Qualitative considerations in Intense Field QED

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Conference on Extremely High Intensity Laser Physics (ExHILP)
Heidelberg, Germany, 21 – 24 July 2015

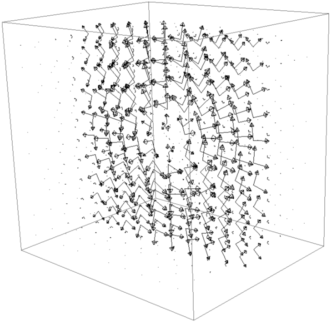
Motivation

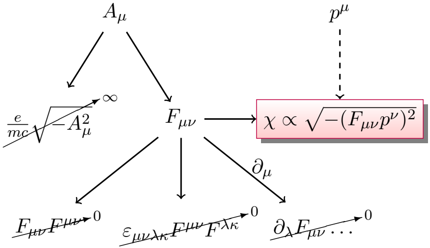
- Intense Field QED is a well-developed (at least, theoretically) research area.
- However, most results were being obtained by **extremely bulky calculations**.
- Merely everybody would agree that **qualitative considerations** always **allow to gain deeper insight** into a problem.
- Surprisingly, **qualitative considerations in IFQED have been almost never discussed in literature** in general setting.

  Notable exceptions (discussions of important selected aspects):
  - A. B. Migdal, "Vacuum Polarization in Strong Inhomogeneous Fields", Sov. Phys. JETP **35**, 845 – 853 (1972) [see also A.B. Migdal, "Fermions and bosons in strong fields" [in Russian] (Nauka, Moscow, 1978)]
  - E. Kh. Akhmedov, "Beta Decay and Other Processes in Strong Electromagnetic Fields", Physics of Atomic Nuclei **74**, 12991315 (2011) [arXiv:1011.3776].
- I am going to demonstrate how at least some of known simple asymptotic expressions for probability rates of basic processes in strong external field could receive a **simple-man explanation** (**analysis of kinematics + uncertainty principle + dimensional arguments**).

Processes seeded by ultra-relativistic particles: **locally constant crossed** field approximation

General fact: **any** EM field in the proper reference frame of ultra-relativistic particle looks as **constant crossed** ($E \approx H$, $\vec{E} \cdot \vec{H} \approx 0$) [Nikishov&Ritus, 1964]



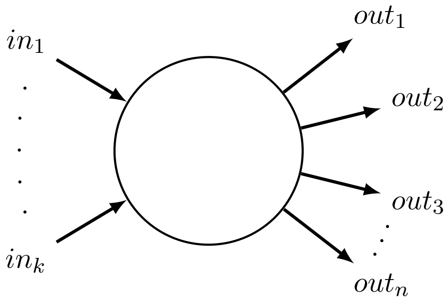


$$\chi = \frac{e\hbar}{m^3c^4}\sqrt{-(F_{\mu\nu}p^\nu)^2} = \frac{\gamma\sqrt{\left(\vec{E} + \frac{\vec{v}\times\vec{H}}{c}\right)^2 - \frac{(\vec{v}\cdot\vec{E})^2}{c^2}}}{E_S} = \frac{E_P}{E_S}$$

–proper acceleration (in Compton units)

$$E_{L\parallel} \sim E_{L\perp} \Rightarrow E_{P\parallel} \sim E_{L\parallel}, \quad E_{P\perp} \sim \gamma E_{L\perp} \Rightarrow E_P \sim \gamma E_{L\perp}$$

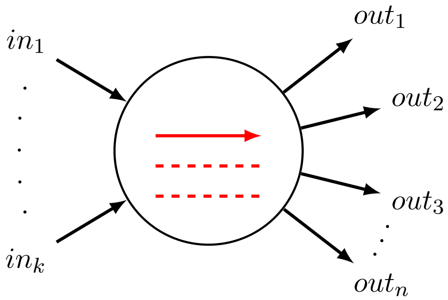
General framework



- Energy release $(-)$:

$$\Delta\varepsilon = \sum \varepsilon_f - \sum \varepsilon_i > 0$$

General framework



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$$t \lesssim t_q \simeq \frac{1}{\Delta\varepsilon}$$

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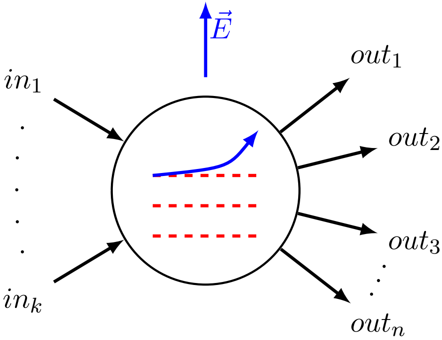

