Quantum Kinematic Approach to Vacuum Polarization and Schwinger Effect

:Spin Resonance and Pair Production in Rotating Electric Fields

Sang Pyo Kim

Kunsan Nat'l Univ. & CoReLS, IBS*

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Outline

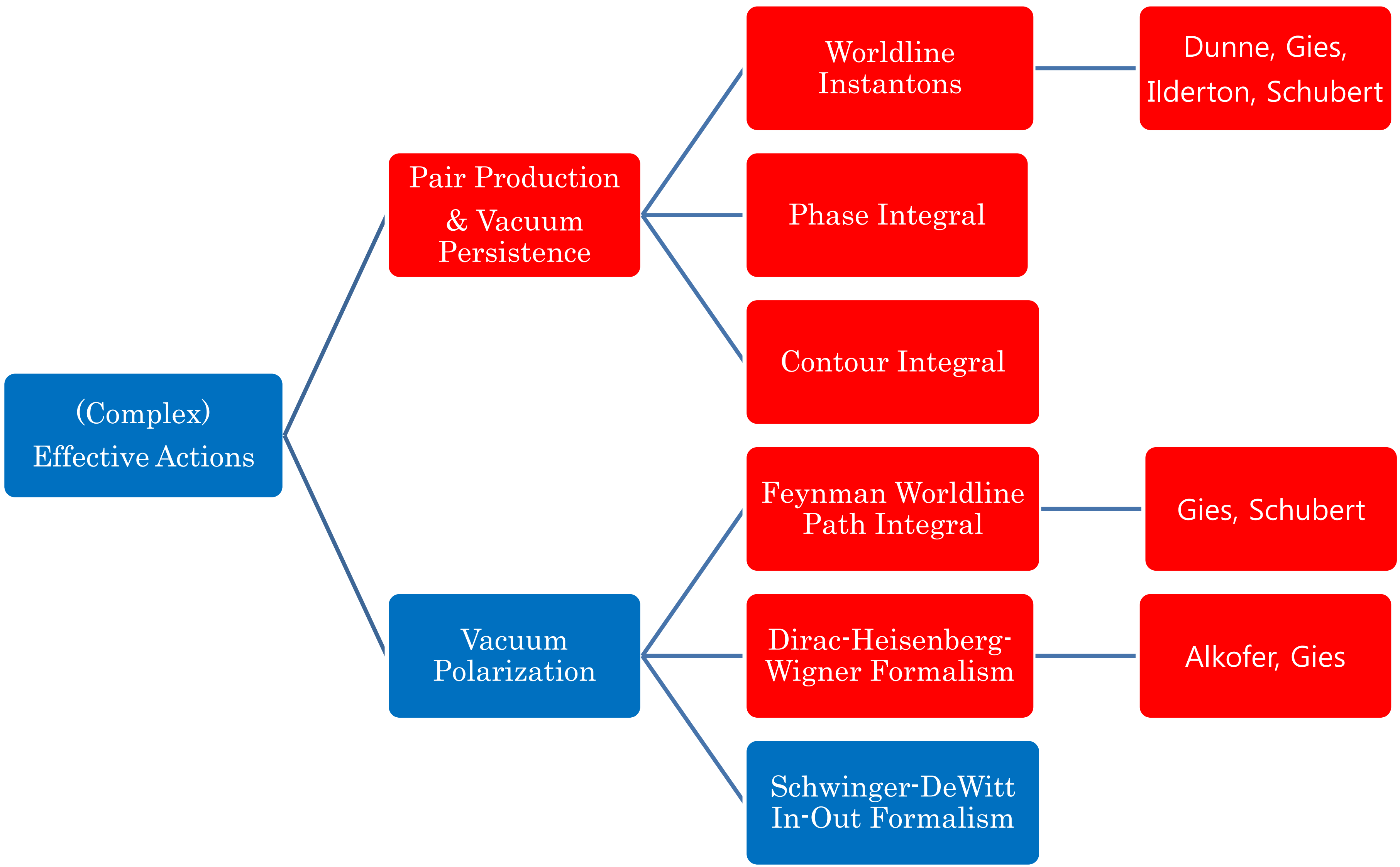
- How to find QED Action and Schwinger Effect in General EM Fields?
- Characterizing Quantum States in EM Fields
- Quantum Kinematic Approach to E(t) or B(t)
- Spin-Resonance and Pair Production
- Summary

QED Actions and Schwinger Effect in General EM Fields?

