

# Strong-field pair production: An introduction with applications to two-color laser fields

**Carsten Müller**

*Institut für Theoretische Physik I  
Heinrich-Heine-Universität Düsseldorf*



# Outline

- **Introduction:**

From quantum mechanics to QED

From “ordinary” QED to strong-field QED

- **$e^+e^-$  pair creation in strong laser fields:**

The three “standard” mechanisms

Theoretical description and experimental observations

Recent developments: More complex field configurations

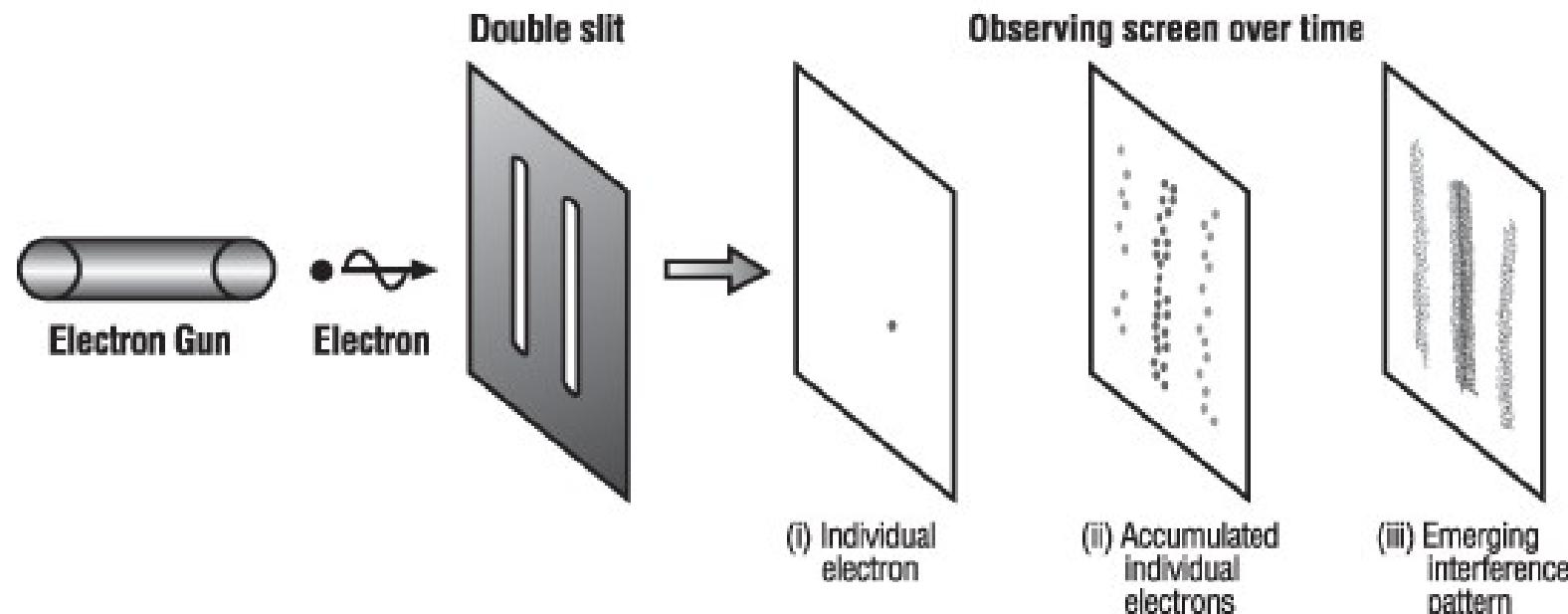
- **$e^+e^-$  pair creation in two-color laser fields:**

Further variants and getting back to where we started

# **Introduction part I:**

From quantum mechanics to QED

# The double-slit experiment with electrons



Double-slit apparatus showing the pattern of electron hits on the observing screen building up over time.

© Perimeter Institute

$$W = |A_1 + A_2|^2$$

**Wave-particle duality  
2-pathway interference**

Davisson & Germer (1927)

“God runs electromagnetics on Monday, Wednesday, and Friday by the wave theory,  
and the devil runs it by quantum theory on Tuesday, Thursday, and Saturday.”

(Sir Lawrence Bragg)

“God runs electromagnetics on Monday, Wednesday, and Friday by the wave theory,  
and the devil runs it by quantum theory on Tuesday, Thursday, and Saturday.”

(Sir Lawrence Bragg)

“Der Gedanke, daß ein Elektron aus freiem Entschluß den Augenblick und die Richtung wählt, in der es fortspringen will, ist mir unerträglich. Wenn schon, dann möchte ich lieber Schuster oder gar Angestellter in einer Spielbank sein als Physiker.”

(Albert Einstein)

“God does not play dice with the universe.” (Albert Einstein)

“God runs electromagnetics on Monday, Wednesday, and Friday by the wave theory, and the devil runs it by quantum theory on Tuesday, Thursday, and Saturday.”

(Sir Lawrence Bragg)

“Der Gedanke, daß ein Elektron aus freiem Entschluß den Augenblick und die Richtung wählt, in der es fortspringen will, ist mir unerträglich. Wenn schon, dann möchte ich lieber Schuster oder gar Angestellter in einer Spielbank sein als Physiker.”

(Albert Einstein)

“God does not play dice with the universe.” (Albert Einstein)

“Einstein, stop telling God what to do with his dice!” (Niels Bohr)

“God runs electromagnetics on Monday, Wednesday, and Friday by the wave theory, and the devil runs it by quantum theory on Tuesday, Thursday, and Saturday.”

(Sir Lawrence Bragg)

“Der Gedanke, daß ein Elektron aus freiem Entschluß den Augenblick und die Richtung wählt, in der es fortspringen will, ist mir unerträglich. Wenn schon, dann möchte ich lieber Schuster oder gar Angestellter in einer Spielbank sein als Physiker.”

(Albert Einstein)

“God does not play dice with the universe.” (Albert Einstein)

“Einstein, stop telling God what to do with his dice!” (Niels Bohr)

We used to think that if we knew one, we knew two, because one and one are two.  
We are finding that we must learn a great deal more about 'and'.

(Sir Arthur Eddington)

# Development of quantum theory in the 1920s

**Schrödinger equation** (1926)

→ Hydrogen atom

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{1}{2m} \left( \hat{\vec{p}} - \frac{e}{c} \vec{A} \right)^2 \Psi$$

Quantized radiation field (1927):

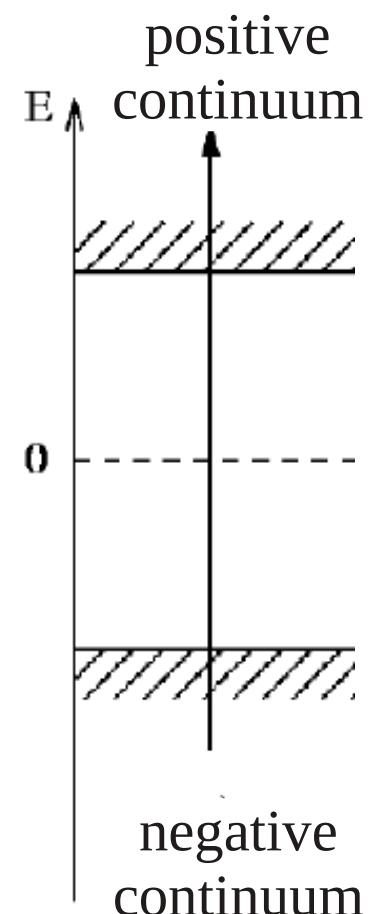
→ absorption & emission of **photons**

$$\hat{\mathbf{A}}_\gamma(\mathbf{r}, t) = \sum_{\mathbf{k}, \rho} \sqrt{\frac{2\pi c^2}{V \omega_k}} \mathbf{e}_{\mathbf{k}, \rho} (c_{\mathbf{k}, \rho}^+ e^{i(\omega_k t - \mathbf{k} \cdot \mathbf{r})} + C.C.)$$

Relativistic generalization:

**Dirac equation** (1928) → antimatter

$$(ic\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu - mc^2) \Psi = 0$$



# Development of QED in the 1940s

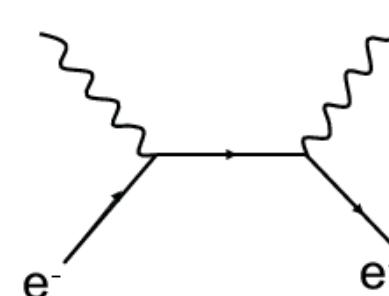
Tomonaga, Schwinger, Feynman, Dyson (1948/49):

- Second-quantize fermion field to describe creation/annihilation of  $e^+$  and  $e^-$
- Introduce propagators (2-point correlation functions of field operators):

$$\left[ \gamma^\mu \left( i\hbar\partial'_\mu - \frac{e}{c} A_\mu(x') \right) - mc \right] \underbrace{\mathcal{G}(x'; x)}_{4 \times 4\text{-Matrix}} = \hbar \delta^4(x' - x) \cdot \mathbb{1}$$

Evolution of fermion state:

$$\Psi_i^{(+)}(x') = \Phi_i(x') + \frac{e}{\hbar c} \int d^4x \mathcal{G}_0(x' - x) (\gamma A(x)) \Psi_i^{(+)}(x)$$

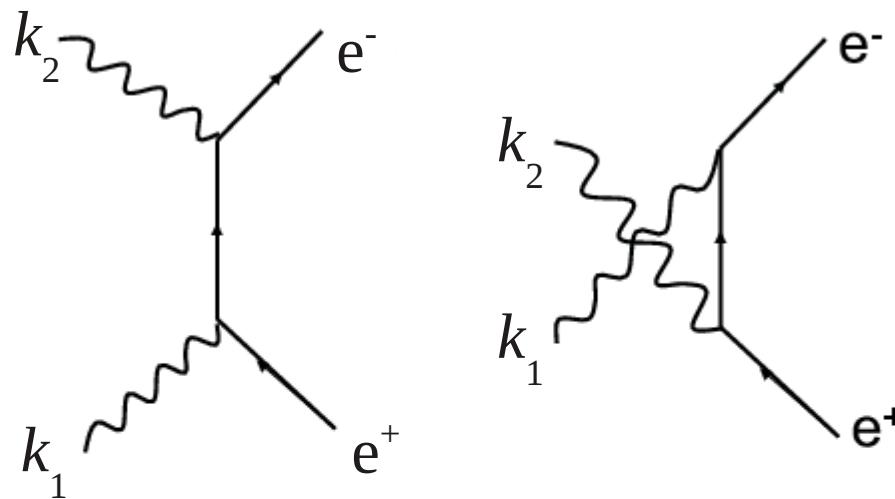


Photon-electron scattering:

