Modelling radiation emission in the transition from the classical to the quantum regime

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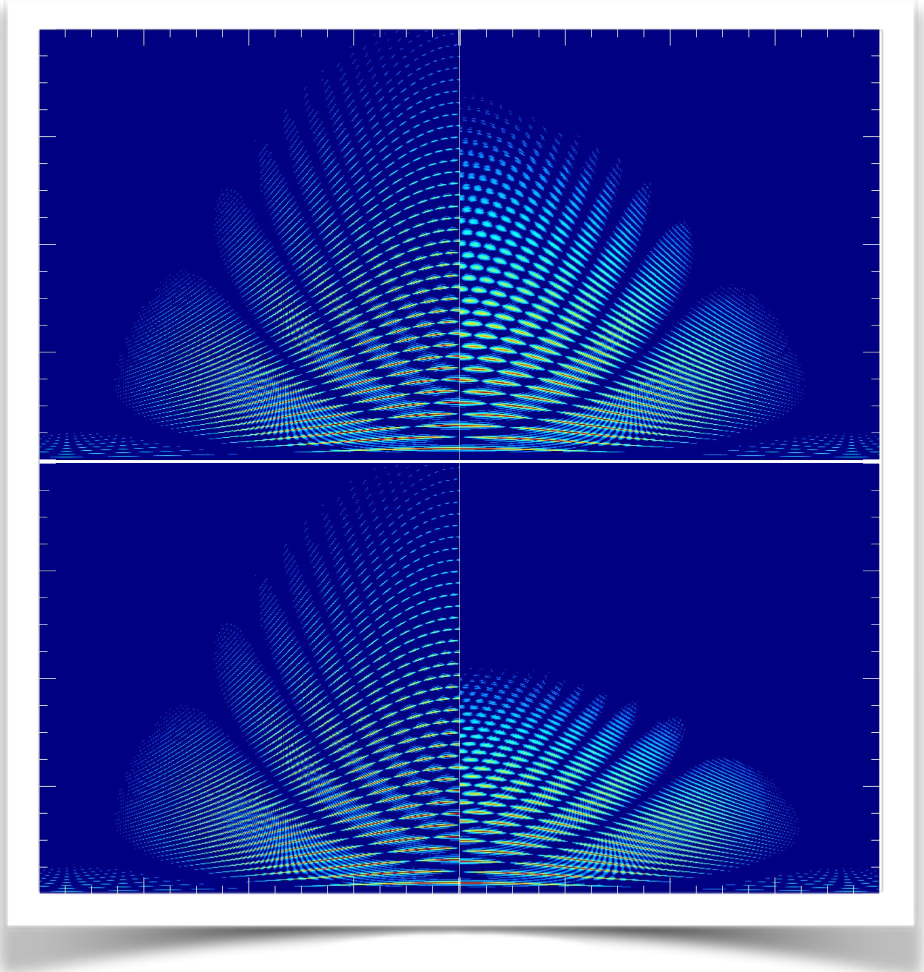
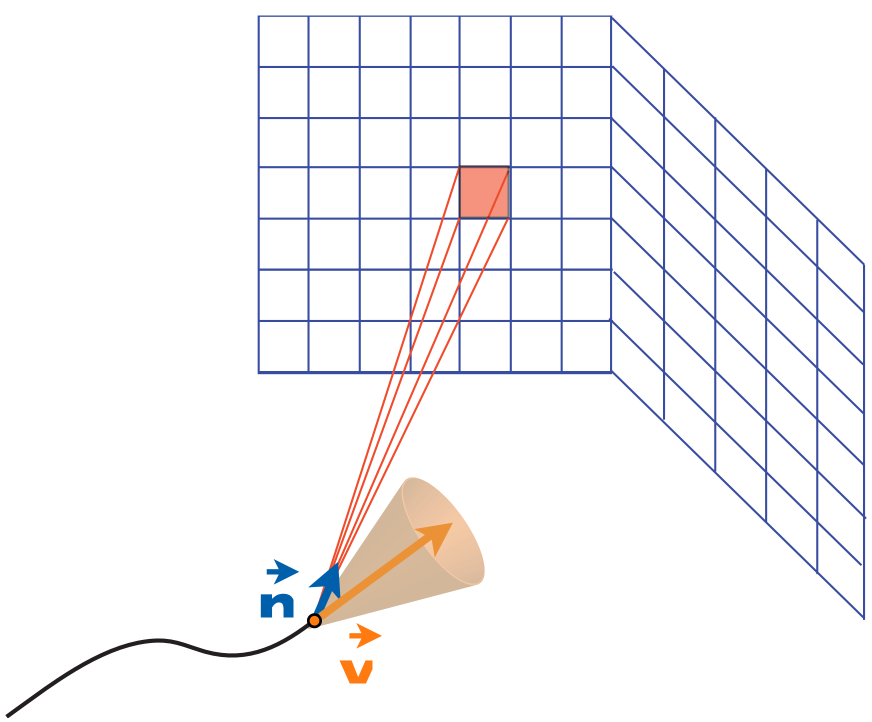


Compton scattering not included in classical treatment of radiation

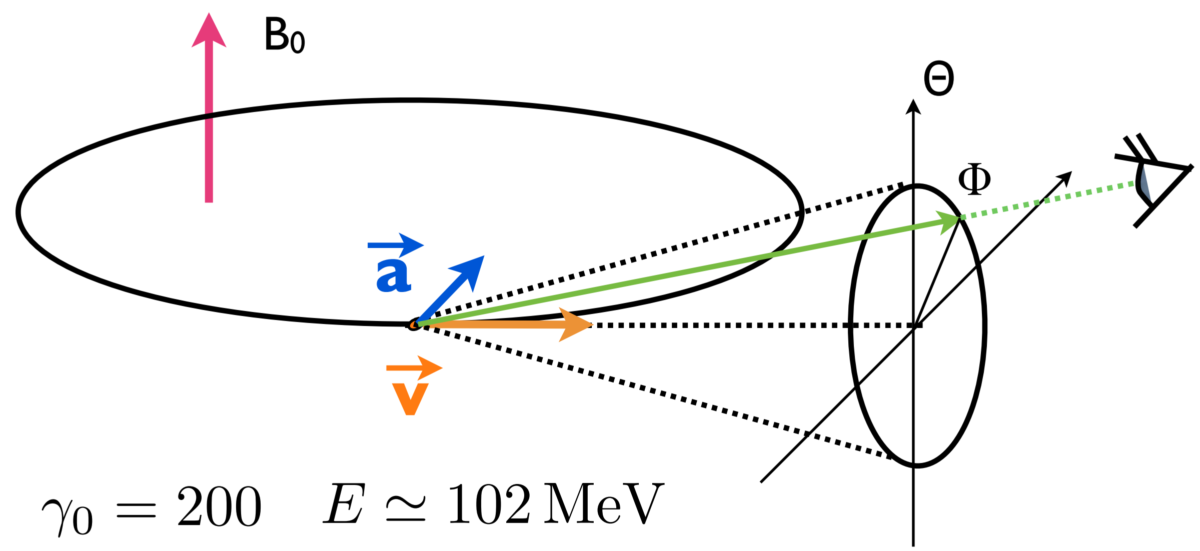
How to model radiation scattering from PIC simulations?

- traditionally: calculate spectrum from Liénard-Wiechert potentials using particle trajectories from simulation (post-processing)...



