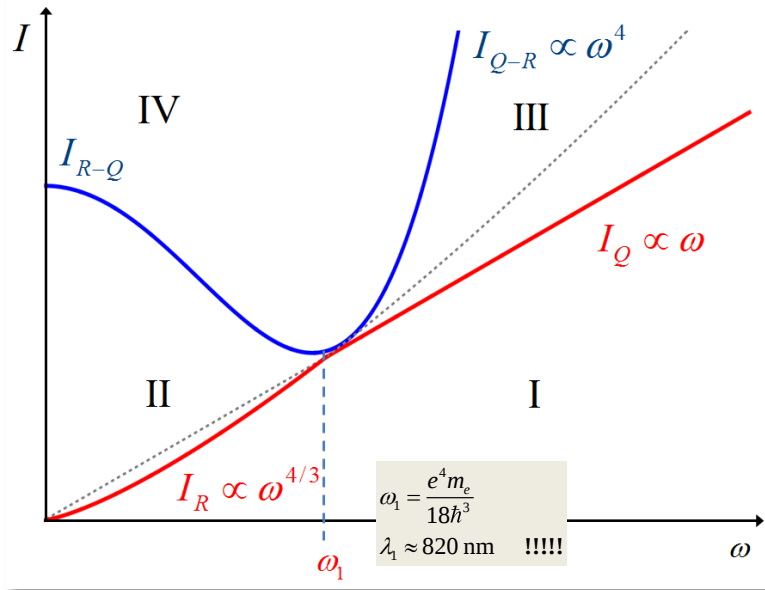
χ_e parameter (Lorentz and gauge inv)

$$\chi_e = (\gamma_e/E_S)[(\mathbf{E} + \boldsymbol{\beta} \times \mathbf{B})^2 - (\boldsymbol{\beta} \cdot \mathbf{E})^2]^{1/2}$$

At $\chi_e = \chi_e^* < 1$ QED effects come into play.

Four Interaction Domains

SVB, T. Zh. Esirkepov, M. Kando, J. Koga, K. Kondo, and G. Korn,
Plasma Phys. Rep. 41, 1-51 (2015)



Curves $I_R(\omega)$, $I_Q(\omega)$ and $I_{R-Q}(\omega)$, $I_{Q-R}(\omega)$ subdivide (I, ω) plane to 4 domains:

- I) Relativistic electron - EM field interaction with neither radiation friction nor QED effects
- II) Electron - EM wave interaction is dominated by radiation friction
- III) QED effects important with insignificant radiation friction effects
- IV) Both QED and radiation friction determine radiating charged particle dynamics in the EM field

$$I_R = \frac{m_e^4 c^5 e^2}{144 \pi \hbar^4} \left(\frac{\omega}{\omega_1} \right)^{4/3} = 3.8 \times 10^{23} \left(\frac{\omega}{\omega_1} \right)^{4/3} \frac{\text{W}}{\text{cm}^2}$$

$$I_Q = \frac{m_e^4 c^5 e^2}{144 \pi \hbar^4} \left(\frac{\omega}{\omega_1} \right) = 3.8 \times 10^{23} \left(\frac{\omega}{\omega_1} \right) \frac{\text{W}}{\text{cm}^2}$$

$$I_{R-Q} = \frac{m_e^4 c^5 e^2}{9 \pi \hbar^4} = 5.6 \times 10^{24} \frac{\text{W}}{\text{cm}^2}$$

$$I_{Q-R} = 87 \frac{c^5 \hbar^8 \omega^4}{e^{14}} = 8.2 \times 10^{21} \left(\frac{\omega}{\omega_1} \right)^4 \frac{\text{W}}{\text{cm}^2}$$

Stationary Rotation in Circularly polarized Standing Wave

$$\dot{\mathbf{p}} = e(\mathbf{E} + \boldsymbol{\beta} \times \mathbf{B}) + G_e \mathbf{f}_{LL}$$

$$\mathbf{a} = -(eE/m_e \omega c)(\mathbf{i}_2 \cos \tau + \mathbf{i}_3 \sin \tau)$$

$$\mathbf{q} = \mathbf{p}/m_e c = \mathbf{i}_2 q_2 + \mathbf{i}_3 q_3$$

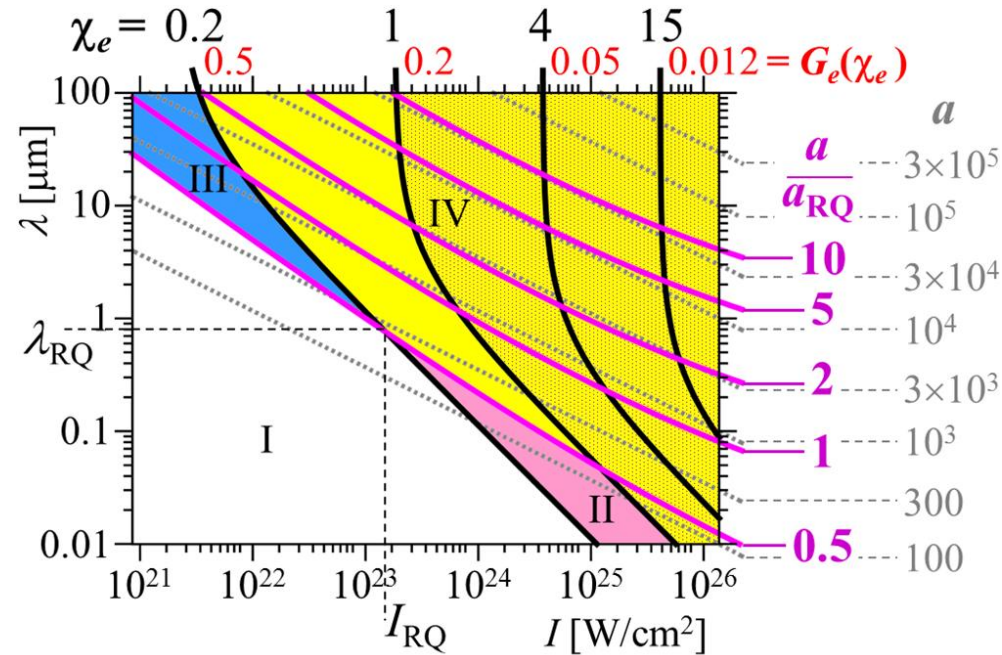
$$\begin{cases} q_{\parallel} = q_2 \cos \tau + q_3 \sin \tau, \\ q_{\perp} = q_2 \sin \tau - q_3 \cos \tau \end{cases}$$

$$\chi_e = (a/a_s)[1 + q_1^2 + q_{\perp}^2]^{1/2}$$

$$q_1 \ll (q_2^2 + q_3^2)^{1/2}$$

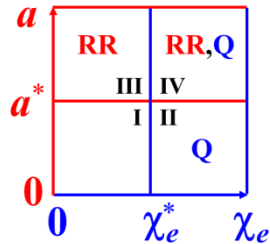
$$\begin{cases} \dot{q}_{\parallel} + q_{\perp} = a - \varepsilon_{\text{rad}} G_e(\chi_e) a^2 \frac{q_{\parallel} q_{\perp}^2}{\gamma_e}, \\ \dot{q}_{\perp} - q_{\parallel} = -\varepsilon_{\text{rad}} G_e(\chi_e) \left[\gamma_e a + a^2 \frac{q_{\perp}(1+q_{\perp}^2)}{\gamma_e} \right]. \end{cases}$$

