Master formula for nonlinear Compton scattering in scalar QED from worldline formalism

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Abstract

We apply the string-inspired worldline formalism to amplitudes in Scalar QED involving open scalar lines. At the tree-level, we obtain a compact Bern-Kosower type master formula for multiphoton Compton scattering, on- and off-shell.

Introduction

One of the main reasons to studying string theory is the fact that it provides us with an efficient mathematical framework to be applied to quantum field theory in the limit of infinite string tension. A systematic investigation of the this limit led Bern and Kosower to introduce a quick and useful way to calculate loop amplitudes in gauge theories by using 4-dimensional heterotic strings [1]. They introduce a master formula, known as the *Bern-Kosower master formula*, to study one-loop N-gluon amplitudes. Later, Strassler recalculated many of their results by using the worldline path integral and he found the same master formula without invoking string theory [2]. In the present work we apply the worldline formalism to the efficient construction of multi-photon amplitudes in scalar QED, both for on- and off-shell cases. Our main motivation and objective in mind is for on-shell amplitudes, the multi-photon generalizations of Compton scattering are becoming important these days for laser physics, see for example [3] and [4] for a review on high-intensity laser QED. We present a Bern-Kosower type master formula for multiphoton Compton scattering in Scalar QED.

Worldline formalism for scalar propagator

In this section, we discuss our method which is based on the worldline formalism, initially developed by Feynman in 1948 through the path integral approach to nonrelativistic quantum mechanics. It was later extended to relativistic quantum field theory, scalar QED and spinor QED. Our starting point is the basic expression for a scalar propagator of mass m, which propagates from point x' to x, in the presence of a background $U(1)$ gauge filed A,

$$\Gamma[x';x] = \int_0^\infty dT \mathrm{e}^{-m^2T} \int_{x(0)=x'}^{x(T)=x} \mathcal{D}x(\tau)\, \mathrm{e}^{-\int_0^T d\tau[\frac{1}{4}\dot{x}^2 + ie\dot{x}\cdot A(x(\tau))]}$$

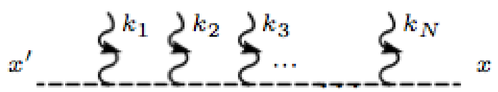
The path integral is computed by splitting $x_\mu(\tau)$ into a background part $x_{\mu\mathrm{bg}}(\tau)$, which encodes the boundary conditions, and a fluctuation part $q_\mu(\tau)$, which has Dirichelet boundary conditions at $\tau = 0, T$, where x and x' are two fixed points.

$$\begin{aligned} x(\tau) &= x_{\mathrm{bg}}(\tau) + q(\tau)\,, \\ x_{\mathrm{bg}}(\tau) &= \frac{(x-x')\tau}{T} + x'\,, \\ \dot{x}(\tau) &= \frac{x-x'}{T} + \dot{q}(\tau)\,, \\ q(0) &= q(T) = 0 \end{aligned}$$

The "worldline propagator" $\Delta(\tau_i,\tau_j)$ is defined as the Wick contraction of two quantum fields

$$\begin{aligned} \langle q^\mu(\tau_1) q^\nu(\tau_2)\rangle &= -2\delta^{\mu\nu}\Delta(\tau_1,\tau_2) \\ \Delta(\tau_1,\tau_2) &= \frac{\tau_1\tau_2}{T} + \frac{|\tau_1-\tau_2|}{2} - \frac{\tau_1+\tau_2}{2} \\ \Delta(\tau,\tau) &= \frac{\tau^2}{T} - \tau \end{aligned}$$

Multiphoton amplitude in scalar QED



The amplitude which is represented by the above diagram can be written as

$$\Gamma[x';x;k_1,\varepsilon_1;\cdots] = (-ie)^N \int_0^\infty dT \mathrm{e}^{-m^2T} \int_{x(0)=x'}^{x(T)=x} \mathcal{D}x(\tau)\, \mathrm{e}^{-\frac{1}{4}\int_0^T d\tau \dot{x}^2}$$
$$\times \int_0^T \prod_{i=1}^N d\tau_i \Big\langle V^A_{\mathrm{scal}}[k_1,\varepsilon_1]\cdots V^A_{\mathrm{scal}}[k_N,\varepsilon_N]\Big\rangle$$

where

$$V^A_{\mathrm{scal}}[k,\varepsilon] = \varepsilon_\mu \int_0^T d\tau \dot{x}^\mu(\tau) \mathrm{e}^{ik\cdot x(\tau)} = \int_0^T d\tau \mathrm{e}^{ik\cdot x(\tau)+\varepsilon\cdot\dot{x}(\tau)}\Big|_{\mathrm{lin}\ \varepsilon}$$

is the scalar vertex operator which represents the coupling of a photon with a scalar particle. In configuration space one gets