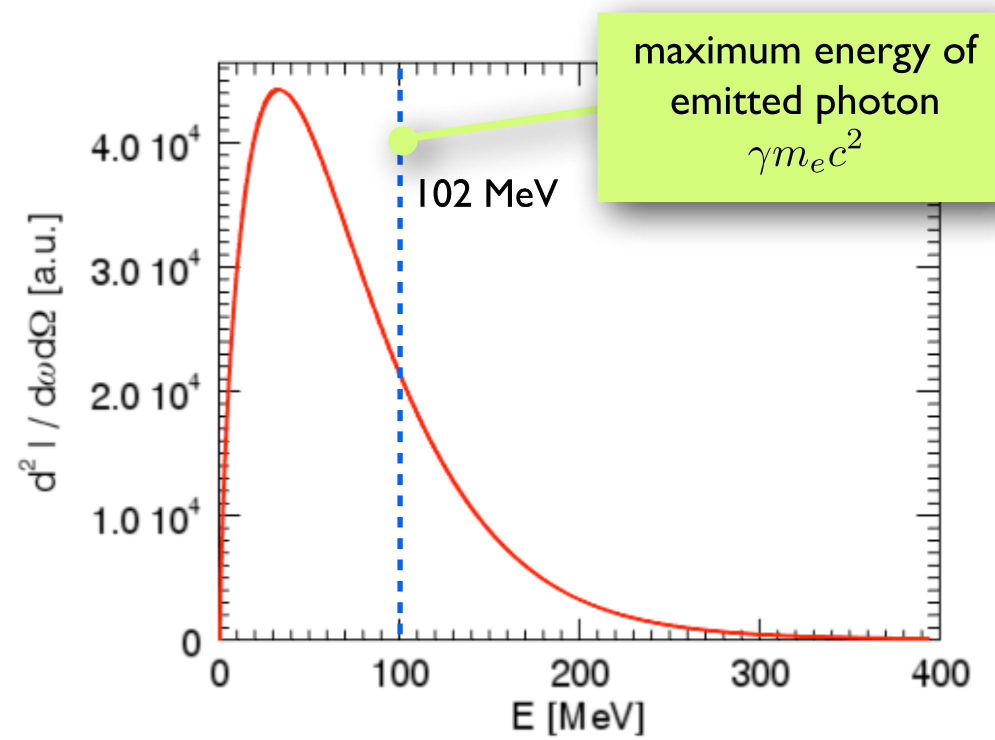
$$\frac{d^2 I}{d\omega dS} = \frac{e^2}{4\pi c}\left| \int_{-\infty}^{-\infty} \frac{\vec{n}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{(1-\vec{n}\cdot\vec{\beta})^2 R(t')^2} e^{i\omega(t'+R(t')/c)} dt \right|^2$$

Synchrotron



$\gamma_0 = 200 \quad E \simeq 102\,\mathrm{MeV}$

$\chi = \gamma B / B_C \simeq 0.26$



Nonlinear Thomson/Compton scattering models

Semiclassical calculations using Volkov solutions

- The electron interacts with an external electromagnetic field described classically.
- Transition amplitudes for nonlinear Compton scattering are determined in terms of Volkov solutions.

Narozhnyi & Fofanov, JETP **83**, 14 (1996)
M. Boca & V. Florescu, PRA **80** 053403 (2009)
D. Seipt & B. Kampfer, PRA **83**, 022101 (2011)
C. Harvey *et al*, Phys. Rev. A **79**, 063407 (2009)

Linear and weakly nonlinear Compton scattering

- Calculations in terms of Klein-Nishina differential scattering cross section and Compton formula. Monte-Carlo methods can be used to assess the occurrence of scattering.[1]
- Weakly nonlinear analytical theory including both nonlinear effects and recoil.[2]

[1]F.V. Hartemann, Nucl. Instr. Meth. Phys. Res. A **608**, S1–S6 (2009)
M. Chen *et al*, Phys. Rev. ST Accel. Beams **16**, 030701 (2013)

[2]F. Albert *et al*, Phys. Plasmas **18**, 013108 (2011)

Emission in QED-strong fields

- The electron motion in strong fields is considered as a sequence of intervals where trajectory is classical and the wave functions are described in a self-contained way in terms of the local quantities (as opposed to using Volkov states).
- Transition between states of the e- and photon emission are determined using QED perturbation theory.

I.V. Sokolov, *et al*, Phys. Rev. E **81**, 036412 (2010)
A. Di Piazza *et al*, PRL **105**, 220403 (2010)
A. Di Piazza *et al*, PRA **83**, 032106 (2011)

g($\mathscr{X}$e) factor

- A function that ensures the radiated photon energy cannot exceed that of the emitting electron is introduced in the radiation emission calculation and in the equation of motion.

A.G.R. Thomas *et al*, PRX, **2**, 041004 (2012)
J. Kirk *et al*, Plasma Phys. Control. Fusion, **51**, 085008 (2009)

Nonlinear Thomson scattering using classical emissivity + radiation damping:

J. Koga *et al*, PoP **12**, 093106 (2005); F.V. Hartemann & A. K. Kerman, PRL 76, 624 (1996)
C. H. Keitel *et al,* J. Phys. B **31,** L75 (1998); Y. Cang *et al,* PoP **13**, 113106 (2006)

Review (including radiation damping and nonlinear Thomson/Compton scattering):

A. Di Piazza *et al*, Rev. mod. Phys **84**, 1177 (2012)

QED Compton scattering modeled in PIC codes

QED algorithm in OSIRIS

- QED probabilistic approach: particle pusher + Monte Carlo module

  - every Δt : probability of photon emission

$$\boxed{\frac{d^2P}{dt d\chi_\gamma}}$$

A.I. Nikishov & V.I. Ritus (1967)
N.P. Kelpikov (1954)
V.N. Baier & V.M. Katkov (1967)

  - Select a photon in QED synchrotron spectrum

  - Update particle momentum due to quantum recoil

$$\boxed{\frac{d\boldsymbol{p}}{dt} = \boldsymbol{F_L} + \frac{d^2P}{dt d\chi_\gamma}}$$

- The QED approach can be generalised to any external EM fields under the conditions of: quasi-static fields and weak fields

OSIRIS-QED T. Grismayer *et al*, Bull. of the APS **57**, (2012)

Particle merging M. Vranic *et al*, Comp. Phys. Comm. **191**, p. 65 (2015)

Other PIC codes with QED:

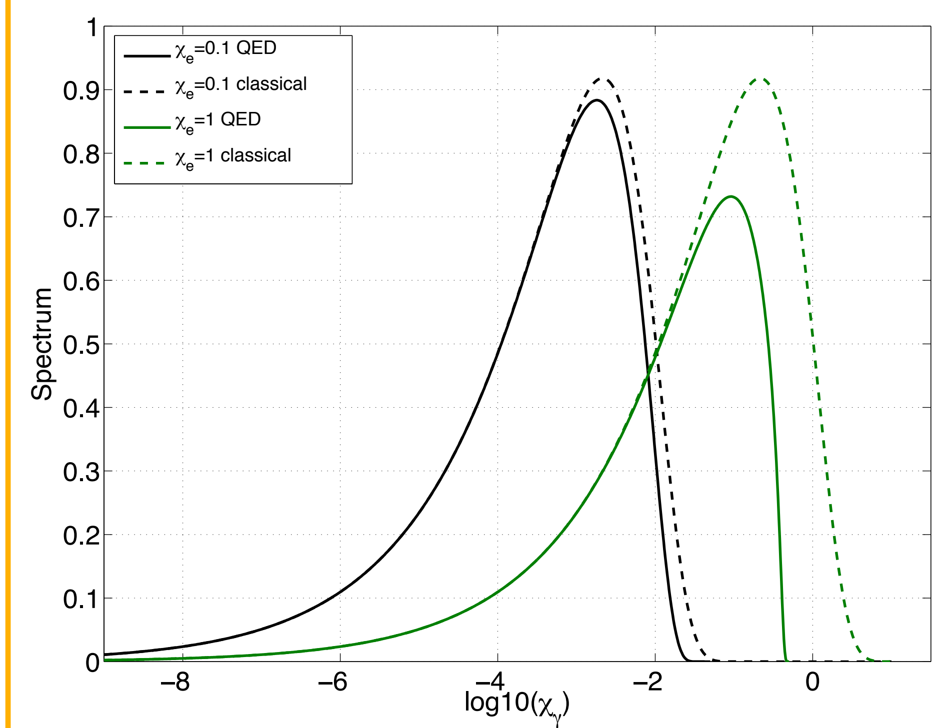
M. Lobet *et al*, ArXiv 1311.1107v2

C. Ridgers *et al*, J. of Comp. Physics, **260**, 273–285 (2012)

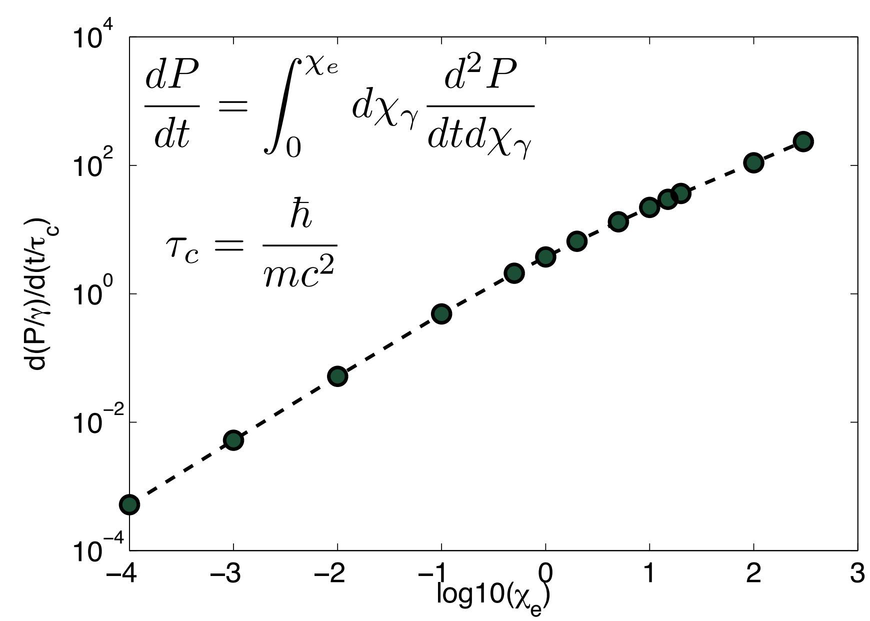
A. N. Timokhin *et al*, MNRA, **408**, 2092 (2010)

E. N. Nerush *et al*, Phys. Rev. Lett. **106**, 035001 (2011)

Synchrotron radiation spectrum

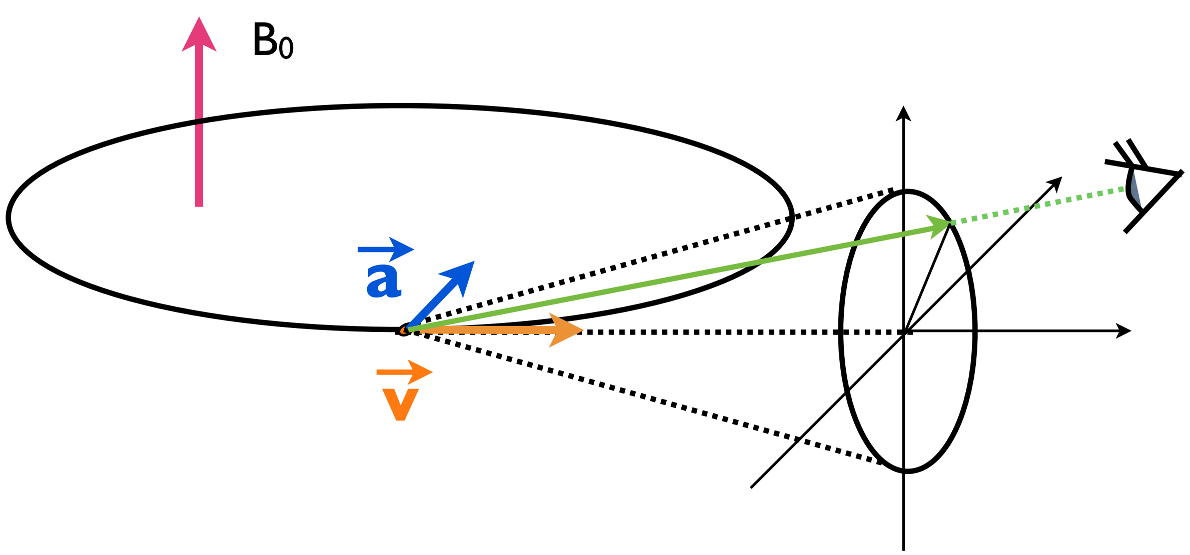


Emission rate



Generalisation of method of virtual photons in synchrotron

Lieu & Axford



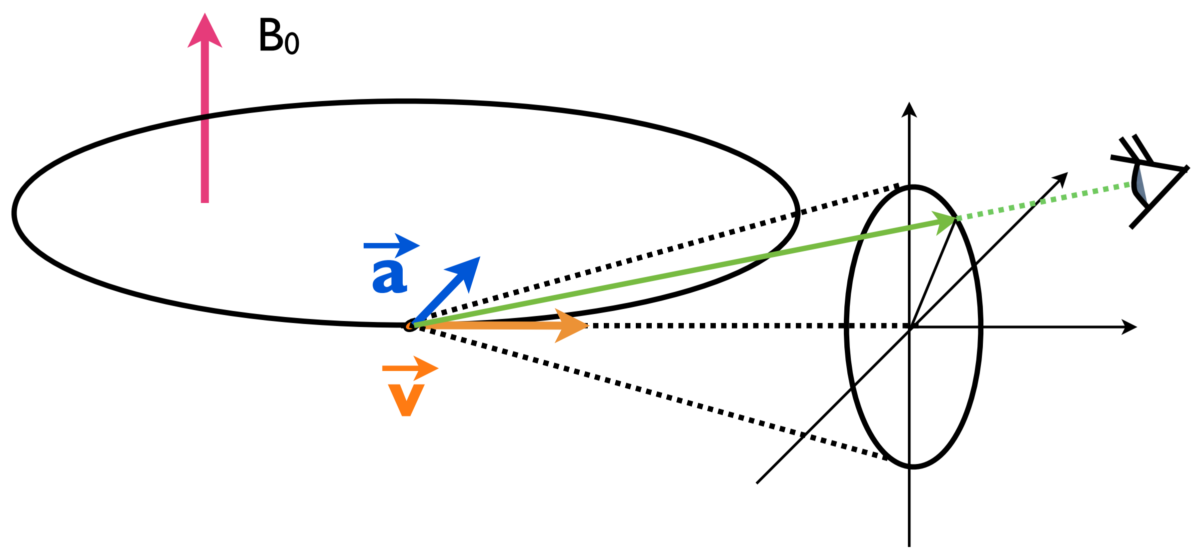
R. Lieu & W.I. Axford, ApJ, 416, 700 (1993)

- Generalisation of FWW can be used to introduce quantum corrections into the classical emissivity formula.

- Synchrotron spectrum is calculated based on Thomson scattering of virtual photons in consecutive instantaneous rest frames.

- Synchrotron QED emissivity is recovered.

