QED multi-dimensional vacuum polarisation solver

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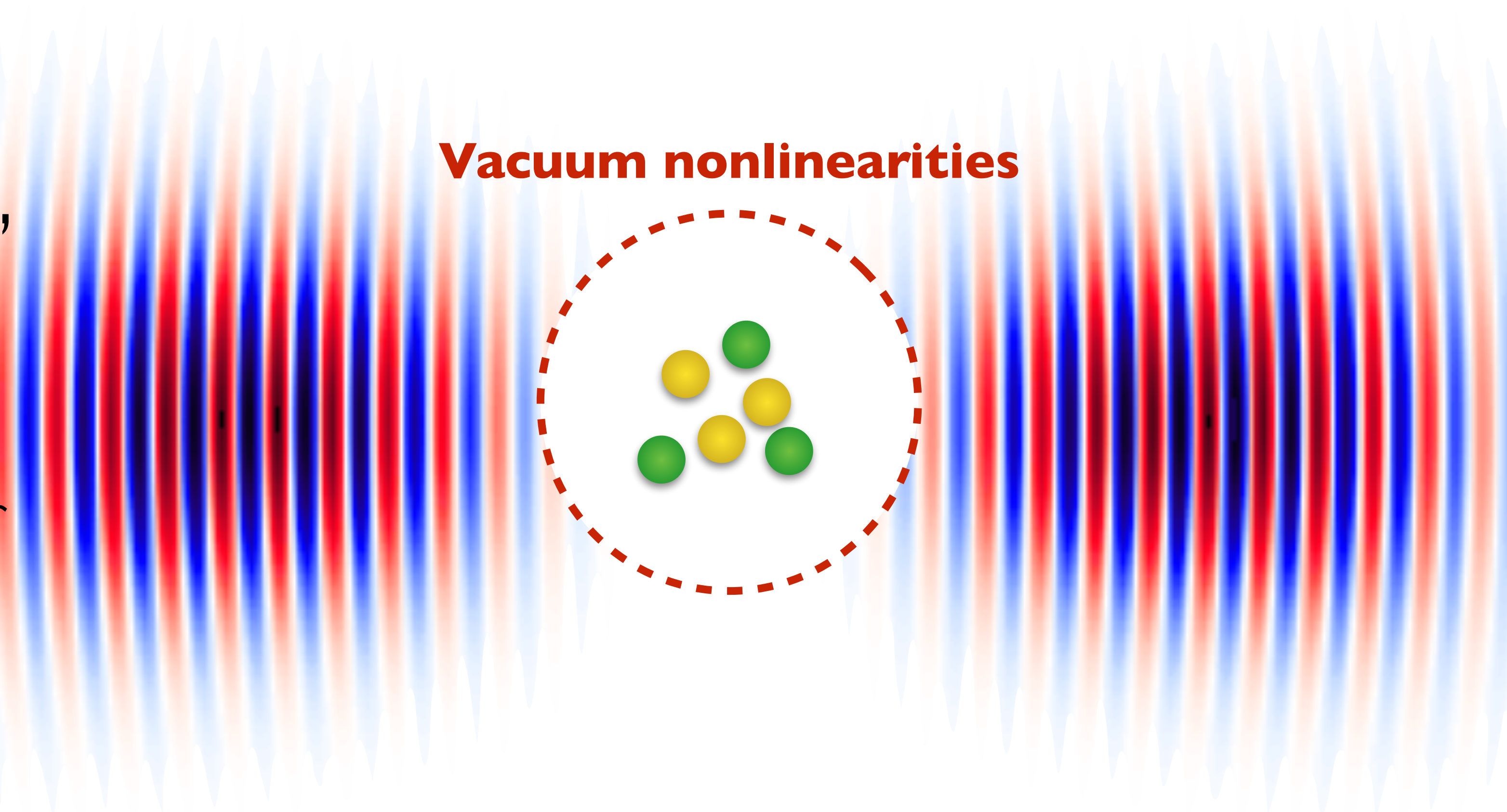


Motivation

Physics bellow Schwinger limit

- Relevance for extreme astrophysical scenarios?
- Effect on laser properties as we reach Schwinger limit?
- Extract observable consequences of fundamental QED predictions.
- ELI energies will allow us to probe the dynamics of Quantum Vacuum.

eli | beamlines

Heisenberg-Euler corrections to Maxwell's Equations*

Electron-positron fluctuations give rise to an effective polarisation and magnetisation of the vacuum which can be treated in an effective form as corrections to Maxwell's equations.

$$\mathcal{L} = \mathcal{L}_{\mathcal{M}} + \mathcal{L}_{HE} + \mathcal{L}_D$$

Valid for static inhomogeneous fields such that

$$E << E_S \qquad \omega << \omega_c$$

$$E_S = \frac{m^2c^3}{e\hbar} \qquad \omega_c = \frac{mc^2}{2\hbar}$$

Effectively, we obtain a highly non linear, non dispersive vacuum

Higher order corrections include spatial and temporal derivatives of these corrections. May be neglected for:

$$\omega << \omega_c \frac{E}{E_S}$$

*W. Heisenberg and H. Euler, Z. Physik 98, 714 (1936).

Heisenberg-Euler modified equations

QED corrections to Maxwell's equations

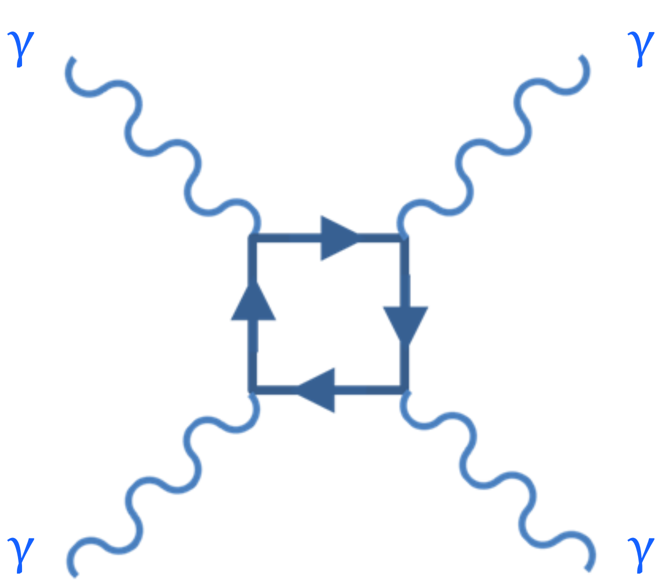
$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \qquad \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{P} = \xi(2(E^2 - c^2B^2)\vec{E} + 7c^2(\vec{E}\cdot\vec{B})\vec{B})$$

$$\vec{M} = -\xi c^2(2(E^2 - c^2B^2)\vec{B} - 7(\vec{E}\cdot\vec{B})\vec{E})$$

$$\xi = \frac{20\alpha^2\varepsilon_0^2\hbar^3}{45m_e^4c^5} \sim 10^{-50}(SI)$$

$$\frac{P}{E} \sim \xi E^2$$



Developed Maxwell's Equations

$$\frac{1}{\mu_0}\vec{\nabla}\times\vec{B} = \varepsilon_0\frac{\partial\vec{E}}{\partial t} + \frac{\partial\vec{P}}{\partial t} + \vec{\nabla}\times\vec{M}$$

