e^+e^- Pair Production in Rotating Fields

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Abstract

We explore Schwinger pair production in time-dependent electric fields of arbitrary polarization using the real-time DHW formalism. We determine the time evolution of the Wigner function as well as asymptotic particle distributions neglecting back-reactions on the electric field. The field rotation leaves characteristic imprints in the momentum distribution that can be interpreted in terms of interference and multiphoton effects.

The Wigner function

The equal-time Wigner function is defined as the vev of a Wigner operator, which is in turn a Fourier transformed two-point commutator [1],

$$\mathcal{W}_{ab} = -\frac{1}{2}\int d\vec{s}\, e^{-\frac{i}{\hbar}\vec{p}\cdot\vec{s}}\, \langle 0|\, \hat{\Phi}\left[\hat{\Psi}_a(t,\vec{x}+\vec{s}/2),\, \hat{\bar{\Psi}}_b(t,\vec{x}-\vec{s}/2)\right]|0\rangle\,.$$

A Wilson line is introduced to achieve gauge invariance

$$\hat{\Phi} = \exp\left(-ie\int_{\vec{x}+\vec{s}/2}^{\vec{x}-\vec{s}/2} \vec{A}(t,\vec{x}')\cdot d\vec{x}'\right),$$

and the function is decomposed into its Fierz components according to

$$\mathcal{W} = \frac{1}{4}\left(\mathbb{1}\mathbb{s} + i\gamma_5\,\mathbb{p} + \gamma^\mu\,\mathbb{v}_\mu + \gamma^\mu\gamma_5\,\mathbb{a}_\mu + \sigma^{\mu\nu}\,\mathbb{t}_{\mu\nu}\right).$$

Those components can in turn be used to find the phase space energy density in the Dirac field, which also gives the 1-particle distribution function

$$\epsilon = \vec{p}\cdot\vec{\mathbb{v}}(t,\vec{x},\vec{p}) + m\,\mathbb{s}(t,\vec{x},\vec{p})\,,$$

$$f[\mathcal{W}] = \frac{1}{2\sqrt{m^2+\vec{p}^2}}\left(\epsilon(t,\vec{x},\vec{p}) - \epsilon_{\text{vac.}}(t,\vec{x},\vec{p})\right).$$

Calculating the Wigner function

In the special case of a spatially homogeneous, purely electric field that is treated classically, inserting the Dirac Equation into the definition leads to a partial differential equation as an equation of motion for the Wigner function. This can be transformed into an ordinary initial value problem by inserting

$$\vec{p} \to \vec{\pi} = \vec{p}_{\vec{q}}(t) = -e\vec{A}(t) + \vec{q}\,.$$

Afterwards the following substitution is applied in order to directly calculate the 1-particle distribution function

$$\begin{aligned}
\mathbb{s}(\vec{\pi},t) &= (1-f(\vec{q},t))\,\mathbb{s}_{\text{vac.}}(\vec{\pi},t) - \vec{\pi}\cdot\vec{v}(\vec{q},t)\\
\vec{\mathbb{v}}(\vec{\pi},t) &= (1-f(\vec{q},t))\,\vec{\mathbb{v}}_{\text{vac.}}(\vec{\pi},t) + \vec{v}(\vec{q},t)\\
\vec{\mathbb{a}}(\vec{\pi},t) &= \vec{a}(\vec{q},t)\\
\vec{\mathbb{t}}(\vec{\pi},t) &= \vec{t}(\vec{q},t)\,.
\end{aligned}$$