Lieu & Axford method (using FWW)



Fermi-Weizäcker-Williams (FWW) method of equivalent photons applied to synchrotron radiation*

✻ **Divide trajectory in segments**

- in the segment, the particle trajectory is approximately straight
- a rest frame can be associated to each segment (in this frame motion is nonrelativistic)

✻ **Emitted radiation in each segment can be determined from Thomson scattering**

- Thomson cross section
- EM equivalent field to the B field from each segment

✻ **Total emitted radiation is obtained from the coherent summation of radiation amplitudes from the successive segments/rest frames.**

*R. Lieu & W.I. Axford, ApJ, 416, 700 (1993)

Introducing quantum corrections into our calculation

When recoil is significant, the emissivity is given by:

$$\Delta\alpha' = \left(\frac{d\sigma'}{d\Omega'}\right)_C \times \Delta S'$$

Compton cross section

$$\left(\frac{d\sigma'}{d\Omega'}\right)_C = \left(\frac{e^2}{mc^2}\right)^2 \eta'^2 \left|\boldsymbol{\epsilon}'_{out} \cdot \boldsymbol{\epsilon}'_{in}\right|^2$$

Compton shift

$$\eta' = \eta'(\omega', \Omega') = 1 - \frac{\hbar\omega'}{mc^2}(1 - \cos\Psi')$$

➡ a photon with **ω'** is scattered from incoming quantum with **ω'/η'**

➡ this leads to corrections given by

$$d(\omega'/\eta')/d\omega' = 1/\eta'^2 \qquad \hbar\omega'/\hbar(\omega'/\eta') = \eta'$$

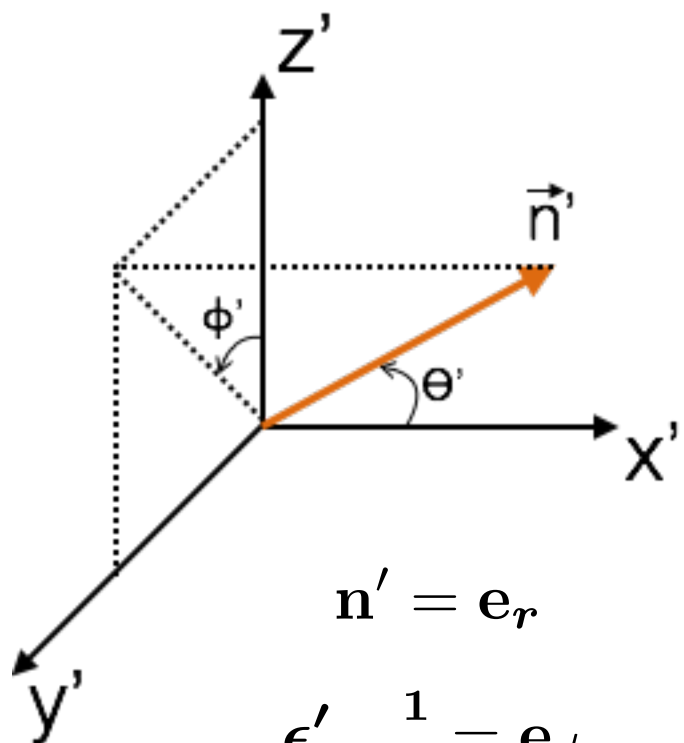
R. Lieu & W.I. Axford, ApJ, 416, 700 (1993)

Generalising the method for arbitrary observation direction

specific emissivity of
outgoing radiation
(rest frame)

$$\Delta\alpha'(\omega', \mathbf{\Omega}') = \frac{e^2\omega'^2}{4\pi^2 c^3} \left| \boldsymbol{\epsilon}'_{\boldsymbol{out}} \cdot \mathbf{v}'_\perp(t') e^{i\omega' t'} \Delta t' \right|^2$$

(outgoing) polarisation vector



$$\mathbf{n}' = \mathbf{e_r}$$

$$\boldsymbol{\epsilon}'_{\boldsymbol{out}}{}^{\mathbf{1}} = \mathbf{e}_\phi$$

$$\boldsymbol{\epsilon}'_{\boldsymbol{out}}{}^{\mathbf{2}} = \mathbf{e}_\theta$$

perpendicular displacement

$$\mathbf{v}'_\perp \Delta t' = (\Delta x', \Delta y', \Delta z')$$

$$\mathbf{v}'_\perp \Delta t' = [\gamma(v_x - v)\Delta t, v_y \Delta t, v_z \Delta t]$$

3D emissivity with quantum corrections

➡ After some algebraic manipulation we obtain:

3D classical specific emissivity

$$\alpha = \frac{d^2 I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c^3}\left|\int \mathbf{n}\times\mathbf{n}\times\boldsymbol{\beta}\exp[i\omega(t-\mathbf{n}\cdot\mathbf{r}/c)]dt\right|^2$$

Equal to the 3D formula derived from Liénard-Wiechert potentials

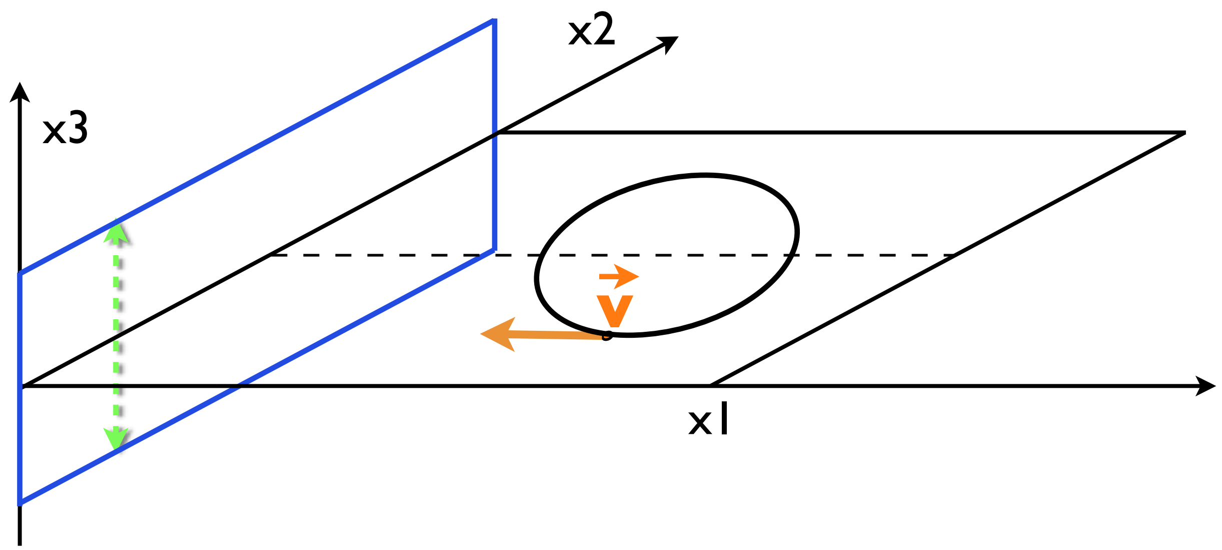
3D classical specific emissivity with quantum corrections*