QED Actions in Some EM Fields

- One-Loop QED Actions
  - Constant EM
    - Heisenberg-Euler/ Schwinger
    - Weisskopf/ Schwinger
  - Sauter-type E or B Field
    - Cangemi, D'Hoker, Dunne, Hall
    - Kim, Lee, Yoon

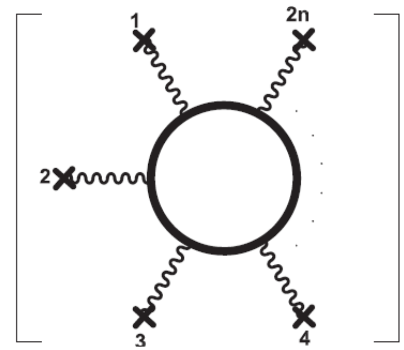
Schwinger Effects & QED Actions



In-Out Formalism for QED Actions

- In the in-out formalism, the vacuum persistence amplitude gives the effective action [Schwinger ('51); DeWitt ('75), ('03)] and is equivalent to the Feynman integral

$$e^{iW} = e^{i\int (-g)^{1/2} d^D x L_{\mathrm{eff}}} = \langle 0, \mathrm{out} \,|\, 0, \mathrm{in} \rangle = \sum_{n=1}^{\infty}$$



- The complex effective action and the vacuum persistence for particle production

$$\left| \langle 0, \mathrm{out} \,|\, 0, \mathrm{in} \rangle \right|^2 = e^{-2\,\mathrm{Im} W} \;, \quad 2\,\mathrm{Im} W = \pm VT \sum_{\mathrm{k}} \ln(1 \pm N_k)$$

QED Actions at T=0 & T

- Zero-temperature QED actions in proper-time integral via gamma-function regularization [SPK, Lee, Yoon, PRD (‘08), (‘10); SPK (‘11)]; gamma-function & zeta-function regularization [SPK, Lee (‘14)]; quantum kinematic approach [Bastianelli, SPK, Schubert, in preparation (‘15)]

$$W = \pm i \sum_{\mathrm{k}} \ln \alpha^{*}_{\mathrm{k}} = \pm i \sum_{l} \sum_{\mathrm{k}} \ln \Gamma\big(a_l + i b_l(\mathrm{k})\big)$$

- finite-temperature effective action [SPK, Lee, Yoon, PRD (‘09), (‘10)]

$$\exp\left[i \int d^3 x dt L_{\mathrm{eff}}\right] = \langle 0, \beta, \mathrm{in} | U^{+} | 0, \beta, \mathrm{in} \rangle = \frac{\mathrm{Tr}(U^{+} \rho_{\mathrm{in}})}{\mathrm{Tr}(\rho_{\mathrm{in}})}$$

Quantum Kinematic Approach

- Quantum kinematic (functional Schrodinger) approach to QED in homogeneous E(t) or B(t): time-dependent quadratic Hamiltonian (Fourier and/or Landau level decomposition)

$$H(t) = \sum_{\alpha\beta} Z_\alpha^+ H_{\alpha\beta}(t) Z_\beta \ , \ Z_\alpha^T = \left(\pi_\alpha , \phi_\alpha^*\right), \ \left[Z_\alpha , Z_\beta^*\right] = \sigma_2 \delta_{\alpha\beta}$$