# Example: $e^+e^-$ pair creation from photons

[Breit & Wheeler (1934)]

$$(k_1 + k_2)^2 \geq (2mc)^2$$



**NOTE that single photon cannot create pairs**

Transition amplitude in momentum space

$$\begin{aligned} S_{\text{fi}} = & -i \left( \frac{e}{\hbar c} \right)^2 \frac{2\pi\hbar c^2}{V\sqrt{\omega_1\omega_2}} \overline{u^{(+)}(\vec{p}_-, s_-)} \left\{ (\gamma\varepsilon_2) \mathcal{G}_0(p_- - \hbar k_2) (\gamma\varepsilon_1) \right. \\ & \quad \left. + (\gamma\varepsilon_1) \mathcal{G}_0(p_- - \hbar k_1) (\gamma\varepsilon_2) \right\} u^{(-)}(-\vec{p}_+, -s_+) \\ & \times (2\pi\hbar)^4 \delta^{(4)}(p_- + p_+ - \hbar k_1 - \hbar k_2) \end{aligned}$$

# **Introduction part II:**

From QED to strong-field QED

# Development of strong-field QED in the 1960s

Maiman (1960): “Stimulated optical radiation in ruby”



Franken et al. (1961): First observation of 2nd harmonic  
“...exploiting extraordinary ruby laser intensities of  $10^6 \text{ W/cm}^2$  “

Intriguing question for theory:

How to generalize newly established QED to electron-laser interactions?

# Development of strong-field QED in the 1960s

Maiman (1960): “Stimulated optical radiation in ruby”

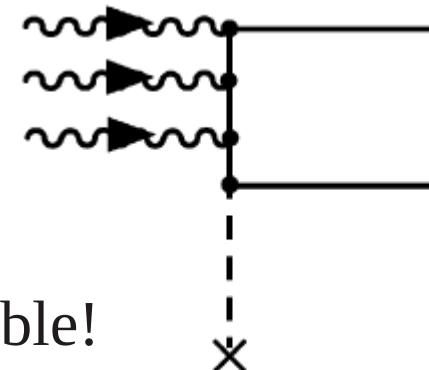


Franken et al. (1961): First observation of 2nd harmonic  
“...exploiting extraordinary ruby laser intensities of  $10^6 \text{ W/cm}^2$  “

Intriguing question for theory:

How to generalize newly established QED to electron-laser interactions?

In principle, one can calculate in ordinary QED processes with  $N > 1$  photons, but becomes tedious very quickly...



Besides, perturbative expansion not always applicable!

# Furry picture (1951)

$$\left[ i c \gamma^\mu \partial_\mu - e \gamma^\mu \left( A_\mu + A_\mu^{(\text{ext})} \right) - mc^2 \right] \Psi = 0$$

**Ordinary QED:** Let free states interact with  $A_\mu + A_\mu^{(\text{ext})}$

# Furry picture (1951)

$$\left[ i c \gamma^\mu \partial_\mu - e \gamma^\mu \left( A_\mu + A_\mu^{(\text{ext})} \right) - mc^2 \right] \Psi = 0$$

**Ordinary QED:** Let free states interact with  $A_\mu + A_\mu^{(\text{ext})}$

Alternatively, IF you can solve

$$\left( i c \gamma^\mu \partial_\mu - e \gamma^\mu A_\mu^{(\text{ext})} - mc^2 \right) \Psi = 0$$

you may use these solutions as basis states which interact with  $A_\mu$  only!

→ **Strong-field QED**

(also applied in “bound-state QED” to calculate, e.g., Lamb shift in high-Z ions)

# Strong-field QED in laser physics

Dirac equation: 
$$\left( i c \gamma^\mu \partial_\mu - e \gamma^\mu A_\mu^{(\text{ext})} - mc^2 \right) \Psi = 0$$

  
**classical plane wave**  
**relative coupling strength  $\xi = eA^{(\text{ext})}/mc^2$**

# Strong-field QED in laser physics

Dirac equation:  $\left( i c \gamma^\mu \partial_\mu - e \gamma^\mu A_\mu^{(\text{ext})} - mc^2 \right) \Psi = 0$

  
**classical plane wave**  
**relative coupling strength  $\xi = eA^{(\text{ext})}/mc^2$**

**Volkov solutions (1935):**

$$\Psi_{p,s}(x) = \left( 1 - \frac{e k A}{2c(kp)} \right) u_{p,s} e^{-i(px)} e^{i f(x)}$$

  
**free solution**

$$f(x) = \frac{e}{c(kp)} \int^{(kx)} \left[ p \cdot A(\eta) + \frac{e}{2c} A^2(\eta) \right] d\eta$$

# How to formulate your transition amplitude

Example: Strong-field Breit-Wheeler process  
(pair production by  $\gamma$ -photon + laser wave)

A Feynman diagram illustrating the Strong-field Breit-Wheeler process. It shows a wavy line representing a quantized  $\gamma$ -photon interacting with a Volkov electron (represented by a solid line) to produce a Volkov positron (another solid line). The diagram is annotated with arrows pointing to each particle type: a red arrow points to the Volkov electron, a blue arrow points to the quantized  $\gamma$ -photon, and another red arrow points to the Volkov positron.

$$S_{fi} = -\frac{ie}{\hbar} \int d^4x \overline{\Psi}_{p-,s-} (\gamma^\mu A_\mu) \Psi_{p+,s+}$$

Volkov electron      Quantized  $\gamma$ -photon      Volkov positron

Produce pair in photon-multiphoton collision:  $(k_\gamma + nk_L)^2 \geq (2m_*c)^2$

# Why laser field not quantized?

Volkov states can also be formulated for a quantized laser field.

However, one can show that in the limit of

- **large initial laser photon numbers**  
and
- **small depletion** of the laser wave

quantum and classical descriptions coincide ( $\rightarrow$  **external-field approx.**)

Note that 800 nm laser pulse of 1 J energy contains  $\sim 10^{20}$  photons

Bergou & Varro, JPA (1980) + (1981)

# $e^+e^-$ pair creation in strong laser fields

# Typical energy scales in laser physics

Photon energy	Electric work	Ponderomotive energy
$\hbar\omega$	$eE\Delta r$	$U_p = e^2 E^2 / 4m\omega^2$

Efficient coupling of a laser field with a quantized system is possible when its level spacing  $\Delta\epsilon$  compares with one of these scales:

$\Delta\epsilon \sim \hbar\omega$  Resonant (multiphoton) transition

$\Delta\epsilon \sim eE\Delta r$  Quasistatic tunneling process

$\Delta\epsilon \sim U_p$  Fast electron-induced reaction

# Typical energy scales in laser physics

Photon energy	Electric work	Ponderomotive energy
---------------	---------------	----------------------

$$\hbar\omega$$

$$eE\Delta r$$

$$U_p = e^2 E^2 / 4m\omega^2$$

Efficient coupling of a laser field with a quantized system is possible when its level spacing  $\Delta\epsilon$  compares with one of these scales:

$$\Delta\epsilon \sim \hbar\omega$$

Resonant (multiphoton) transition

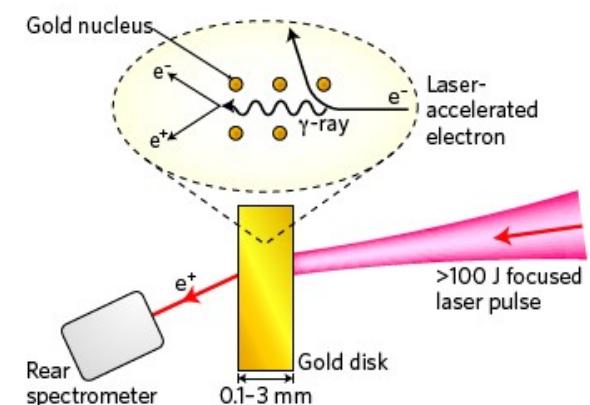
$$\Delta\epsilon \sim eE\Delta r$$

Quasistatic tunneling process

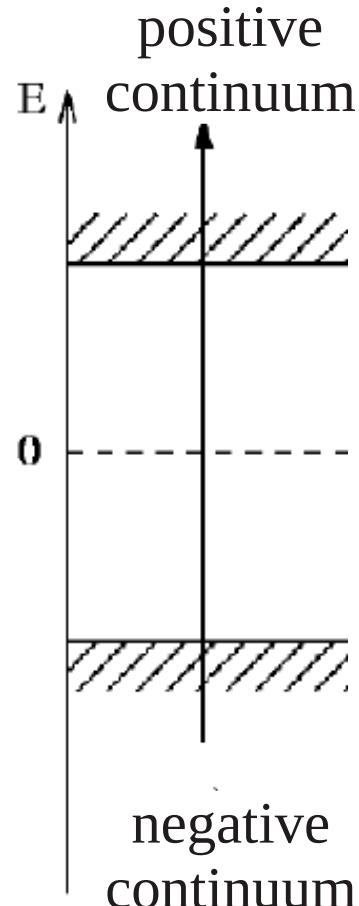
$$\Delta\epsilon \sim U_p$$

Fast electron-induced reaction

**Pair creation** experiments by powerful laser-solid interaction (Gahn/Chen/Sarri) where  $\Delta\epsilon \sim 1 \text{ MeV}$  have relied on  $U_p$  of secondary electrons