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$$e \int_0^{t_e} \vec{E} \cdot d\vec{s} \simeq \Delta\varepsilon$$

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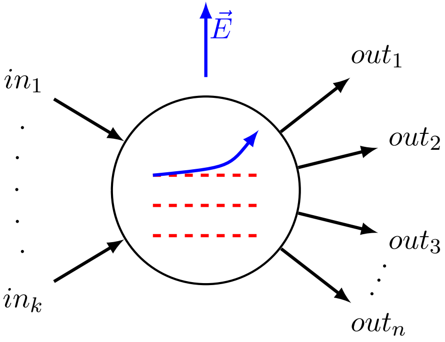

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$$e \int_0^{t_e} \vec{E} \cdot d\vec{s} \simeq \Delta\varepsilon$$

- $t_e \lesssim t_q$ $\rightarrow$ process is 'allowed' (quantum regime!)
- $t_e \gtrsim t_q$ $\rightarrow$ process is 'suppressed' $\propto e^{-t_e/t_q}$ (quasiclassical regime!)

Precise evaluation of exponential suppression factor in quasiclassical regime

- Purely electric constant field, time gauge: $\vec{A}(t) = -\vec{E}t$
- Quasiclassical solutions ($E \ll E_S = m^2/e$):

$$\Psi(\vec{r},t) \propto \exp\left\{ i\vec{p}\vec{r} - i\int_0^t \varepsilon(t')\,dt' \right\},\ \varepsilon(t) = \sqrt{\left(\vec{p} - e\vec{A}(t)\right)^2 + m^2}$$

- Quantum amplitude of the process:

$$c_{i\to f} = -i\int_{-\infty}^{+\infty} dt\, V_{fi}\exp\left\{ i\int_0^t \Delta\varepsilon(t')\,dt' \right\}, \quad V_{fi} \propto \delta^{(3)}\left(\Delta\vec{p}\right)$$

- Landau (1932), but $t \leftrightarrow x$:

$$c_{i\to f} \propto \exp\left\{ -\int_0^{t_*} \Delta\varepsilon(it')\,dt' \right\}, \quad \Delta\varepsilon(it_*) = 0, \quad \Delta\vec{p} = 0$$

- It turns out that $t_* \simeq t_e$, so that $c_{i\to f} \simeq e^{-t_e/t_q}$!

Spontaneous pair creation in constant electric field

- Characteristic time scales:

$$\Delta\varepsilon = 2m, \quad eEt_e \simeq 2m \quad \Longrightarrow \quad \boxed{t_e \simeq \frac{m}{eE}}$$

$$\boxed{t_q \simeq \frac{1}{\Delta\varepsilon} \simeq \frac{1}{m}}, \quad eE\tilde{t}_q \simeq \frac{1}{\tilde{t}_q} \quad \Longrightarrow \quad \boxed{\tilde{t}_q \simeq \frac{1}{\sqrt{eE}} \gg t_q}$$

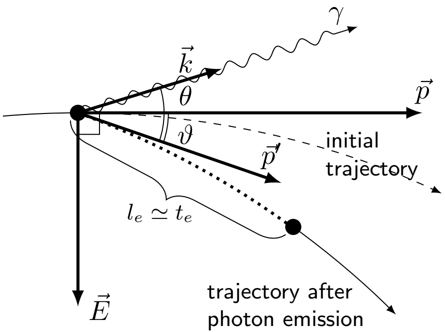
- For $E \ll E_S = m^2/e$ we have $t_e \gg \tilde{t}_q \gg t_q, \quad \Longrightarrow$ process is 'suppressed' (quasiclassical regime).
- Expected suppression factor $e^{-t_e/t_q} \simeq e^{-E_S/E}$.
- More precisely, for $\vec{p}_\perp = 0$ (for sake of simplicity only):

$$\Delta\varepsilon(t) = 2\sqrt{m^2 + e^2E^2t^2}, \quad \Delta\varepsilon(it_*) = 0 \implies t_* = \frac{m}{eE} \simeq t_e,$$

$$W_{e^-e^+} = \left| \exp\left\{ -2 \int\limits_0^{m/eE} \sqrt{m^2 - e^2E^2t'^2}\, dt' \right\} \right|^2 = e^{-\pi m^2/eE}$$

- Correct pre-exponential factor $N_{loops} \simeq VT/\tilde{t}_q^4 \simeq e^2E^2VT$!

