Nonlinear Compton scattering of ultrashort laser pulses by a superposition of Volkov states

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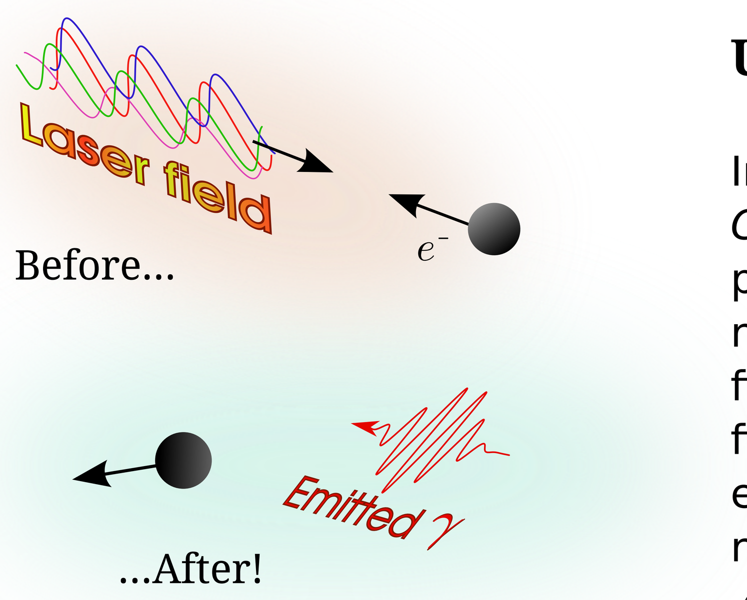
Nonlinear Compton scattering

Nonlinear Compton scattering indicates the process of inelastic scattering of an electron by an intense electromagnetic laser wave. There are two gauge- and Lorentz-invariant parameters that characterize this process[1,2]:

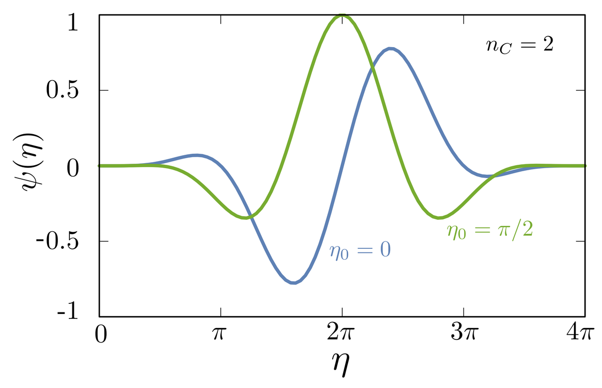


$$\xi = \frac{|e|\mathcal{E}}{\omega m c} \qquad \chi = \frac{(pk)}{m\,\omega}\,\frac{\mathcal{E}}{\mathcal{E}_{cr}}$$

Here, $\mathcal{E}$ is the peak value of the electric field of the laser, ω is its central angular frequency and k^μ its four-wavevector. p^μ is the initial four-momentum of the electron, e and m its charge and mass, respectively. $\mathcal{E}_{cr} = m^2c^3/\hbar|e| \approx 1.3\times10^{16}\ \mathrm{V/cm}$ is the so-called critical field of QED, that is, an electric field whose value is so high that it can transfer to the electron an energy comparable to its rest energy in an electron Compton wavelength $\lambda_C = \hbar/mc \approx 3.9\times10^{-11}\ \mathrm{cm}$. Notice that $\mathcal{E}_{cr}$ contains $\hbar$, and thus one can see that χ will be related to quantum effects, like photon recoil. On the other hand, ξ is the work, in units of the laser photon energy, performed by the field in a Compton wavelength, and it is related to the probability amplitude of multiphoton processes.

