# Narrowband Inverse Compton Scattering X and $\gamma$-Ray Sources at High Laser Intensities

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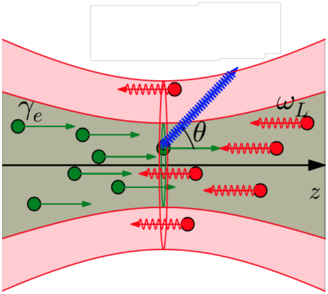
D. Seipt, S. G. Rykovanov et al, Phys. Rev. A**91**, 033402 (2015)

# Inverse Compton Scattering as Hard x- and $\gamma$-ray Source




[Rykovanov et al J. Phys. B 47, 234013 (2014)]

- x($\gamma$)-ray frequency:

$$\omega'_\ell = \frac{4\gamma^2\omega_L\ell}{1+\gamma^2\theta^2+2\ell\rho+a_0^2/2}$$

- Doppler upshift of laser frequency $4\gamma^2$
- Harmonics: $\ell$-photon processes
- Red-shifts due to: off-axis scattering $\theta$, quantum recoil $\rho$, laser intensity $a_0^2$

## Dimensionless Interaction Parameters

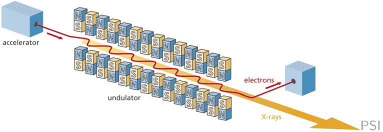
- Dimensionless laser amplitude: $a_0 \quad \sim 1$
- Recoil parameter $\rho = 2\gamma\omega_L/m \quad < 1$
- Quantum nonlinearity parameter $\chi = \rho a_0/2 \quad < 1$

# Photon Yield

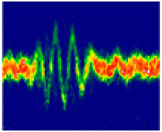
## Scattering Probability (of the fundamental line $\ell = 1$)

- $P \propto \frac{a_0^2}{1+a_0^2} \Rightarrow$ Laser intensity should be as large as possible
- Higher $a_0$ is more efficient: one photon per electron costs 1.6 Joules of laser energy at $a_0 = 0.15$, but only 0.4 J at $a_0 = 0.3$. [Rykovanov et al J Phys B (2014)]
- Analogy to magnetic undulators $a_0 \leftrightarrow K \sim 1$

## BUT:




Undulator has constant $K$ over its whole length



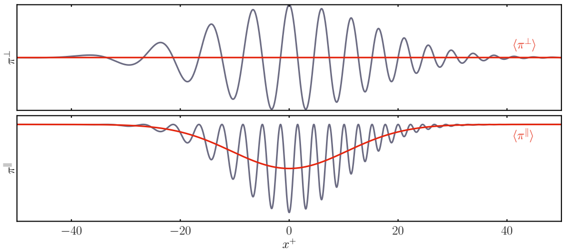
MPIPKS (atto07)

Laser pulses ramp on and off smoothly: $a_0 \to a(t)$ changes with time.

Laser pulse ramp-on leads to additional broadening of the spectral lines: Ponderomotive Broadening $\Rightarrow$ Limits allowed laser intensity

# Classical Electron in a Strong Laser Pulse

## Electron 4-momentum: $\pi^\mu(x^+) = p^\mu + p^\mu_{\mathrm{osc}}[A(x^+)] + U^\mu[A(x^+)]$




$p^\mu$: asymptotic electron momentum

$A^\mu$: laser vector potential

$x^+ = t + z$: light-front time

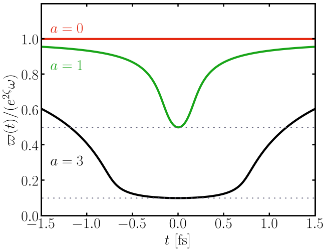
## Averaged four-momentum

Ponderomotive four-potential $U^\mu$: relativistic generalization of $U_p$ [H. R. Reiss, PRA **89**, 022116 (2014)]

$$\langle\pi^\mu(x^+)\rangle = p^\mu + U^\mu(x^+) \qquad U^\mu(x^+) = \frac{e^2\langle -A\cdot A\rangle}{2n\cdot p} n^\mu$$