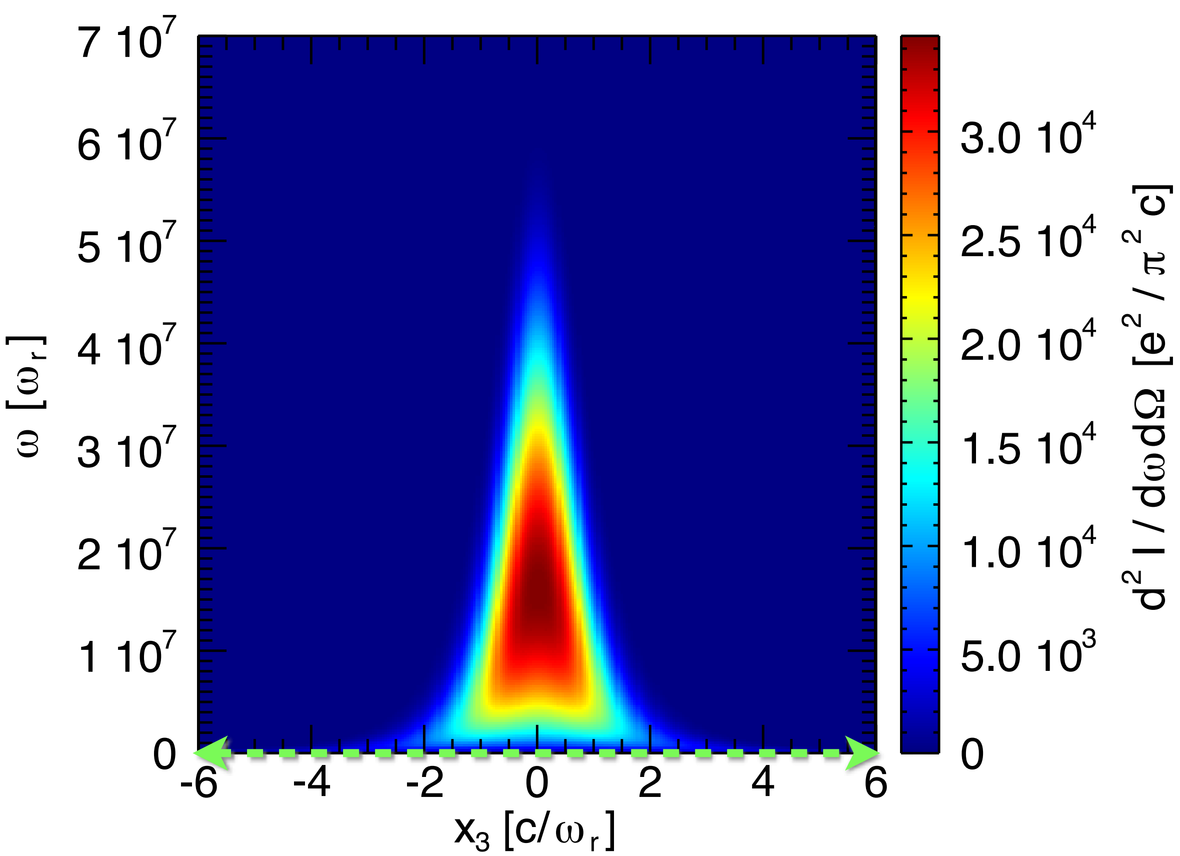
$$\left(\frac{d\sigma}{d\Omega}\right)_C = \left(\frac{e^2}{mc^2}\right)^2\left(\frac{k'}{k}\right)^2 \left|\boldsymbol{\epsilon_{out}}\cdot\boldsymbol{\epsilon_{in}}\right|^2 \qquad \frac{k'}{k} = \eta = 1 - \frac{\hbar\omega}{\gamma m_e c^2}$$

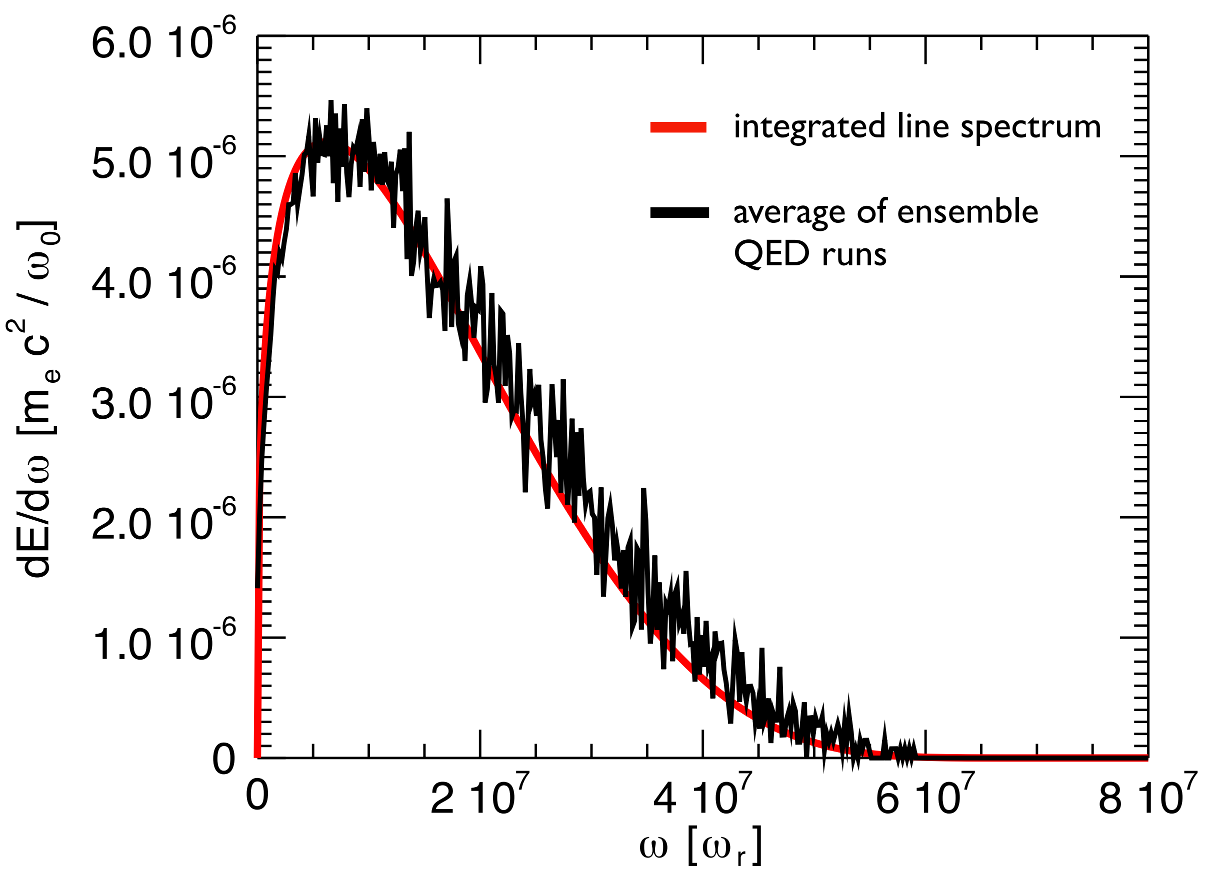
$$\frac{d^2 I(\omega)}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c^3}\left|\int_{-\infty}^{+\infty}\frac{\mathbf{n}\times(\mathbf{n}\times\boldsymbol{\beta})}{\sqrt{\eta}}\exp\left[i\frac{\omega}{\eta}(t-\mathbf{n}\cdot\mathbf{r}/c)\right]dt\right|^2$$

*Recovers result of I.V. Sokolov, *et al*, Phys. Rev. E 81, 036412 (2010)