# Direct laser-induced $e^+e^-$ pair creation from vacuum?



Pair creation requires

$$\hbar\omega \approx mc^2 \sim 1 \text{ MeV}$$

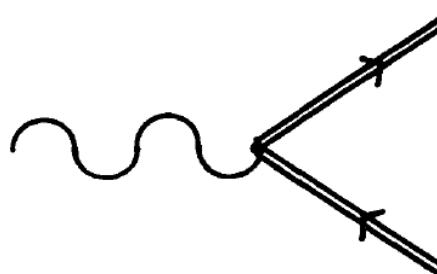
or

$$E \approx E_{cr} = mc^2/e\lambda_C \approx 10^{16} \text{ V/cm}$$

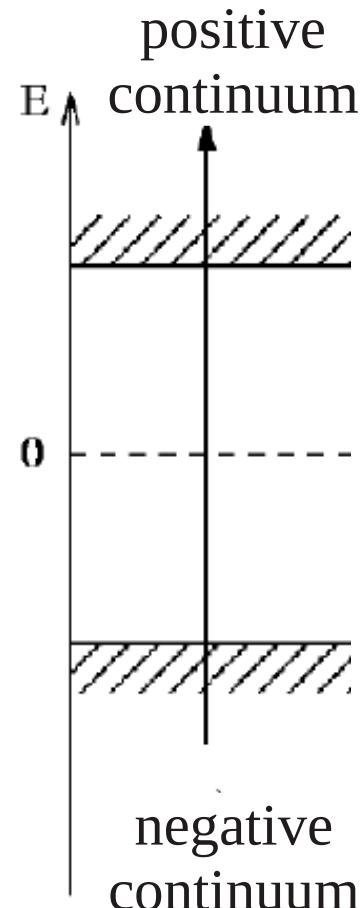
A single (plane) laser wave  
does not polarize the vacuum

since  $\mathbf{E}^2 - \mathbf{B}^2 = \mathbf{E}\mathbf{B} = 0$

(J. Schwinger, 1951)



# Direct laser-induced $e^+e^-$ pair creation from vacuum?



Pair creation requires

$$\hbar\omega \approx mc^2 \sim 1 \text{ MeV}$$

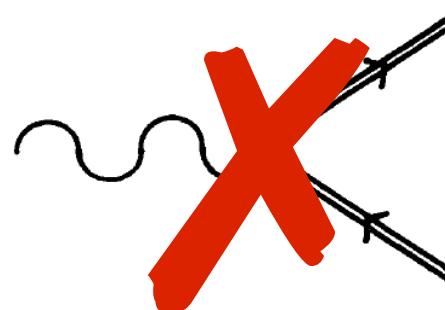
or

$$E \approx E_{cr} = mc^2/e\lambda_C \approx 10^{16} \text{ V/cm}$$

A single (plane) laser wave  
does not polarize the vacuum

since  $E^2 - B^2 = EB = 0$

(J. Schwinger, 1951)



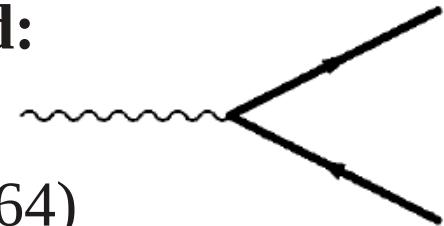
# Early theoretical investigations:

## The three “standard” mechanisms

**Pair creation by high-energy photon + laser field:**

Reiss, J. Math. Phys. 3, 59 (1962)

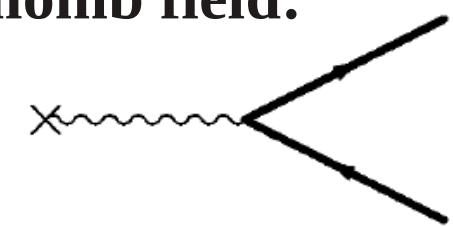
Nikishov & Ritus, Zh. Eksp. Teor. Fiz. 46, 776 (1964)



**Pair creation in combined laser and nuclear Coulomb field:**

Yakovlev, Zh. Eksp. Teor. Fiz. 49, 318 (1965)

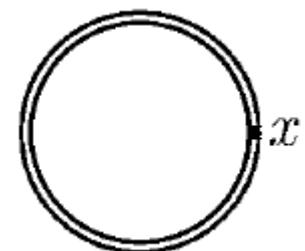
Mittleman, Phys. Rev. A 35, 4624 (1987)



**Pair creation in a standing laser wave:**

Brezin & Itzykson, Phys. Rev. D 2, 1191 (1970)

Popov, Pis'ma Zh. Eksp. Teor. Fiz. 13, 261 (1971)

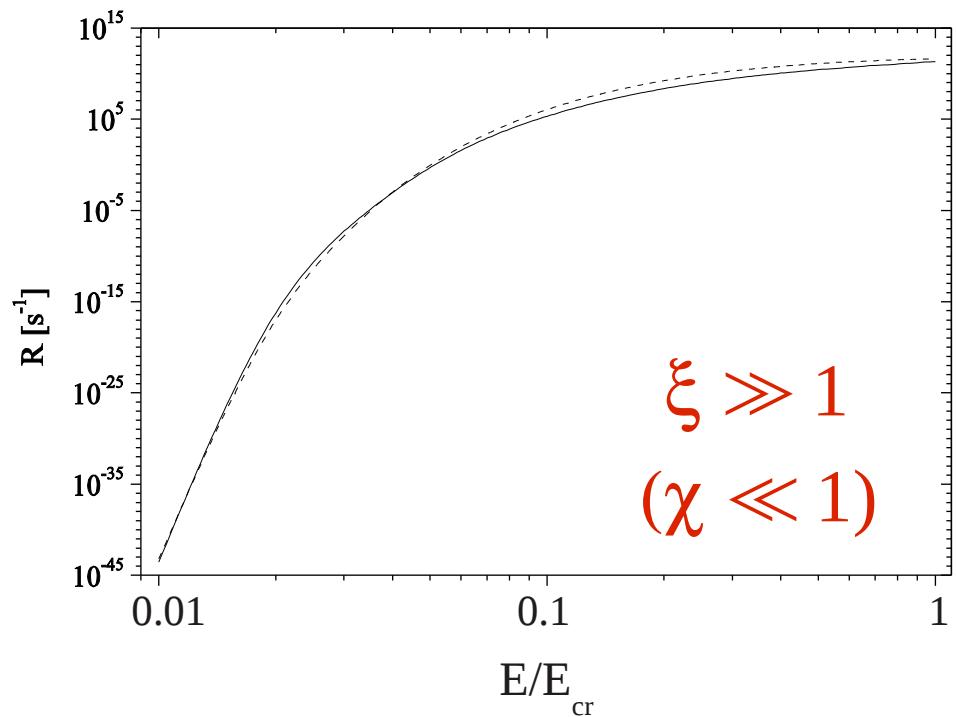


# Tunneling and multiphoton pair creation

inverse Keldysh parameter for pair creation:

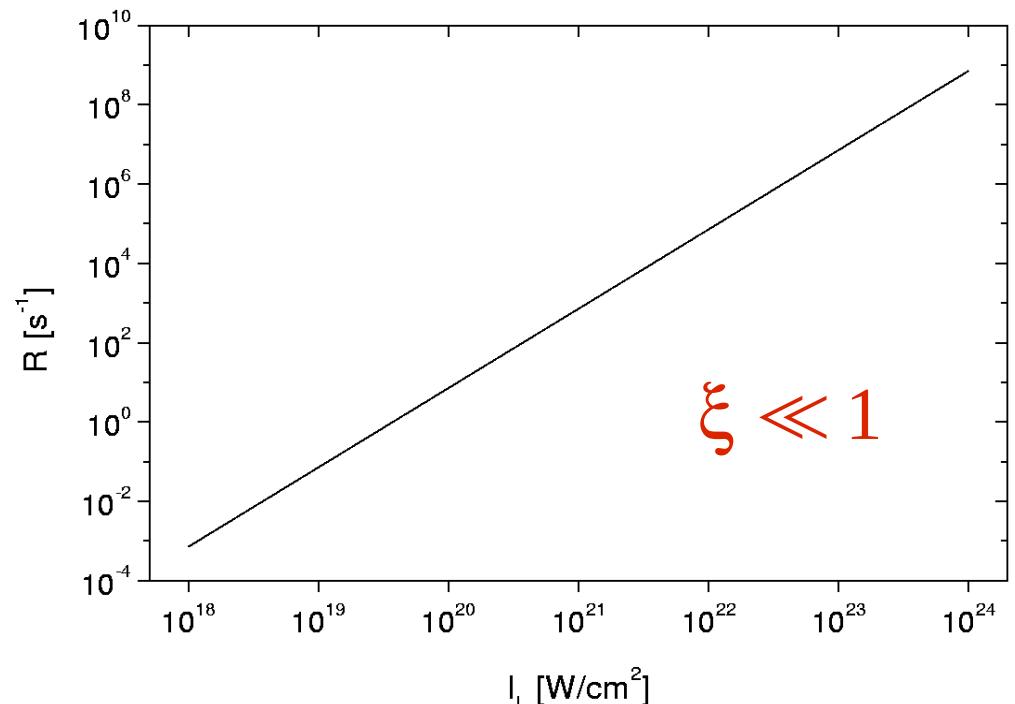
$$\xi = eA / mc^2$$

Tunneling rate:  $R \sim \exp(-E_{\text{cr}}/E)$



Nonperturbative tunneling regime

$n$ -photon rate:  $R \sim \xi^{2n} \sim I^n$

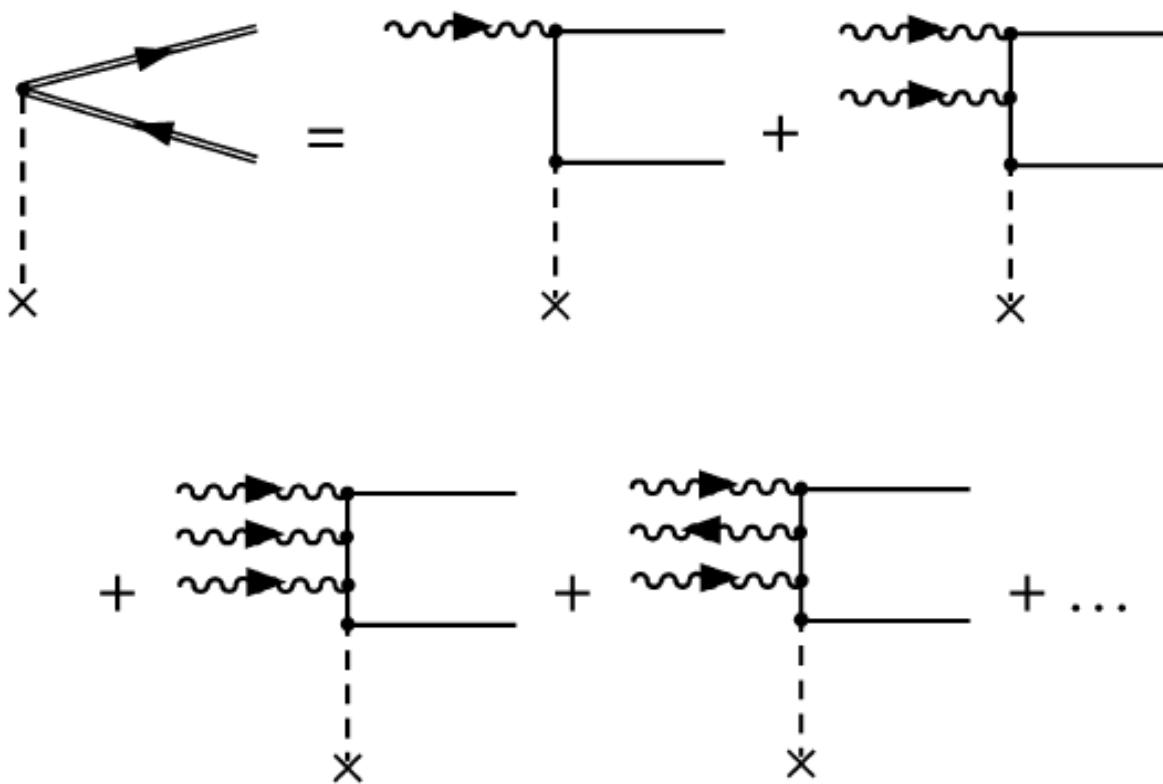


Perturbative multiphoton regime

# Example of pair creation amplitude

$$S_{fi} = -\frac{ie}{\hbar} \int d^4x \overline{\Psi}_{p_-, s_-} (\gamma^\mu A_\mu) \Psi_{p_+, s_+}$$

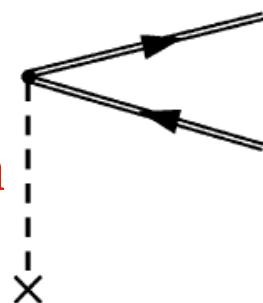
Furry-Feynman  
diagram:



In the weak-field limit, equivalent to perturbative expansion in  $\xi = eA/mc^2 \ll 1$

# Example of pair creation amplitude

$$S_{fi} = -\frac{ie}{\hbar} \int d^4x \overline{\Psi}_{p_-, s_-} (\gamma^\mu A_\mu) \Psi_{p_+, s_+}$$



Furry-Feynman  
diagram:

Plug in Volkov states and  
Fourier-expand oscillating terms:

$$S_{fi} \sim \sum_n \mathcal{M}_n(p_+, p_-) \int d^4x e^{i(q_+ + q_- - k_\gamma - nk) \cdot x}$$

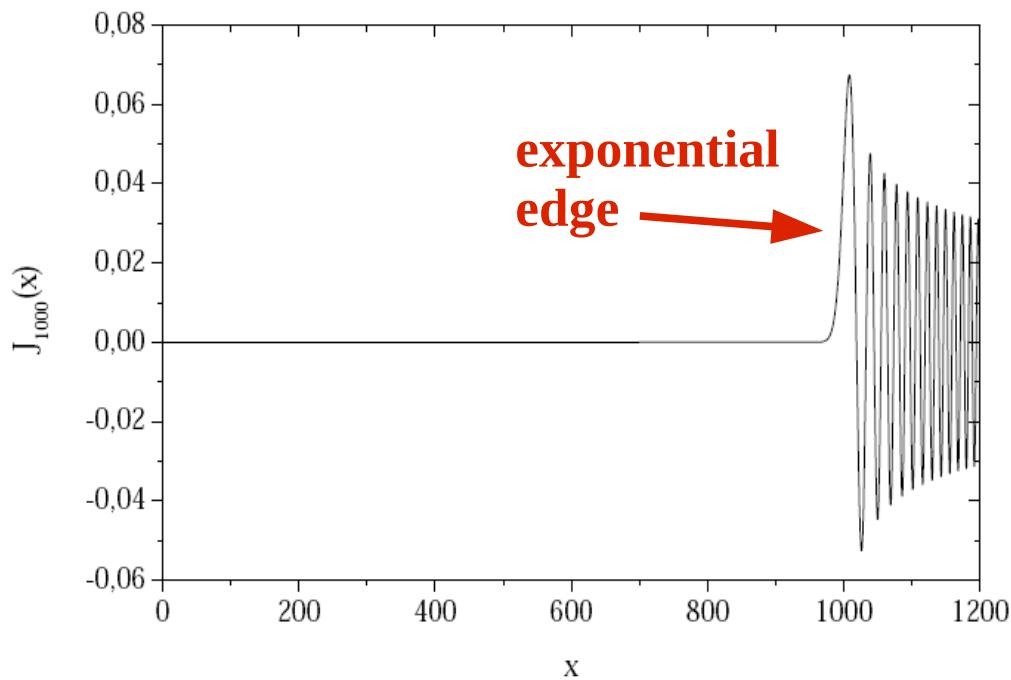
summation index counts  
number of laser photons!

$$\delta(q_+^\mu + q_-^\mu - k_\gamma^\mu - nk^\mu)$$

# Fourier coefficients = Bessel functions

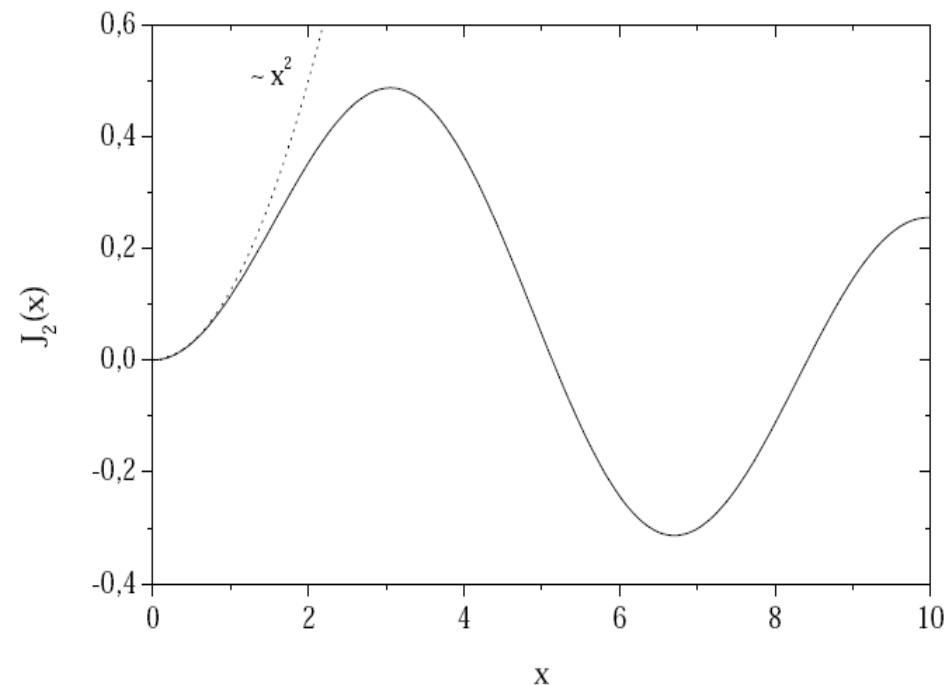
$$e^{if(x)} = \sum_n J_n(\alpha) e^{in(kx)}$$

number of photons!



tunneling process

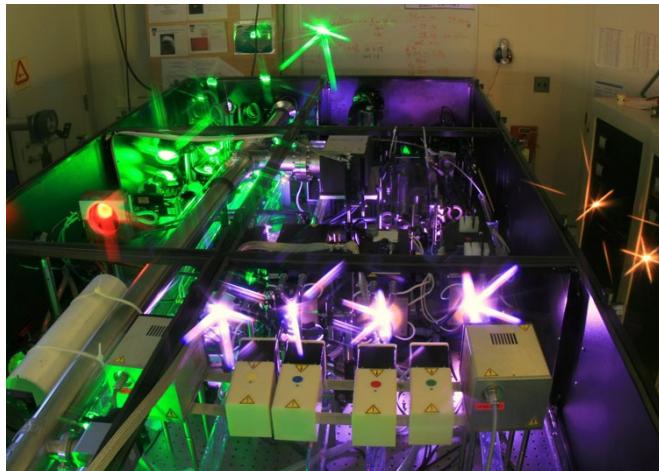
rate  $\sim \exp(-E_{\text{cr}}/E_L)$



multiphoton process

rate  $\sim I^n$

# Can it be measured?



HERCULES Petawatt laser:  
 $10^{22} \text{ W/cm}^2$  at 800 nm

$$E \sim 10^{-4} E_{\text{cr}}$$

Tunneling regime:

$$R \sim \exp(-E_{\text{cr}} / E) \sim 10^{-5000}$$



Free Electron Laser FLASH:  
 $100 \text{ eV}$  at  $10^{17} \text{ W/cm}^2$

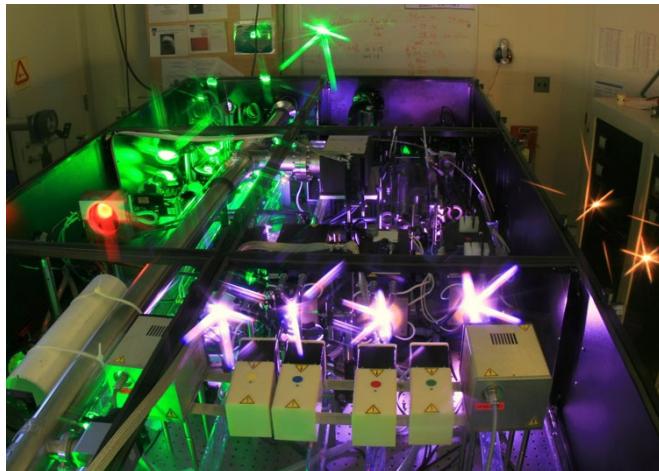
$$\hbar\omega \sim 10^{-4} mc^2$$

Multiphoton regime:

$$R \sim \xi^{4mc^2/\hbar\omega} \sim 10^{-100000}$$

Available laser field strengths and frequencies by 4 orders too small...