In this context the electric Field $\vec{E}$ is given by $\vec{E}(t) = -\dot{\vec{A}}(t)$ and the modified Quantum Kinetic equations [2, 3] read

$$\begin{aligned}
\dot{f} &= \frac{e}{2\omega}\vec{E}\cdot\vec{v}\\
\dot{\vec{v}} &= \frac{e}{2\omega^3}\left(\vec{p}(\vec{E}\cdot\vec{p}) - \omega^2\vec{E}\right)(f-1) - \frac{e}{\omega^2}\vec{p}(\vec{E}\cdot\vec{v}) - \vec{p}\times\vec{a} - 2\vec{t}\\
\dot{\vec{a}} &= -\vec{p}\times\vec{v}\\
\dot{\vec{t}} &= 2\left(\vec{v} + \vec{p}(\vec{p}\cdot\vec{v})\right).
\end{aligned}$$

This system can be solved numerically with Runge-Kutta type solvers using double precision arithmetic for pulse durations of up to 1000 Compton times. For long pulses some noise suppression techniques are used when integrating over the spectra to obtain the total particle yield.

Spin, charge and chirality

Spin, charge and chirality projections are given by

$$P_{(a,b,c)} = 1/2\left(\mathbb{1} + \sigma_{(a,b,c)}\right) \quad \sigma_{(a,b,c)} = ia\gamma^2\gamma^3 + ib\gamma^3\gamma^1 + ic\gamma^1\gamma^2$$

$$P_{\mp\text{Q}} = 1/2\left(\mathbb{1} \pm \gamma^0\right) \qquad P_{\text{l/r}} = 1/2\left(\mathbb{1} \pm \gamma_5\right).$$

The projectors can then be used to distiguish created particles according to their chirality or direction of the magnetic moment

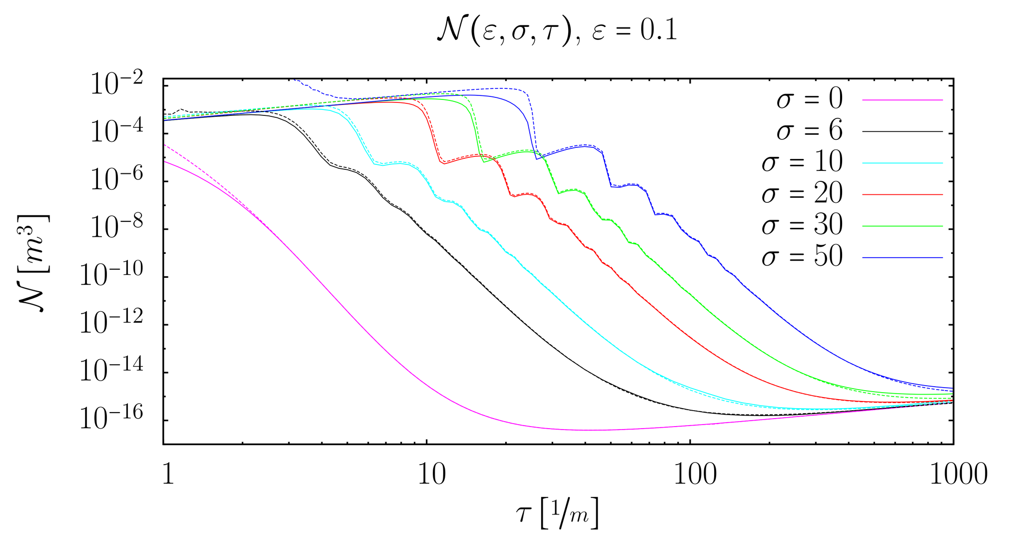
$$\begin{aligned}
f_{\text{l/r}} &:= f[P_{l/r}\mathcal{W}] = 1/2\left(f \pm \frac{\vec{a}\cdot\vec{p}}{2\omega(\vec{p})}\right)\\
f_{\mu_z^\pm} &:= f\left[\left(P_{(0,0,\pm1)}P_{-Q} + P_{(0,0,\mp1)}P_Q\right)\mathcal{W}\right]\\
&= 1/2\left(f \pm \frac{m\,a_z + \left(\vec{p}\times\vec{t}\right)_z}{2\omega(\vec{p})}\right).
\end{aligned}$$