- Quantum states via quantum invariants

$$\left|\Psi(t)\right\rangle = \sum_\lambda C_\lambda e^{-i\int^t dt' \langle \lambda, t' | H(t') - i\partial/\partial t' | \lambda, t' \rangle} \left|\lambda, t\right\rangle \ , \ I(t)\left|\lambda, t\right\rangle = \lambda\left|\lambda, t\right\rangle$$

- QED action in terms of the mode solutions

$$\langle \mathrm{out} | \mathrm{in} \rangle = \prod_\alpha \mathrm{Tr}\left(\Psi_{\alpha,0}^*[\varphi_{\mathrm{out}}(t)]\Psi_{\alpha,0}[\varphi_{\mathrm{in}}(t)]\right) \Leftrightarrow W \text{ and Worldline}$$

Characterizing Quantum States of Charge in EM Fields

Homogeneous EM Fields

- $E(t)$ along a fixed direction
  - Klein-Gordon: sum of harmonic oscillators with real time-dependent frequencies
  - Dirac: harmonic motion with complex frequencies due to spin states
- $B(t)$ with a fixed direction ($\vec{A} = \vec{B} \times \vec{r}/2$)
  - Klein-Gordon: Landau levels continuously change
  - Dirac: Landau levels and eigenspinors continuously change

More General EM Fields

- $\vec{E}(t)$ in 2 or 3 dimensions
  - Klein-Gordon: sum of harmonic oscillators with real time-dependent frequencies in each direction
  - Dirac: harmonic motion with complex frequencies due to spin states and eigenspinors continuously change
- $\vec{B}(t)$ in 2 or 3 dimensions?
  - Klein-Gordon: Landau levels and directions continuously change
  - Dirac: Landau levels, directions, and eigenspinors continously change
- Generic $\vec{A}(t, \vec{x})$ in 2 or 3 dimensions? Yes, convolution theorem in Fourier transformation.

Quantum Kinematic Approach to E(t) and B(t)

Wheeler-DeWitt (WDW) Equation vs Klein-Gordon (KG) Equation in B

- Wheeler-DeWitt equation for quantum cosmology of a Friedmann-Robertson-Walker universe with a massive scalar field & Cauchy initial value problem [SPK ('91); SPK, Page, PRD ('92); SPK, PRD ('92)]

$$\Big[-\underbrace{\pi_a^2}_{\text{time}} + \underbrace{V_G(a)}_{\text{time-dependent mass}} + \overbrace{\frac{1}{a^2}\left(\pi_\varphi^2 + a^6 m^2 \varphi^2\right)}^{\text{time-dependent oscillator}} \Big]\Psi(a,\phi) = 0$$

- Transverse motion of a charged scalar in time-dependent, homogeneous, magnetic field $\vec{A}(t,\vec{r}) = \vec{B}(t) \times \vec{r}/2$

$$\left[\frac{\partial^2}{\partial t^2} + \left(\vec{p}_\perp^{\,2} + \left(\frac{qB(t)}{2}\right)^2 \vec{x}_\perp^{\,2} - qB(t)L_z\right) + \left(m^2 + k_z^2\right)\right]\Phi_\perp(t,\vec{x}_\perp) = 0$$

Landau Levels in Scalar QED in B(t)

- Landau levels in $\vec{A}(t,\vec{r}) = \vec{B}(t) \times \vec{r}/2$ for a time-dependent magnetic field with a fixed direction continuously make transitions among themselves [SPK, AP 344 ('14)]:

$$\frac{d}{dt}\vec{\Phi}(t) = \Omega(t)\vec{\Phi}(t), \quad \vec{\Phi}(t) = \begin{pmatrix} |0,t\rangle \\ |1,t\rangle \\ \vdots \end{pmatrix}, \quad \Omega(t) = \frac{\dot{B}(t)}{4B(t)}\left(\hat{c}^2 - \hat{c}^{+2}\right)$$