$$\frac{\partial\vec{B}}{\partial t} + \vec{\nabla}\times\vec{E} = 0$$

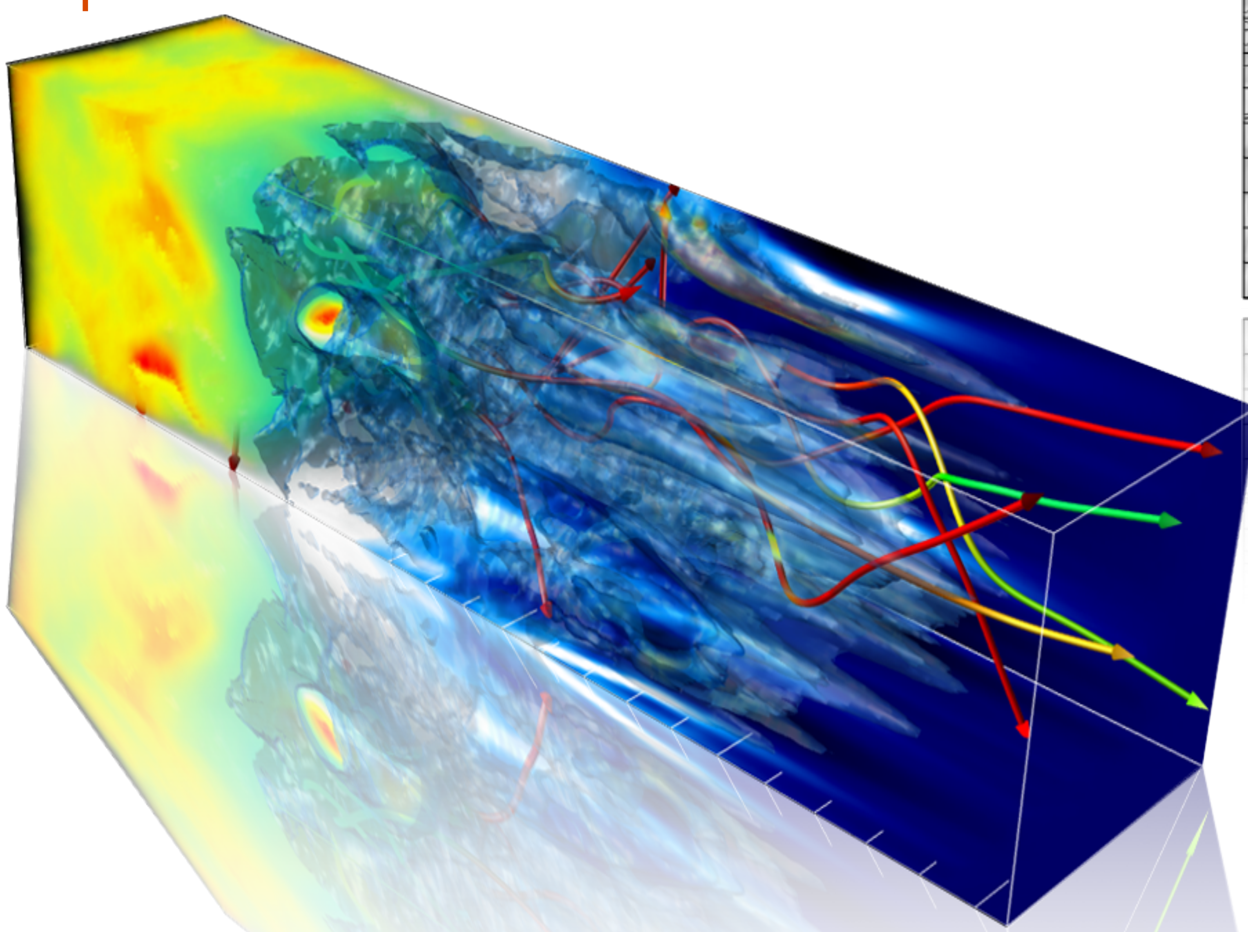
Key Features

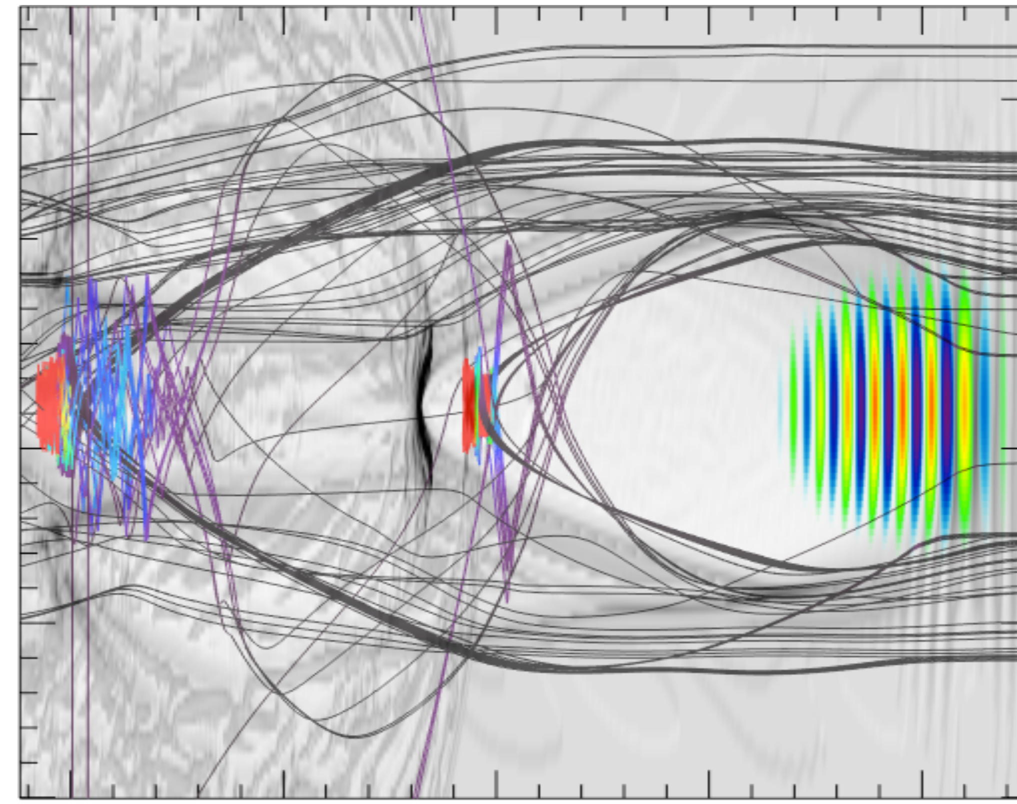
- Non linear polarisation and magnetisation.
- Dependence on EM invariants.
- Relative ordering allow to treat these terms as perturbations to Linear ME.

OSIRIS 2.0

osiris framework

- Massivelly Parallel, Fully Relativistic Particle-in-Cell (PIC) Code
- Visualization and Data Analysis Infrastructure
- Developed by the osiris.consortium
  ⇒ UCLA + IST





New Features in v2.0

- Bessel Beams
- Binary Collision Module
- Tunnel (ADK) and Impact Ionization
- Dynamic Load Balancing
- PML absorbing BC
- Optimized higher order splines
- Parallel I/O (HDF5)
- Boosted frame in 1/2/3D
- QED module + Merging algorithm
- **QED Maxwell Solver**

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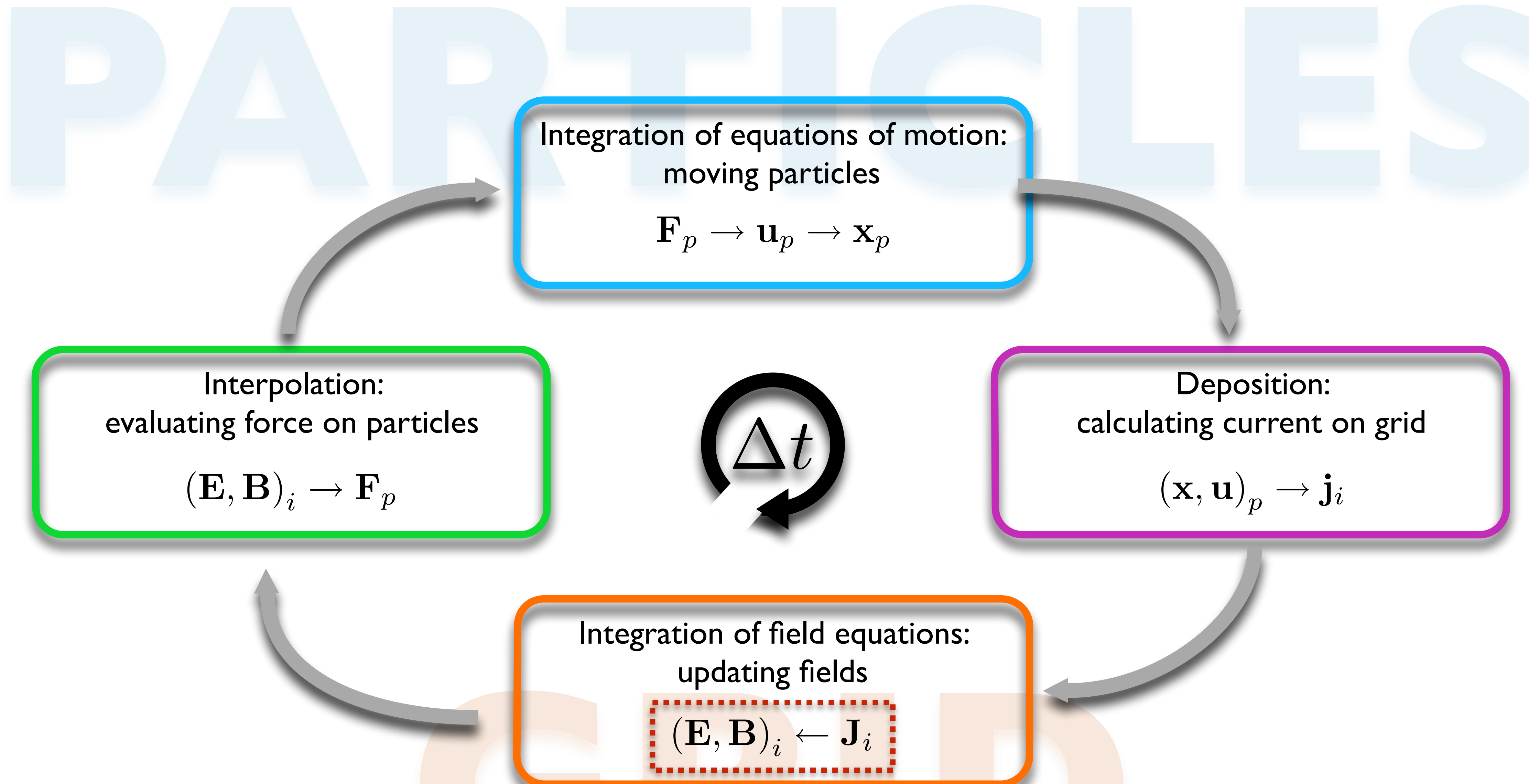
OSIRIS PIC LOOP