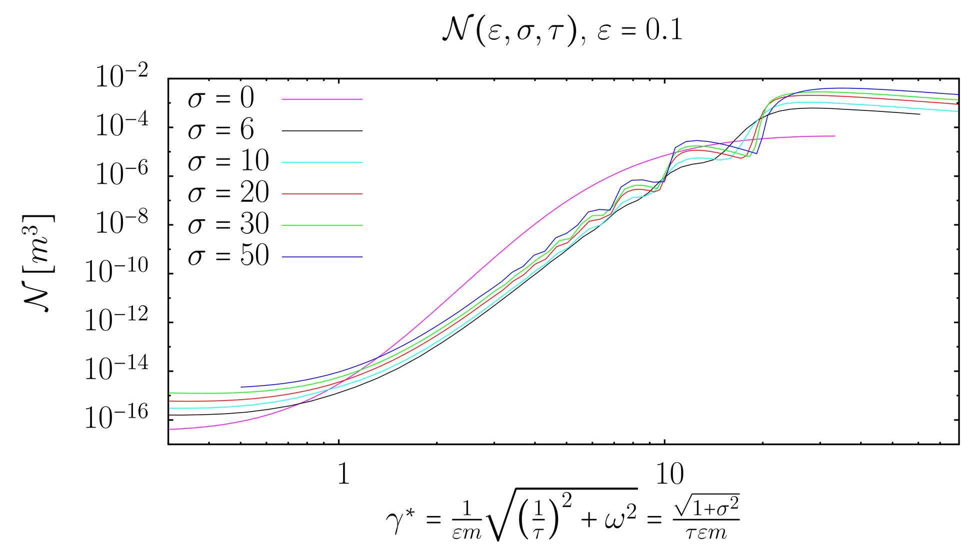
Total particle yield in Rotating Fields

We consider a rotating pulse of the form

$$\vec{E}(t) = \frac{\varepsilon\,E_{\text{cr.}}}{\cosh^2(t/\tau)}\begin{pmatrix}\cos(\frac{\sigma}{\tau}t)\\ \sin(\frac{\sigma}{\tau}t)\\ 0\end{pmatrix}.$$

In order to calculate the total particle yield, a complete spectrum has to be calculated, point by point. If the number of rotations is large enough ($\sigma \geq 20$), the spectra show a cylindrical symmetry around the z axis and thus only a 2D spectrum in the $x-z$ plane has to be calculated in order to obtain the full picture. However, in the case of a low number of oscillations, the spectrum does not yet have a cylindrical symmetry, so a 3D spectrum has to be calculated, which needs much more time.



