# Semi-classical Schwinger pair production in rotating electric fields


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## Abstract


We investigate Schwinger pair production for time dependent fields with more than one component using a semiclassical saddlepoint approximation. In this framework it is possible to study rotating electric fields. We find that the momentum and spin spectra of different pulses show characteristic shapes.


## The idea

1. Reformulate the equation of motion in terms of equations for the mode functions $\alpha(t)$, $\beta(t)$.
2. Perform a multiple integral iteration to compute $|\beta(t)|^2$.
3. Calculate the integrals with a semiclassical saddle point approximation to derive the number of created pairs for each momentum mode $\vec{k}$.

## 1 Equations for $\alpha(t), \beta(t)$

We start from the Dirac equation

$$([i\hbar\partial_\mu - eA_\mu(x)]\,\gamma^\mu - mc)\,\Psi(\vec{x},t) = 0.$$

and decompose the spinor operator as

$$\Psi(x,t) \sim e^{\frac{i}{\hbar}\vec{k}\vec{x}}\psi(t).$$

For two component fields solely depending on time we now make the ansatz

$$\psi_\uparrow(t) = \begin{pmatrix} \psi_1^+(t) \\ \psi_2^+(t) \\ -\frac{ck_z+\epsilon_\perp}{mc^2}\psi_1^+(t) \\ \frac{ck_z+\epsilon_\perp}{mc^2}\psi_2^+(t) \end{pmatrix}, \psi_\downarrow(t) = \begin{pmatrix} -\frac{ck_z+\epsilon_\perp}{mc^2}\psi_1^-(t) \\ \frac{ck_z+\epsilon_\perp}{mc^2}\psi_2^-(t) \\ \psi_1^-(t) \\ \psi_2^-(t) \end{pmatrix}$$

where

$$\epsilon_\perp^2 := c^2k_z^2 + m^2c^4$$

and

$$\psi_i^\pm(t) = \alpha_\pm(t)F_{\alpha,i}^\pm(t)\frac{e^{-\frac{i}{2}K_0(t)}}{\sqrt{\omega_{\vec{k}}(t)}} + \beta_\pm(t)F_{\beta,i}^\pm(t)\frac{e^{\frac{i}{2}K_0(t)}}{\sqrt{\omega_{\vec{k}}(t)}}$$

with

$$K_0(t) := \frac{2}{\hbar}\int_0^t \omega_{\vec{k}}(t')dt',$$

and

$$\omega_{\vec{k}}(t)^2 := c^2\vec{p}(t)^2 + m^2c^4, \qquad \vec{p}(t) := \vec{k} + ieA(x).$$

Here we want to choose $F_{\alpha/\beta,i}^\pm(t)$ such that $\dot{\alpha}_\pm(t)$ is only a function of $\beta_\pm(t)$ and vice versa. One finds [1]

$$\dot{\alpha}_s(t) = \frac{\dot{\omega}_{\vec{k}}(t)}{2\omega_{\vec{k}}(t)}G_+^s(t)e^{iK_s(t)}\beta_s(t),$$

$$\dot{\beta}_s(t) = \frac{\dot{\omega}_{\vec{k}}(t)}{2\omega_{\vec{k}}(t)}G_-^s(t)e^{-iK_s(t)}\alpha_s(t),$$

where

$$K_s(t) := K_0(t) - s\epsilon_\perp\int_0^t \frac{\dot{p}_x(t')p_y(t') - \dot{p}_y(t')p_x(t')}{\omega_{\vec{k}}(t')p_\parallel(t')^2}dt',$$

$$G_\pm^s(t) = is\frac{\epsilon_\perp}{cp_\parallel(t)} \pm \frac{\dot{p}_x(t)p_y(t) - \dot{p}_y(t)p_x(t)}{\dot{p}_x(t)p_x(t) + \dot{p}_y(t)p_y(t)}\frac{\omega_{\vec{k}}(t)}{cp_\parallel(t)}.$$

## Results

- good agreement with numerical DHW-results
- numerical method: complementary to DHW-method
- approximate method: faster than numerical ones

For two component fields:

- dependence on spin
- interference effects in total particle number

## 2 Multiple Integral Iteration

The transmission probability

$$W^s(\vec{k}) := \lim_{t\to\infty}|\beta_s(t)|^2,$$

can be interpreted as the number of produced electron-positron pairs as a function of the momentum $\vec{k}$. Posing appropriate boundary conditions, i.e. $\beta_s(-\infty) = 0$ and $\alpha_s(-\infty) = 1$, we find

$$\beta^s(\infty) = \sum_{m=0}^{\infty}\int_{-\infty}^{\infty}dt_0G_-(t_0)\frac{\dot{\omega}_{\vec{k}}(t_0)}{2\omega_{\vec{k}}(t_0)}e^{-iK_s(t_0)}\prod_{n=1}^{m}\int_{-\infty}^{t_{n-1}}d\tau_nG_+(\tau_n)\frac{\dot{\omega}_{\vec{k}}(\tau_n)}{2\omega_{\vec{k}}(\tau_n)}e^{iK_s(\tau_n)}\int_{-\infty}^{\tau_n}dt_nG_-(t_n)\frac{\dot{\omega}_{\vec{k}}(t_n)}{2\omega_{\vec{k}}(t_n)}e^{-iK_s(t_n)}$$

## 3 Saddlepoint approximation

Using the fact that the integrals are dominated by regions around the classical turning points

$$\omega_{\vec{k}}(t_p^\pm) := 0$$

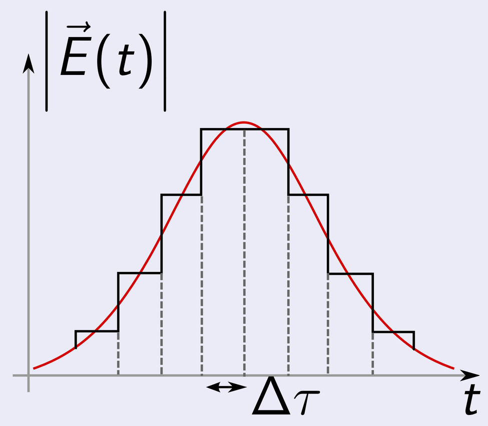
and assuming that we only have simple turning points, one can use a semiclassical saddlepoint approximation to find the momentum spectrum and total pair creation rate [1, 2]

$$W_{SC}^s(\vec{k}) = \left|\sum_{t_p}e^{-iK_s(t_p)}\right|^2,$$

$$\frac{\Gamma^s}{V} = \int\frac{d^3k}{(2\pi\hbar)^3}W_{SC}^s(\vec{k}).$$

## Approximative method

We can use the fact that there is an analytic solution for the rectangular pulse to approximate every rotating pulse like this:




Approximation for $\Delta\tau \to 0$

$$W_{SC}^{\approx}(\vec{k}) = W_{SC}^{\text{const}}(\vec{k}, E(t_p(\vec{k})))$$

- $W_{SC}^{\text{const}}(\vec{k}, E_0)$: $W_{SC}(\vec{k})$ of constant rotating field $E_0$
- $E(t) = |\vec{E}(t)|$: form of the pulse
- $t_p(\vec{k})$: turning points of constant rotating field

## Rectangular Pulse

Since a constant electric field would create an infinite amount of pairs we look at the rectangular pulse

$$\vec{E} = E_0\,\text{Rect}\,(t/\tau)\,(\cos(\Omega t), -g\sin(\Omega t), 0), \qquad \text{Rect}(x) = \Theta(x) - \Theta(x-1).$$

We can find analytical results for $K_\pm(t_p)$ [1]:

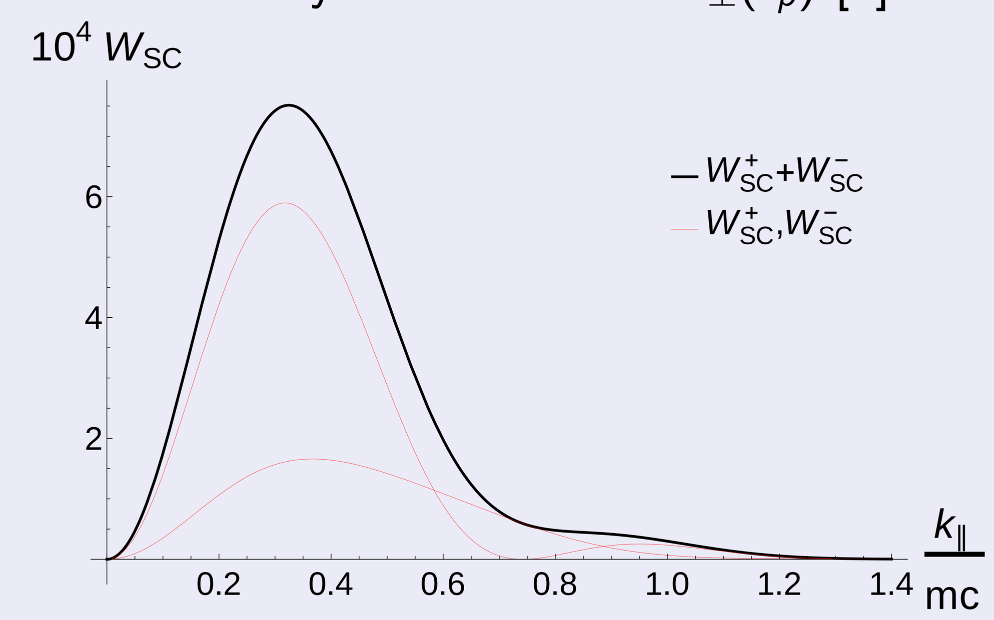



Figure : Momentum spectrum of pairs produced for $E = 0.1E_c$, $\tau = 4\pi\lambda_c/c$, $\Omega\tau = 4\pi$ and $k_z = 0$. The two different spin states are plotted as a thin red line.

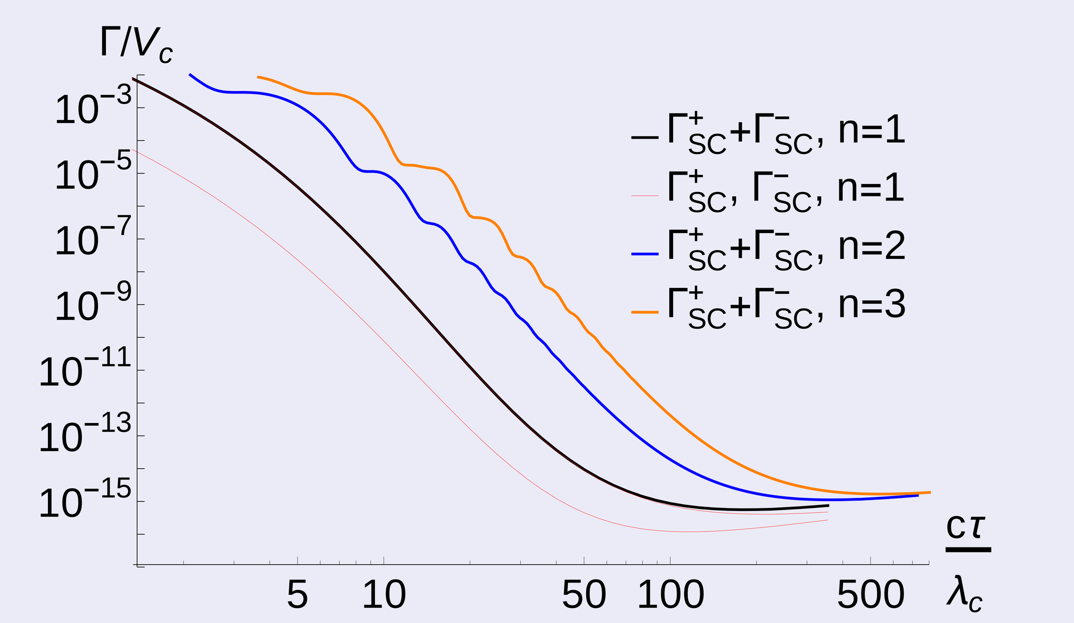



Figure : Total particle number per Compton volume $V_c$ for $E = 0.1E_c$ and $\Omega\tau = 2\pi\,n$. For $n = 1$ we also plotted the pair creation in the respective spin states.

## Comparison to DHW-results: Sauter Pulse

We can also look at the rotating Sauter Pulse.

$$\vec{E}(t) = \frac{E_0}{\cosh^2\left(\frac{t}{\tau}\right)}(\cos(\Omega t), \sin(\Omega t), 0)$$

Compare to the DHW method [3, 4]:

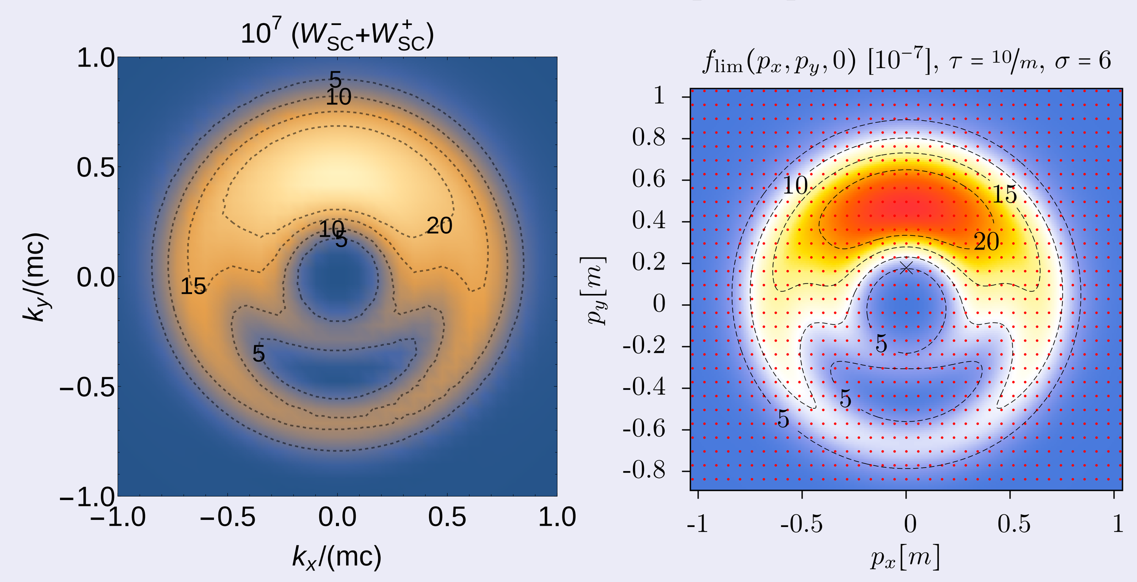



Figure : Momentum spectrum for $E = 0.1E_c$, $\tau = 4\pi\lambda_c/c$, $\sigma = \Omega\tau = 6$ and $k_z = 0$.The semiclassical result (left) agrees with the one of the DHW method of [3] (right).

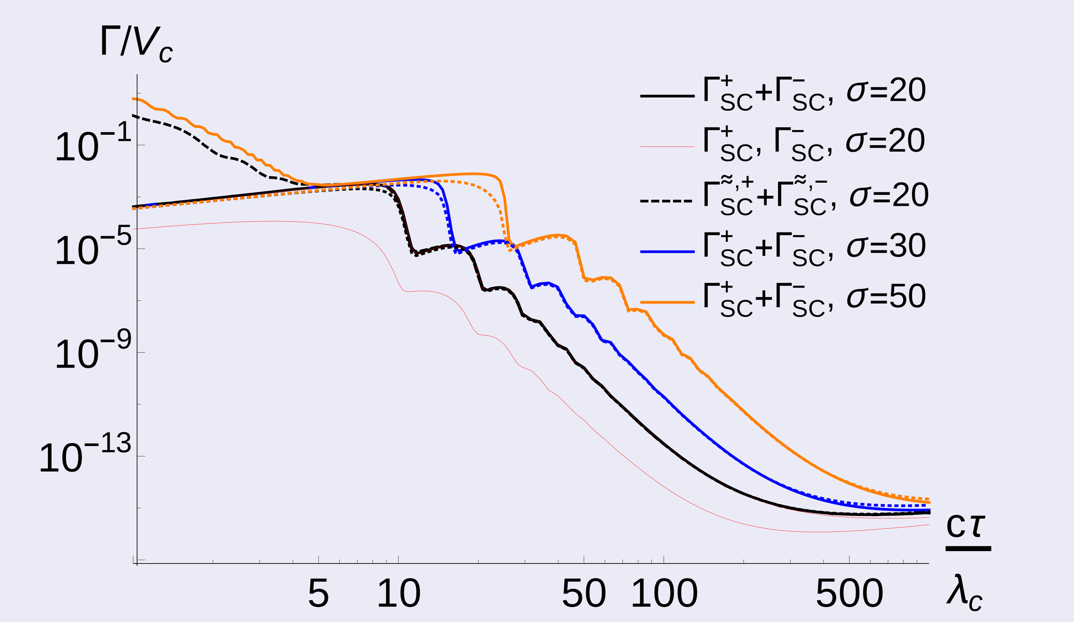



Figure : Total particle number for $E = 0.1E_c$ and different $\sigma = \Omega\tau$. We compare to the approximative (dashed) and DHW-result (dotted).

## References and Acknowledgements


[1] E. Strobel and S.-S. Xue, Phys. Rev. D **91**, 045016 (2015).

[2] M. Berry, J. Phys. A **15**, 3693 (1982).

[3] A. Blinne and H. Gies, Phys. Rev. D **89**, 085001 (2014).

[4] A. Blinne and E. Strobel, *in preparation*.



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