osiris v2.0



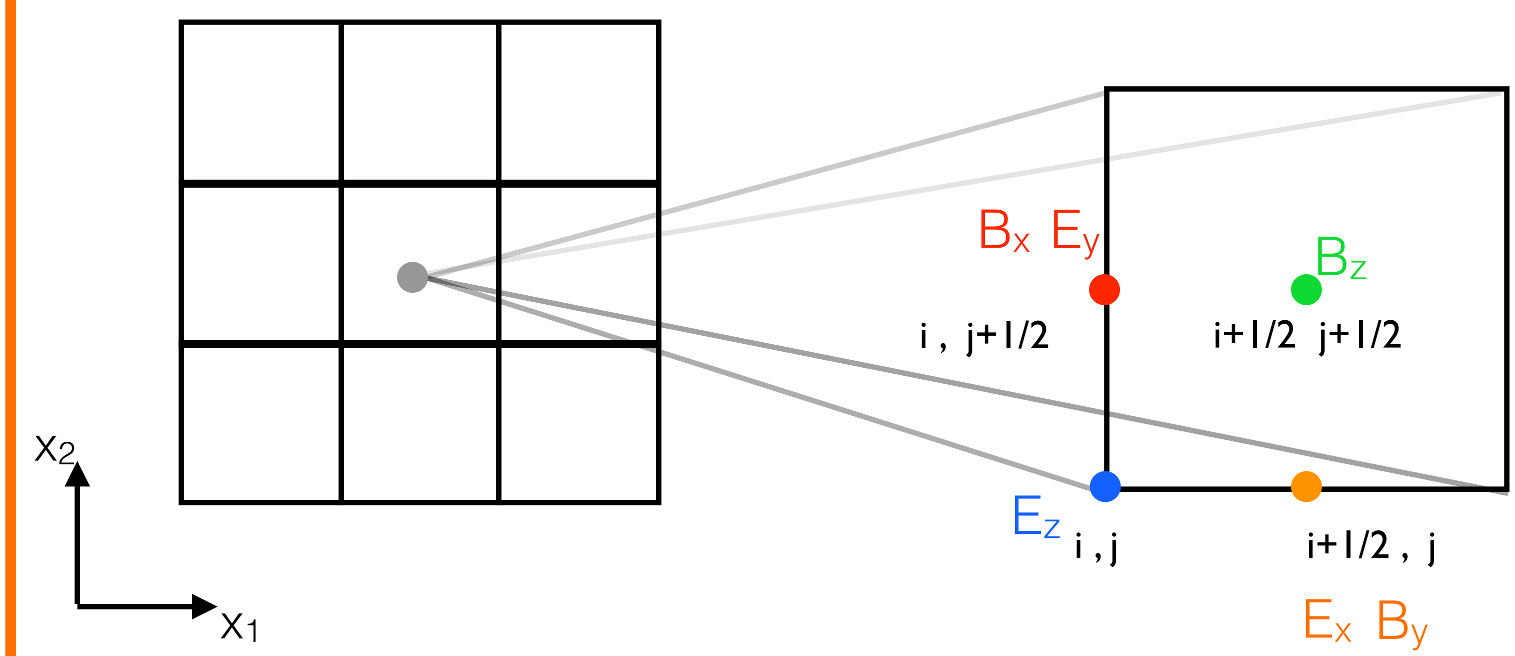
Fields are Integrated self consistently with QED vacuum non linear terms

NL Scheme requires field values at all grid points

Standard Yee Solver

1. advance B field Δt/2
2. advance E field Δt
3. advance B field another Δt/2

Spatial Grid 2D visualisation



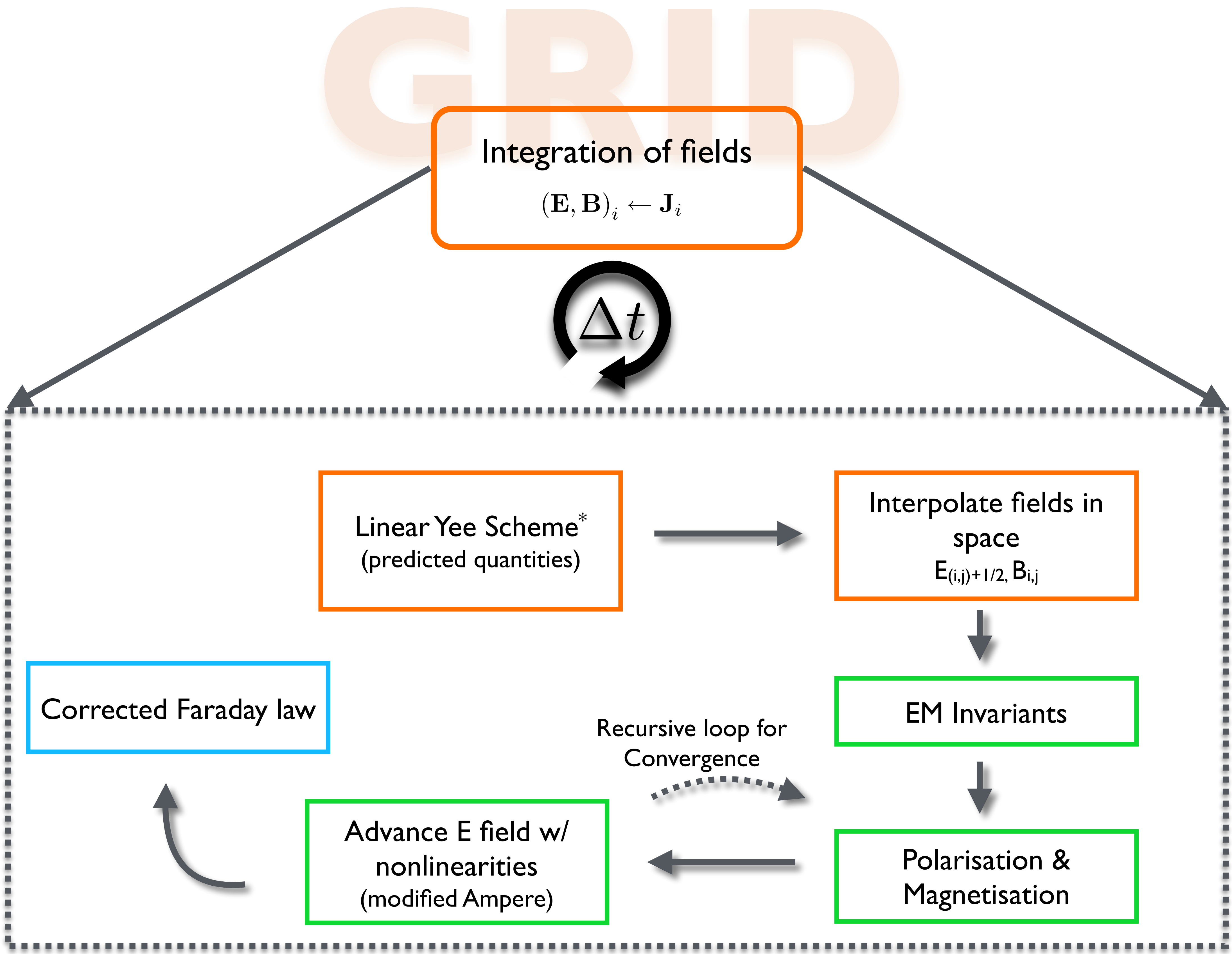
Main features of linear Solver

- Uses Faraday's and Ampère's Equation rather than wave equation to advance fields.
- Staggered grid: E & B fields are decentered from each other allowing second order precision.
- Linear coupling between fields allows straightforward temporal evolution.

Non Linear Solver

- Ampère's law is corrected by nonlinear polarisation & magnetisation.
- EM invariants couple all components of all fields → necessary to calculate them at all grid points.
- Temporally the loss of linearity does not allow fields to be straightforwardly advanced.

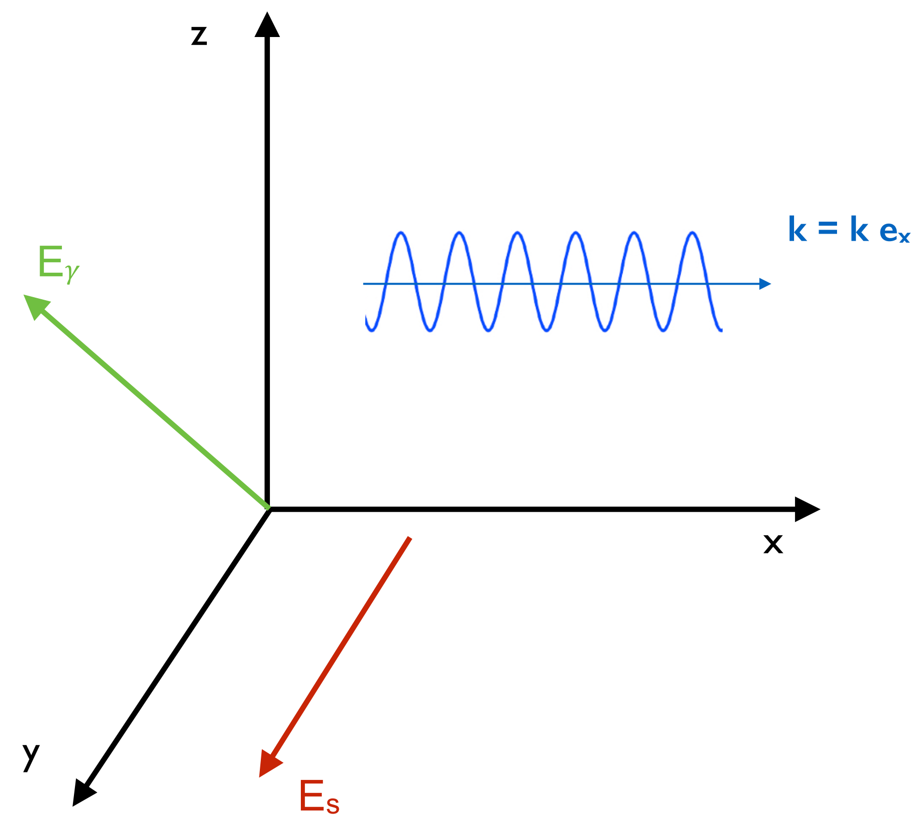
NL Yee Solver



* K. S. Yee, *IEEE Trans. Antennas Propagat.*, vol. 14, pp. 302–307, 1966.