Synchrotron benchmark with QED module

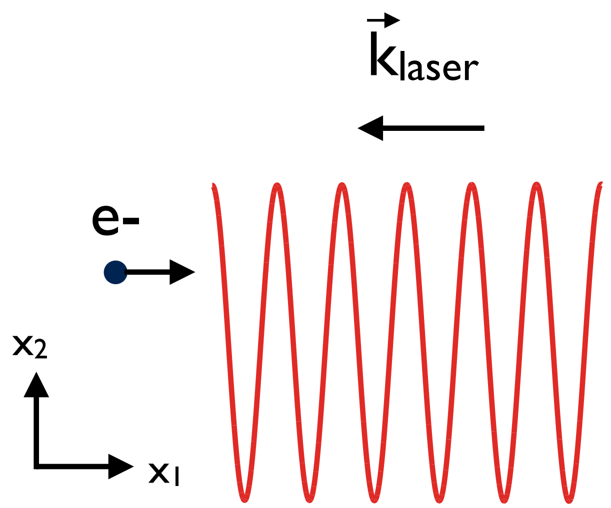






Both radiation reaction and quantum corrections change spectrum

Setup



$a_0 = 2$

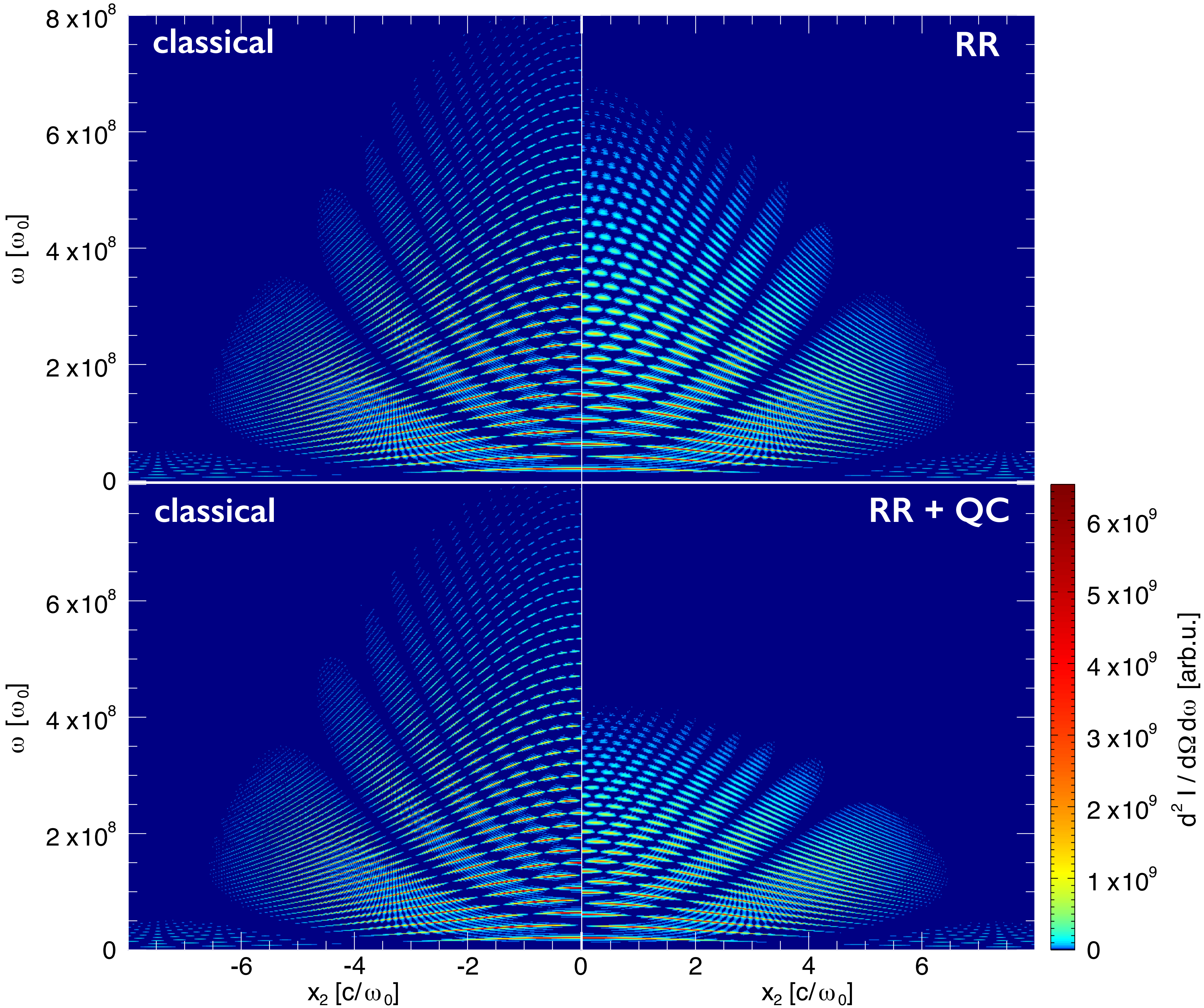
$\lambda = 1\ \mu m$

$10\,T_0$

$\gamma = 4000$

$\Delta E \sim -1\%$

Spectra spatially resolved in polarisation direction

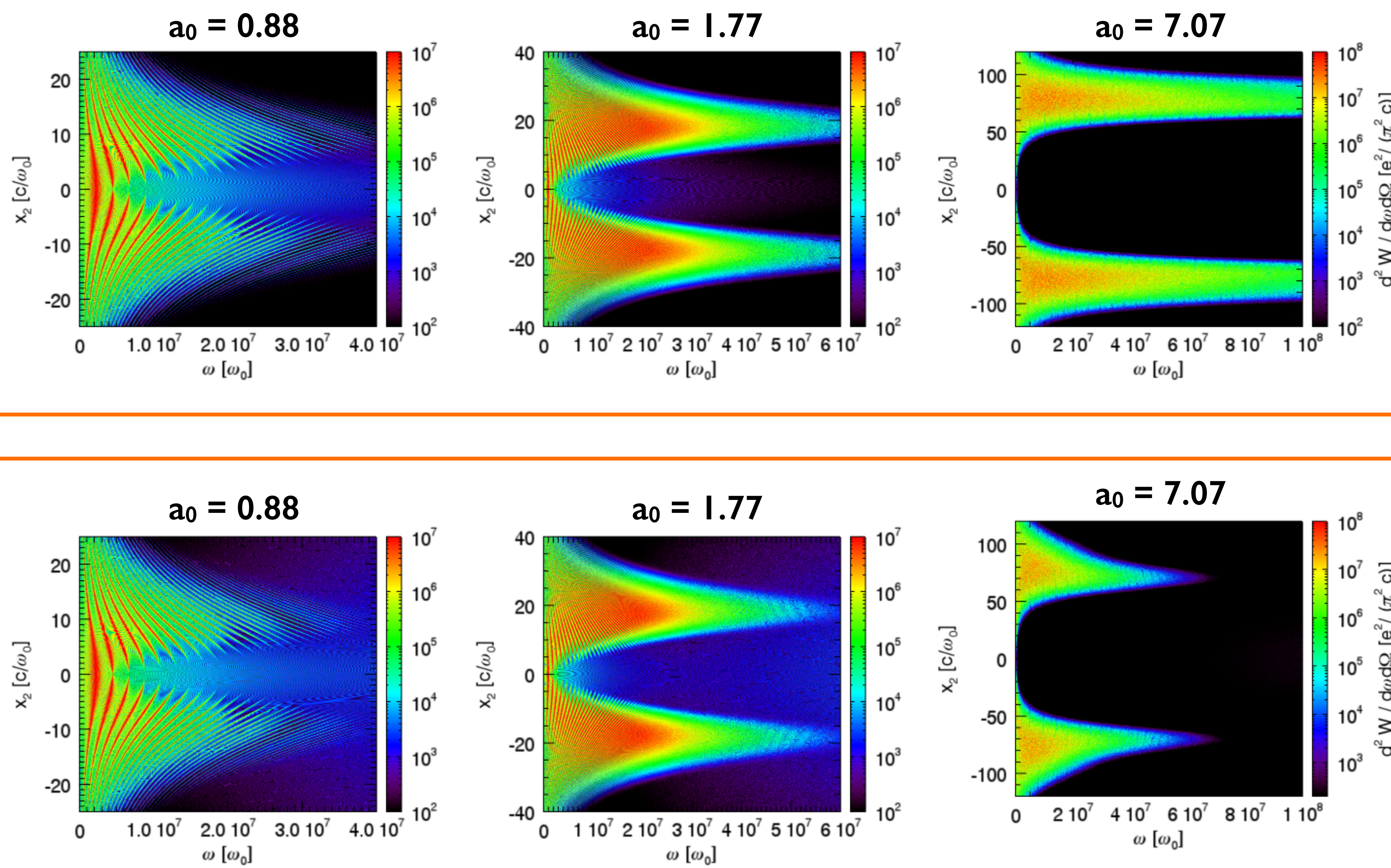


Quantum corrections effect increases with a_0

Spatially resolved spectra for CP plane wave (g=1000)