# Can it be measured?



HERCULES Petawatt laser:  
 $10^{22} \text{ W/cm}^2$  at 800 nm

$$E \sim 10^{-4} E_{\text{cr}}$$



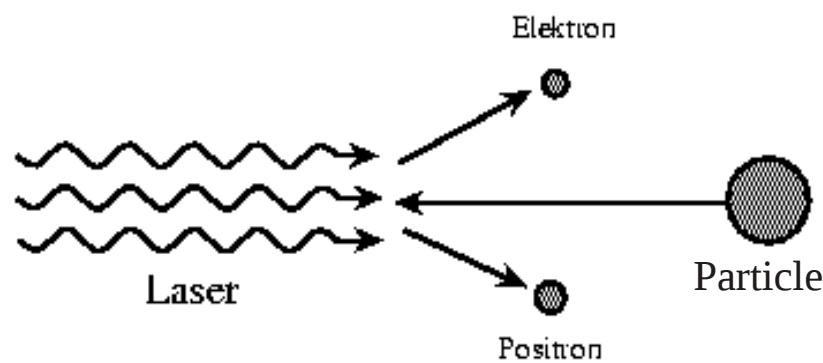
Free Electron Laser FLASH:  
 $100 \text{ eV}$  at  $10^{17} \text{ W/cm}^2$

$$\hbar\omega \sim 10^{-4} mc^2$$

*“The cross section for this process at optical frequencies or below is so small at any laser intensity as to make it completely negligible. It may be the smallest (nonzero) cross section on record.”*

(M. Mittleman, 1987)

# Relativistic particle beam colliding with laser pulse

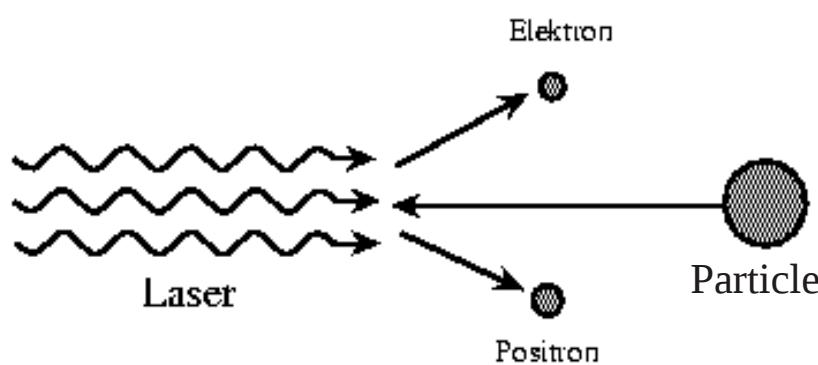


**Exploit relativistic Doppler shift**

lab frame:  $\hbar\omega \approx 100 \text{ eV}$ ,  $E \approx 10^{12} \text{ V/cm}$

rest frame:  $\hbar\omega'$  and  $E'$  enhanced by  $2\gamma$

# Relativistic particle beam colliding with laser pulse

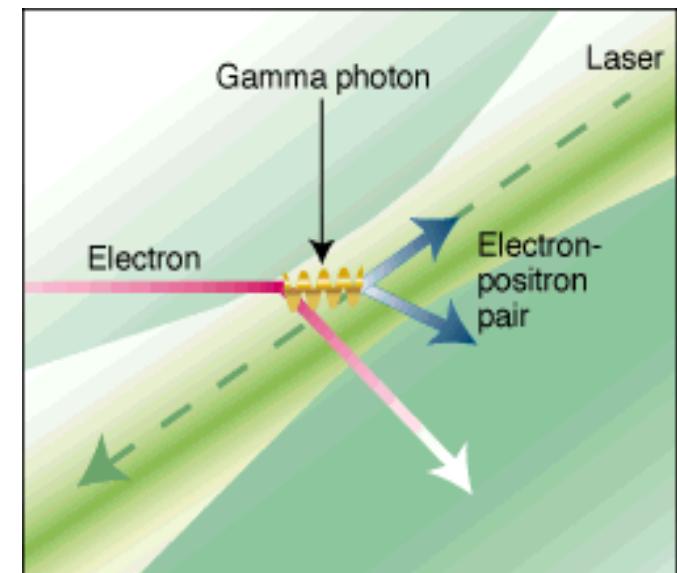


SLAC experiment:  
46 GeV electron + optical laser pulse  
(D. Burke et al., PRL 1997)

## Exploit relativistic Doppler shift

lab frame:  $\hbar\omega \approx 100$  eV ,  $E \approx 10^{12}$  V/cm

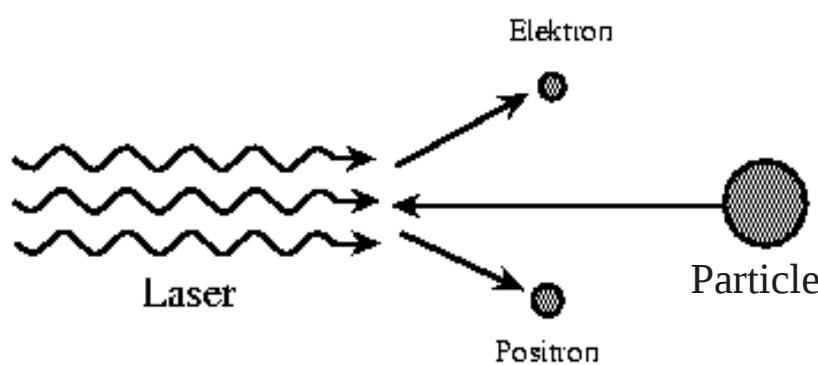
rest frame:  $\hbar\omega'$  and  $E'$  enhanced by  $2\gamma$



Pairs were produced in two-step process through an intermediate high-energy Compton photon:



# Relativistic particle beam colliding with laser pulse

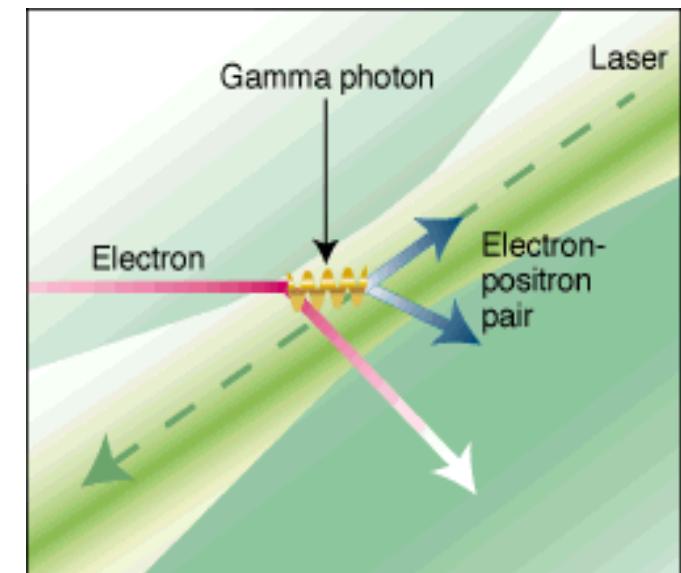
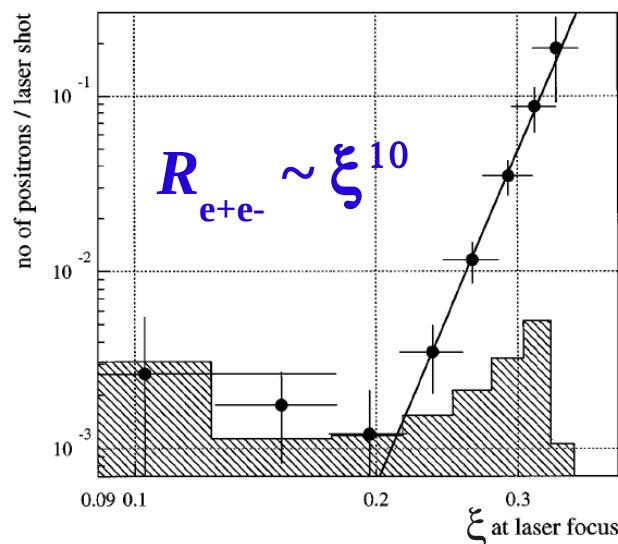


SLAC experiment:  
46 GeV electron + optical laser pulse  
(D. Burke et al., PRL 1997)

## Exploit relativistic Doppler shift

lab frame:  $\hbar\omega \approx 100 \text{ eV}$ ,  $E \approx 10^{12} \text{ V/cm}$

rest frame:  $\hbar\omega'$  and  $E'$  enhanced by  $2\gamma$

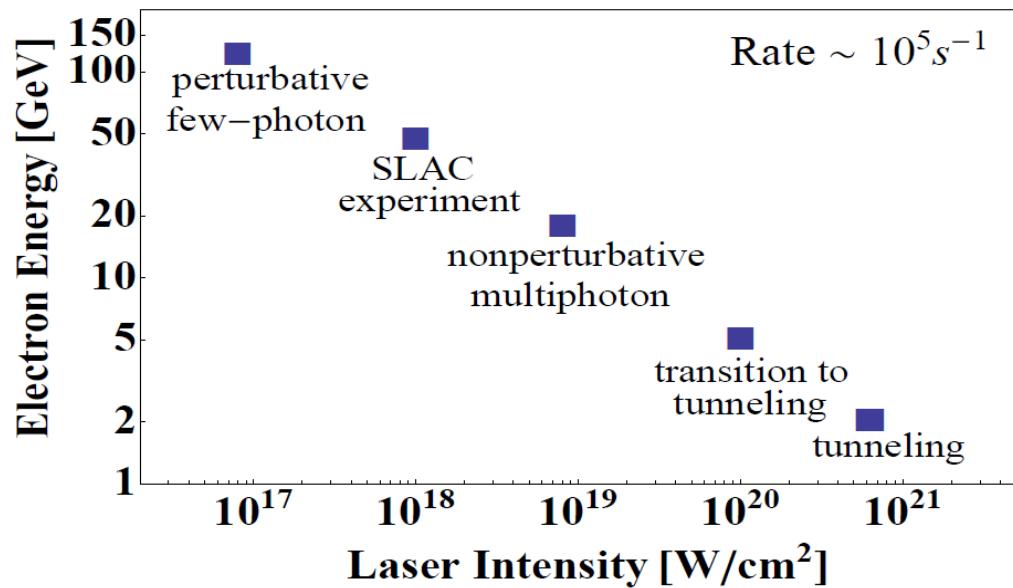


Pairs were produced in two-step process through an intermediate high-energy Compton photon:

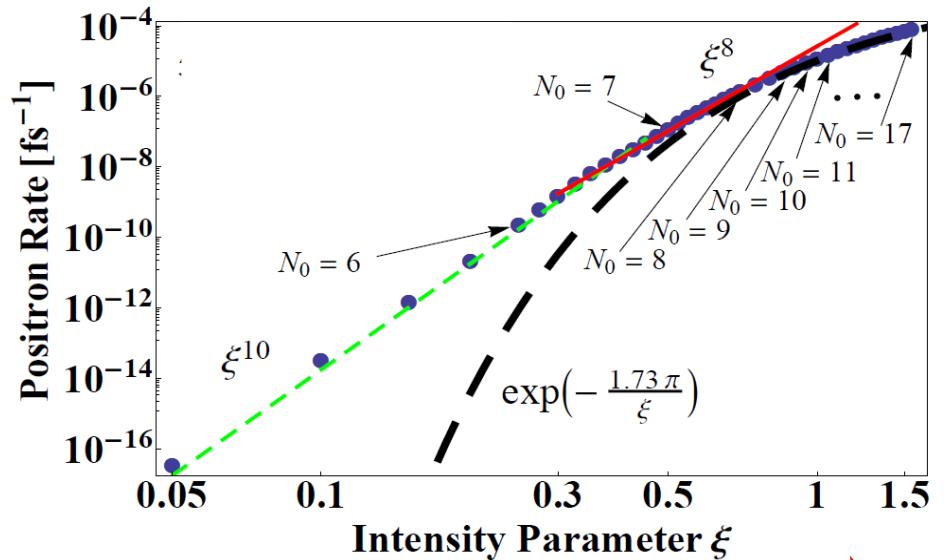


(strong-field Breit-Wheeler process)

# Transition from perturbative to nonperturbative regime



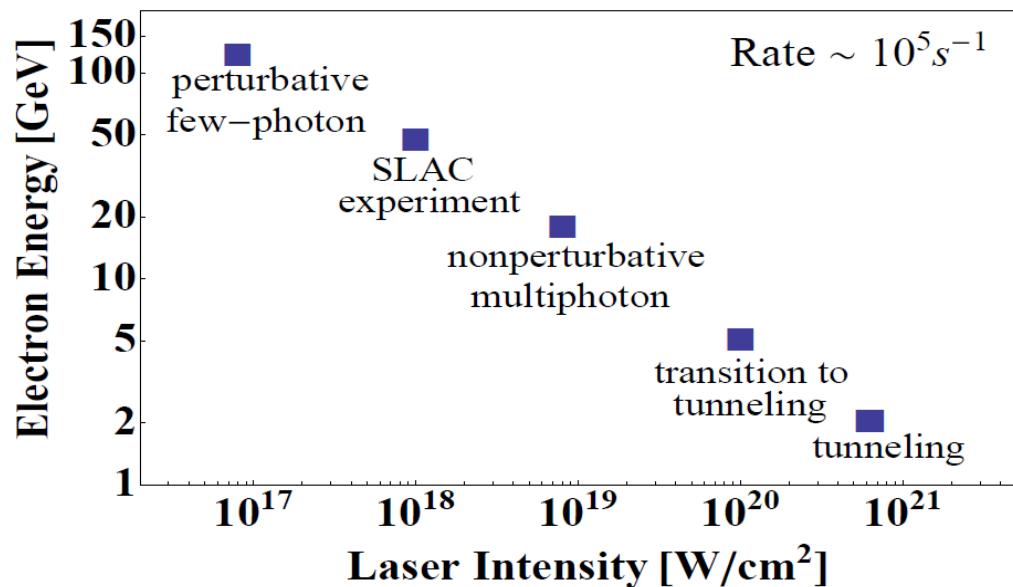
Hu, Müller & Keitel,  
PRL **105**, 080401 (2010)



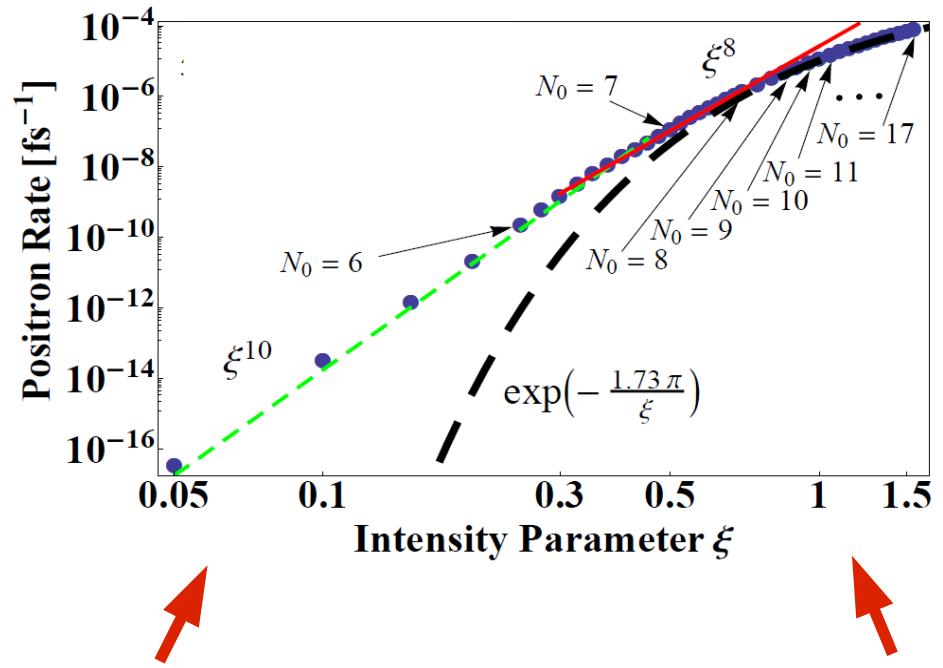
perturbative  
domain

nonperturbative  
domain

# Transition from perturbative to nonperturbative regime



Hu, Müller & Keitel,  
PRL **105**, 080401 (2010)

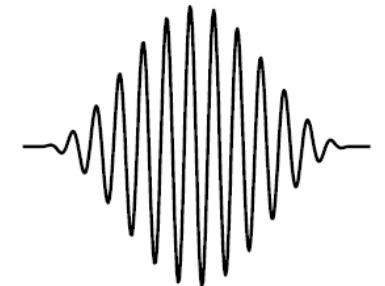


Combine laser-accelerated electron beam  
with second counter-propagating laser pulse:  
All-optical realization of SLAC experiment  
to probe the tunneling regime