Strong static field changes vacuum refractive index

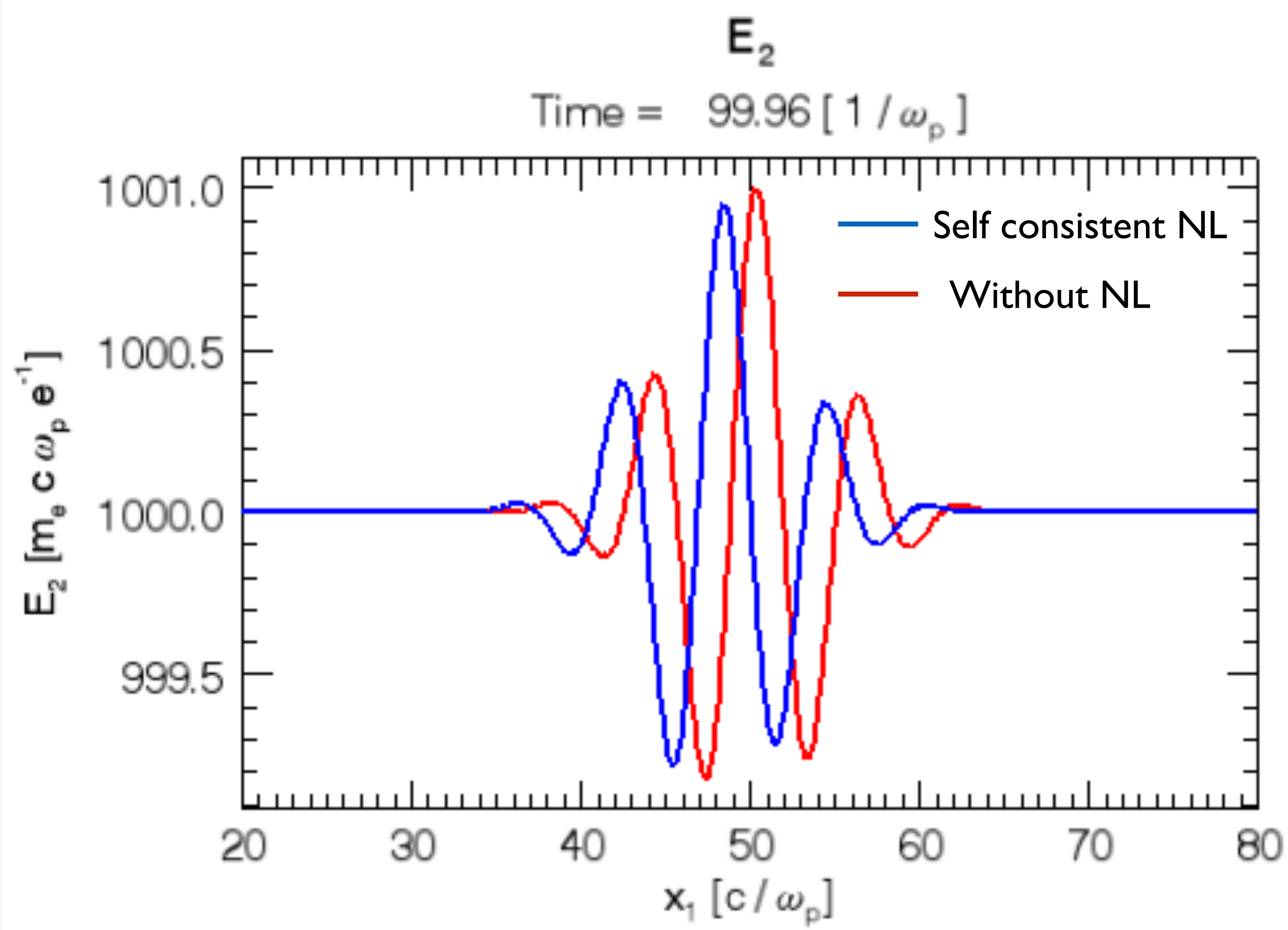
With Static Electric Field:*



$$E_y = \tilde{E}_y + E_s \qquad E_z = \tilde{E}_z$$

$$n_{\parallel} = \left(\frac{1 + 6\xi E_s^2}{1 + 2\xi E_s^2} \right)^{1/2} \approx 1 + 2\xi E_s^2$$

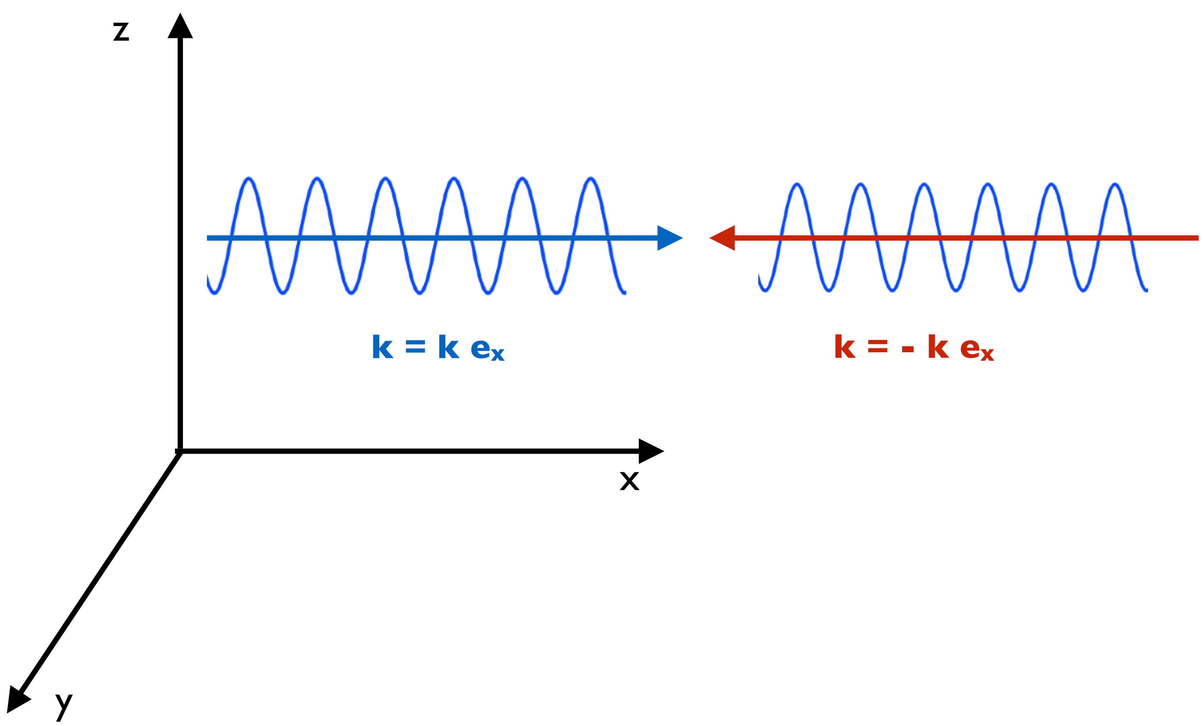
$$n_{\perp} = \left(\frac{1 + 2\xi E_s^2}{1 - 5\xi E_s^2} \right)^{1/2} \approx 1 + \frac{7}{2}\xi E_s^2$$



ξ	E	ξ	n	n
2× 10	1000	0.02	1.0377	1.0749

*Della Valle F, et.al, arXiv:1301.4918 [quant-ph]

Counter propagating plane waves give rise to HHG



In the absence of NL

$$E_y = E_0(\cos(x-t) + \cos(x+t))$$

$$B_z = B_0(\cos(x+t) - \cos(x-t))$$

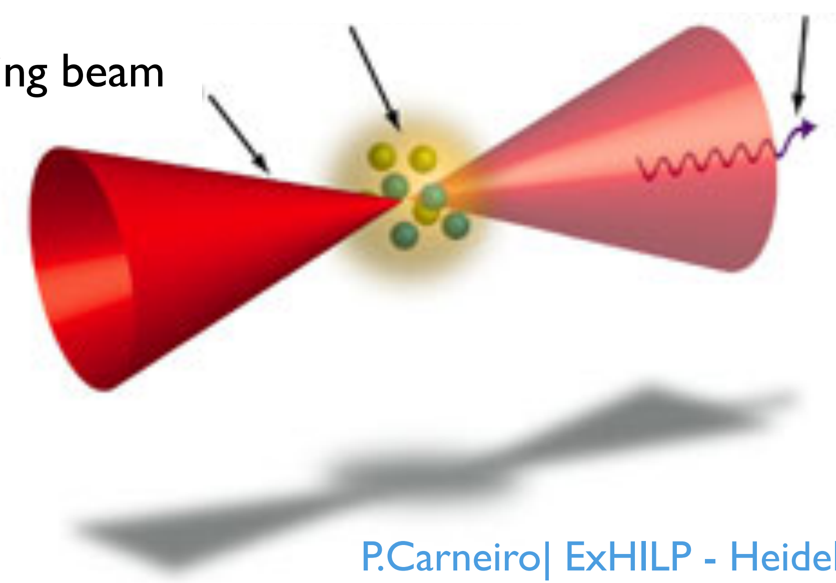
We would simply have a standing wave

The first EM invariant couples both fields leading to NL polarisation and magnetisation → generation of higher harmonics

$$P_y^{(0)} \sim 2\xi(\cos(3x-t) + \cos(x-3t) + ...)$$

Do the QED corrections change the dynamics?

