Spin evolution in classical versus quantum laser pulses

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Justification of the single-mode approximation for a finite-duration laser pulse interacting with an electron

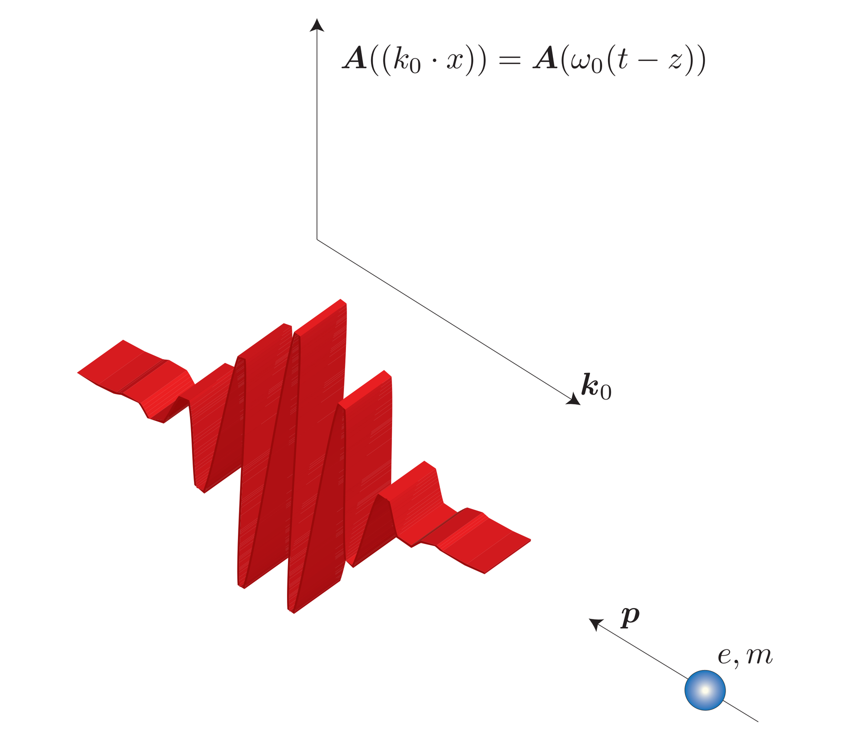

Fig. 1: Collision of an electron with a plane-wave laser pulse

The equation for the state vector of a relativistic electron interacting with a multi-mode transversal quantized laser pulse reads ($\hbar = c = 1$):

$$i\frac{\partial \Psi}{\partial t} = \left(\sum_{\boldsymbol{k}} \omega_{\boldsymbol{k}} a^\dagger_{\boldsymbol{k}} a_{\boldsymbol{k}} + \boldsymbol{\alpha}\cdot(\boldsymbol{p} - e\boldsymbol{A}) + \beta m\right)\Psi, \tag{1}$$

with the vector potential

$$\boldsymbol{A} = \sum_{\boldsymbol{k}} \frac{\boldsymbol{e}(\boldsymbol{k})}{\sqrt{2\omega_{\boldsymbol{k}} V}}\left(a_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + a^\dagger_{\boldsymbol{k}} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\right). \tag{2}$$

Non-monochromaticity parameters

$$\sigma_2 = \frac{\delta\omega}{\omega_0} \approx \frac{1}{\omega_0 \tau}, \quad \sigma_1 = \frac{\delta \boldsymbol{k}_0}{k_0^2} \approx \frac{1}{\omega_0^2 S}. \tag{3}$$