Photon emission by relativistic electron -I



- Assume: $p, k, p-k \gg m, eEt$
- Initial energy ($\vec{p} \perp \vec{E}$ for simplicity):

$$\varepsilon_{\vec{p}}(t) = \sqrt{(\vec{p}-e\vec{A})^2+m^2} = \\ = \sqrt{p^2+e^2E^2t^2+m^2} \approx \\ \approx p + \frac{e^2E^2t^2+m^2}{2p}$$

- Final momentum: $\vec{p}' = \vec{p} - \vec{k}$
- ${p'}^2 = p^2 + k^2 - 2pk\cos\theta = (p-k)^2 + 4pk\sin^2(\theta/2)$
- Final energy (let $\vec{k} \in \mathrm{Span}\,(\vec{p}, \vec{E})$ for simplicity):

$$\varepsilon_{\vec{p}'}(t) = \sqrt{(\vec{p}'-e\vec{A})^2+m^2} = \sqrt{{p'}^2 - 2e\vec{E}\cdot\vec{k}t + e^2E^2t^2+m^2} \approx \\ \approx p - k + \frac{e^2E^2t^2 + 2eEkt\sin\theta + m^2 + 4pk\sin^2(\theta/2)}{2(p-k)}$$

Photon emission by relativistic electron -II

- Energy release (-):

$$\Delta\varepsilon(t) = \varepsilon_{\vec{p}'}(t) + k - \varepsilon_{\vec{p}}(t) \approx$$

$$\approx \not{p} - \not{k} + \frac{e^2E^2t^2 + 2eEkt\sin\theta + m^2 + 4pk\sin^2(\theta/2)}{2(p-k)} +$$

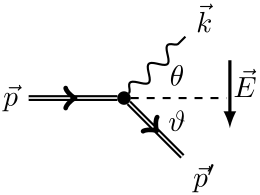
$$+ \not{k} - \left(\not{p} + \frac{e^2E^2t^2 + m^2}{2p}\right) =$$

$$= \boxed{\frac{k\left[e^2E^2t^2 + 2eEpt\sin\theta + m^2 + 4p^2\sin^2(\theta/2)\right]}{2p(p-k)}}$$

Photon emission – case (i): $t \lesssim m/eE$

$$\Delta\varepsilon = \frac{k\left[\cancel{e^2E^2t^2} + \cancel{2eEpt\sin\theta} + m^2 + \cancel{4p^2\sin^2(\theta/2)}\right]}{2p(p-k)} \simeq \frac{km^2}{p(p-k)}$$

- Estimate of angles:



$$\theta \lesssim \frac{m}{p} = \frac{1}{\gamma} \ll 1$$

$$k_\perp = p'_\perp, \quad p' \approx p - k$$

$$\vartheta \simeq \frac{k}{p-k}\theta \lesssim \frac{km}{(p-k)p} \ll 1$$

- Characteristic times scales:

$$\boxed{t_q \simeq \frac{1}{\Delta\varepsilon} \simeq \frac{p(p-k)}{m^2k}} \gtrsim \boxed{t_e \simeq \frac{\Delta\varepsilon}{eE\vartheta} \simeq \frac{m}{eE}}$$

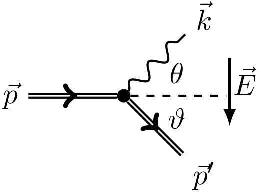
- Radiation frequency range: $k \lesssim \frac{eEp}{m}p = \chi p \lesssim p \Longrightarrow \boxed{\chi \lesssim 1}$
- Emission probability and radiation reaction:

$$\boxed{W_\gamma \overset{(?)}{\simeq} e^2/t_e \sim (e^2m^2/p)\chi}, \quad \boxed{F_{RR} \simeq kW_\gamma \sim e^2m^2\chi^2}$$

Photon emission – case (ii): $t \gg m/eE$

$$\Delta\varepsilon = \frac{k\left[e^2E^2t^2 + \cancel{2eEpt\sin\theta} + \cancel{m^2} + \cancel{4p^2\sin^2(\theta/2)}\right]}{2p(p-k)} \simeq \frac{e^2E^2t^2}{p}$$

- Estimate of angles:



$$k \sim p$$

$$\vartheta, \theta \lesssim \frac{eEt}{p} \ll 1$$

$eE\vartheta t_e \simeq \Delta\varepsilon$ identically!!!