Study the interaction between two counter propagating photons in the presence of HE non linearities!



*Source: ELI Consortium; http://www.eli-laser.eu/highfieldphysics.html

Analytical solution in 1D setup

Theoretical Analysis

$$E = E_0 + E_1 + E_2 + \dots$$

$$B = B_0 + B_1 + B_2 + \dots$$

$$\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial t^2}\right) E_1 = S_1(x,t)$$

With the source term given by:

$$S(x,t)_1 = 16\xi E_0^3 \cos(t)\cos(x)(3\cos(2t) - \cos(2x))$$

Formally, the solution is given by:

$$E_1(x,t) = \int_0^L \int_0^t dt' dx' G(x,x',t,t') S(x',t')$$

The first order correction to the field becomes:

$$E_1(x,t) = -2\xi E_0^3 \sin(t)\cos(x)(2\sin(2t)(\cos(2x) - 2)\boxed{- 4t)}$$

Resonant signature

Fourier Transform yields easier comparison

$$\tilde{E}^{(1)}(k=1) = 4\xi E_0^3 t \sin(t) + 3\xi E_0^3 \sin(t)\sin(2t)$$

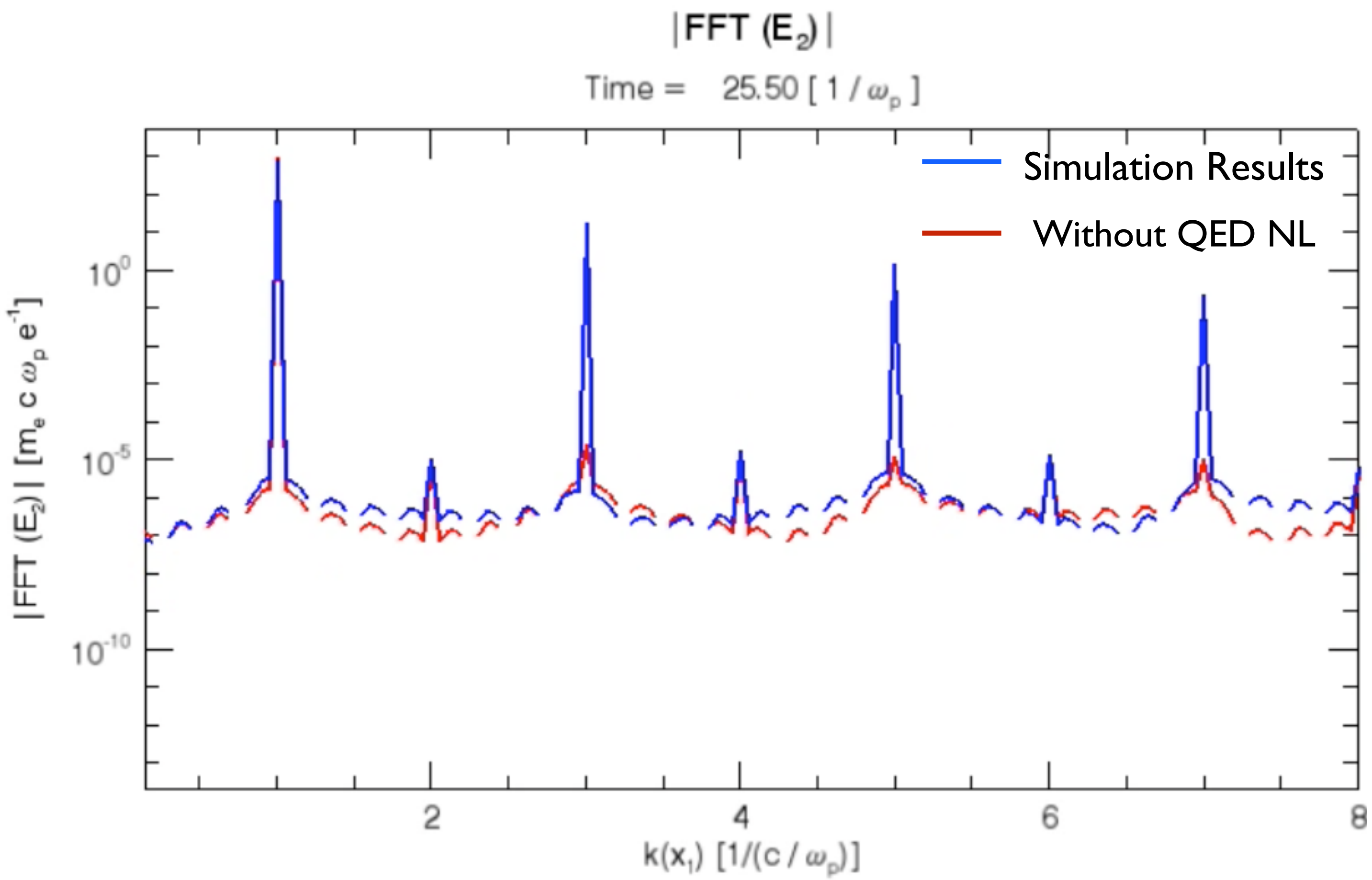
$$\tilde{E}^{(1)}(k=3) = -\xi E_0^3 \sin(t)\sin(2t)$$

Expression for the amplitude of a given (odd) harmonic

Simulations show generation of odd harmonics in the fields, with the following relative amplitude

$$\tilde{E}(k=2n+1) = (E_0^2\xi)^n \tilde{E}(k=1)$$

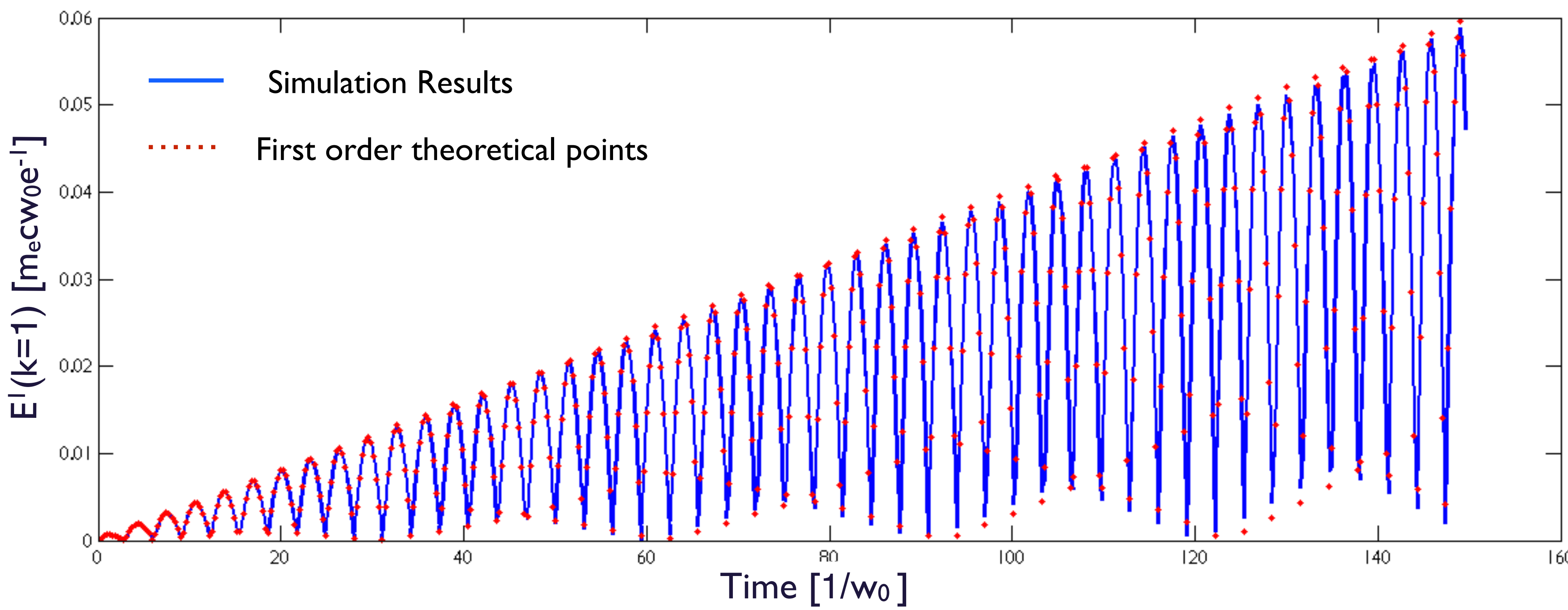
Simulation results match theoretical predictions