The canonical transformations of the field variables

$$Q = \sum_{\boldsymbol{k}<\Delta} q_{\boldsymbol{k}}, \quad y_{\boldsymbol{k}} = q_{\boldsymbol{k}} - \frac{1}{N}Q,$$

$$q_{\boldsymbol{k}} = y_{\boldsymbol{k}} + \frac{1}{N}Q, \ \sum_{\boldsymbol{k}<\Delta} y_{\boldsymbol{k}} = 0, \quad N = \sum_{\boldsymbol{k}<\Delta} 1,$$

and

$$p_{\boldsymbol{k}} = P + p_{y\boldsymbol{k}}, \quad \sum_{\Delta k} p_{yk} = 0, \quad P = -i\frac{\partial}{\partial Q},$$

$$p_{yk} = -i\frac{\partial}{\partial y_{\boldsymbol{k}}} + \frac{i}{N}\sum_{\Delta l}\frac{\partial}{\partial y_l}.$$

lead to the representation of the Hamiltonian as the sum

$$H \equiv H_A + H_1 + H_2,$$

where H_A and $H_{1,2}$ describe the single-collective mode and fluctuations relative to the collective mode, respectively. We build a perturbation theory over $H_{1,2}$ and find the equation for the energy of the system

$$(\hat{q}^{(0)} - m - H_A)|\Psi^{(0)}\rangle = 0,$$
$$\hat{q}^{(1)}|\Psi^{(0)}\rangle + (\hat{q}^{(0)} - m - H_A)|\Psi^{(1)}\rangle = (H_1 + H_2)|\Psi^{(0)}\rangle.$$

This allows us to find dimensionless parameters which determine the applicability of the single-mode approximation ($\lambda \sim \sigma_2/\sigma_1^2$)

$$\frac{\Delta\omega_0}{\omega_0} = \frac{\pi v_\perp^2}{4}\left[\lambda \tan^{-1} 4\lambda - \frac{1}{4}\right] \ll 1, \tag{4}$$

$$\frac{\Gamma}{\omega_0} = \frac{\pi^2 v_\perp^2}{4}\cdot\begin{cases}\lambda, & \lambda > 1\\ \frac{1}{\lambda}, & \lambda < 1\end{cases} \ll 1, \tag{5}$$

$$\mu = e^2 \frac{\delta^5 \sigma_1^2 \sigma_2 \omega_0 \left(\sigma_2^2\left(v^2 - v_z^2\right)^2 + 16\sigma_1^2 v_\perp^2 v_z^2\right)}{96(2\pi)^3 v^2 I S \tau \Phi^3(\delta)} \ll 1. \tag{6}$$

[1] O. D. Skoromnik and I. D. Feranchuk, J. Phys. B: At. Mol. Opt. Phys. **47**, 115601 (2014)

Collapse-revival dynamics of strongly laser-driven electrons

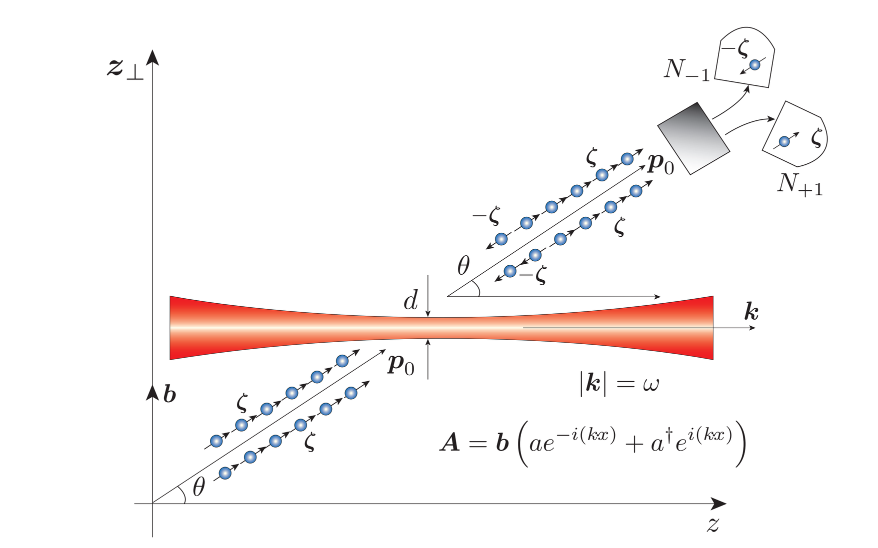

Fig. 2: Interaction of electrons with a single-mode quantized field

We analyze the influence of quantum effects coming from a laser field on an electron spin four-vector:

$$s^\mu(\boldsymbol{x}, t) = \frac{\langle\psi|\gamma^0\gamma^5\gamma^\mu\delta(\boldsymbol{x} - \boldsymbol{r}')|\psi\rangle}{\langle\psi|\psi\rangle}, \tag{7}$$

where $|\psi\rangle$ is the solution of the Dirac equation in a single-mode quantized field. We consider that at the initial time the electron is free and the field is in a coherent state.