$$\dot{q}_{\parallel} = 0, \dot{q}_{\perp} = 0$$



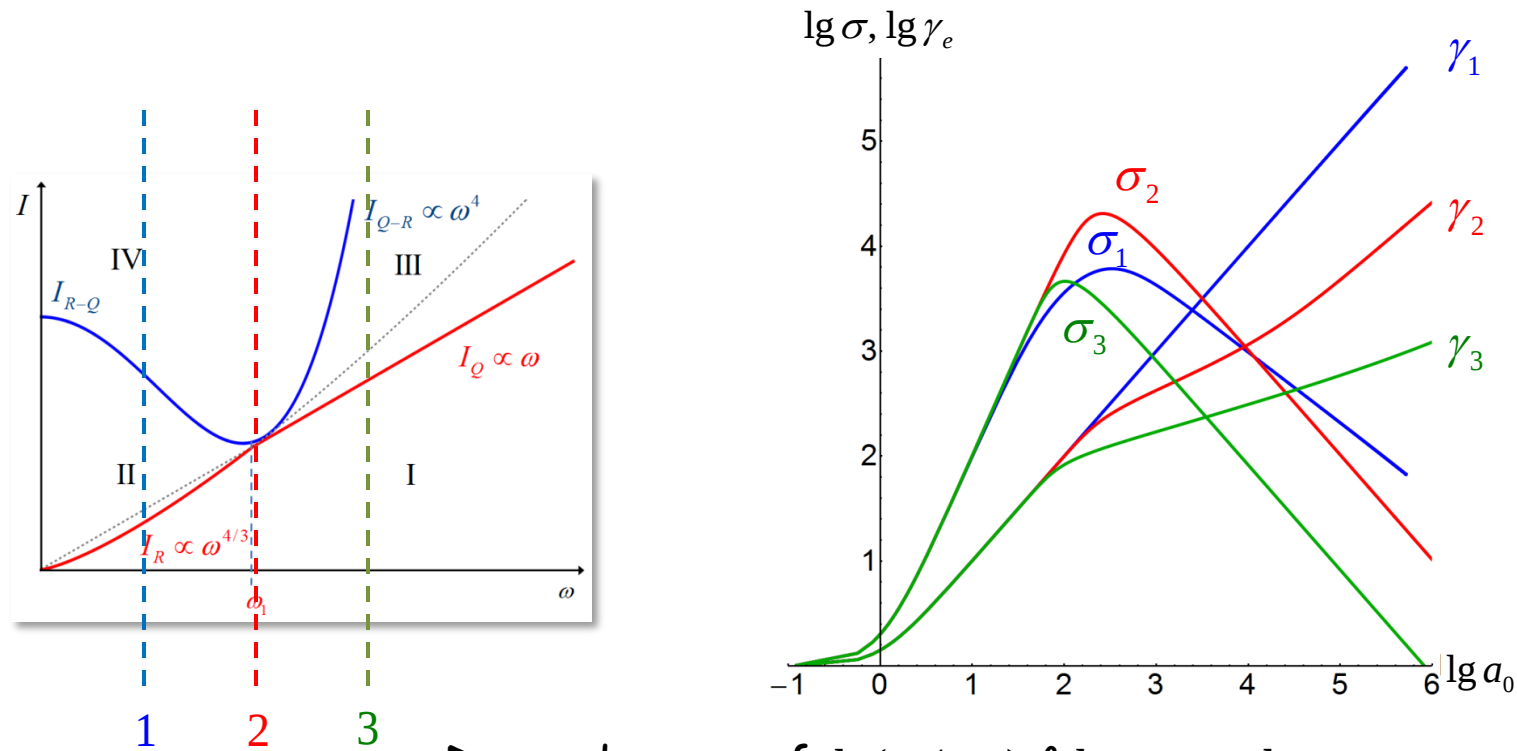
$$\lambda_{RQ} \approx 0.76 \mu\text{m}$$

$$I_{RQ} \approx 1.5 \times 10^{23} \text{ W/cm}^2$$



What Can be Measured?

Cross Section σ and Electron Energy γ_e v.s. EM Wave Amplitude:



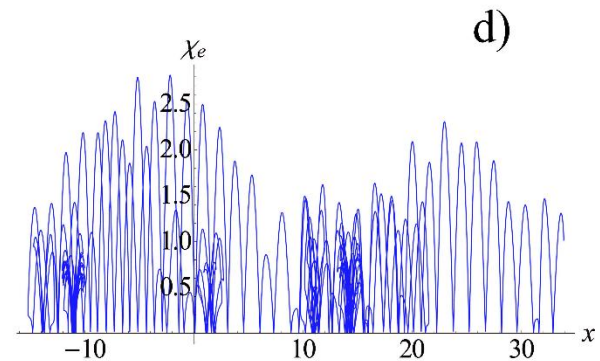
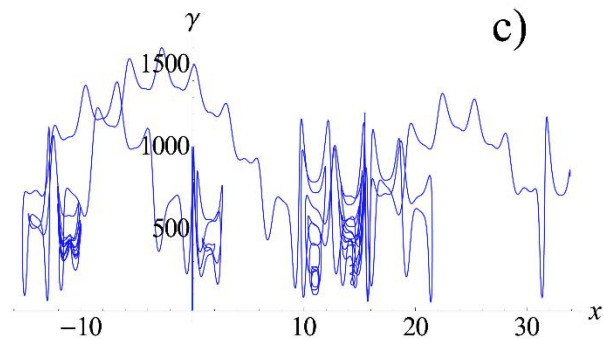
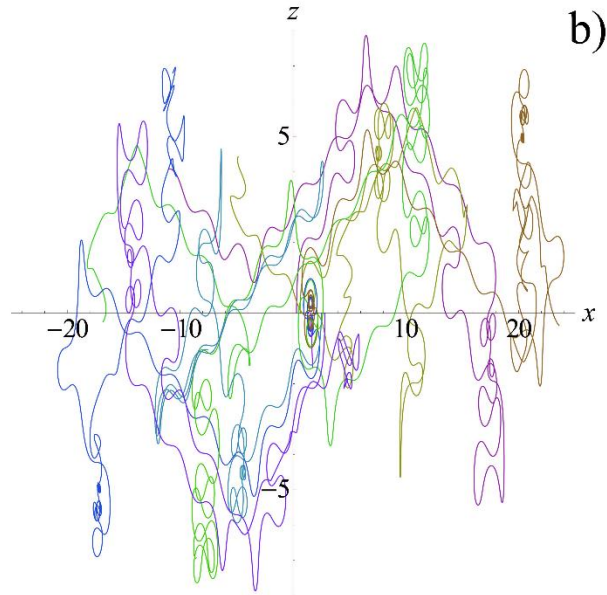
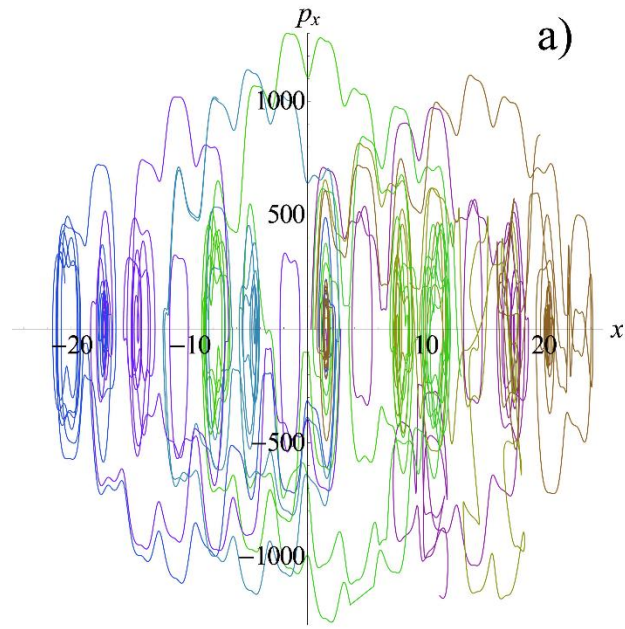
Dependences of $\lg(\sigma / \sigma_T)$ & $\lg \gamma_e$ on $\lg a_0$

1) $\omega = \omega_1 / 12.5$ (I $\rightarrow$ II)