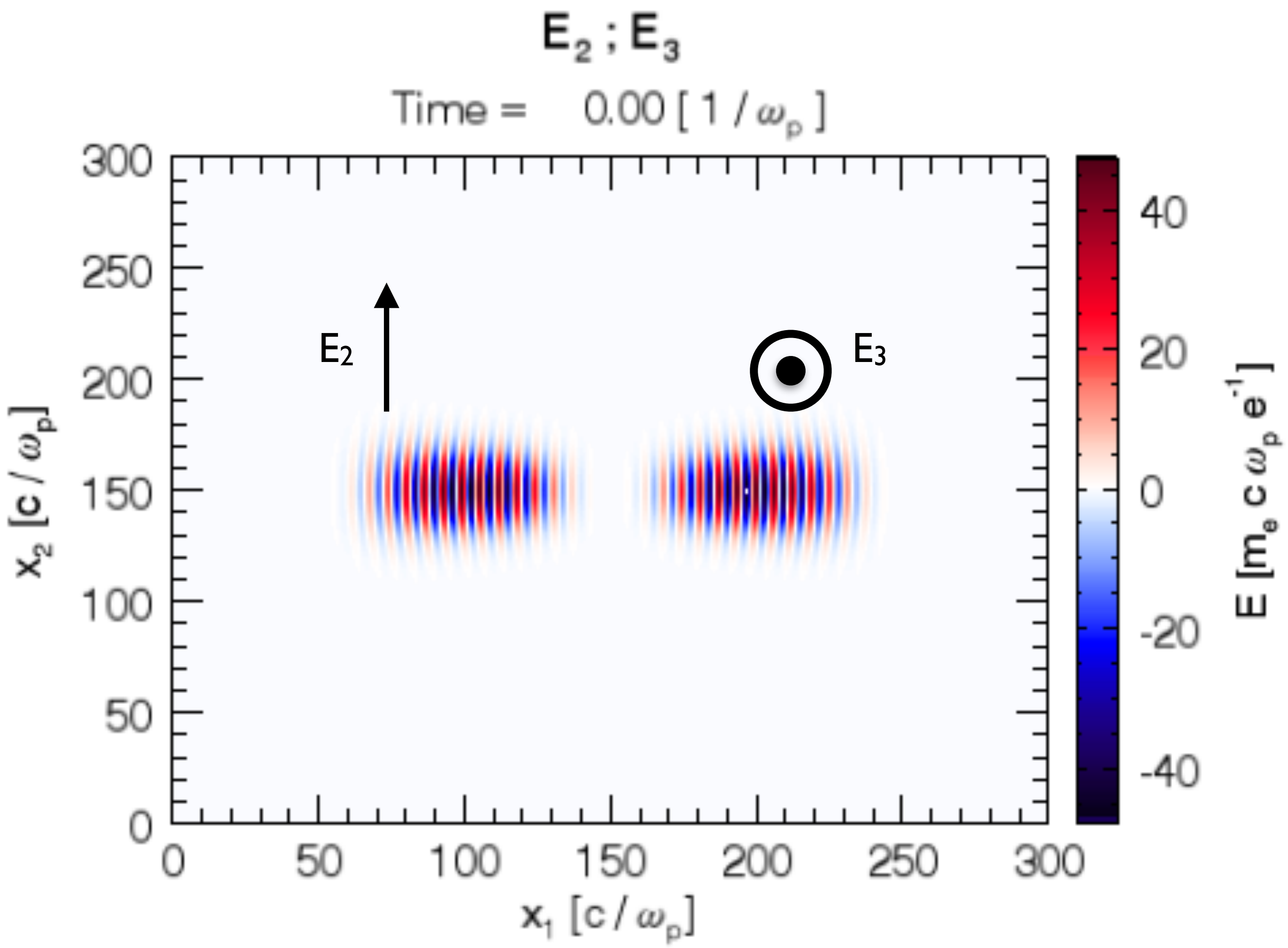
Fourier Transform of Electric field for two simulations, with and without QED corrected Maxwell's Equations.

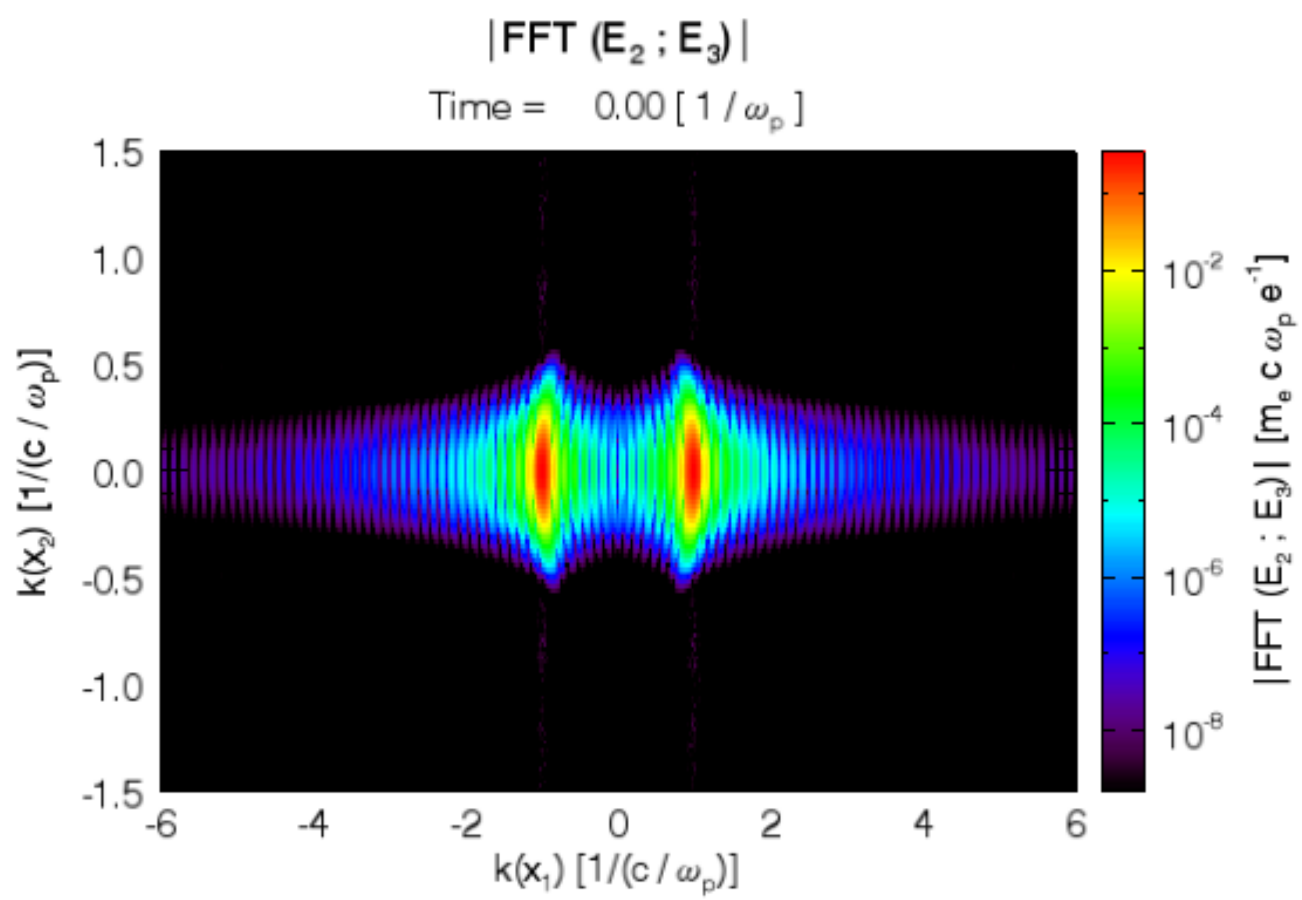
To verify theory we compared the temporal evolution of <u>subtracted</u> k = 1 FFT mode, with theoretical result.