$$\Gamma[x';x;k_1,\varepsilon_1;\cdots] = (-ie)^N \int_0^\infty dT \mathrm{e}^{-m^2T} \mathrm{e}^{-\frac{1}{4T}(x-x')^2} (4\pi T)^{-\frac{D}{2}}$$
$$\int_0^T \prod_{i=1}^N d\tau_i\, \mathrm{e}^{\sum_{i=1}^N \left(\varepsilon_i\cdot\frac{(x-x')}{T} + ik_i\cdot(x-x')\frac{\tau_i}{T} + ik_i\cdot x'\right)}$$
$$\times \mathrm{e}^{\sum_{i,j=1}^N \left[\Delta_{ij} k_i\cdot k_j - 2i {}^\bullet\Delta_{ij}\varepsilon_i\cdot k_j - {}^\bullet\Delta^\bullet_{ij}\varepsilon_i\cdot\varepsilon_j\right]}\Big|_{\mathrm{lin}(\varepsilon_1\varepsilon_2\cdots)}$$

and in momentum space

$$\Gamma[p;p';k_1,\varepsilon_1;\cdots] = (-ie)^N (2\pi)^D \delta^D(p+p'+\sum_i k_i)$$
$$\times \int_0^\infty dT\, \mathrm{e}^{-T(m^2+p^2)} \int_0^T \prod_{i=1}^N d\tau_i\, \mathrm{e}^{\sum_{i=1}^N(-2k_i\cdot p\tau_i + 2i\varepsilon_i\cdot p)}$$
$$\times \mathrm{e}^{\sum_{i,j=1}^N \left[(\frac{|\tau_i-\tau_j|}{2} - \frac{\tau_i+\tau_j}{2})k_i\cdot k_j - i(\mathrm{sign}(\tau_i-\tau_j)-1)\varepsilon_i\cdot k_j + \delta(\tau_i-\tau_j)\varepsilon_i\cdot\varepsilon_j\right]}\Big|_{\mathrm{lin}(\varepsilon_1\varepsilon_2\cdots)}$$

The momentum space version of this master formula is implicit in [5].

Some special cases

- N=1

$$\Gamma[p;p';k_1,\varepsilon_1] = (-ie)(2\pi)^D\delta^D\Big(p+p'+k_1\Big) \int_0^\infty dT\, \mathrm{e}^{-T(m^2+p^2)}$$
$$\times \int_0^T d\tau_1\, \mathrm{e}^{(-2k_1\cdot p\tau_1 - k_1^2\tau_1 + i\varepsilon_1\cdot k_1 + 2i\varepsilon_1\cdot p + \delta(0)\varepsilon_1\cdot\varepsilon_1)}\Big|_{\text{linear in }\varepsilon_1}$$
$$= e(2\pi)^D\delta^D\big(p+p'+k_1\big)\frac{\varepsilon_1\cdot(p-p')}{(m^2+p^2)(m^2+k_1^2+p^2+2k_1\cdot p)}$$
$$= -e(2\pi)^D\delta^D\big(p'+p+k_1\big)\varepsilon_1\cdot(p'-p)$$

- N=2 (Compton scattering)

$$\Gamma[p';p;k_1,\varepsilon_1;k_2,\varepsilon_2] = 2e^2(2\pi)^D\delta^D\big(p'+p+k_1+k_2\big)$$
$$\times\Bigg\{\frac{(\varepsilon_1\cdot p+\varepsilon_1\cdot k_1)(2\varepsilon_2\cdot p+\varepsilon_2\cdot k_2) + (\varepsilon_1\cdot k_2)(2\varepsilon_2\cdot p+\varepsilon_2\cdot k_2)}{m^2+(k_1+p)^2}$$
$$+\frac{(\varepsilon_2\cdot k_2)(2\varepsilon_1\cdot p+\varepsilon_1\cdot k_1)}{m^2+(k_1+p)^2} - \varepsilon_1\cdot\varepsilon_2\Bigg\}\Big|_{\mathrm{off-shell}}$$

$$\Gamma[p';p;k_1,\varepsilon_1;k_2,\varepsilon_2] = -2e^2(2\pi)^D\delta^D\big(p'+p+k_1+k_2\big)$$
$$\times\Bigg\{\frac{(\varepsilon_1\cdot p')(\varepsilon_2\cdot p)}{p\cdot k_2} + \frac{(\varepsilon_1\cdot p)(\varepsilon_2\cdot p')}{p'\cdot k_2} + (\varepsilon_1\cdot\varepsilon_2)\Bigg\}\Big|_{\mathrm{on-shell}}$$

- N=3 (Nonlinear Compton scattering) $\cdots$

Conclusion and outlook

- Advanced technology especially in the field of laser physics renewed interest in non-linear Compton scattering.
- In this work we present a general Bern-Kosower-type formula by using worldline formalism for scalar QED.
- Our master formula:
  - Combines the various orderings.
  - The seagull vertex is implicit in the delta function appearing in the second derivative of Delta.
  - Being off-shell, the master formula can be used to construct higher-loop amplitudes.
- Extension to spinor QED, under study [6].
- Extension to non-abelian gauge theory (scalar case), to appear soon [7].

References

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