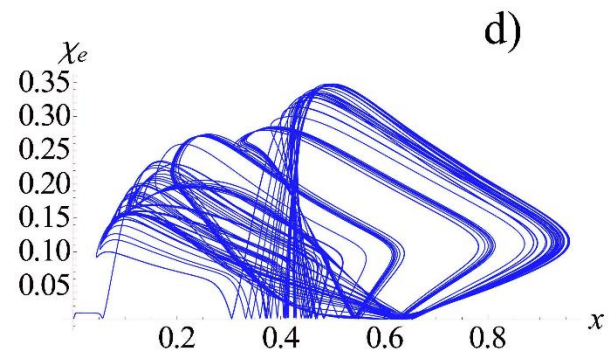
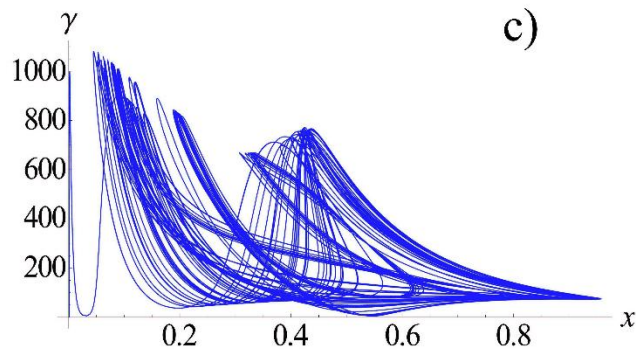
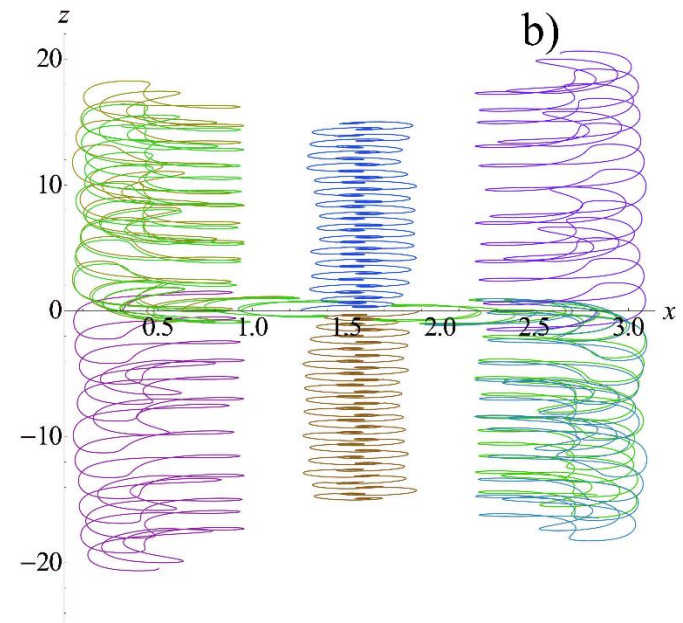
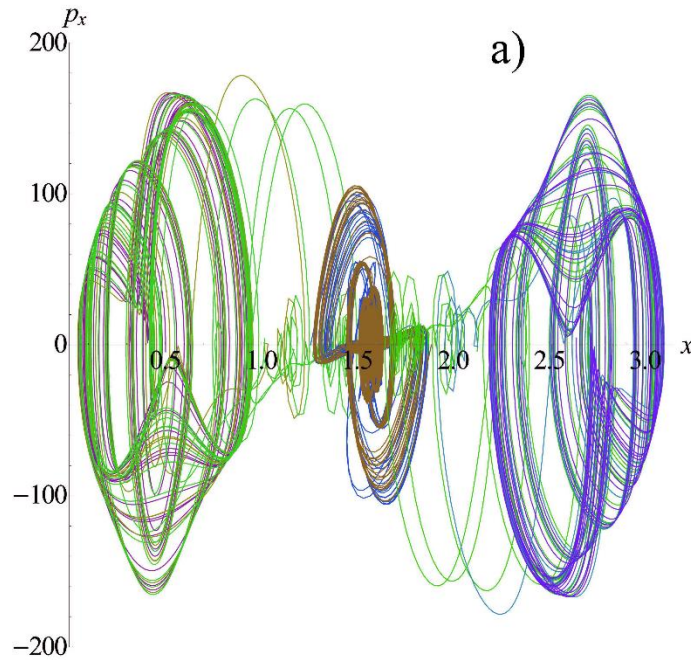
2) $\omega = \omega_1$ (I $\rightarrow$ IV)

3) $\omega = 12.5\omega_1$ (I $\rightarrow$ III)

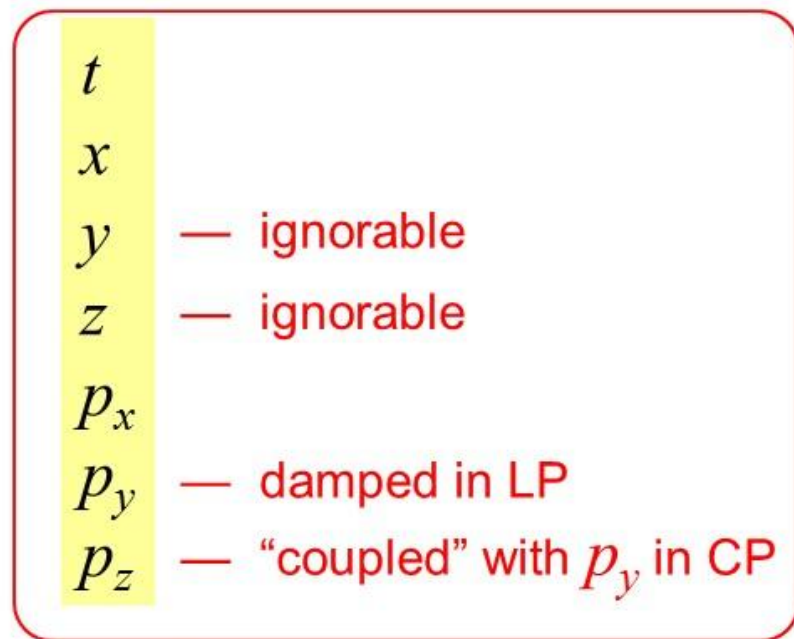
Typical Trajectories for $\alpha=1000$ and $\varepsilon_{\text{rad}}=10^{-9}$



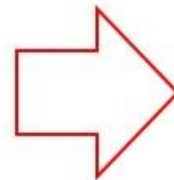
Typical Trajectories for $\alpha=1000$ and $\varepsilon_{\text{rad}}=10^{-7}$



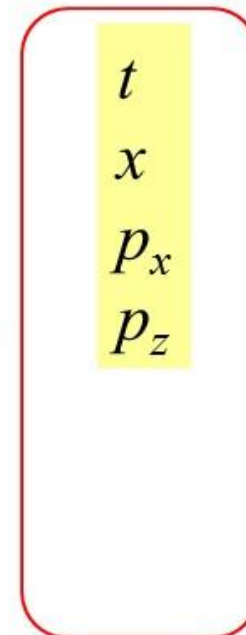
Electron Phase Space



6+1D

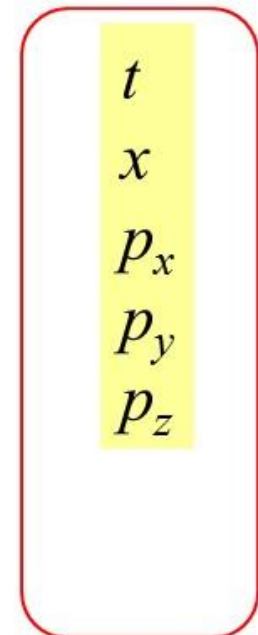


LP



3+1D

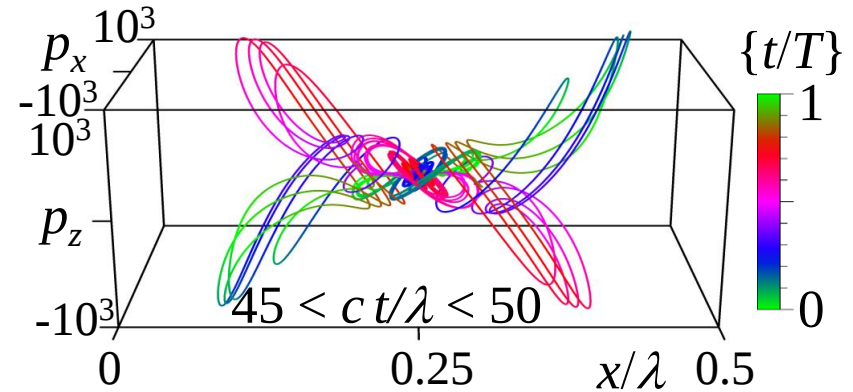
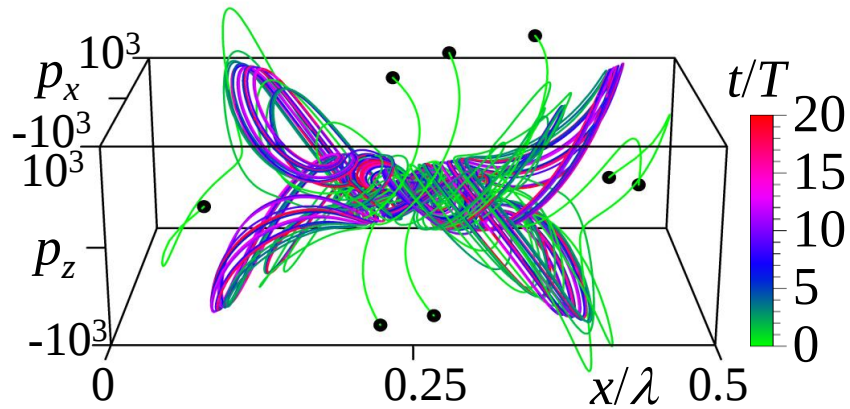
CP