- Time scales analysis:

$$t_q \simeq \frac{1}{\Delta\varepsilon} \simeq \frac{p}{e^2E^2t_q^2} \implies t_q \simeq \left(\frac{p}{e^2E^2}\right)^{1/3} = \frac{m}{eE}\chi^{1/3} \quad (\chi \gg 1)$$

- Emission probability and radiation reaction:

$$W_\gamma \overset{(?)}{\simeq} e^2/t_q \sim (e^2m^2/p)\chi^{2/3}, \qquad F_{RR} \simeq kW_\gamma \sim e^2m^2\chi^{2/3}$$

Pair photoproduction by hard photon -I

- Energy release (-):

$$\Delta\varepsilon(t) = \varepsilon_{\vec{k}-\vec{p}}(t) + \varepsilon_{\vec{p}}(t) - k \overset{\text{1D}}{\approx}$$

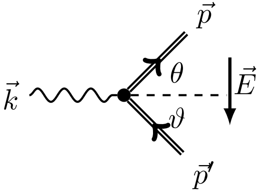
$$\overset{\text{1D}}{\approx} \sqrt{(k-p)^2 + e^2E^2t^2 + m^2} + \sqrt{p^2 + e^2E^2t^2 + m^2} - k \approx$$

$$\approx \not{k} - \not{p} + \frac{e^2E^2t^2 + m^2}{2(k-p)} + \not{p} + \frac{e^2E^2t^2 + m^2}{2p} - \not{k} =$$

$$= \frac{k\left(e^2E^2t^2 + m^2\right)}{2p(k-p)} \gtrsim \boxed{\frac{2\left(e^2E^2t^2 + m^2\right)}{k}}$$

$$\left(\text{minimum is attained at } p = p' = \frac{k}{2}\right)$$

Pair photoproduction: case (i) $t \lesssim m/eE$



$$\boxed{\Delta\varepsilon(t) = \frac{2\left(\cancel{e^2E^2t^2} + m^2\right)}{k}}$$

- Estimation for angles: $\boxed{\theta, \vartheta \simeq m/k \ll 1}$
- Characteristic time scales:

$$\boxed{t_q \simeq \frac{1}{\Delta\varepsilon} \simeq \frac{k}{m^2}} \quad \boxed{t_e \simeq \frac{\Delta\varepsilon}{eE\vartheta} \simeq \frac{m}{eE}}$$

- Thus process is suppressed ($\propto e^{-t_e/t_q}$) for $\varkappa = \frac{eEk}{m^3} \lesssim 1$
- Check in (primed) RF moving along with the incoming photon with $\gamma \simeq k/m$: $\Delta\varepsilon' \simeq k' \simeq k/\gamma \simeq m$, $E' \simeq \gamma E$, $\vartheta \simeq 1$,

$$t_e' \simeq \frac{\Delta\varepsilon'}{eE'} \simeq \frac{m}{eE\gamma} \simeq \frac{1}{m\varkappa} \implies t_e \simeq \gamma t_e' \sim \frac{m}{eE}$$

$$t_q' \simeq \frac{1}{\Delta\varepsilon'} \simeq \frac{1}{m} \implies t_q \simeq \gamma t_q' \simeq \frac{k}{m^2}$$

Pair photoproduction: evaluation of exponential suppression factor

- Seeking for a stationary point:

$$\Delta\varepsilon(it_*) = \frac{2\left(-e^2E^2t_*^2 + m^2\right)}{k} = 0 \implies t_* = \frac{m}{eE} \simeq t_e \ (!!!)$$

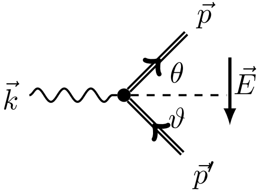
- Suppression factor:

$$W_{e^-e^+} \propto \left|\exp\left(-\int\limits_0^{t_*} \Delta\varepsilon(it)\,dt\right)\right|^2 =$$

$$= \left|\exp\left(-\int\limits_0^{m/eE} \frac{2\left(-e^2E^2t^2 + m^2\right)}{k}\,dt\right)\right|^2 =$$

$$= \left|e^{-4m^3/3eEk}\right|^2 = \boxed{e^{-8/3\varkappa}} \quad !!!$$

Pair photoproduction: case (ii) $t \gg m/eE$



$$\Delta\varepsilon(t) = \frac{2\left(e^2E^2t^2 + \cancel{m^2}\right)}{k}$$

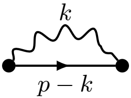
- Estimation for angles: $\boxed{\theta, \vartheta \simeq eEt/k \ll 1}$
- t_e is arbitrary ($eE\vartheta t \simeq \Delta\varepsilon(t)$ identically)
- Time scale analysis:

$$t_q \simeq \frac{1}{\Delta\varepsilon(t_q)} \simeq \frac{k}{e^2E^2t_q^2} \implies \boxed{t_q \simeq \left(\frac{k}{e^2E^2}\right)^{1/3} \simeq \frac{m}{eE}\varkappa^{1/3}}$$