Ultrashort laser pulses

In experiments, high laser intensities are reached with the aid of a technique, *Chirped Pulse Amplification*[3], that provides ultrashort intense laser pulses. Such pulses contain an enormous number of photons, all emitted by the same coherent source; in order to study QED processes in the presence of such intense fields one can adopt the Furry picture of electrodynamics[1,2], and treat the laser field as a classical background field, not affected by the interaction with the electron. In particular, we assume that the background field is a linearly polarized plane wave and thus that its four-vector potential is $A^\mu(\eta) = A^\mu\psi(\eta)$, where A^μ is a constant four-vector related to the amplitude of the field and its polarization, and ψ is a function that gives the shape of the pulse. $A^\mu(\eta)$ depends on the spacetime variables x^μ only via $\eta = kx$.



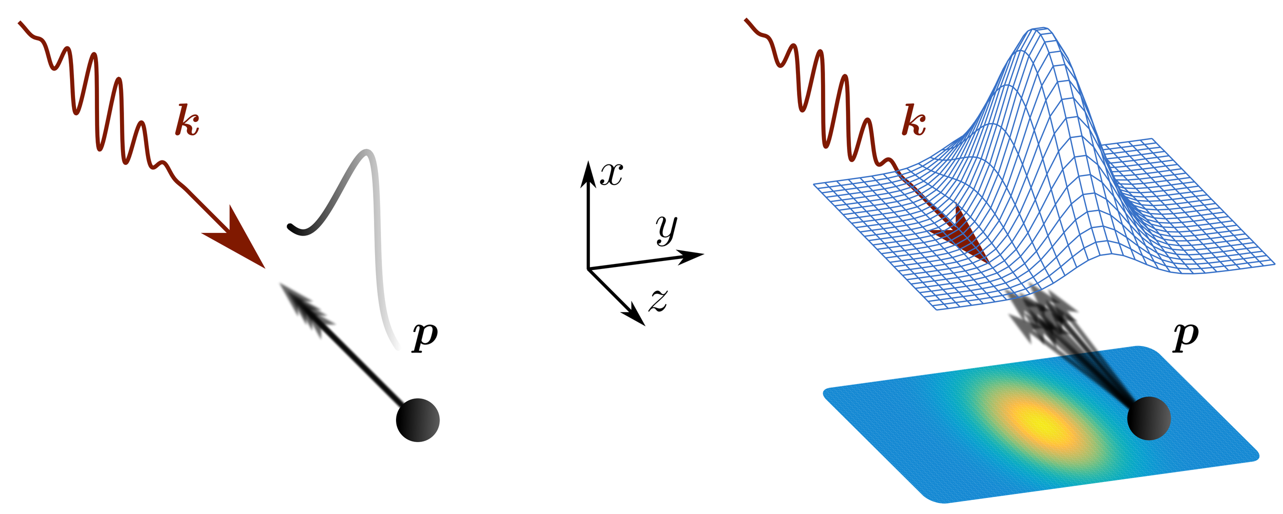
In our simulations, we have often used few-cycles pulses, characterized by the following shape function:

$$\psi(\eta) = \begin{cases} \sin^4\left(\dfrac{\eta}{2n_C}\right)\sin(\eta+\eta_0) & \text{if } \eta\in[0, 2\pi n_C], \\ 0 & \text{otherwise.}\end{cases}$$

Number of cycles (n_C) — Carrier-envelope phase (η_0)

Volkov states and their superpositions

The application of the Furry picture implies the knowledge of the solutions of the Dirac equation in the presence of the external field. These states in the case of a background plane wave are the so-called *Volkov*[4] *states* $\psi_{p,\sigma}(x)$ (where σ is a spinorial index). We studied nonlinear Compton scattering by a superposition of Volkov states with different momenta. In particular, we analyzed two cases:



Gaussian superposition in p_z — Gaussian superposition in p_x, p_y, p_z

Conservation rules forbid interference effects

From the S-matrix of nonlinear single Compton scattering, where the adjective single means that the electron emits only one photon (angular frequency ω' and four-wavevector k'^μ), one can deduce[5] some conservation rules between the initial state and the final one; they are, in natural units (from now on $\hbar = c = 1$):

$$p_y = p'_y + k'_y \qquad p_x = p'_x + k'_x \qquad \epsilon - p_z = \epsilon' - p'_z + \omega' - k'_z$$

where p'^μ is the final four-momentum of the electron.
With this set of independent equations, together with the on-shell conditions $p^2 = p'^2 = m^2$ and $k'^2 = 0$, one can determine all the components of the initial four-momentum as a function of the final state; thus

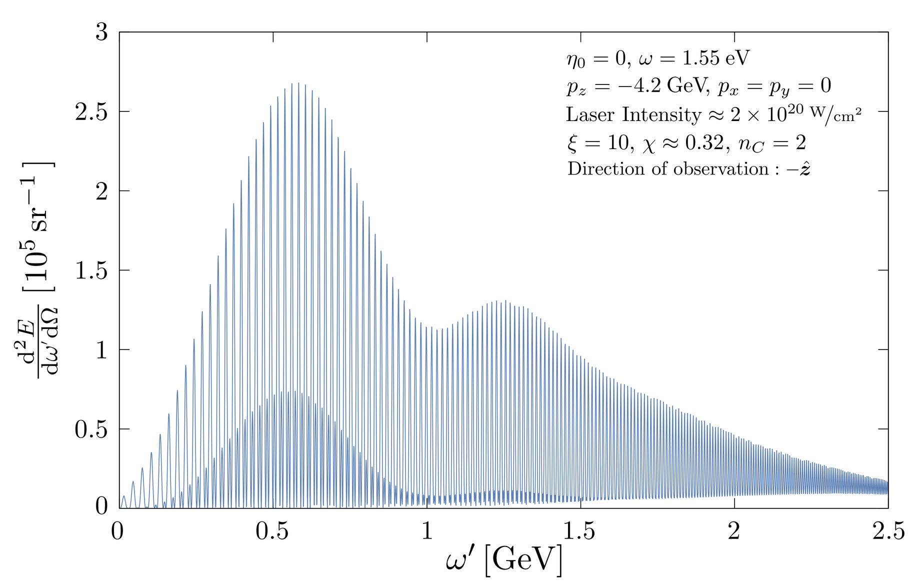
The knowledge of the final state uniquely determines the initial state!

→ There is no quantum interference between two initial Volkov states with different momenta!