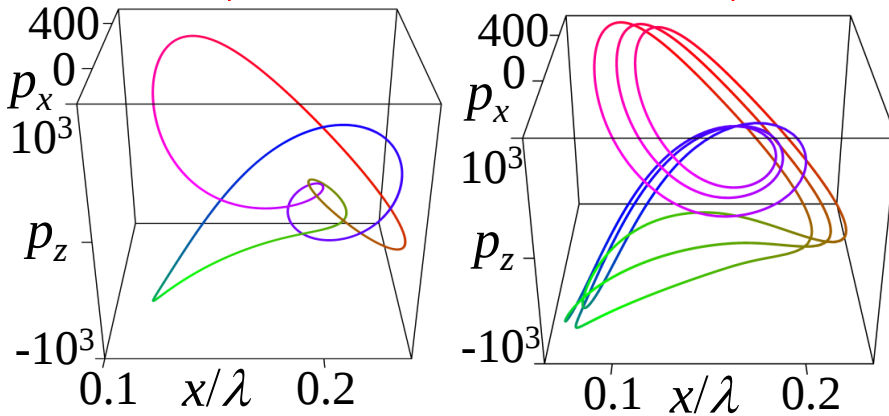
4+1D

Electron Motion in Superstrong Standing LP Wave

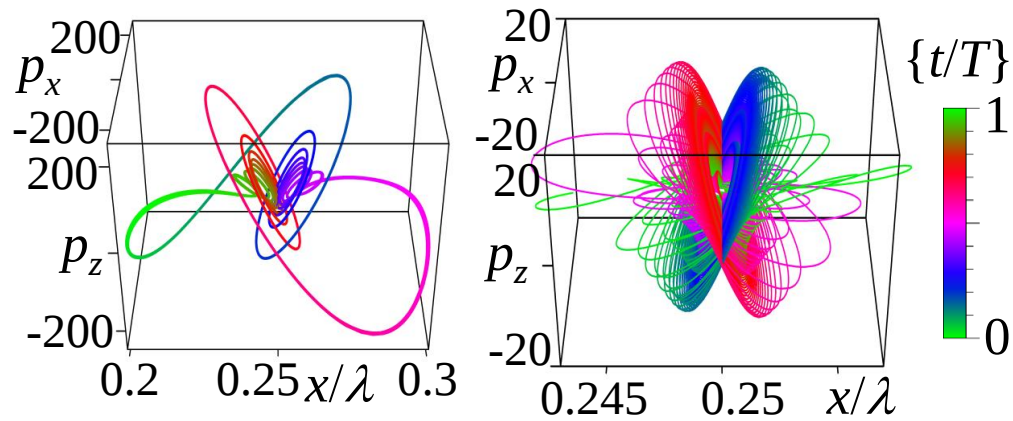
Strong Radiation Reaction (LP, $\lambda = 1\ \mu\text{m}$, $I = 1.37 \times 10^{24}\ \text{W/cm}^2$)



Limit cycles

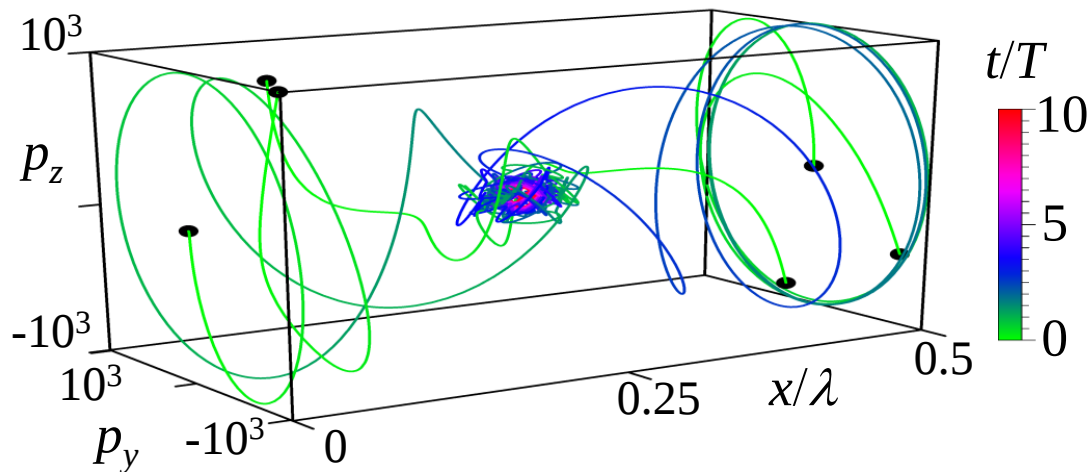


Strange attractor

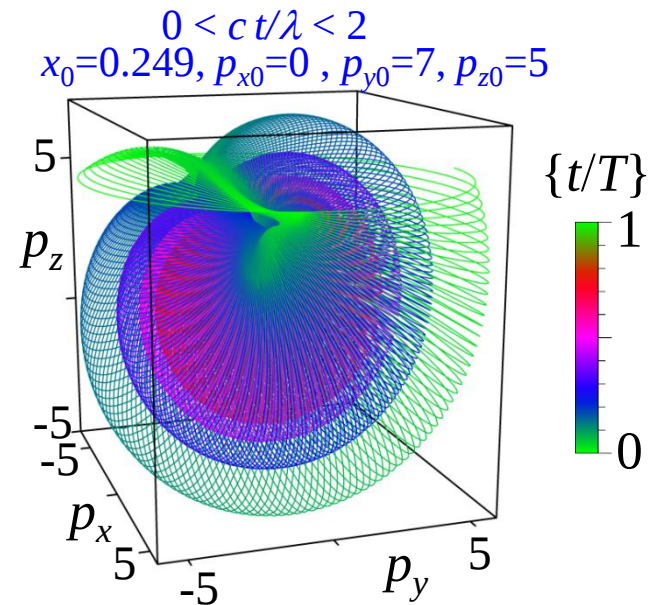
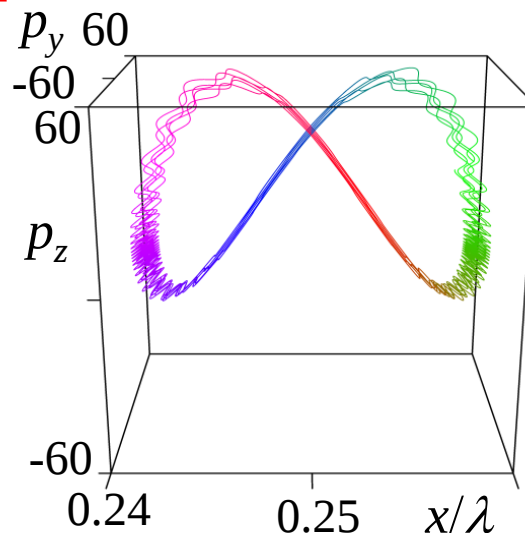
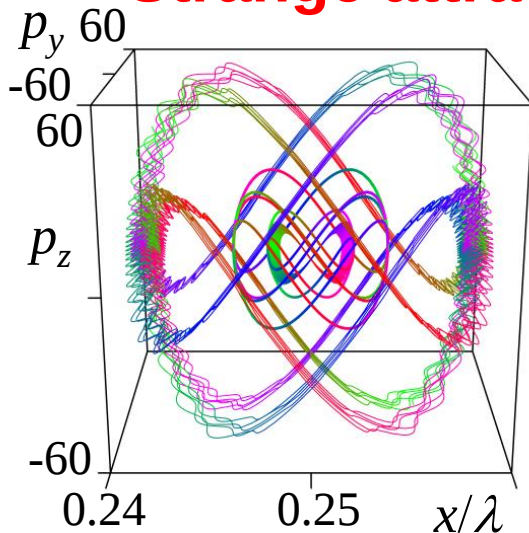