$$\begin{aligned}\langle s^\mu(t)\rangle = {} & \frac{m}{\varepsilon_0}a_0^\mu - \frac{m}{\varepsilon_0}k^\mu(a_0\cdot k)\frac{\beta^2 b^2}{(p_0\cdot k)^2}(1 + \operatorname{Re}\Pi_2)\\ & + \left[\frac{m}{\varepsilon_0}\frac{\beta}{(p_0\cdot k)}\left(k^\mu(a_0\cdot b) - b^\mu(a_0\cdot k)\right)\right]2\operatorname{Re}\Pi_1,\end{aligned} \tag{8}$$

$$\Pi_l = (-i)^l e^{i\omega l t\left(1 - \frac{p_{0z}}{\varepsilon_0}\right)} J_l\left(4\alpha\beta \sin\frac{\omega^2 l t}{2\varepsilon_0}\right), \quad l = 1, 2,$$

where $\varepsilon_0 = \sqrt{\boldsymbol{p}_0^2 + m^2}$, a_0 is the initial four-vector of the electron spin, $p_0 = (\varepsilon_0, \boldsymbol{p}_0)$, and J_l the Bessel function of order l.

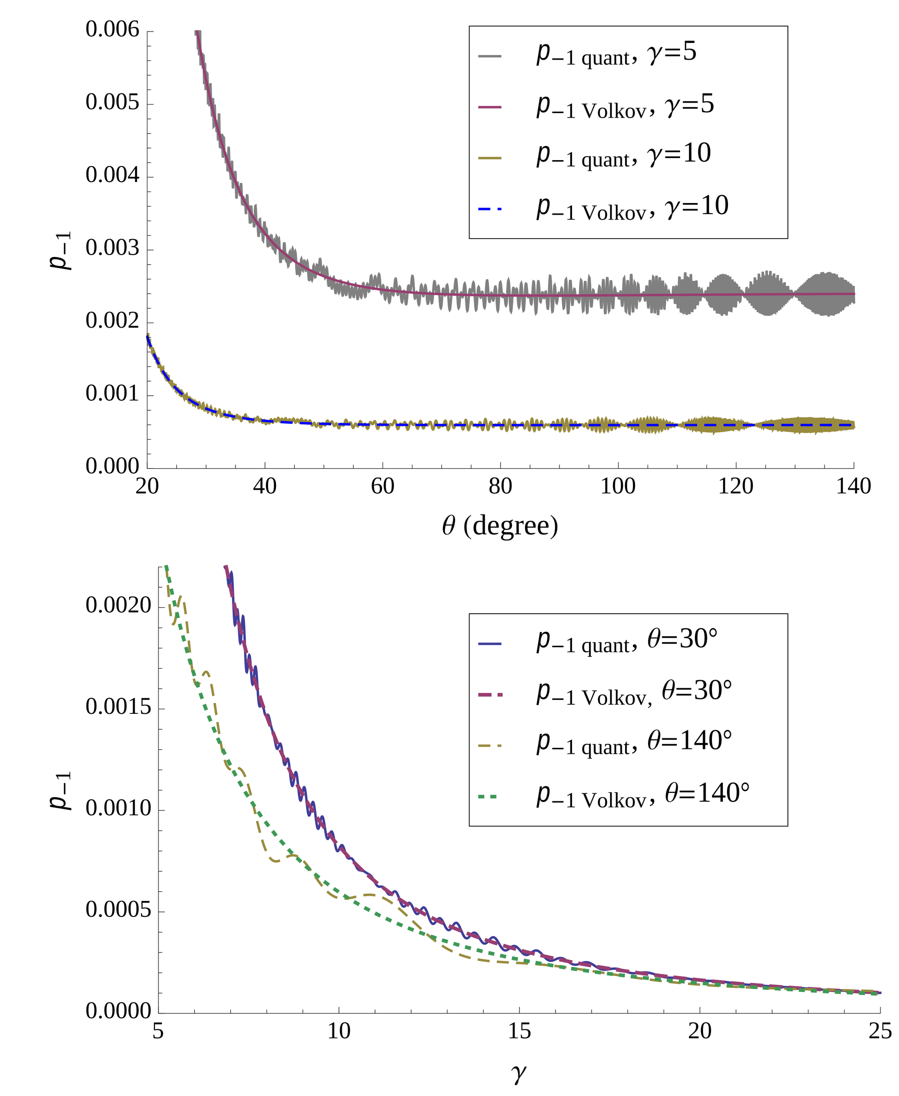

Fig. 3: Probability to find an electron with an oppositely directed spin

[2] O. D. Skoromnik, I. D. Feranchuk and C. H. Keitel, Phys. Rev. A **87**, 052107 (2013)

Spin-dependent Compton scattering in a strong and short laser pulse

The field strength is characterized through the invariant parameter

$$\xi = \frac{|e|A}{m}. \tag{9}$$

When $\xi \gg 1$ all processes include many photons. The second parameter

$$\chi = \frac{|e|\sqrt{-F_{\mu\nu}F^{\mu\delta}p^\nu p_\delta}}{m^3}, \tag{10}$$

