Control of single and collective particle dynamics via radiation reaction

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Introduction and motivations

Current state-of-the-art optical laser systems can deliver intensities up to 10^{22} W/cm² [1] and intensities up to 10^{24-25} W/cm² are expected at the Extreme Light Infrastructure (ELI) [2]. The motion of an electron in such an extremely intense laser field becomes quickly ultrarelativistic, necessitating the inclusion of RR effects in the description of laser electron interaction. The RR force describes the back-action of the radiation emitted by an accelerated electron on the electron itself, and accounts for the energy and momentum loss due to the emission of such radiation [3]. In the case of a plasma, the RR force can be included in the collisionless transport equation for the relativistic distribution function $f(\mathbf{q}, \mathbf{p}, t)$ [4]:

$$\frac{\partial f}{\partial t} + \nabla_{\mathbf{q}} \cdot (f\mathbf{v}) + \nabla_{\mathbf{p}} \cdot (f\mathbf{F}) = 0, \qquad (1)$$

where ${f q}$ are the spatial coordinates, ${f v}={f p}/\gamma m_e$ is the three-dimensional velocity, $m_e~(e)$

is the electron mass (charge), $\gamma = \sqrt{1 + \mathbf{p}^2/m_e^2 c^2}$, and $\mathbf{F} = \mathbf{F}_L + \mathbf{F}_R$ is the mean force due to external and collective fields, where $\mathbf{F}_L = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}/c)$ is the Lorentz force and \mathbf{F}_R is the RR force, whose dominant term is [5]:

$$\mathbf{F}_{R} = -\frac{2e^{4}}{3m_{e}^{2}c^{5}}\gamma^{2}\left[\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right)^{2} - \left(\frac{\mathbf{v}}{c} \cdot \mathbf{E}\right)^{2}\right]\mathbf{v}.$$
(2)

In this work we show that RR effects can provide a route to the control of the single and collective electron dynamics. This is achieved in a setup where ultrarelativistic electrons are exposed to a strong either few-cycle [6] or bichromatic laser pulse via the nonlinear interplay between the Lorentz and radiation reaction forces [7]. Another potentially controllable result of RR effects is coherent Raman scattering due the collective particle dynamics when a strong laser pulse propagates in an underdense plasma [8].

To date the research has focused on revealing RR effects and understanding their fundamental features rather than exploiting them in a possibly beneficial and controlled way. Here, we show that RR effects can provide a route to the control of the electron dynamics via the nonlinear interplay between the Lorentz and the RR force. This is achieved, e.g., in a setup where an ultrarelativistic electron is exposed to a strong bichromatic laser pulse [7]. Physically, if a laser pulse contains two frequency modes, their relative phase can be tuned to manipulate the photon emission process, and to control the RR induced modulation due to the photon emission recoil (see Fig. 1). The deflection angle with respect to the initial propagation direction is [7]:



$$\zeta \approx -\arctan\left(\frac{2r_R\Delta}{1-r_R^2\Delta^2}\right) \tag{3}$$

if $r_R|\Delta| < 1$, $\zeta + \pi$ if $r_R\Delta < -1$ and $\zeta - \pi$ if $r_R\Delta > 1$ independently of the initial electron energy. Here $r_R = 4\pi e^2/3m_ec^2\lambda \approx 1.18 \times 10^{-8}/\lambda_{\mu m}$, with λ being the longer wavelength mode, and $\Delta \propto \xi_1^2 \xi_2 N \cos(\theta_2 - 2\theta_1)$ where ξ_1 , ξ_2 are the field amplitudes of each frequency component, and θ_1 , θ_2 are two constant initial phases. Figure 2 reports the electron density distribution $n_e(p_z, p_x)$ for the interaction of 400 electrons with a 70 fs duration focused laser pulse with $\xi_1 = 40$, $\xi_2 = 28$, and 5 μ m waist radius (the total intensity and power are 5.1×10^{21} W/cm² and 2 PW, respectively). Initially, the electron bunch has mean momentum $\vec{p_0} = (0, 0, -165 m_e c)$, and 3×10^{15} cm⁻³ mean electron density.