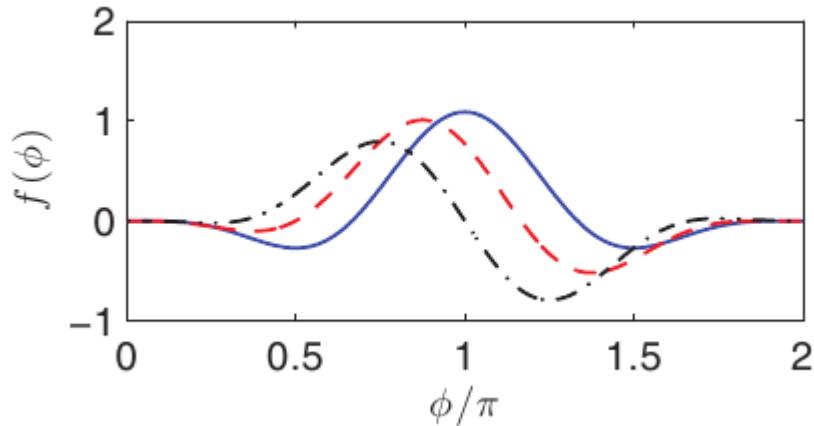


# **Recent developments in laser-induced $e^+e^-$ pair creation**

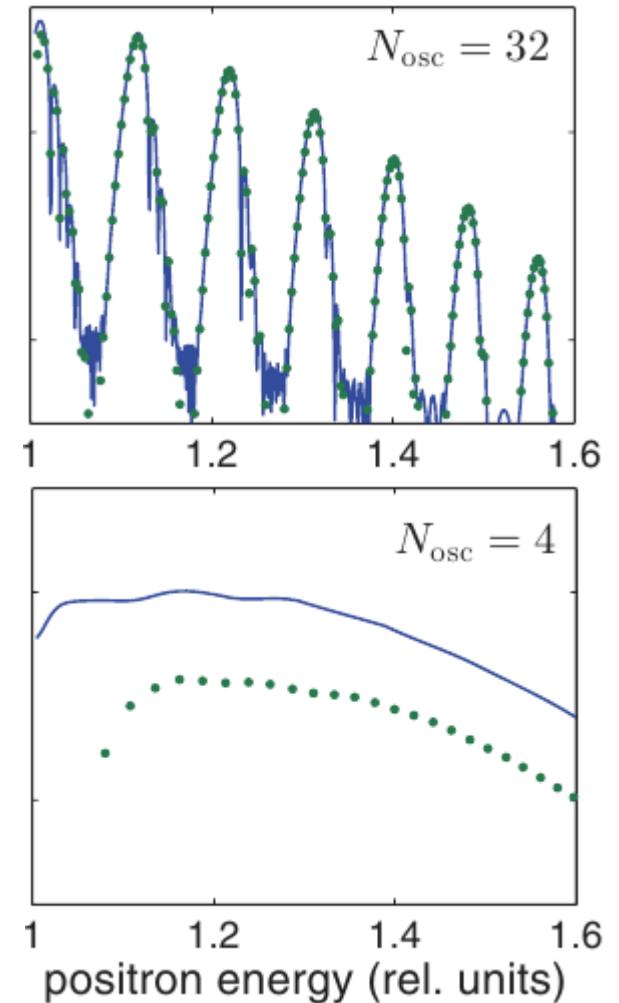
# Pair creation in finite laser pulses



$$\vec{A}(k \cdot x) = A_0 \vec{\varepsilon} f(k \cdot x) \cos(k \cdot x)$$

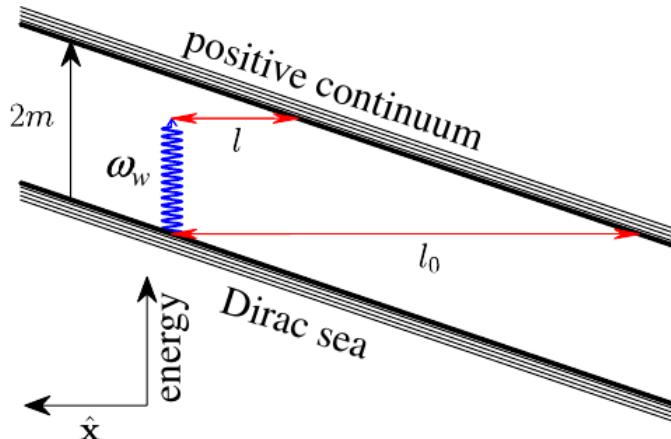


- Boca & Florescu, PRA (2009)  
Mackenroth, Di Piazza & Keitel, PRL (2010)  
Heinzl, Seipt & Kämpfer, PRA (2010)  
Heinzl, Ilderton & Marklund, PLB (2010)  
Ipp, Evers, Keitel & Hatsagortsyan, PLB (2011)  
Titov, Takabe, Kämpfer & Hosaka, PRL (2012)  
Krajewska & Kaminski, PRAs (2012) + (2013)  
Jiang, Lv, Liu, Grobe & Su, PRA (2014)  
Meuren, Hatsagortsyan, Keitel & Di Piazza, PRD (2015)



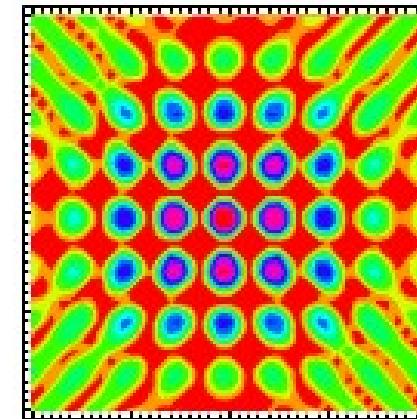
# Enhancement of pair creation

Dynamical assistance:  
add high-frequency component



Schützhold, Gies & Dunne, PRL (2008)  
Di Piazza, Lötstedt, Milstein & Keitel, PRL (2009)  
Orthaber, Hebenstreit & Alkofer, PLB (2011)  
Jiang, Su, Lv, Lu, Li, Grobe & Su, PRA (2012)

Multiple-beam or  
multiple-center setups

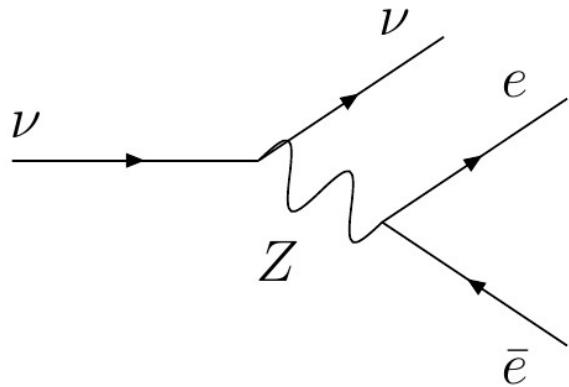


Bulanov, Mur, Narozhny, Nees & Popov, PRL (2010)  
Fillion-Gourdeau, Lorin & Bandrauk, PRL (2013)

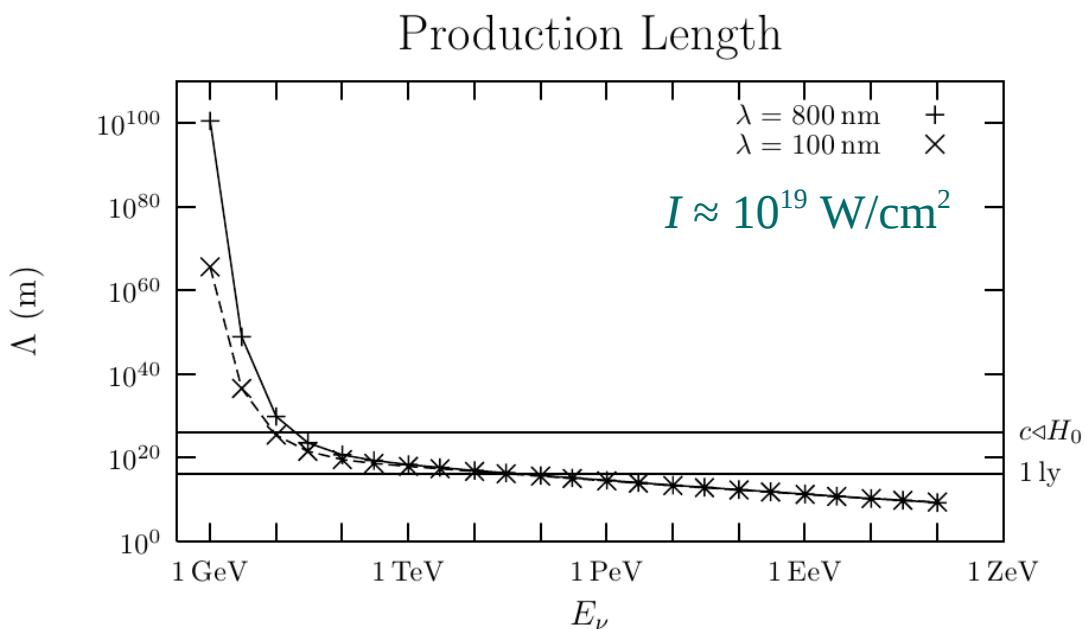
QED cascades

Bell & Kirk, PRL (2008)  
Nerush, Kostyukov,  
Fedotov, Narozhny,  
Elkina & Ruhl, PRL (2011)

# Pair creation in neutrino-laser collisions



$$\begin{aligned} \hat{S} = & -\frac{\imath}{\hbar^8} \frac{4\pi\alpha}{2^2\cos^2\theta_W\sin^2\theta_W} \\ & \times \int d^4x \bar{\psi}_e(x) \gamma^\mu \left( -\frac{1}{2} + 2\sin^2\theta_W + \frac{1}{2}\gamma^5 \right) \psi_e(x) Z_\mu(x) \\ & \times \int d^4y \bar{\psi}_\nu(y) \gamma^\sigma \left( \frac{1}{2} - \frac{1}{2}\gamma^5 \right) \psi_\nu(y) Z_\sigma(y) \end{aligned}$$



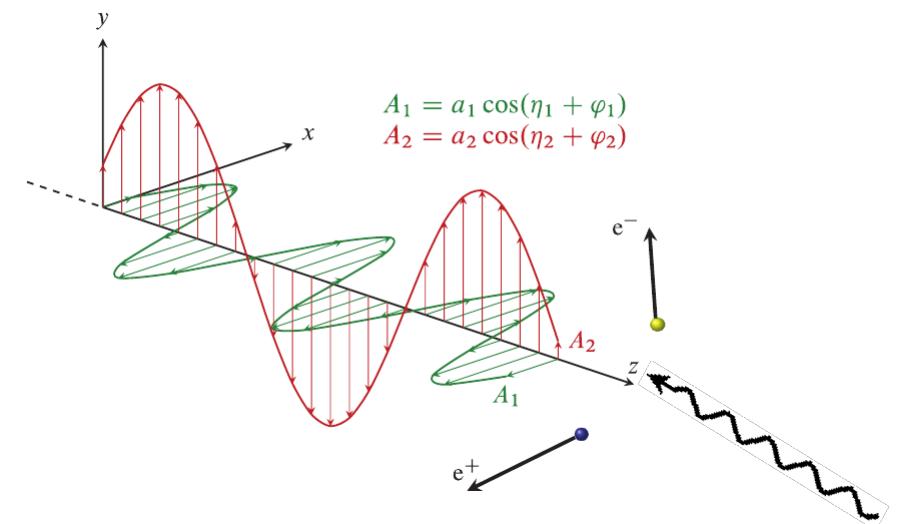
creation length at  $E_\nu \sim 10 \text{ GeV}$ :  
size of the universe....

**e<sup>+</sup>e<sup>-</sup> pair creation  
in two-color laser fields**

# Dynamically assisted Breit-Wheeler pair creation

$$\mathcal{R} = \frac{3^2 \alpha m^2}{2^7 \sqrt{2\pi} \omega_\gamma} \left( \frac{E'}{E_{\text{cr}}} \right)^{3/2} \exp \left( -\frac{8}{3} \frac{E_{\text{cr}}}{E'} \right)$$