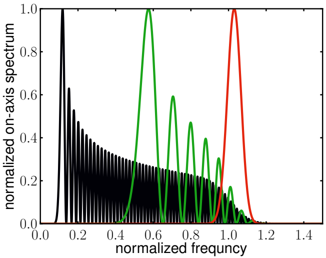
Fast electron slows down as it enters the laser pulse: lower Doppler upshift

## Time dependent X-ray frequency




[T. Heinzl, DS and B. Kämpfer, PRA 2010]

## Broadened spectra



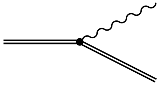

- ▸ Bandwidth increases with increasing laser intensity because of ramp-on and off
- ? What can be done to operate narrowband CS at high laser intensity?
- ! "Time-frequency correlation": Compensate slower electrons by higher laser frequency
  [I. Ghebregziabher et al, PRSTAB 16, 030705 (2013)] [B. Terzić et al, PRL 112, 074801 (2014)] [D. Seipt et al, PRA 91 033402 (2015)]
- ▸ Employ chirped laser pulses: What is the optimal frequency modulation $\omega(x^+)$

# NL Compton Scattering in Chirped Laser Pulses

Formulation of strong-field QED in the Furry picture with chirped laser pulses:

$A = a(x^+) \cos \Phi(x^+)$, with $\omega(x^+) = \dot{\Phi}(x^+)$, therefore also $k^\mu(x^+) \neq const.$

- Volkov states $\Psi(x, A)$ in strong chirped light pulse
- Non-linear Compton scattering Feynman diagram:



$$\Longrightarrow \qquad S = -ie \int d^4x \, \bar{\Psi}(x, A) \, \gamma_\mu \mathcal{A}^\mu_{k'}(x) \, \Psi(x, A)$$

- Classical four-momentum appears in exponent of scattering amplitude

$$M \sim \exp\left\{ i \int^{x^+} d\xi \, \frac{k' \cdot \pi(\xi)}{n \cdot p'} \right\}$$

- Separation of fast and slow electron motion $\pi$ in the scattering amplitude
- The line-shapes are determined by the ponderomotive four-potential $U^\mu(x^+)$
- Details: D. Seipt, S. G. Rykovanov et al, Phys. Rev. A**91**, 033402 (2015).

# Determination of the Optimal Frequency Modulation

## *Local* Four-Momentum Conservation

$$p^\mu + U^\mu(x^+) + \ell\omega(x^+)n^\mu = p'^\mu + U'^\mu(x^+) + \omega'_\ell n'^\mu$$

Solve for the instantaneous X-ray frequency:

$$\omega'_\ell(x^+) = \frac{\ell\omega(x^+)n\cdot p}{n'\cdot p + n'\cdot U(x^+) + \ell\omega(x^+)n'\cdot n}$$

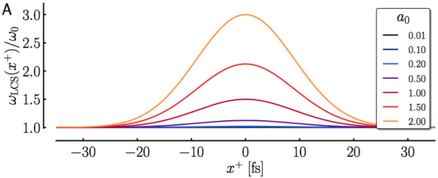
## Compensation Condition

$$\frac{d\omega'_\ell(x^+)}{dx^+} \stackrel{!}{=} 0 \qquad \Rightarrow \qquad \frac{\dot\omega(x^+)}{\omega} = \frac{n'\cdot \dot U(x^+)}{n'\cdot p + n'\cdot U(x^+)}$$

Provides differential equation for the optimal frequency modulation

# Optimal Frequency Modulation

$$\omega(x^+) = \omega_0 \left(1 + \frac{n' \cdot U(x^+)}{n' \cdot p}\right) = \omega_0 \left(1 + \frac{1}{1+\gamma^2\vartheta^2}\frac{a^2(x^+)}{2}\right)$$




[D. Seipt et al, PRA 91 033402 (2015)]

$$A(x^+) = a(x^+)\cos\Phi(x^+), \qquad \omega(x^+) = \frac{\partial\Phi(x^+)}{\partial x^+}$$

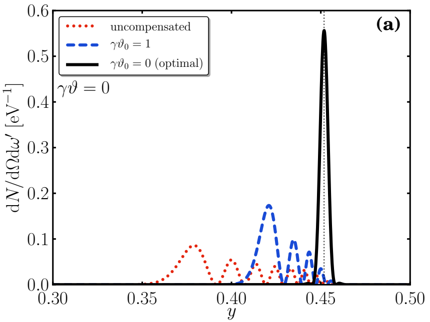
## Properties of the Optimal Frequency Modulation