Fig. 1: If a plane-wave like laser pulse contains a second frequency mode, its relative phase can be tuned to control the photon emission process. As a result, it is possible to induce the electron to emit prevalently along a specific direction, which results in a net transverse momentum gain and a controllable deflection induced by RR effects. Panel (a): An electron collides head-on with a bichromatic laser pulse with zero relative phase between the higher and the lower frequency modes. Panel (b): Same as panel (a) but with $\pi/2$ relative phase between the two frequency modes.

Fig. 2: Electron density distribution $n_e(p_z, p_x)$ as a function of the longitudinal p_z and transverse p_x momentum after the interaction of 400 electrons with a bichromatic laser pulse. Panel (a): $\cos(\theta_2 - 2\theta_1) = 0$ without RR. Panel (b): $\cos(\theta_2 - 2\theta_1) = 0$ with RR. Panel (c): $\cos(\theta_2 - 2\theta_1) = 1$ without RR. Panel (d): $\cos(\theta_2 - 2\theta_1) = 1$ with RR.

Collective particle dynamics with radiation reaction effects

The RR force, in the perturbative approximation, causes phase slippage on the quiver momentum of the electrons in the laser fields. Due to this dispersion relation for the electronic parametric instabilities in an underdense plasma is modified [8].

$$\left(\frac{R_+}{D_+} + \frac{R_-}{D_-}\right) = 1,\tag{4}$$

$$D_{\pm} = \omega_{\pm}^{2} - \omega_{p}^{'2} \left(1 - \frac{i\varepsilon a_{0}^{2}\gamma_{0}\omega_{0}}{\omega_{\pm}} \right) - \left[(k_{z} \pm k_{0})^{2} + k_{\perp}^{2} \right] c^{2},$$

$$R_{\pm} = \frac{\omega_{p}^{2}a_{0}^{2}}{4\gamma_{0}^{3}} \left[\frac{k_{z}^{2}c^{2}}{D_{e}} \left(1 \mp i\varepsilon a_{0}^{2}\gamma_{0} + \frac{2i\varepsilon a_{0}^{2}\gamma_{0}\omega\omega_{0}}{k_{z}c} - \omega_{\pm} \right) - \left(1 \mp i\varepsilon a_{0}^{2}\gamma_{0}\frac{\omega}{\omega_{\pm}} + 4i\varepsilon\gamma_{0}^{3}\frac{\omega_{0}}{\omega_{\pm}} \right) \right], \quad (5)$$

where $\varepsilon = e^2 \omega_0 / 3m_e c^3$ (ω_0 being the laser frequency and $\omega_{\pm} = \omega \pm \omega_0$) denotes the influence of the RR force. From the dispersion relation, the growth rate of the forward

We term the nonlinear mixing of the two modes due to RR force as the manifestation of this accumulation of phase shifts, and it leads to the enhanced growth rate of the FRS instability. One can also intuitively imagine this growth enhancement occurring due to the availability of an additional channel of RR force induced laser energy decay and its efficient utilization by both the Stokes and the anti-Stokes modes.



Raman scattering instability can be calculated and it is plotted in Fig.3. One can immediately notice that radiation reaction force significantly enhances the growth rate of the FRS at lower plasma densities $\omega'_p/\omega_{0r} \ll 1$ and higher laser amplitude $a_0 \gg 1$. This enhancement occurs due to the mixing between the Stokes and the anti-Stokes modes mediated by RR force. In the absence of RR force, nonlinear currents that drive the Stokes and the anti-Stokes modes have opposite polarizations. Consequently, the phase shift induced by RR force - as seen from the expression of R_{\pm} in Eq.(5) - is opposite for these modes. This results in the interaction between the nonlinear current terms, culminating into phase shifts accumulation in Eq.(4).

Fig. 3: Normalized growth rate $(\Gamma_{frs} - \delta\omega_0)/\omega_{0r}$ of the FRS as a function of the normalized plasma density $\Omega_p \equiv \omega_p/\omega_{0r}$ and normalized pump laser amplitude $a_0 = eA_0/mc^2$ (a) including the RR force, (b) without the RR force. The normalized growth rate is plotted on Log₁₀ scale.

The RR force enhances the growth of parametric instabilities in plasmas



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