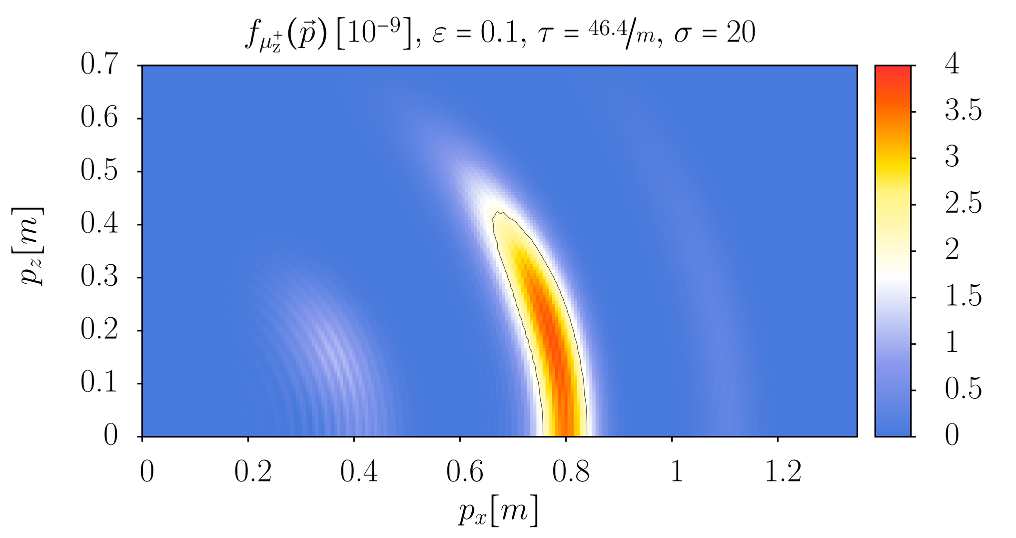


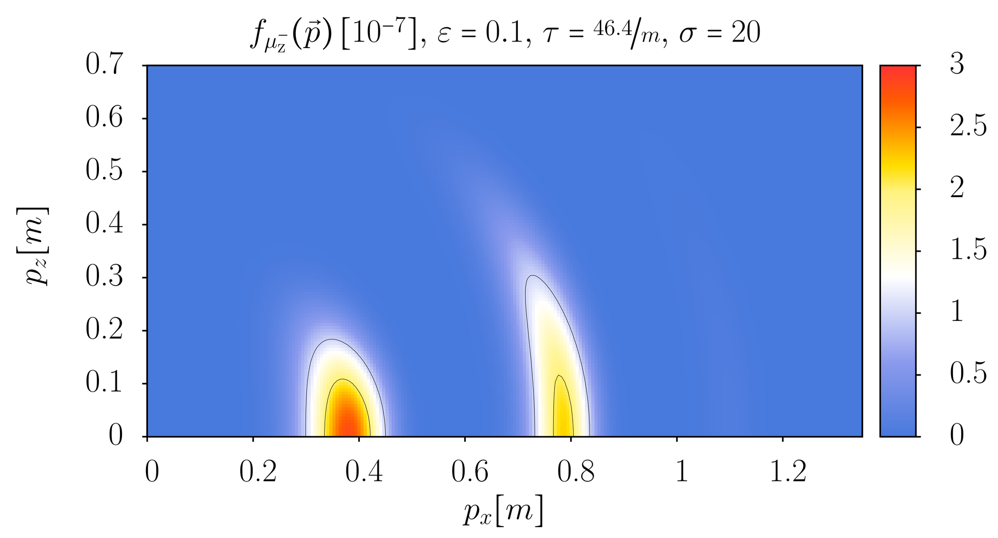
These plots show the resulting total particle yield for Rotating Fields. The dashed lines in the left plot show results obtained by a semiclassical method as derived in [4], for comparison [5].

The Keldysh adiabacity parameter is used to discriminate multiphoton or Schwinger-like pair production. This field configuration has two distinct time scales, one from the oscillation of the field and one from the envelope. Using a combined Keldysh parameter as shown in the right hand plot shows that the results from different σ are not that different. We interpret this by saying that the presence of the rotation time scale shifts the pair production towards the multiphoton regime.

Magnetic moment

The spectra of created particles can differ quite a lot w. r. t. the alignment of the magnetic moment. Different peaks can be more or less pronounced. In general, spectra show more complex interference patterns for longer pulses, the onset of which can be seen in the leftmost peak in the left figure.