$$\tilde{E}^{(1)}(k=1) = 4\xi E_0^3 t \sin(t) + 3\xi E_0^3 \sin(t)\sin(2t)$$



2D Results - Beams with different polarisations



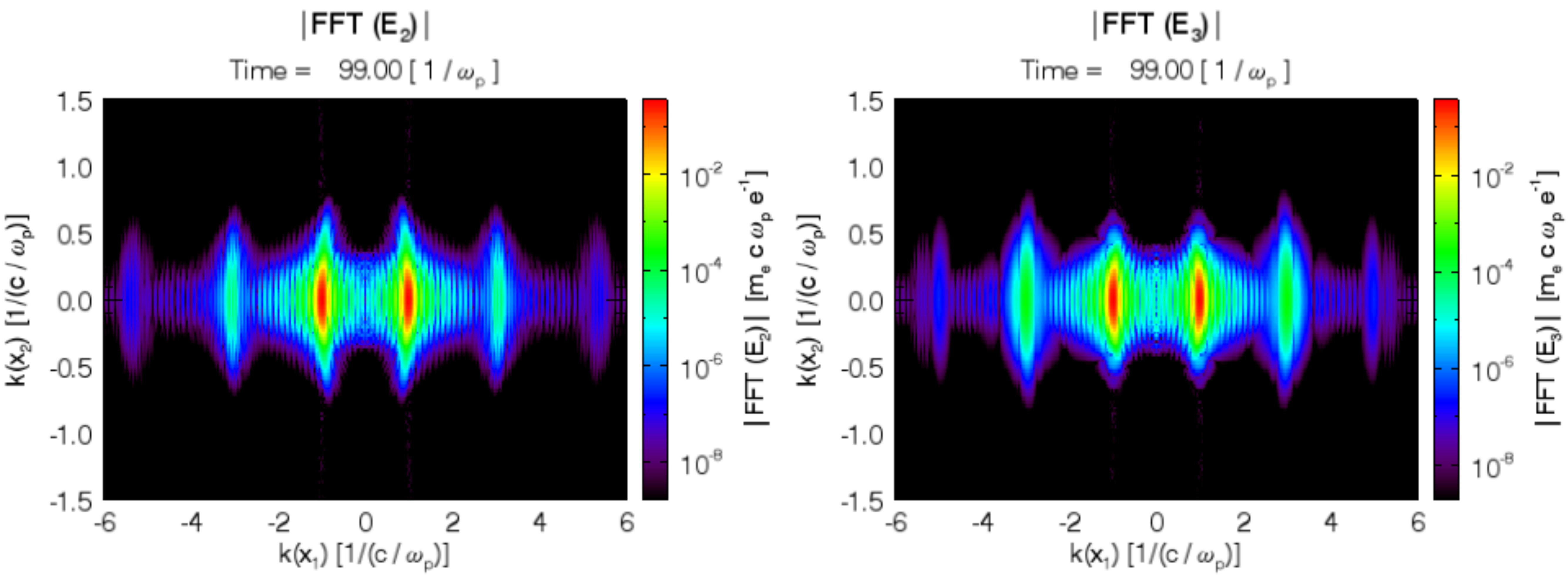


Setup with 2 counter propagating Gaussian pulses polarised in x_2 and x_3 respectively.

Fourier transform of beams before nonlinear interaction.

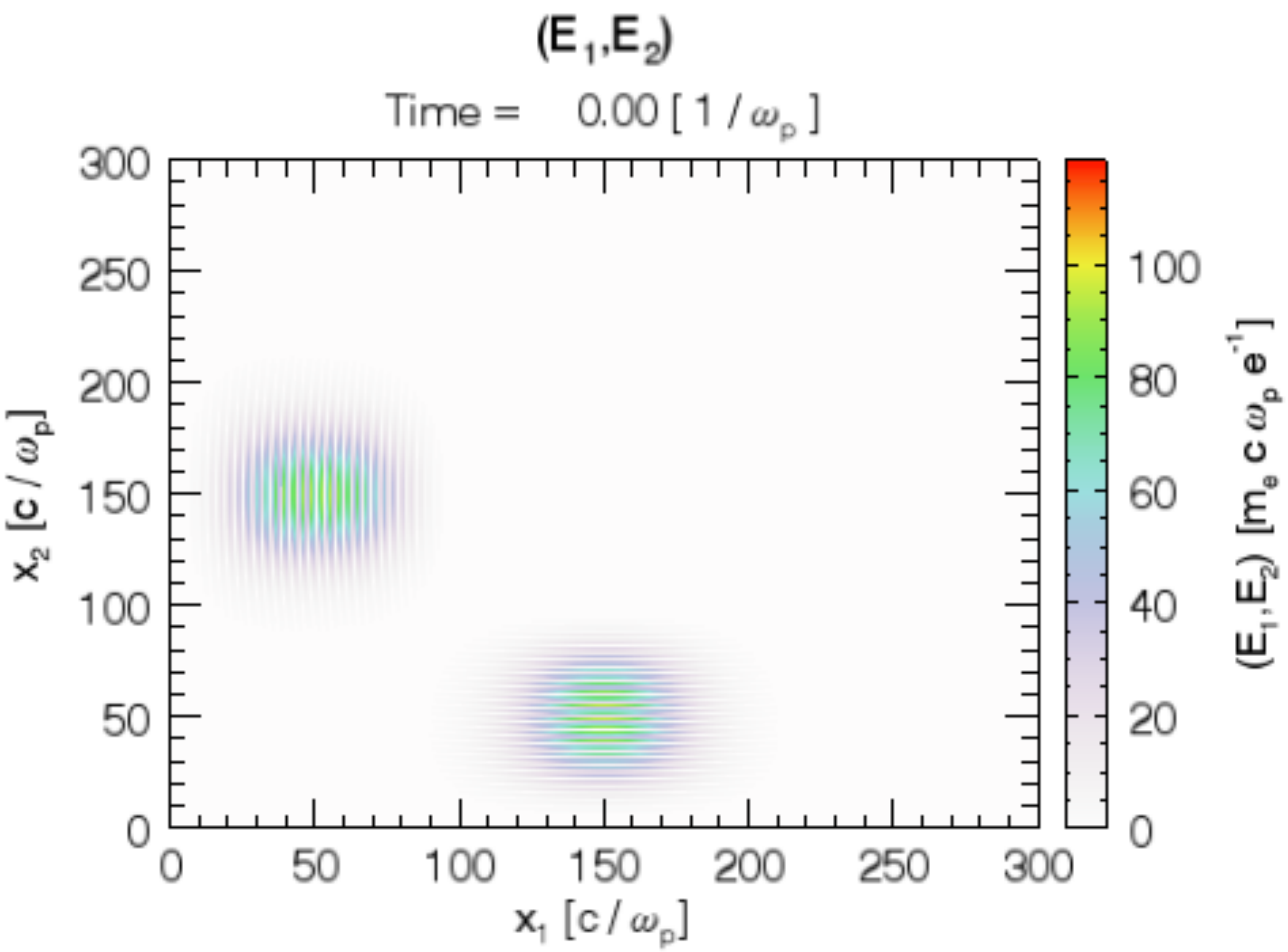
$a_0 = 50$
$\xi = 1.0 \times 10^{-7}$
$\lambda = 1.0 \times 10^{-6}$

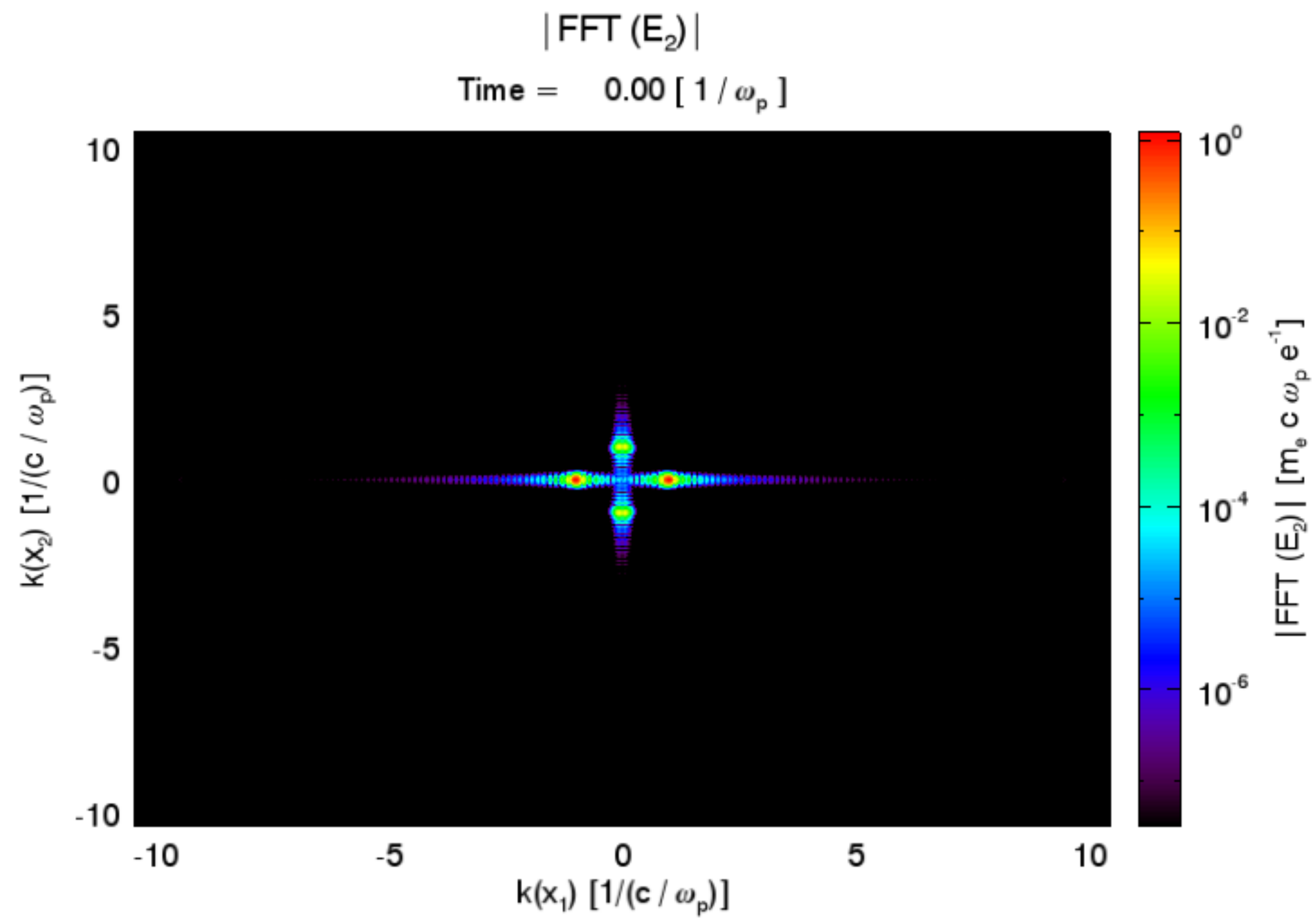
2D Results - HHG for both fields



Odd Harmonics are generated for fields in both directions!
Relative amplitude of FFT seem to agree with 1D prediction