- The Cauchy data for KG equation

$$\begin{pmatrix} \Psi_\perp(t,\vec{x}_\perp) \\ \dot{\Psi}_\perp(t,\vec{x}_\perp) \end{pmatrix} = \begin{pmatrix} \vec{\Phi}^T(t,\vec{x}_\perp) & 0 \\ 0 & \vec{\Phi}^T(t,\vec{x}_\perp) \end{pmatrix} \cdot T\exp\left[\int_{t_0}^{t} \begin{pmatrix} \Omega(t') & I \\ -\omega^2(t') & \Omega(t') \end{pmatrix} dt'\right] \begin{pmatrix} \vec{\Psi}_\perp(t_0) \\ \dot{\vec{\Psi}}_\perp(t_0) \end{pmatrix}$$

$$\omega^2(t) = qB(t)\left(2\hat{c}^+\hat{c} + 1\right) + m^2 + k_\parallel^2$$

Classification of Quantum Motions

- Dimensionless measure that characterizes the quantum motion of nth Landau level during any time interval (t_i, t_f)

$$R_n = \frac{(n/4)\left|\ln(B(t_f)/B(t_i))\right|}{\int_{t_i}^{t_f}\omega(t',n)dt'}$$

- Classification of quantum motions

$$\begin{cases} R_n << 1: \text{adiabatic motion} \\ R_n >> 1: \text{sudden change} \\ R_n \approx \mathrm{O}(1): \text{nonadiabatic change} \end{cases}$$

- Schwinger pair production due to the change of the in-vacuum or induced electric field

$$\left|\mathrm{in}, t_0\right\rangle = \frac{e^{-i\omega_{\mathrm{in}}t}}{\sqrt{2\omega_{\mathrm{in}}}}\vec{\Phi}(t_0)$$

Second Quantized Scalar QED in E(t)

- Scalar action in an EM field

$$S = \int dt d^3\vec{x} \left[\eta^{\mu\nu} \left(\partial_\mu + iqA_\mu \right) \phi^* \left(\partial_\mu - iqA_\mu \right) \phi - m^2 \phi^* \phi \right]$$

- Time-dependent Hamiltonian in E(t): time-dependent oscillators [SPK, AP 351 ('14)]

$$H(t) = \int \frac{d^3\vec{k}}{(2\pi)^3} \left[\pi^*_{\vec{k}} \pi_{\vec{k}} + \omega^2_{\vec{k}}(t) \phi^*_{\vec{k}} \phi_{\vec{k}} \right], \ \omega^2_{\vec{k}}(t) = \left(k_\parallel - qA_\parallel \right)^2 + \vec{k}^2_\perp + m^2$$

- Quantum invariant approach to quantum states, the in-vacuum and out-vacuum, and quantum Vlasov equation for Schwinger pair production [Schubert's talk]

Second Quantized Scalar QED in B(t)

- Time-dependent Hamiltonian in Landau levels in B(t): time-dependent coupled oscillators

$$H(t) = \int \frac{dk_{\parallel}}{2\pi} \left[\sum_{n} \pi_n^* \pi_n + \omega_n^2(t) \phi_n^* \phi_n + \sum_{mn} \pi_m^* \Omega_{mn} \phi_n + \pi_m \Omega_{mn} \phi_n^* \right]$$

$$\omega_n^2(t) = |qB(t)|(2n+1) + k_{\parallel}^2 + m^2$$

- Quantum invariant approach to annihilation and creation operators, quantum states, the in-vacuum and out-vacuum [SPK, AP 351 ('14)].
- Classification of quantum motions: (i) adiabatic change, (ii) sudden change and (iii) nonadiabatic change.