Electron Motion in Superstrong Standing CP Wave

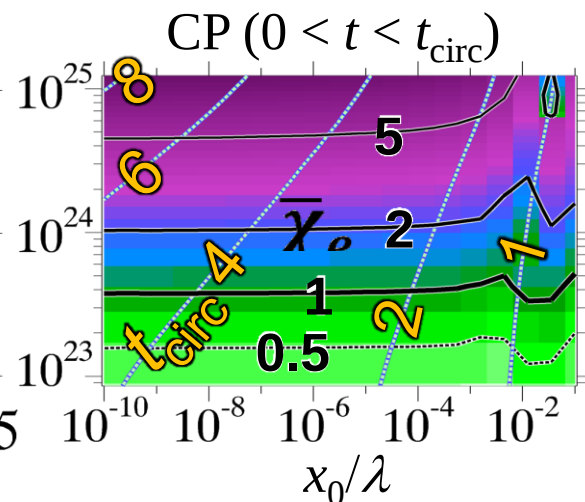
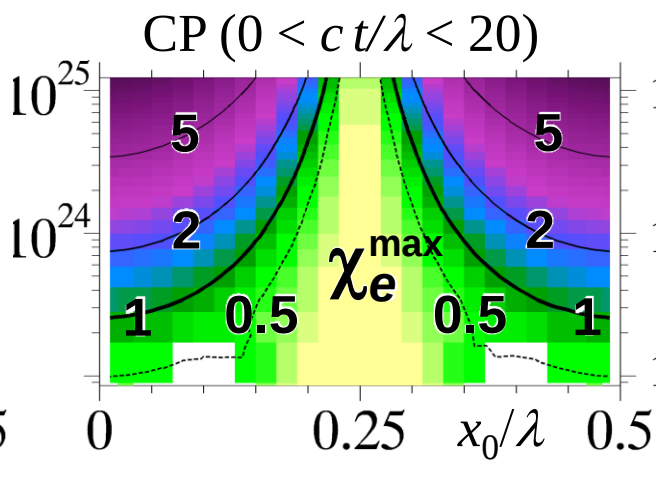
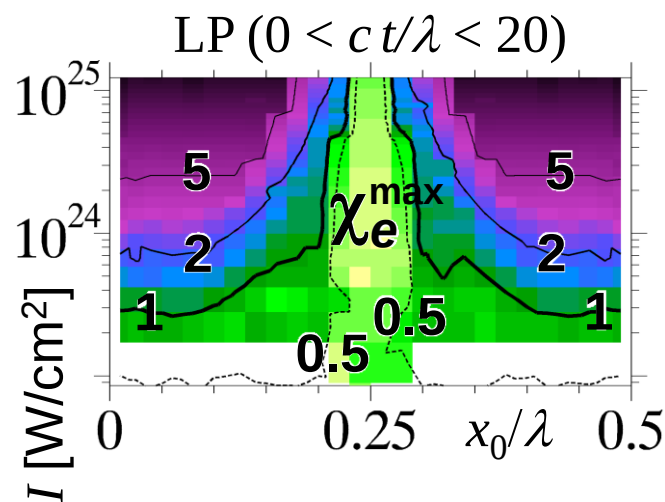
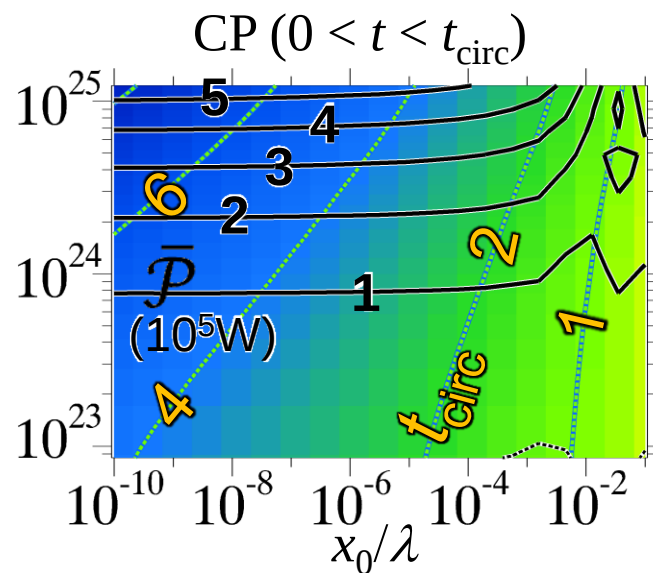
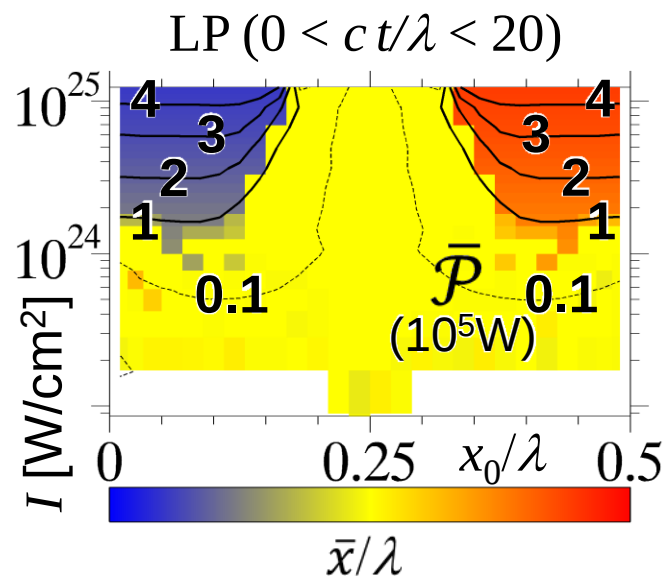
Strong Radiation Reaction (CP, $\lambda = 1 \mu\text{m}$, $I = 1.37 \times 10^{24} \text{ W/cm}^2$)



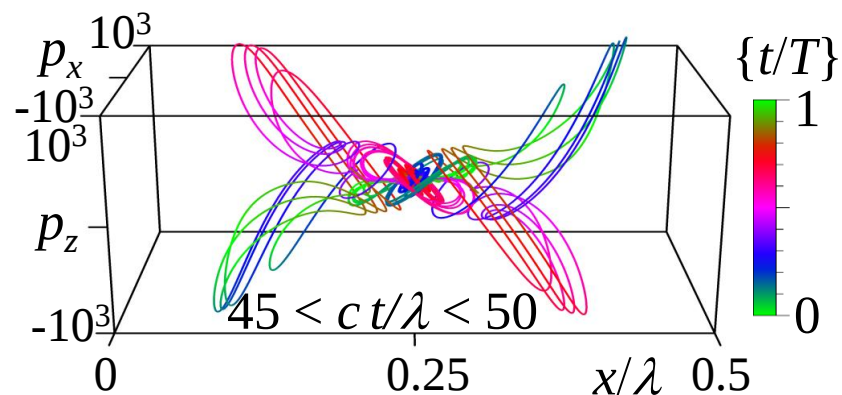
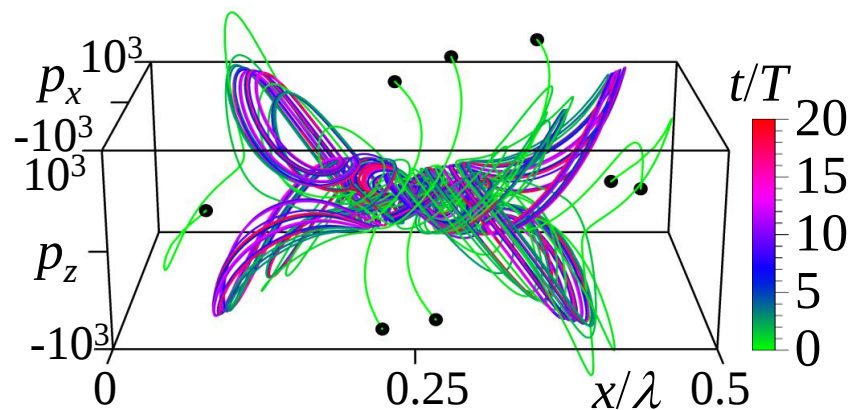
Strange attractor



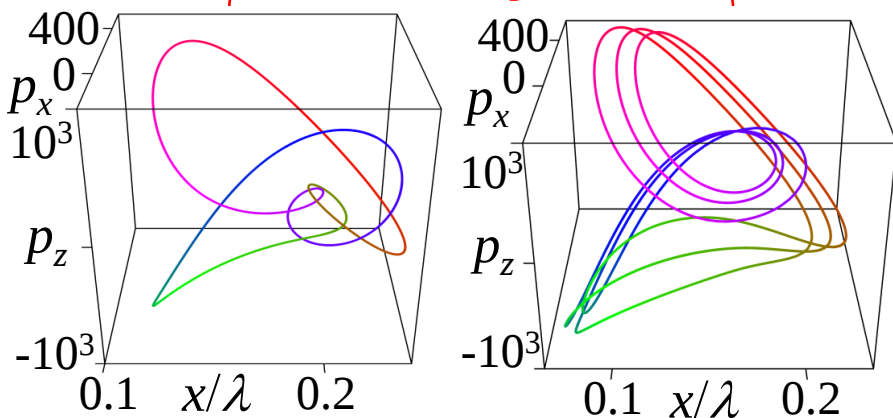
Average Location, Emission Power and χ_e Parameter for $\lambda=1\mu\text{m}$