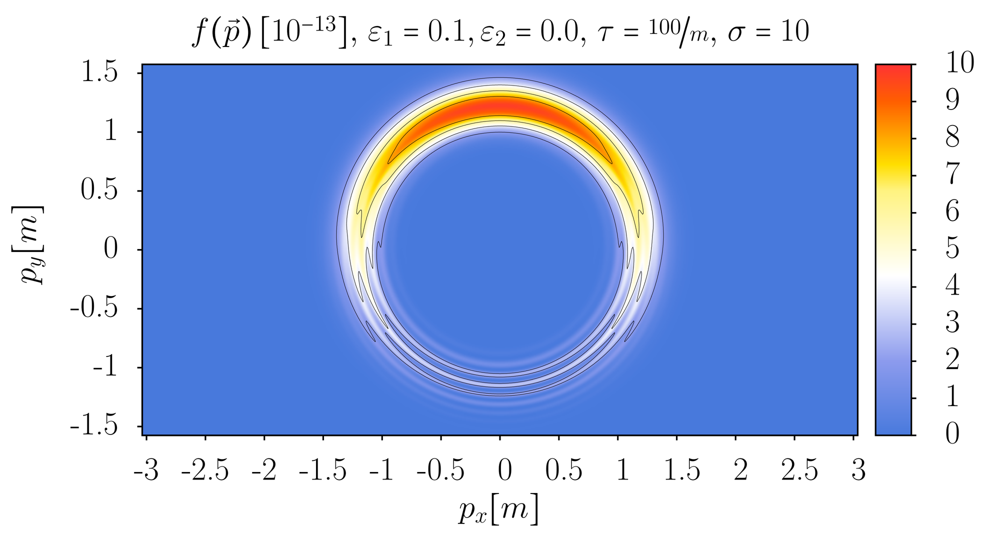


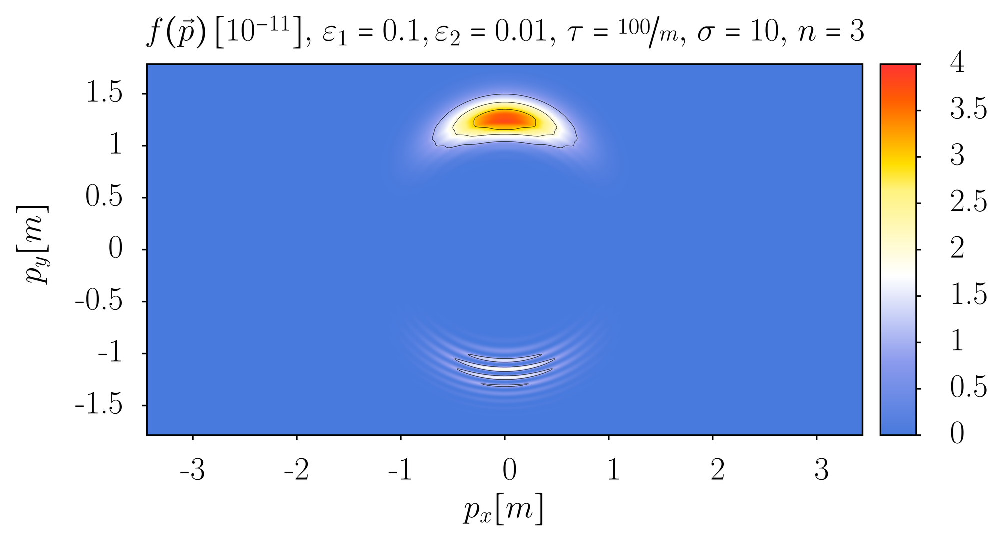
The semiclassical method [4] as presented by Eckhard Strobel also gives access to similar degrees of freedom, which can be related to a linear combination of the above and the chiral projections [5].

Particle yield in Bifrequent Fields

Particle pair production can be greatly enhanced by mixing different frequencies. Consider

$$\vec{E}(t) = \frac{E_{\text{cr.}}}{\cosh^2(t/\tau)}\left(\varepsilon_1\begin{pmatrix}\cos(\frac{\sigma}{\tau}t)\\ \sin(\frac{\sigma}{\tau}t)\\ 0\end{pmatrix} + \varepsilon_2\begin{pmatrix}\cos(n\frac{\sigma}{\tau}t)\\ \sin(n\frac{\sigma}{\tau}t)\\ 0\end{pmatrix}\right).$$





The higher frequencies introduce photons of higher energy into the game. As can be seen in these spectra, not only the particle yield is greatly increased, but the spectrum has a totally different shape. The produced pairs in this case are much more collimated when a harmonic field is added. This is only one example for a number of effects that can be observed in pair production spectra in these kinds of fields [6].

Literature

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