Quantum Invariants

- Time-dependent Hamiltonian in homogeneous E(t) or B(t)

$$H(t) = \sum_{\alpha\beta} Z_\alpha^+ H_{\alpha\beta}(t) Z_\beta \ , \ H_{\alpha\beta}(t) = \begin{pmatrix} \delta_{\alpha\beta} & \Omega_{\alpha\beta} \\ -\Omega_{\alpha\beta} & \omega_\alpha^2 \delta_{\alpha\beta} \end{pmatrix}$$

$$Z_\alpha = \begin{pmatrix} \pi_\alpha \\ \phi_\alpha^* \end{pmatrix}, Z_\alpha^* = \begin{pmatrix} \pi_\alpha^* \\ \phi_\alpha \end{pmatrix}, \left[Z_\alpha, Z_\beta^*\right] = \sigma_2 \delta_{\alpha\beta}, \left[Z_\alpha, Z_\beta\right] = \left[Z_\alpha^*, Z_\beta^*\right] = 0$$

- Quantum invariants for annihilation and creation operators [Lewis, Riesenfeld ('69); SPK, Page ('01)]

$$i\frac{\partial I(t)}{\partial t} + [I(t), H(t)] = 0, \ I = \left(U(t), V(t)\right)\frac{Z + Z^*}{\sqrt{2}}$$

- Quantum states

$$\left|\Psi(t)\right\rangle = \sum_\lambda C_\lambda e^{-i\int^t dt' \langle \lambda, t' | H(t') - i\partial/\partial t' | \lambda, t' \rangle} \left|\lambda, t\right\rangle \ , \ I(t)\left|\lambda, t\right\rangle = \lambda \left|\lambda, t\right\rangle$$

Spin Resonance and Pair Production in Rotating E Fields

with Chul Min Kim (CoReLS/IBS)

Rotating Electric Fields

- Pair production in a rotating E field in Dirac-Heisenberg-Wigner (DHW) formalism [Blinne, Gies, PRD ("14); Blinne, poster ExHILP], Dirac in WKB approximation [Strobel, Xue, NPB ('14); PRD ('15); Strobel, poster ExHILP]:

$$\vec{E}(t) = \left(E(t)\cos\Omega t,\ E(t)\sin\Omega t,\ 0\right)$$

- Spin-diagonal, two-component, second-order equation for Dirac equation

$$\left[\left(\frac{\partial^2}{\partial t^2} + \left(i\vec{\nabla} + e\vec{A}(t)\right)^2 + m^2\right) I_{4\times 4} + ie\begin{pmatrix} 0 & \vec{\sigma}\cdot\vec{E}(t) \\ \vec{\sigma}\cdot\vec{E}(t) & 0 \end{pmatrix}\right]\Psi(t,x) = 0$$

Instantaneous Eigenspinors

- One set of eigenspinors v_λ with eigenvalue i^λ:

$$v_0(t) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\Omega t/2} \\ 0 \\ 0 \\ e^{i\Omega t/2} \end{pmatrix}, \quad v_2(t) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\Omega t/2} \\ 0 \\ 0 \\ -e^{i\Omega t/2} \end{pmatrix}$$

- Another set of eigenspinors v_λ with eigenvalue $i^{\lambda-1}$:

$$v_1(t) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ e^{i\Omega t/2} \\ e^{-i\Omega t/2} \\ 0 \end{pmatrix}, \quad v_3(t) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ e^{i\Omega t/2} \\ -e^{-i\Omega t/2} \\ 0 \end{pmatrix}$$

Transformation of Eigenspinors

- Orthonormality $v_i{}^+ v_j = \delta_{ij}$ and the rate of the change of eigenspinors

$$\frac{d}{dt}\begin{pmatrix} v_0(t) \\ v_2(t) \end{pmatrix} = -i\frac{\Omega}{2}\sigma_1\begin{pmatrix} v_0(t) \\ v_2(t) \end{pmatrix}, \quad \frac{d}{dt}\begin{pmatrix} v_1(t) \\ v_3(t) \end{pmatrix} = i\frac{\Omega}{2}\sigma_1\begin{pmatrix} v_1(t) \\ v_3(t) \end{pmatrix}$$