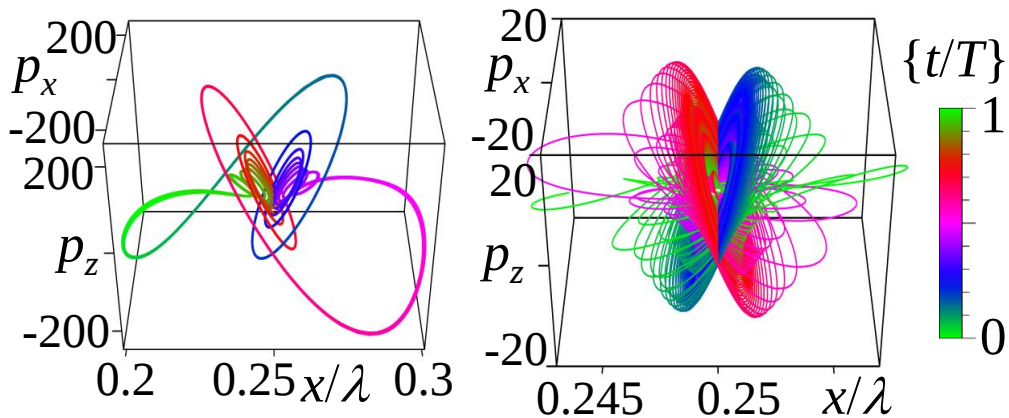
Strong Radiation Reaction (LP, $\lambda = 1\ \mu\text{m}$, $I = 1.37 \times 10^{24}\ \text{W/cm}^2$)



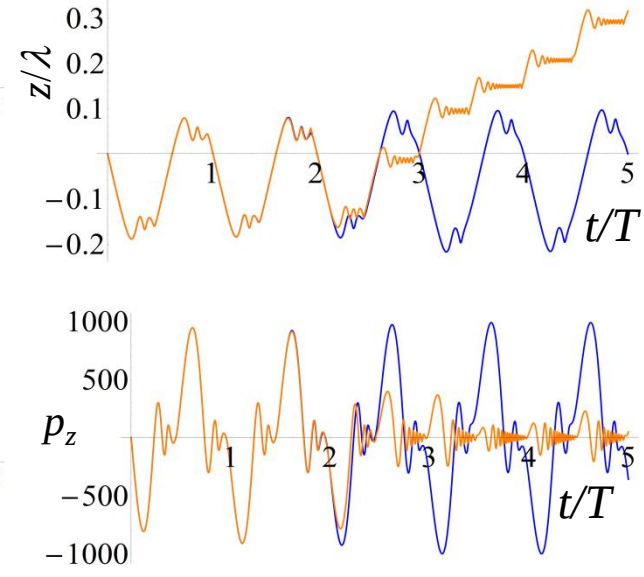
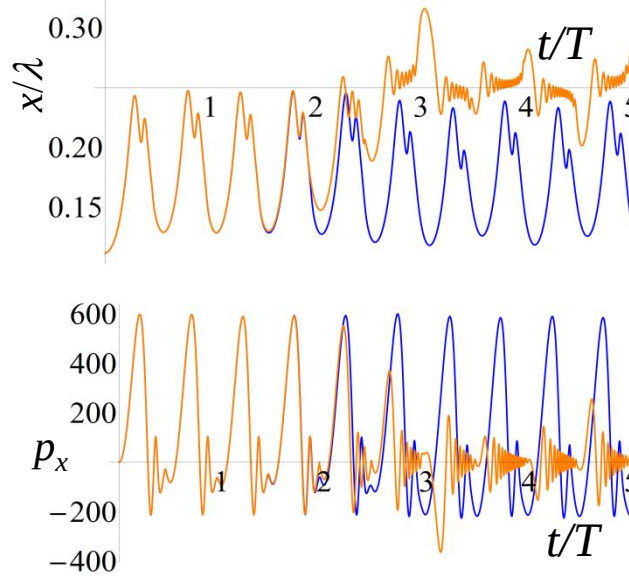
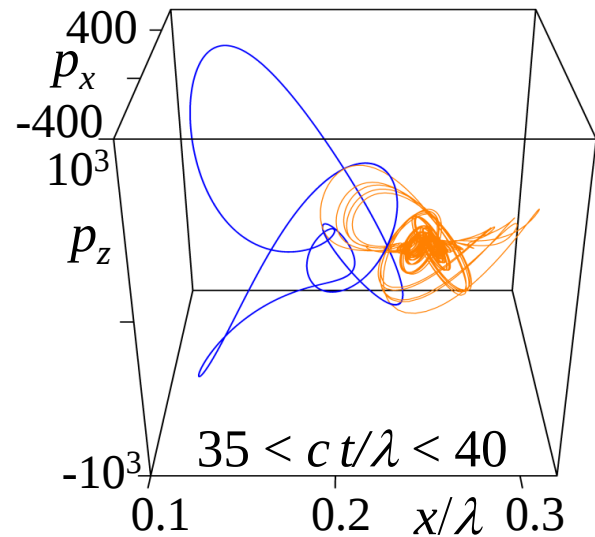
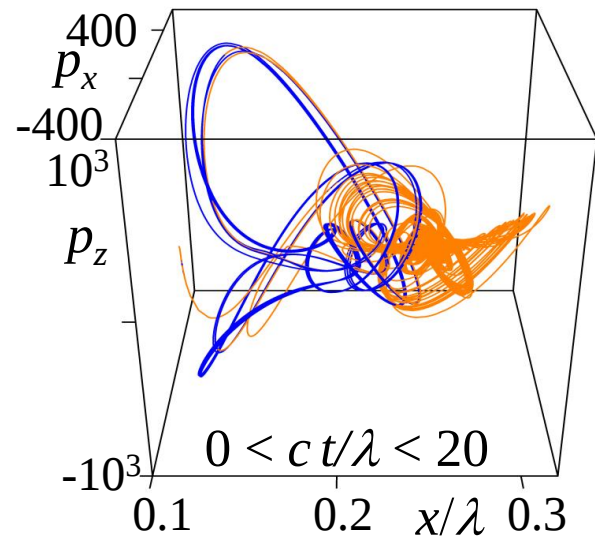
Limit cycles



Strange attractor



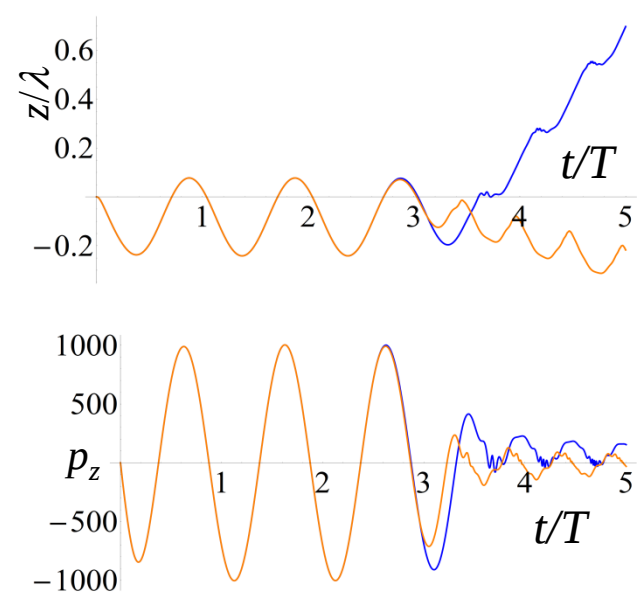
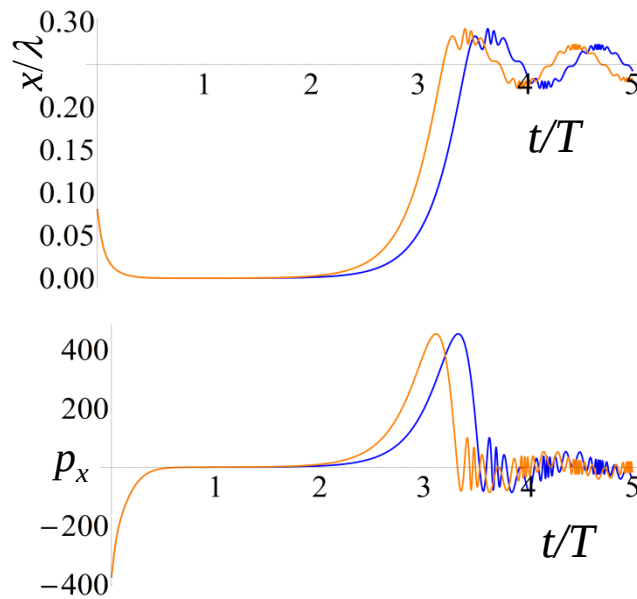
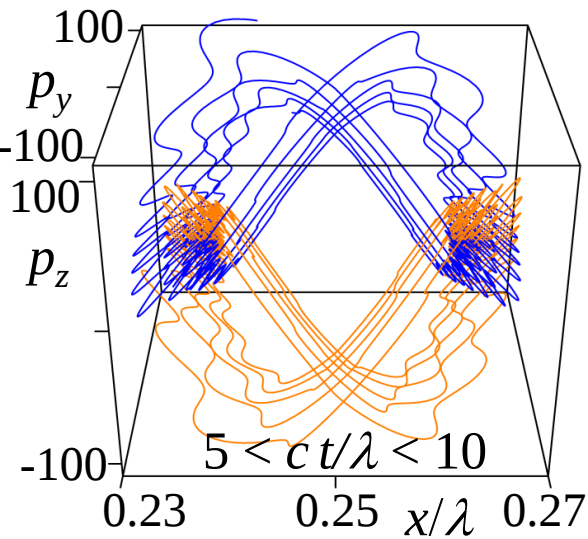
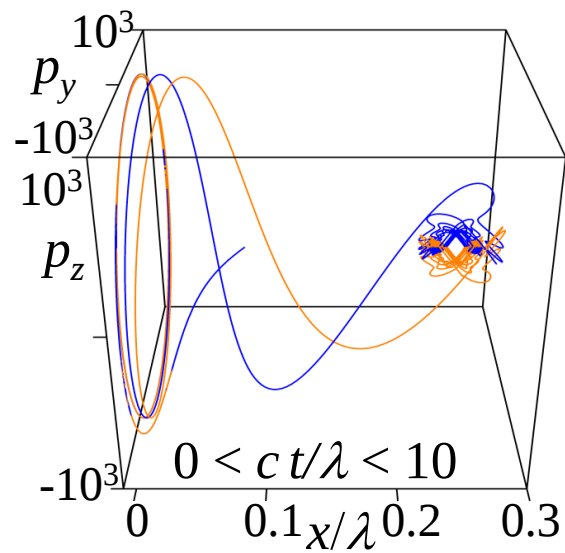
LP Strange Attractor: High Sensitivity to Initial Conditions