- Frequency modulation depends on the pulse envelope and the laser intensity
- No influence of the electron recoil
- Full compensation is possible for *one emission angle only*: The influence of the slow-down of the electrons depends on the emission angle

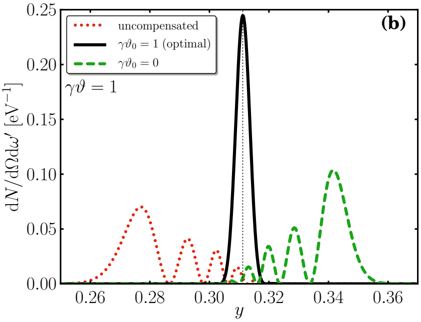
Conclusion: Given a laser pulse that fulfills the above relation $\omega(x^+) = \omega[a(x^+)]$ the ponderomotive broadening is compensated completely for radiation that is emitted under the angle $\vartheta$.

# Compensated Nonlinear Compton Source

## On-Axis: $\vartheta = 0$




## Off-Axis: $\vartheta = 1/\gamma$



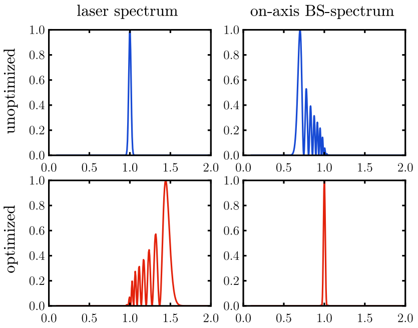

$y = \omega'/4\gamma^2\omega_0$ is the scaled frequency of the scattered x-rays

$a_0 = 1,\ E_{el} = 51$ GeV, $\gamma = 10^5 \longrightarrow y = 1 \Leftrightarrow 62$ GeV

# Realizing Optimized Laser Pulses

## Problem for generation of optimized chirped pulses: $\omega(x^+) = \omega[a(x^+)]$.

Given an envelope $a(x^+)$ we have to add chirp to compensate ponderomotive broadening. But adding a chirp to a laser pulse changes the envelope $a(x^+)$.



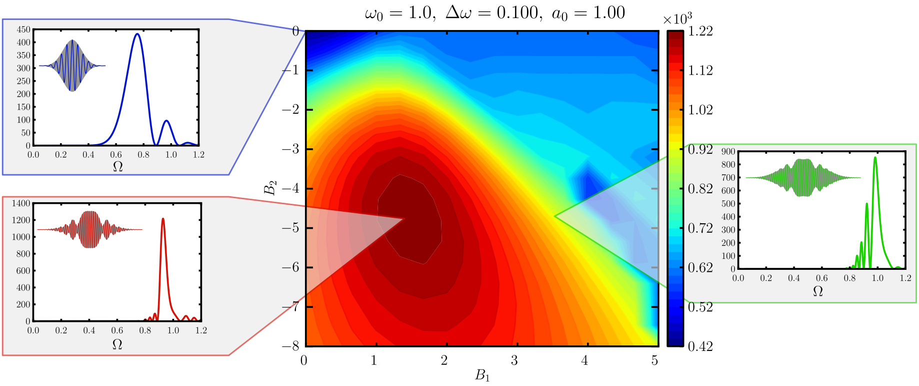

## Spectral approach

Optimize laser spectrum $\tilde{A}(\omega, \vec{B})$ by tuning control parameters $\vec{B}$

$\Rightarrow$ Derive instantaneous frequency $\omega(x^+, \vec{B})$ and intenstiy $a(x^+, \vec{B})$

$\Rightarrow$ They should fulfil "as close as possible" the optimal condition $\omega(x^+) = 1 + a^2(x^+)/2$.