2D Results - Perpendicular beam collision



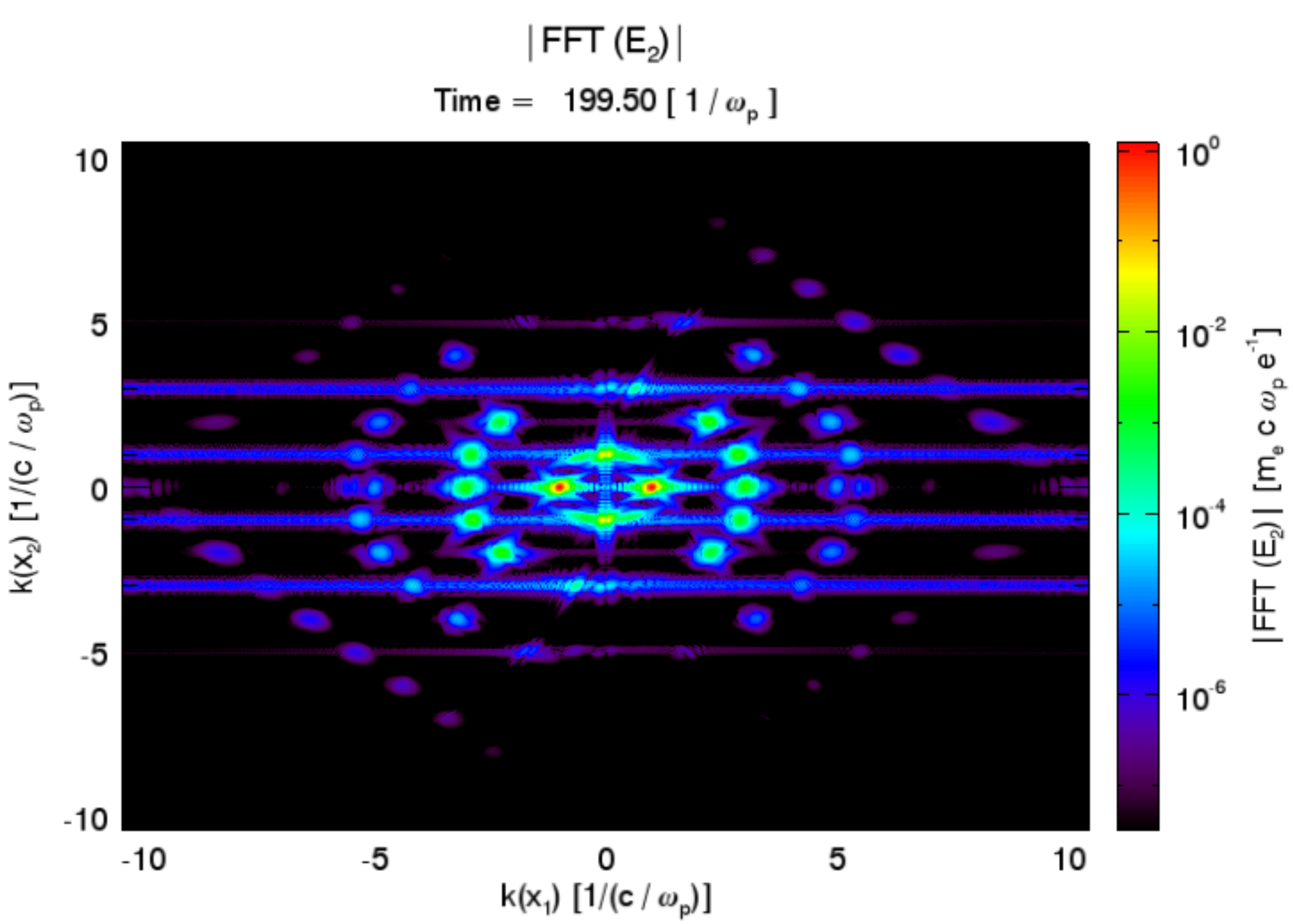


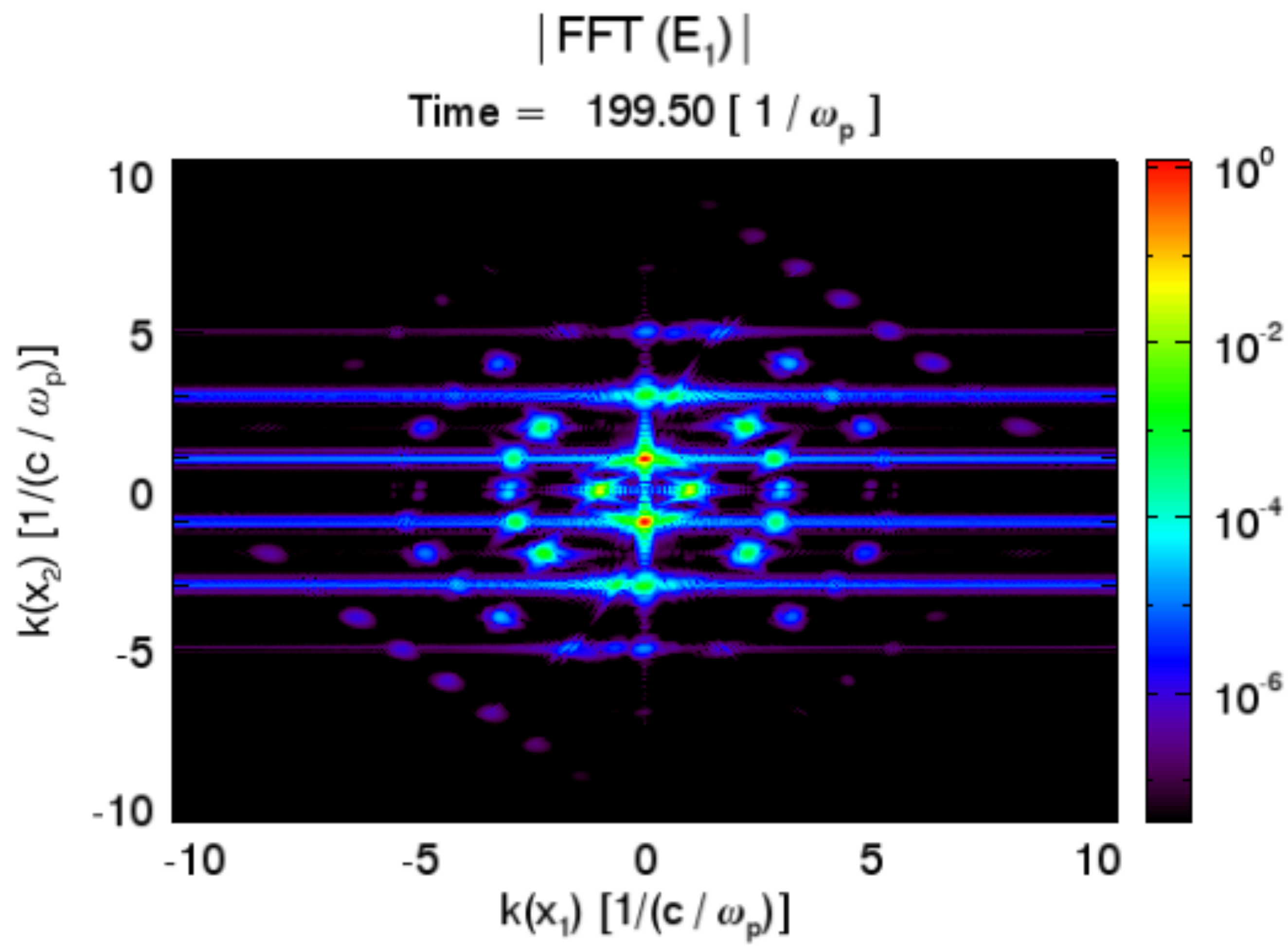
Setup with 2 Gaussian pulses propagating in perpendicular directions

Fourier transform of beam before nonlinear interaction.

$a_0 = 100$
$\xi = 1.0 \times 10^{-6}$
$\lambda = 1.0 \times 10^{-6}$ m

2D Results - HHG for perpendicular beam collision





Combination of odd and even harmonics is generated; **After interaction**, imprint is left in both pulses as they now freely propagate.

Conclusions

New multi-dimensional Maxwell QED Solver implemented in OSIRIS

◉ Modified Yee scheme includes nonlinear polarisation and magnetisation terms due to e^-e^+ fluctuations

◉ Nontrivial solver requires evaluation of EM fields at all grid points and a recursive loop to advance them in time

Simulations in excellent agreement with theoretical predictions

◉ Birefringence of vacuum verified and present in many setups.

◉ Counter propagating plane waves: results in 1D in perfect agreement with theory

◉ 2D counter propagating pulses reveals expected qualitative behaviour.

Future work

◉ Broad range of experimental 2D setups may be tested

◉ Can these corrections alter plasma dynamics in extreme environments?

◉ 3D generalisation to verify predictions available in literature.