Maximal Lyapunov Exponent

$$\Lambda = \lim_{t \rightarrow \infty} \frac{1}{t} \lim_{\delta(0) \rightarrow 0} \ln \frac{\delta(t)}{\delta(0)} \gtrsim 5$$

CP Strange Attractor: High Sensitivity to Initial Conditions



Maximal Lyapunov Exponent

$$\Lambda = \lim_{t \rightarrow \infty} \frac{1}{t} \lim_{\delta(0) \rightarrow 0} \ln \frac{\delta(t)}{\delta(0)} \gtrsim 1$$

Ergodicity, Lyapunov index, Stochasticity, ...

■ **Measure** $d\mu = f(x)dx$ $\mu(X) = 1$

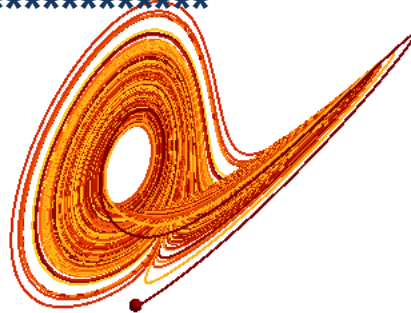
■ **Ergodicity** $\langle b \rangle = \lim_{T \rightarrow \infty} \frac{1}{T - t_0} \int_{t_0}^T b dt = \int b d\mu$

■ **Exponential instability** $\lim_{t \rightarrow \infty} \frac{1}{t} \lim_{\delta(0) \rightarrow 0} \ln \frac{\delta(t)}{\delta(0)} > 0$

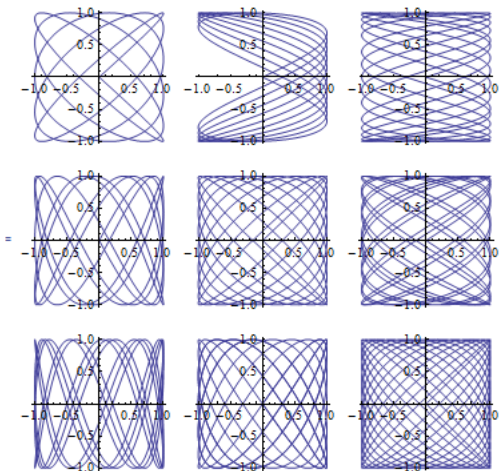
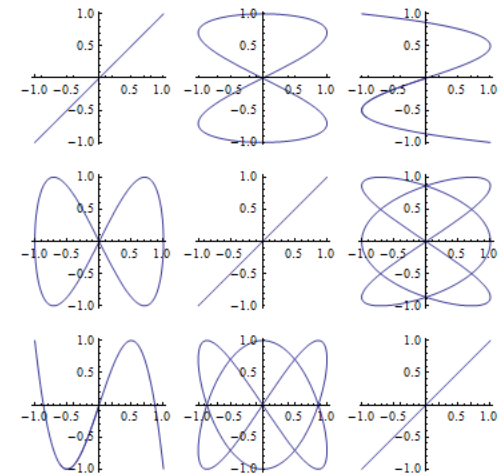
■ **Exponential decay of correlations** $\langle b(t)b(t + \Delta t) \rangle \underset{\Delta t \rightarrow \infty}{\propto} \exp(-\Delta t / \tau)$

■ *****

■ **Attractors**

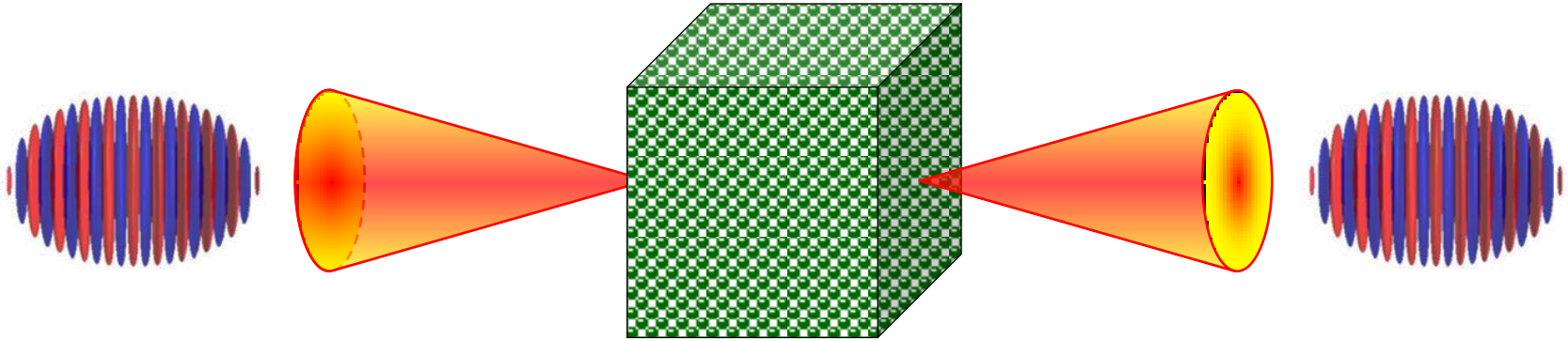


Lissajous's curves



Y. G. Sinai (2001)

Transient Standing Wave Formed by Two Colliding Pulses

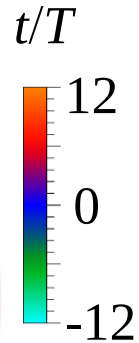
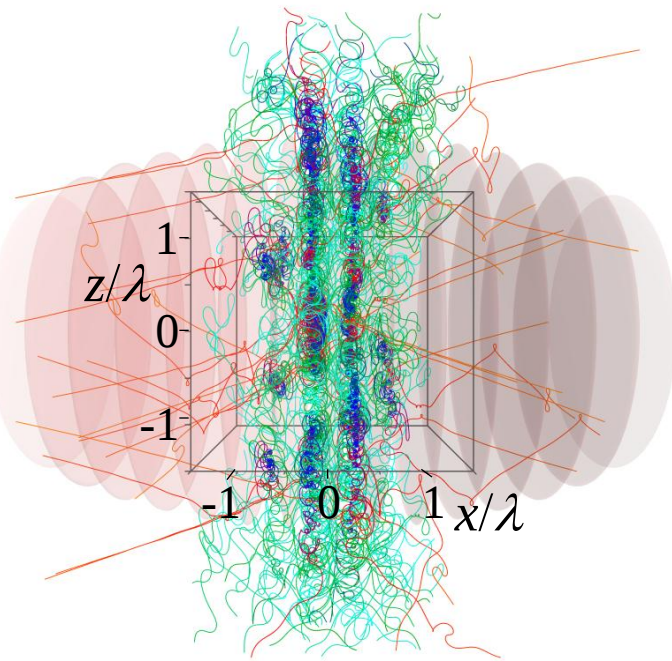


Laser pulse (each):
intensity $I = 1.37 \times 10^{24} \text{ W/cm}^2$,
wavelength $\lambda = 1 \mu\text{m}$,
duration $10\lambda/c = 33 \text{ fs}$,
focal spot $3 \mu\text{m}$,
power $P = 123 \text{ PW}$.