- Expand the Dirac spinor by eigenspinors and Fourier component

$$\Psi(t,\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^2} e^{i\vec{k}\cdot\vec{x}} \vec{v}^{\,T}(t)\cdot S^+(t)\cdot \vec{\varphi}_{\vec{k}}(t)$$

$$\vec{v}(t) = \begin{pmatrix} v_0(t) \\ v_2(t) \end{pmatrix}, \quad S(t) = e^{-i\frac{\Omega}{2}\sigma_1(t-t_0)}, \quad \vec{\varphi}_{\vec{k}}(t) = \begin{pmatrix} \varphi_{0\vec{k}}(t) \\ \varphi_{2\vec{k}}(t) \end{pmatrix}$$

Evolution Equations

- Evolution equation for the Fourier component

$$\begin{pmatrix} \vec{\varphi}_{\vec{k}}(t) \\ \dot{\vec{\varphi}}_{\vec{k}}(t) \end{pmatrix} = S(t) \cdot T \exp\left[\int_{t_0}^{t} \begin{pmatrix} i\Omega\sigma_1/2 & I_{2\times 2} \\ -\pi_{\vec{k}}^2(t') & i\Omega\sigma_1/2 \end{pmatrix} dt' \right] \begin{pmatrix} \vec{\varphi}_{\vec{k}}(t_0) \\ \dot{\vec{\varphi}}_{\vec{k}}(t_0) \end{pmatrix}$$

$$\pi_{\vec{k}}^2(t) = \left[\left(\vec{k}_\perp - e\vec{A}(t) \right)^2 + k_z^2 + m^2 \right] I_{2\times 2} + ieE(t)\sigma_3$$

- Evolution equation for the Dirac spinor

$$\begin{pmatrix} \Psi_{\vec{k}}(t) \\ \dot{\Psi}_{\vec{k}}(t) \end{pmatrix} = \vec{v}^{+}(t) \cdot T \exp\left[\int_{t_0}^{t} \begin{pmatrix} i\Omega\sigma_1/2 & I_{2\times 2} \\ -\pi_{\vec{k}}^2(t') & i\Omega\sigma_1/2 \end{pmatrix} dt' \right] \begin{pmatrix} \vec{\varphi}_{\vec{k}}(t_0) \\ \dot{\vec{\varphi}}_{\vec{k}}(t_0) \end{pmatrix}$$

Two-Dimensional Electric Fields

- Two-dimensional, homogeneous, time-dependent electric field
  $\vec{E}(t) = \left(E_x(t),\ E_y(t),\ 0\right),\ \ \varepsilon(t) = E_x(t) + iE_y(t) = E(t)e^{i\theta(t)}$
- Eigenspinors

$$v_0(t) = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta(t)/2} \\ 0 \\ 0 \\ e^{i\theta(t)/2} \end{pmatrix}, v_2(t) = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta(t)/2} \\ 0 \\ 0 \\ -e^{i\theta(t)/2} \end{pmatrix}$$

$$v_1(t) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ e^{i\theta(t)/2} \\ e^{-i\theta(t)/2} \\ 0 \end{pmatrix}, v_3(t) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ e^{i\theta(t)/2} \\ -e^{-i\theta(t)/2} \\ 0 \end{pmatrix}$$

Conclusion

- Quantum kinematic approach to charged scalar field in time-dependent electric or magnetic fields.
  - $E(t)$: sum of time-dependent oscillators
  - $B(t)$: sum of time-dependent coupled oscillators
- Rotating or two-dimensional electric field induces continuously changing eigenspinors and leads to a spin-resonance effect.
- Quantum kinematic approach may apply to Klein-Gordon or Dirac equation in $\vec{A}(t, \vec{x})$ with proper modifications and a hope is that it may be a new facet, not extravagance, to strong QED phenomena.