no QC

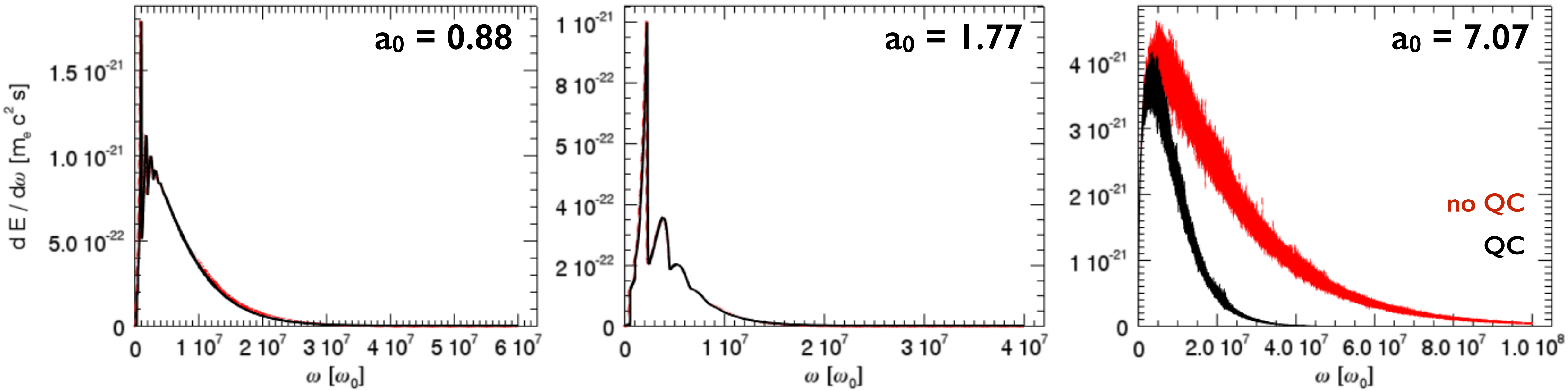
QC



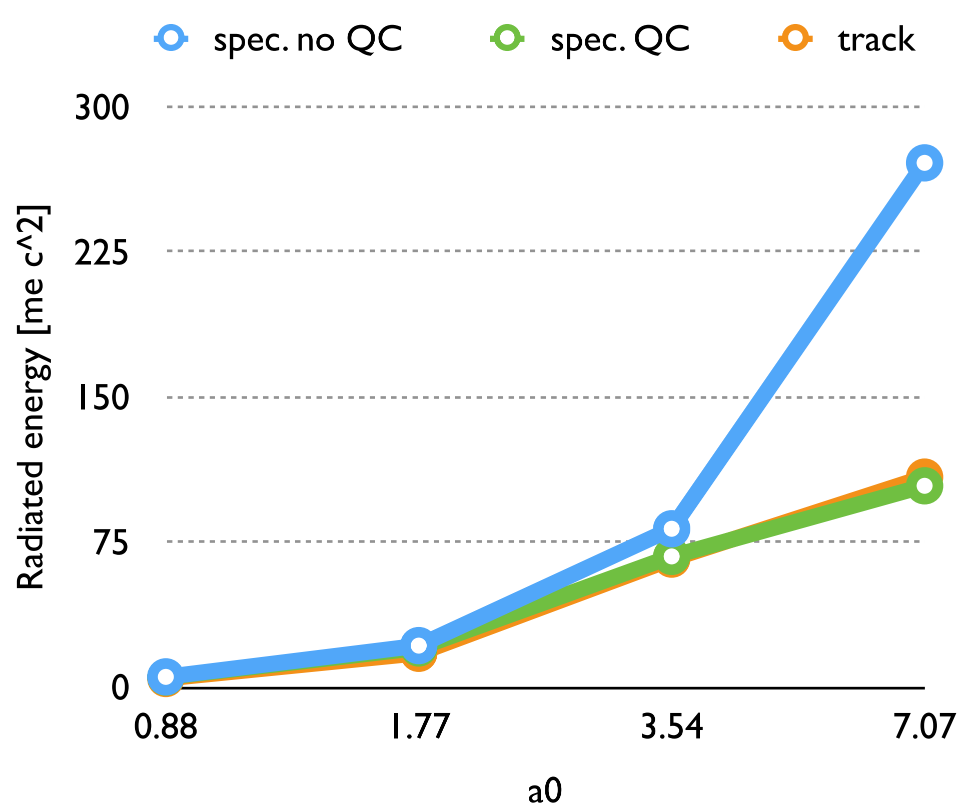
increasing a0

Quantum corrections needed to obtain correct energy loss

Spectra for CP plane wave integrated in solid angle

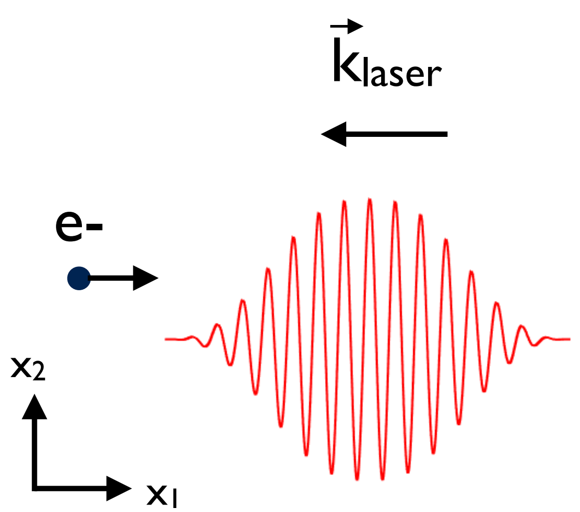


Total radiated energy



Quantum corrections lead to spike structure in laser pulse scattering

Setup

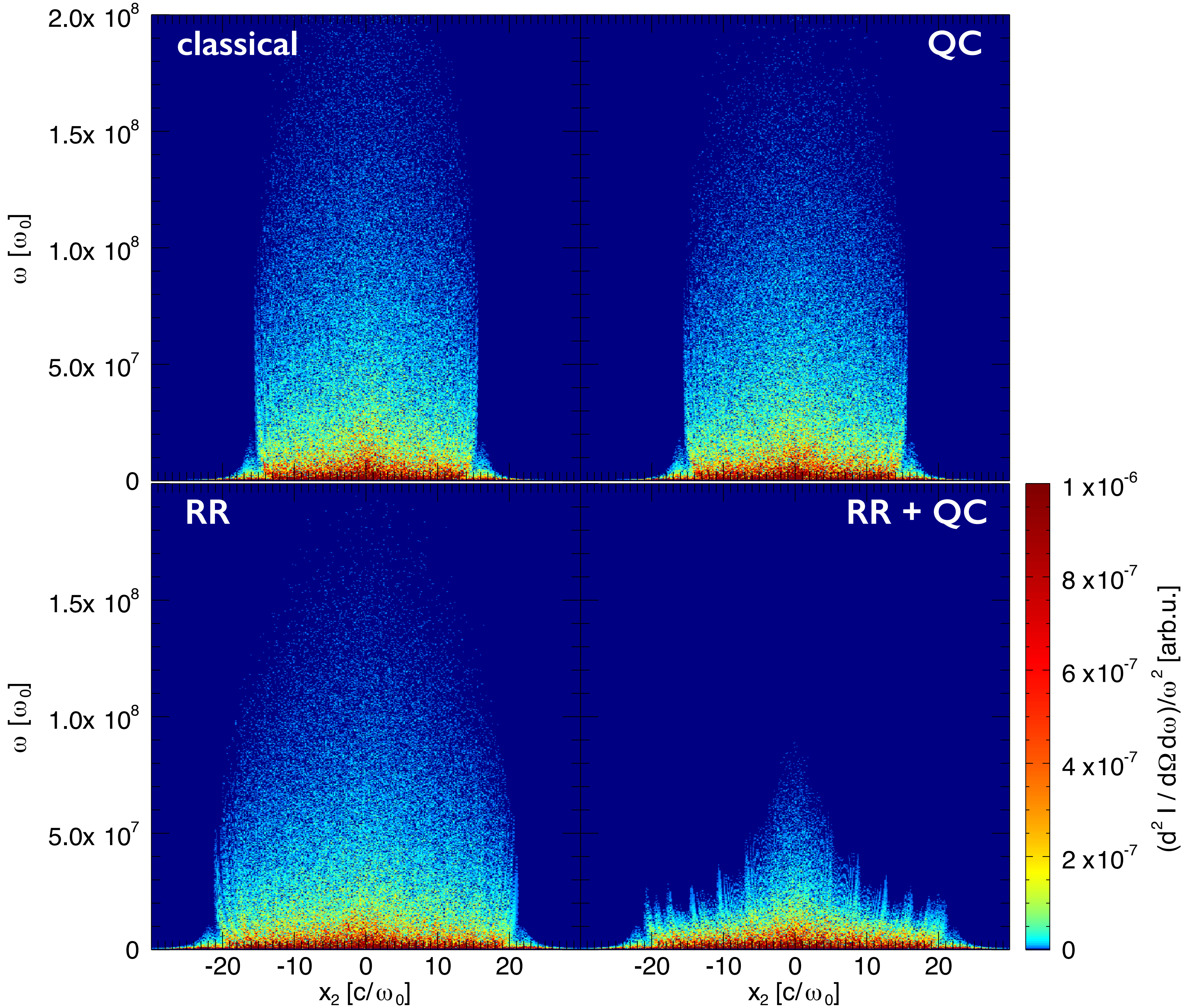


$a_0 = 27$ $\gamma = 1700$

$\lambda = 1\ \mu m$ $\Delta E \sim -35\%$

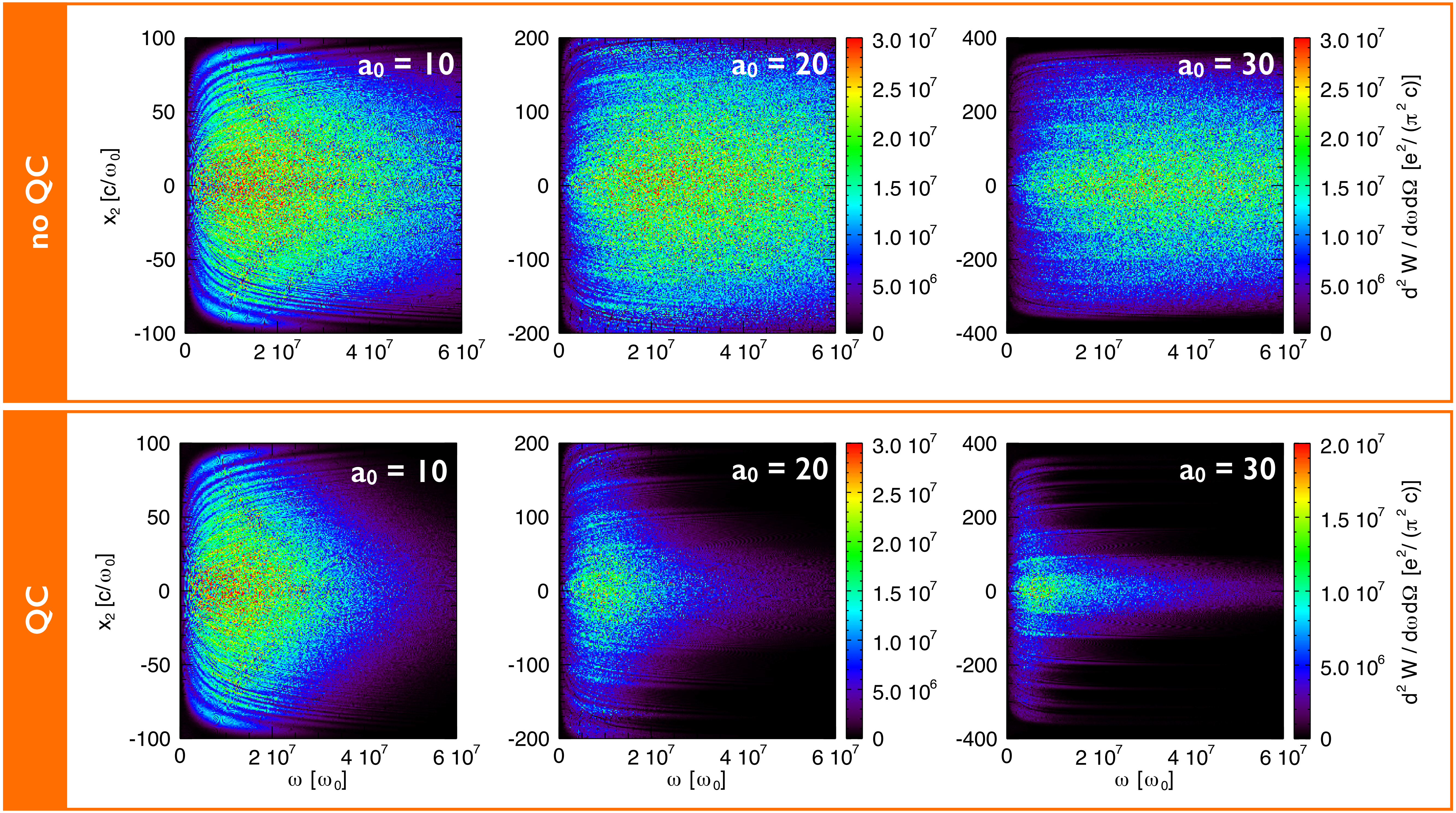
$\tau = 26.5$ fs $\chi = 0.26$

Spectra spatially resolved in polarisation direction



Higher a_0 leads to spike structure in spectra from linearly polarised laser pulse scattering

Spatially resolved spectra for laser pulse (γ = 1000, τ = 26.5 fs, λ = 1 μm)



Conclusions & Future work

Generalisation of FWW method allows introduction of quantum corrections to emissivity

- Lieu & Axford method of emissivity with quantum corrections for arbitrary directions of observation was extended
- quantum corrected emissivity consistent with QED results from literature

Inclusion of radiation cooling in the electron motion is not enough to obtain correct spectrum

- reduced energy emission at high frequencies and shifting and broadening of harmonics are observed consistently with previous results
- integrated radiated energy with quantum corrections is consistent with energy loss from particle trajectory with radiation damping
- post-processing diagnostic with quantum corrections combined with radiation reaction force allow exploration of transition from classical regime to radiation cooling regime
- strong radiation damping in laser pulses may lead to signatures but these will most likely be smeared out in an electron bunch scenario