contains the Planck's constant $\hbar$ and consequently determines the importance of quantum effects.

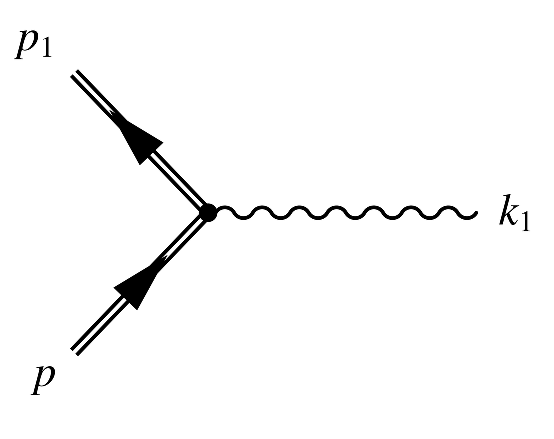

Fig. 4: Feynman diagram of the Compton effect in a strong laser field

The scattering amplitude in the Furry picture reads:

$$S_{fi} = -ie\int \bar{\psi}_{\mu_1 p_1}(x)\frac{\hat{e}^* e^{i(k_1\cdot x)}}{\sqrt{2V\omega_1}}\psi_{\mu p}(x) d^4 x,$$

where $\psi_{\mu p}$ are Volkov solutions of the Dirac equation.
In our study we consider circularly polarized strong and short laser pulses, such that an electric field is equal to

$$\boldsymbol{E}(x) = \frac{m\xi\omega}{e}f(\phi)(\boldsymbol{e}_x\cos\phi + \boldsymbol{e}_y\sin\phi),$$

where $\phi = (k\cdot x)$ is the field phase and $f(\phi)$ is the envelope function.
Since there is a preferable direction in this problem, namely the wave vector $\boldsymbol{k}$ of the external field, the conservation of momentum is different from the free case

$$|S_{fi}|^2 \sim \delta^{(\perp)}(\boldsymbol{p}^\perp - \boldsymbol{k}_1^\perp - \boldsymbol{p}_1^\perp)\delta(p^- - k_1^- - p_1^-) \tag{11}$$

$\gamma = 2\cdot 10^5$	$\gamma = 2\cdot 10^5$
$\xi = 5$	$\xi = 5$
$\sigma = 7$	$\sigma = 1$
$\chi = 3.92$	$\chi = 3.92$