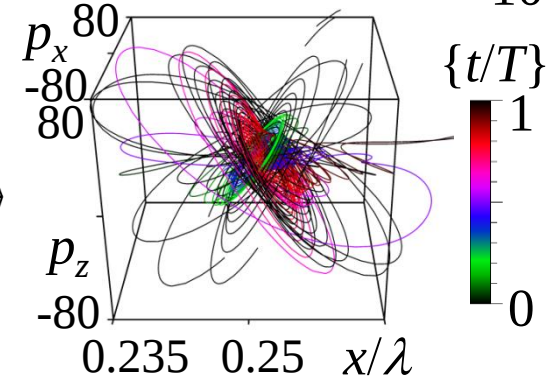
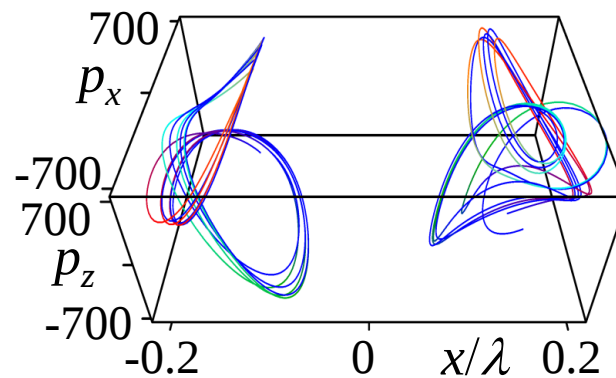
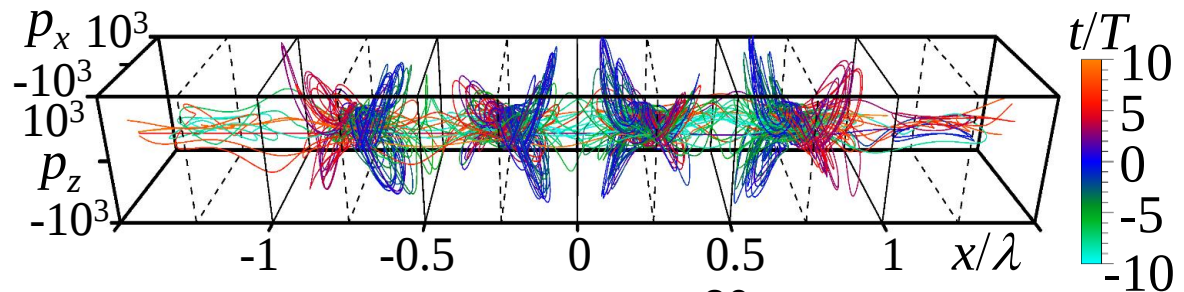
Initially, 10^3 electrons are distributed randomly in a $(3 \mu\text{m})^3$ box at $t = -20\lambda/c$.

Standing wave is formed at $t = 0$ for $\approx 3 \lambda/c$ (pulse duration/3).

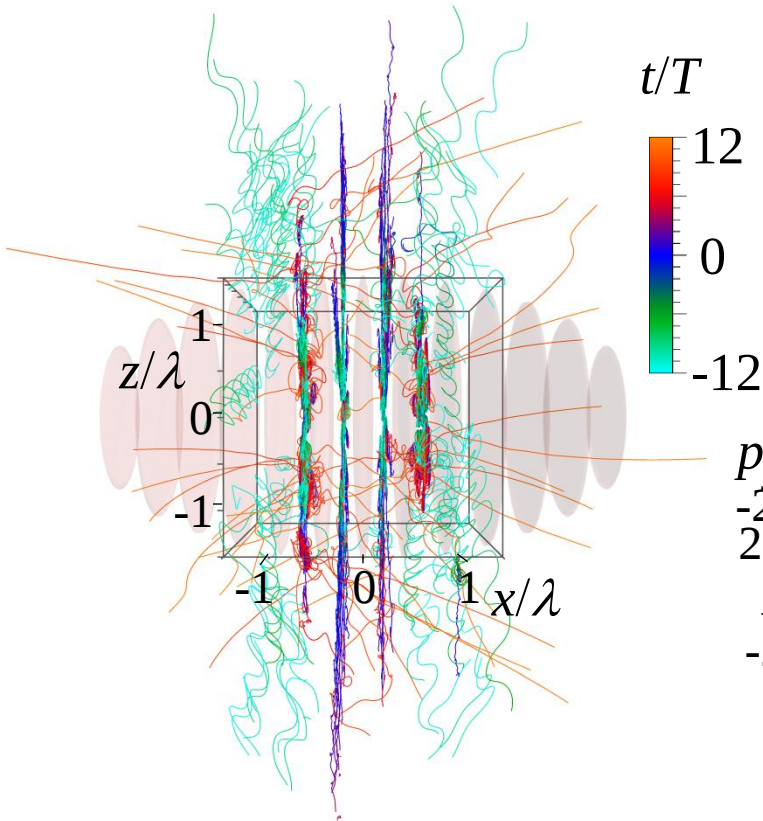
Transient LP Standing Wave ($I = 1.37 \times 10^{24}$ W/cm²)



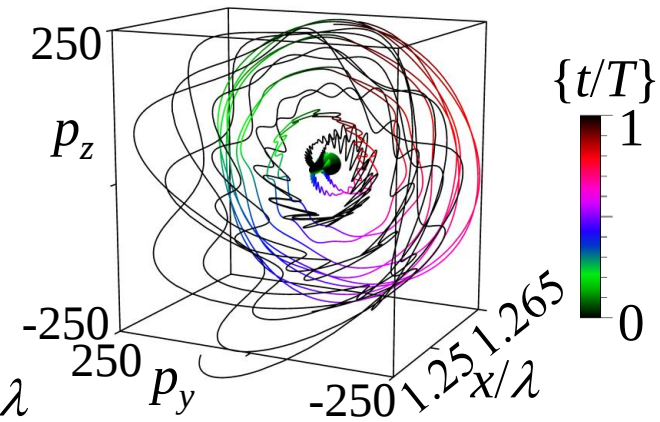
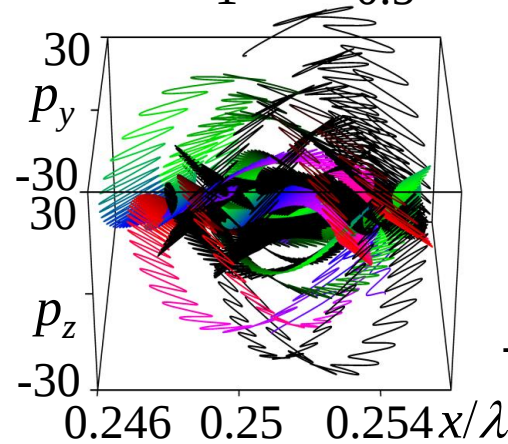
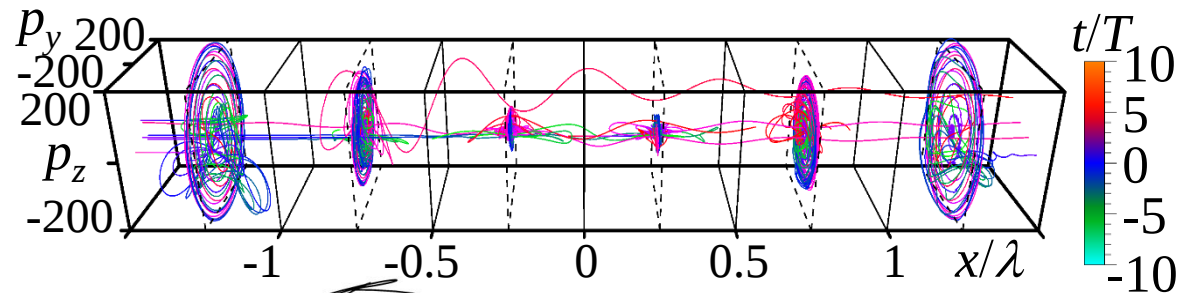
10% of particles remain in the initial box for >25 laser cycles.



Transient CP Standing Wave ($I = 1.37 \times 10^{24}$ W/cm²)



15% of particles remain in the initial box for >25 laser cycles.



Conclusion

- **LAD vs LL**
- **1+1 D Electrodynamics**
- **High Power Gamma-Ray Source**
- **3D EM Field Configurations**
- **Chaos in Electron-EM Wave Interaction**

Acknowledgment

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Thank you for listening to me!



The 15th International Conference on X-ray Lasers *May 22-27, 2016, Nara Kasugano International Forum*

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