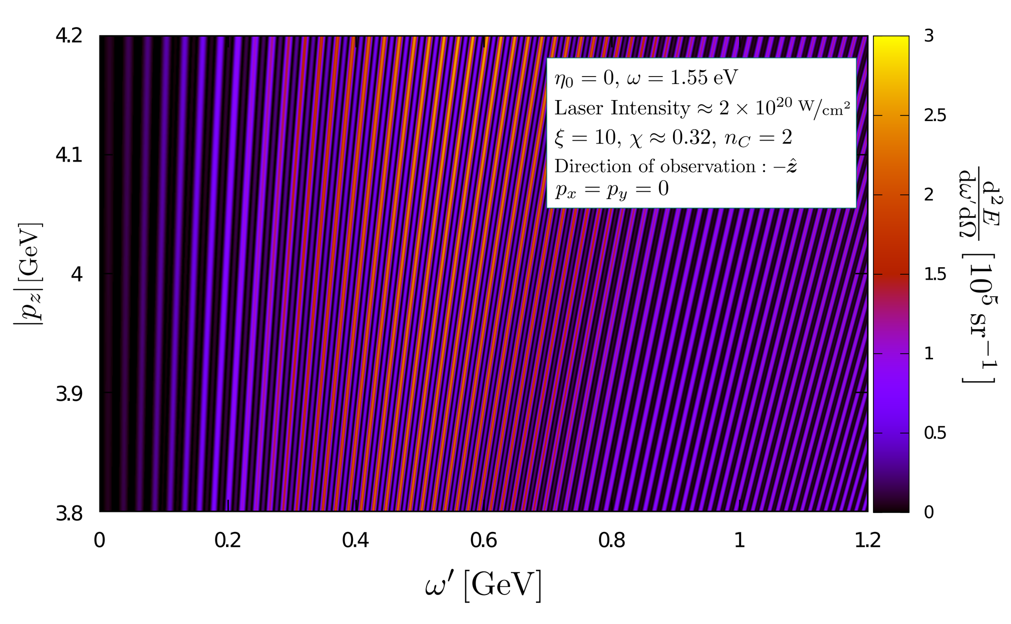
Results

Monochromatic emission spectrum

Since there are no interference effects, the photon emission spectrum of a superposition of Volkov states with different $\boldsymbol{p}$ is an incoherent sum of spectra with definite initial electron momentum.

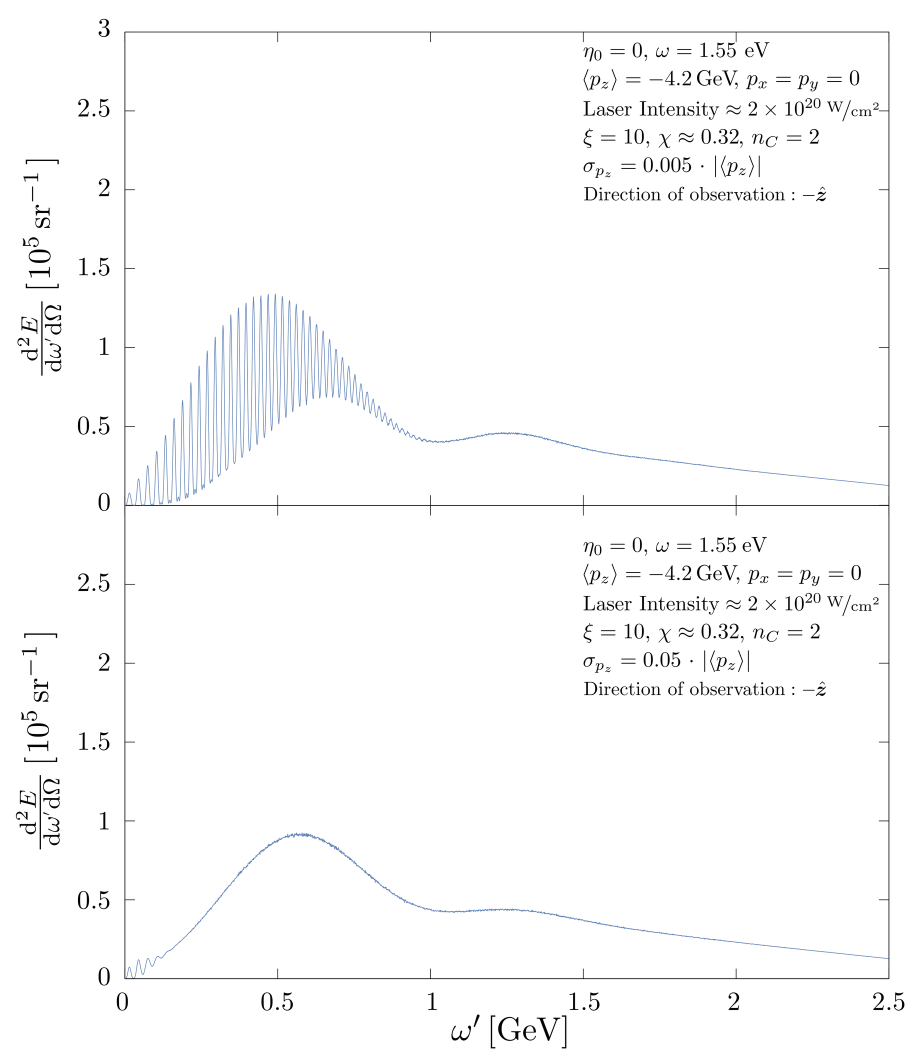


These spectra, in the regime $\xi \gg 1$, $\chi < 1$ are characterized by the presence of many peaks (we can see an example in the figure above). The position of the peaks varies when varying the components of $\boldsymbol{p}$; in the following figure one can see how the spectrum is altered by changing p_z:



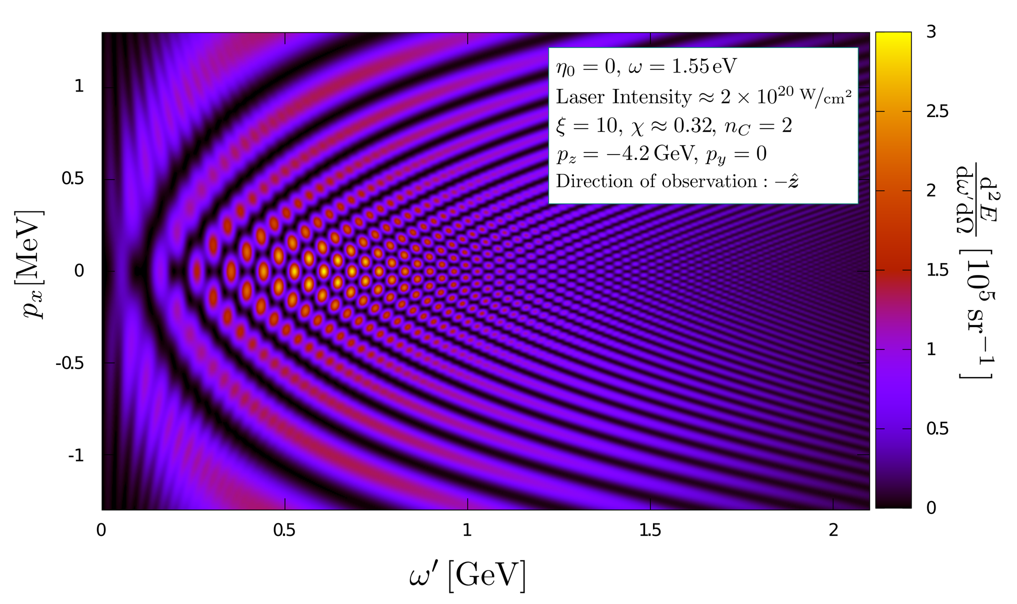
Gaussian superposition of different p_z

As the emission peaks are shifted for different values of p_z they will tend to fill the neighbouring valleys between two successive peaks; the net effect of this, when averaging all these spectra, is a smoothing and a lowering of the final spectrum.



General Gaussian superposition

When the transverse components $p_T = \sqrt{p_x^2+p_y^2}$ of the initial momentum of the electron are not zero, the monochromatic spectra are changed in a non-trivial way, as we can see in the following figure.



The effect of the indeterminacy in p_T is in general to smooth and lower the emission spectrum even more than the indeterminacy in p_z. The reason is that, as we have checked analytically[5], the relative variation $\delta\omega'/\omega'$ as a function of δp_T is much steeper than as a function of δp_z.
However, in experiments usually it is $\delta p_T \ll \delta p_z$, such that the two effects can be comparable.

References:
[1] V. I. Ritus, J. Russ. Laser Res. **6**, 497 (1985).
[2] A. Di Piazza, C. Müller, K. Z. Hatsagortsyan and C. H. Keitel, Rev. Mod. Phys. **84**, 1177 (2012).
[3] D. Strickland and G. Mourou, Opt. Commun. **56**, 219 (1985).
[4] D. Volkov, Z. Phys. **94**, 250 (1935).
[5] A. Angioi, F. Mackenroth and A. Di Piazza, *In preparation*.

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