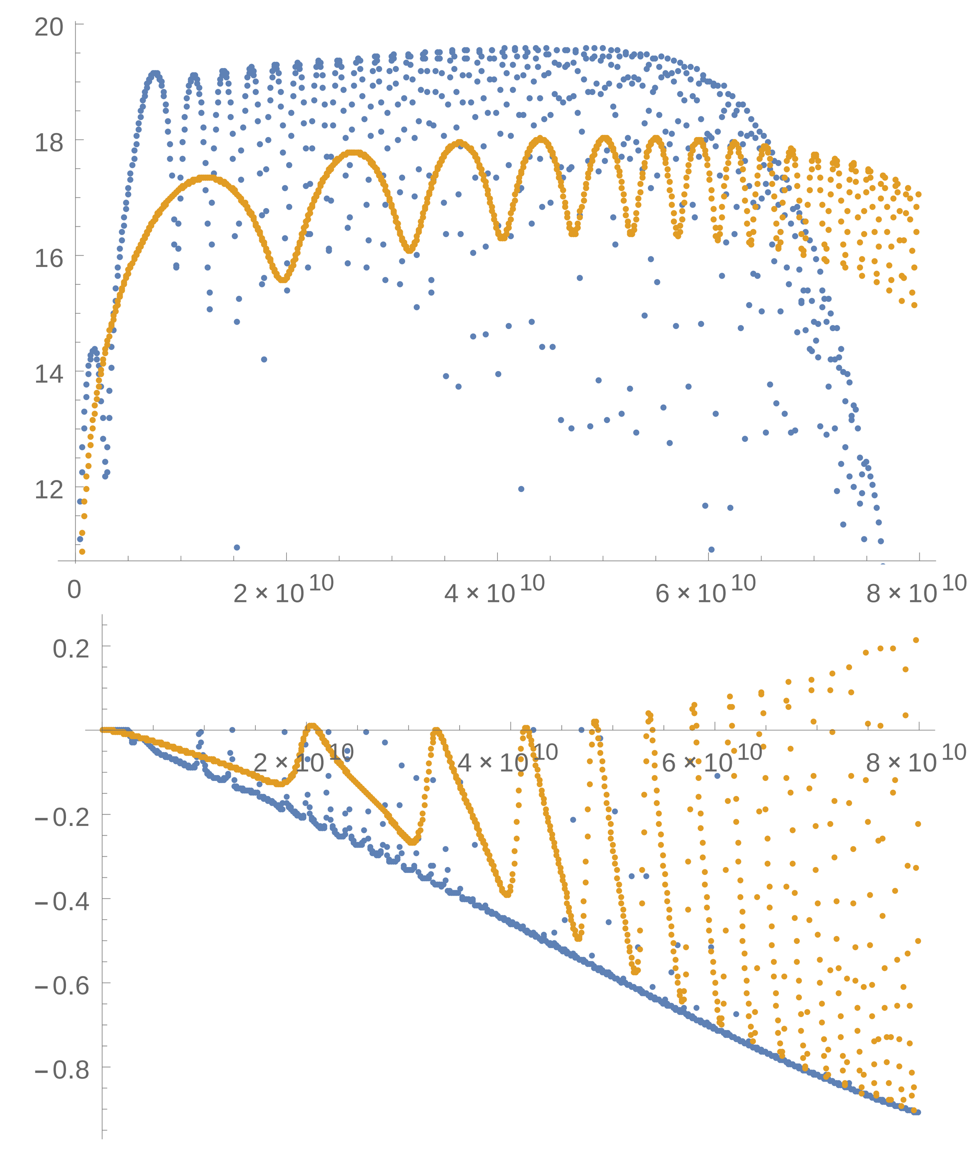

Fig. 5: Photon emission spectrum and asymmetry of scattering in a fully quantum regime as a function of emitted photon frequency and pulse length

[3] O. D. Skoromnik, K. Z. Hatsagortsyan, A. Di Piazza and C. H. Keitel, In preparation