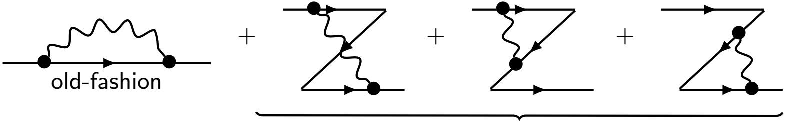
- Hence, for $\varkappa \gg 1$

$$\boxed{W_{e^-e^+} \overset{(?)}{\simeq} \frac{e^2}{t_q} \sim \frac{e^2m^2}{k}\varkappa^{2/3}}$$

Mass operator

- $M^{(2)} = e^2 \int \frac{d^4k}{(2\pi)^4} \gamma_\mu S(p-k) \gamma^\mu D(k) =$ 

- Introduce a regularization parameter R: $r \gtrsim R$, $k \lesssim R^{-1}$.
- Then in terms of 'old-fashion' PT:

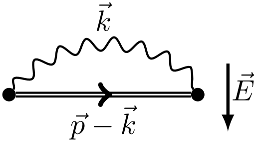


- Hence [V. Weisskopf, Zeitschrift für Physik **89**, 27 (1934); A.V. Vilenkin and P.I. Fomin, Sov. Phys. JETP **40**, 6 (1974)]:

$$M \simeq \begin{cases} e^2/R, & R \gg \dfrac{1}{m} \\ e^2/R + e^2/R_{e_1^- e_2^-} - e^2/R_{e_1^- e^+} - e^2/R_{e_2^- e^+} \simeq \boxed{e^2 m \ln \dfrac{1}{mR}}, & R \ll \dfrac{1}{m} \end{cases}$$

- In presence of a strong enough field virtual $e^-_{1,2}$ and e^+ get shifted $(R, R_{e_1^- e_2^-} \ll R_{e^- e^+})$ and thus $M \simeq e^2/R$ is restored!

Mass operator for $\chi \gg 1$



As discussed above,

$$t_q \simeq \left(\frac{p}{e^2E^2}\right)^{1/3} \simeq \frac{p}{m^2}\chi^{-2/3}$$

- In a proper RF ($\gamma = p/m$):

$$R \simeq \tau_q = \frac{t_q}{\gamma} \simeq \frac{1}{m\chi^{2/3}}, \qquad \boxed{M \overset{(?)}{\simeq} \frac{e^2}{\tau_q} \simeq \alpha m \chi^{2/3}}$$

- Validity of perturbation theory: radiation corrections ($m \to m + M$) must be small:

$$\frac{M}{m} \simeq \alpha\chi^{2/3} \ll 1$$

- More evidence for that proper PT expansion parameter for $\chi \gg 1$ is $\alpha\chi^{2/3}$ is given in

  NB Narozhny, "Expansion parameter of perturbation theory in intense-field quantum electrodynamics", Physical Review D **21**, 1176 (1980).

Reference values for $\alpha\chi^{2/3} \sim 1$

ε_{in}, GeV	10^3	10^2	10	1
E/E_S	10^{-3}	10^{-2}	0.1	1
I_L,W/cm^2	10^{23}	10^{25}	10^{27}	10^{29}

(note: the table assumes **transverse** propagation in the field. For A-type cascade $\chi \sim (E/\alpha E_S)^{3/2}$ [AMF et al, PRL 2010] and $\alpha\chi^{2/3} \sim E/E_S \ll 1$).

Discussion

- Qualitative arguments (kinematical + uncertainty principle + dimensional analysis) suffice for deeper understanding of origination of various formulas of IFQED previously obtained by formal manipulations.
- Key parameter is formation time (or length) of the process.
- For vacuum processes, $r \sim \tilde{t}_q \sim (eE)^{-1/2}$, while for ultrarelativistic particles-induced processes

$$r_\perp \sim t_q \vartheta \sim \frac{eEt_q^2}{p} \sim \frac{1}{(eEp)^{1/3}} \sim \boxed{\frac{1}{m}\chi^{-1/3}}$$

- Concept of formation length quantifies the relation [V. I. Ritus, "Connection between strong-field quantum electrodynamics with short-distance quantum electrodynamics", Zh. Eksp. Teor. Fiz. **73**, 807-821 (1977)]:

- For $\alpha\chi^{2/3} \sim 1$ (i.e., $\chi \sim 10^3$) **IFQED is expected to become non-perturbative**.

THANK YOU FOR ATTENTION!