## Spectral brightness as a function of the control parameters $\vec{B}$




D. Seipt, S. G. Rykovanov et al, to be published

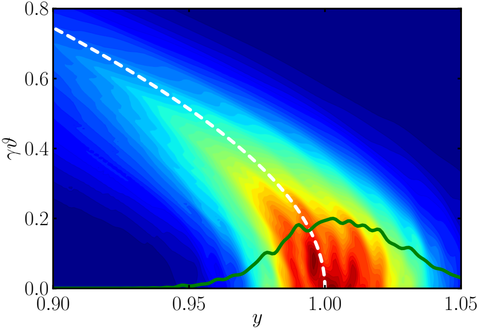
# Summary

- High-intensity laser slows down electrons: Ponderomotive four-potential and spectral broadening
- Compensation of ponderomotive broadening by proper frequency modulation of laser pulse
- Quantum recoil does not affect the form of the optimal chirp
- Simultaneous compensation for all harmonics
- Electron beam effects & focussing geometry: Compensated Nonlinear Compton Source works for realistic scenarios
- Spectral brightness increase by optimizing laser spectrum

[D. Seipt et al, Phys. Rev. A **91**, 033402 (2015)]

# BACKUP

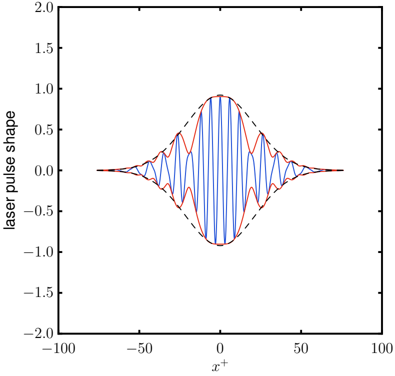
# Compensated Nonlinear Compton Source: Realistic Scenario

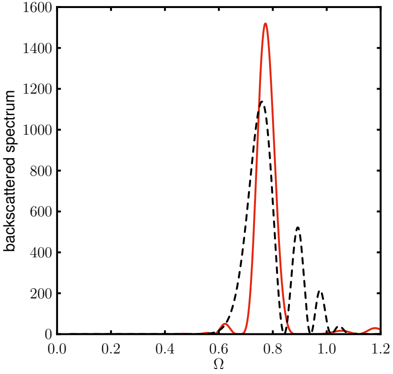


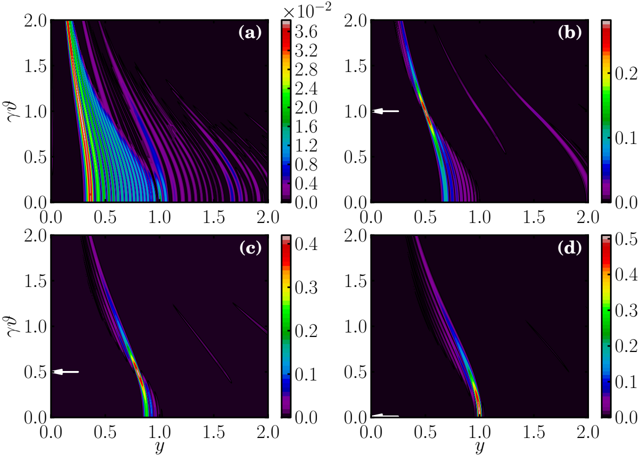

Focused laser pulse ($w_0 = 30$ μm), peak intensity $a_0 = 2.83$
Finite size electron bunch (1.8 μm),
energy spread (2.2 %) and emittance (0.2 mm mrad)

Optimized FWHM bandwidth $\kappa < 5$ % (vs. unoptimized 80 %)

# Equivalent Pulse





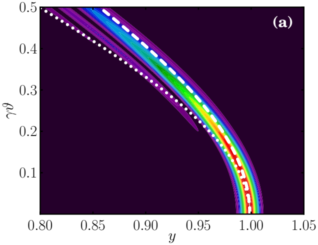

$y = \omega'/4\gamma^2\omega_0$ is the scaled frequency of the scattered x-rays

$a_0 = 2,\ E_{el} = 51$ MeV, $\gamma = 100 \longrightarrow y = 1 \Leftrightarrow 62$ keV

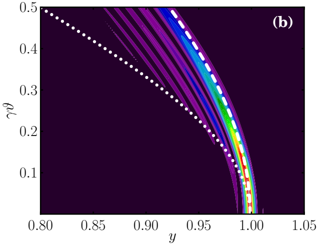
# Modified Angular Distribution

Reduced effective $\gamma$ factor of the electron: $\gamma_\star = \gamma/\sqrt{1 + a_0^2/2}$

$$y = \frac{1}{1 + \gamma_\star^2 \vartheta^2}$$




$(a_0 = 1)$




$(a_0 = 2)$

- Photons can be collected with a larger collimation angle $\theta_c \to \theta_c \sqrt{1 + a_0^2/2}$
- Increased photon yield: $N_X = \pi\alpha a_0^2$ instead of $N_X = \pi\alpha a_0^2/(1 + a_0^2/2)$.
- Relaxed criterion for the influence of electron beam emittance

# Strong Field QED in the Furry Picture

- Lagrangian of QED with background field $A$:

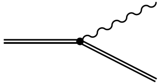
$$
\begin{aligned}
L_e &= \bar{\Psi}(i\partial\!\!\!/ - m)\Psi \\
L_r &= -\tfrac{1}{4}F^2 \\
L_i &= -e\bar{\Psi}\gamma_\mu\Psi(\mathcal{A}^\mu + A^\mu)
\end{aligned}
\qquad \rightarrow \qquad
\begin{aligned}
L_e^{(F)} &= \bar{\Psi}(i\partial\!\!\!/ - eA\!\!\!/ - m)\Psi \\
L_r^{(F)} &= -\tfrac{1}{4}F^2 \\
L_i^{(F)} &= -e\bar{\Psi}\gamma_\mu\Psi\mathcal{A}^\mu
\end{aligned}
$$

  - Laser field $A^\mu$ is taken into account nonperturbatively
    - Volkov States [D. M. Volkov, Z. Phys (1935)]:

$$
\begin{aligned}
\Psi(x, A) &= \left(1 + \frac{e n\!\!\!/ A\!\!\!/(x^+)}{2np}\right) \exp\left\{-i\frac{1}{2np}\int^{x^+} d\xi(2epA - e^2A^2) - ipx\right\} u_p \\
&= \Omega[A(x^+)]\psi(x)
\end{aligned}
$$

# Nonlinear Comtpon Scattering in Strong Field QED

- Furry Picture Feynman rules:



$$\Longrightarrow \qquad S = -ie \int d^4x \, \bar{\Psi}(x) \, \gamma_\mu \mathcal{A}^\mu_{k'}(x) \, \Psi(x)$$

- Scattering amplitude:

$$M = c_j \int dx^+ f_j(x^+) \, \exp\left\{ i \int^{x^+} d\xi \, \frac{k' \cdot \pi(\xi)}{n \cdot p'} \right\}$$

- Separation of fast and slow electron motion $\pi$ in the scattering amplitude
- Sum of harmonics $\ell$

$$M \propto \sum_{\ell=1}^{\infty} \int dx^+ \, (\ldots) \, \exp\left\{ i \int^{x^+} d\xi \, \frac{k' \cdot (p + U(\xi))}{n \cdot p'} - \ell\omega(\xi) \right\}$$

- The line-shapes are determined by the ponderomotive four-potential $U^\mu(x^+)$

# Bandwidth of the Inverse Compton Source

Laser pulse ramp-on leads to additional broadening of the spectral lines:
$\Rightarrow$ Limits the allowed laser intensity depending on bandwidth requirements.
$\Rightarrow$ Limits the photon yield and source efficiency.

**Spectral Brightness $=$ $\gamma$-ray yield per Bandwidth**

Bandwidth: $\left(\frac{\Delta\omega'}{\omega'}\right)^2 = \left(2\frac{\Delta\gamma_e}{\gamma_e}\right)^2 + \left(\frac{\gamma^2\Delta\theta_e^2}{4}\right)^2 + \left(\frac{\Delta\omega_L}{\omega_L}\right)^2 + \left(\frac{a_0^2}{2}\right)^2 + \ldots$

Yield: $N_\gamma \sim \frac{a_0^2}{1+a_0^2/2}$

Bandwidth requirement sets upper limit for laser intensity, limits the spectral brightness of the source. Even $a_0 = 0.2 \rightarrow 1\%$ bandwidth can be a significant contribution.

The goal is to find a way to efficiently operate narrowband inverse Compton sources with $a_0 \sim 1$.