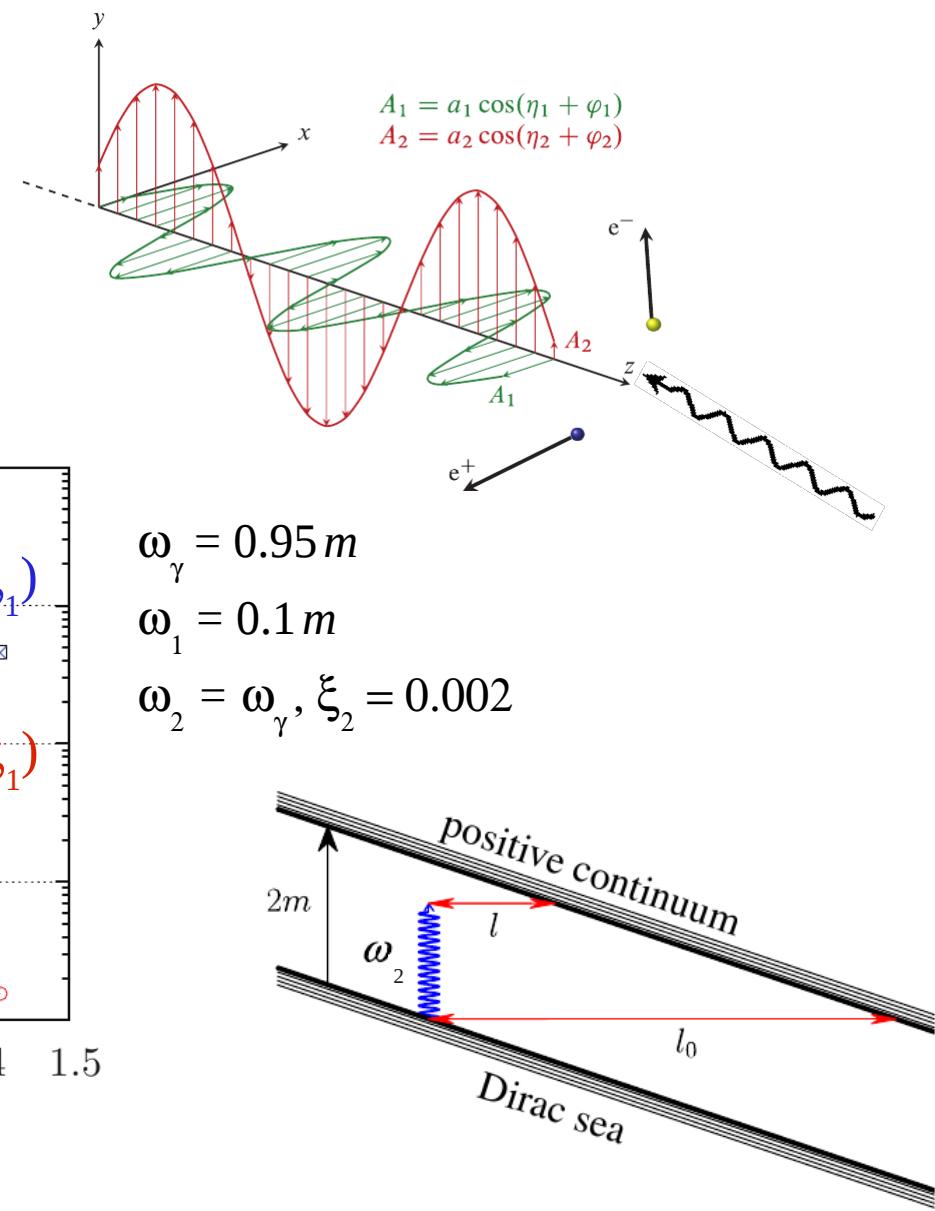
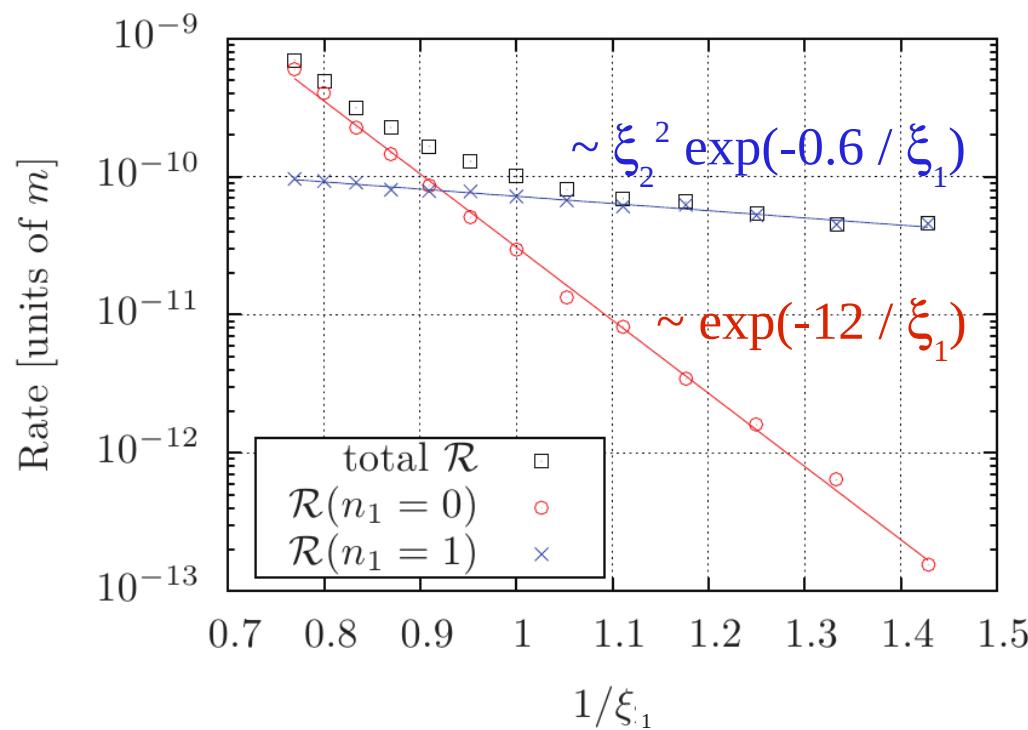
(Reiss 1962)



# Dynamically assisted Breit-Wheeler pair creation

$$\mathcal{R} = \frac{3^2 \alpha m^2}{2^7 \sqrt{2\pi} \omega_\gamma} \left( \frac{E'}{E_{\text{cr}}} \right)^{3/2} \exp \left( -\frac{8}{3} \frac{E_{\text{cr}}}{E'} \right)$$

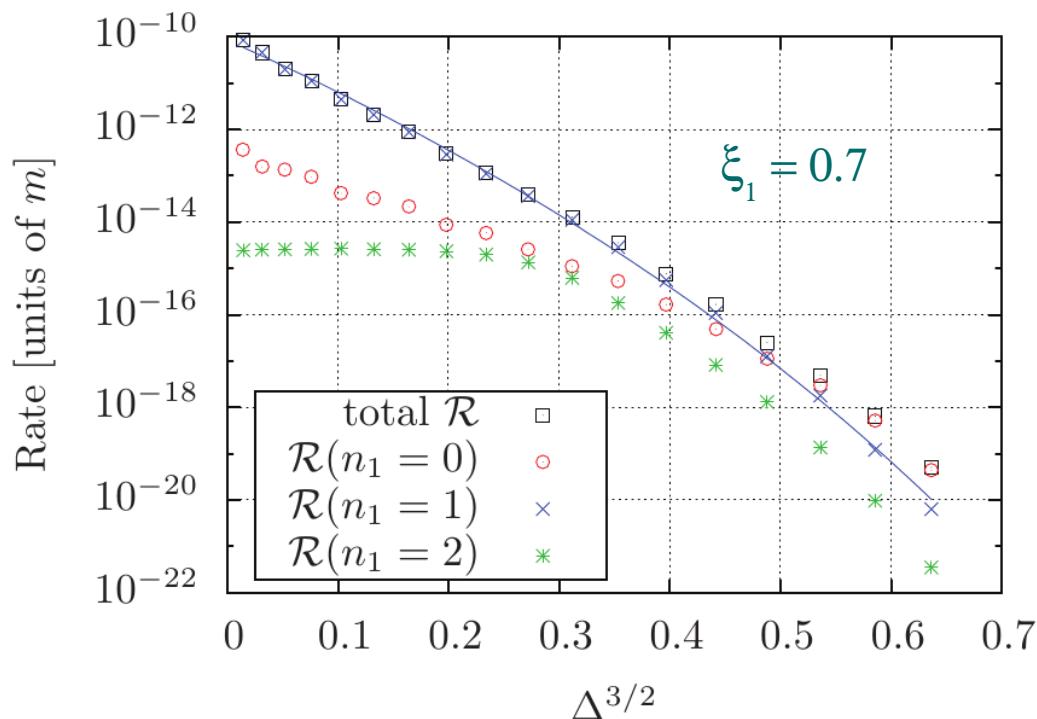
(Reiss 1962)



unassisted and assisted channels compete

# Dynamically assisted Breit-Wheeler pair creation

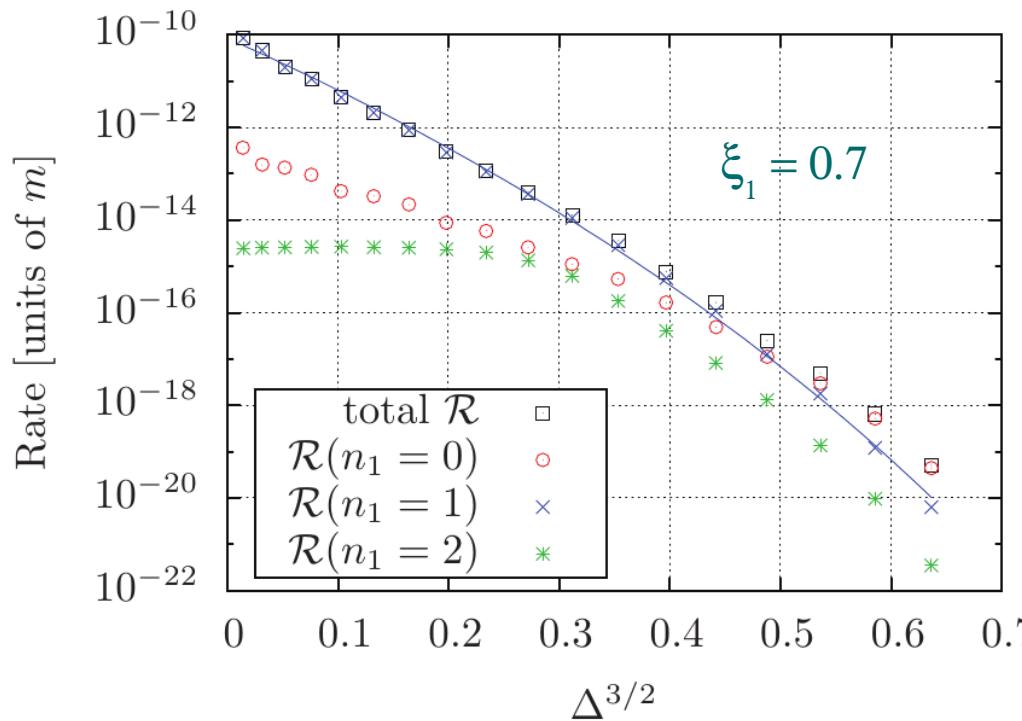
Depencence on energy gap



$$(k_2 + k_\gamma)^2 = 4m^2(1 - \Delta)$$

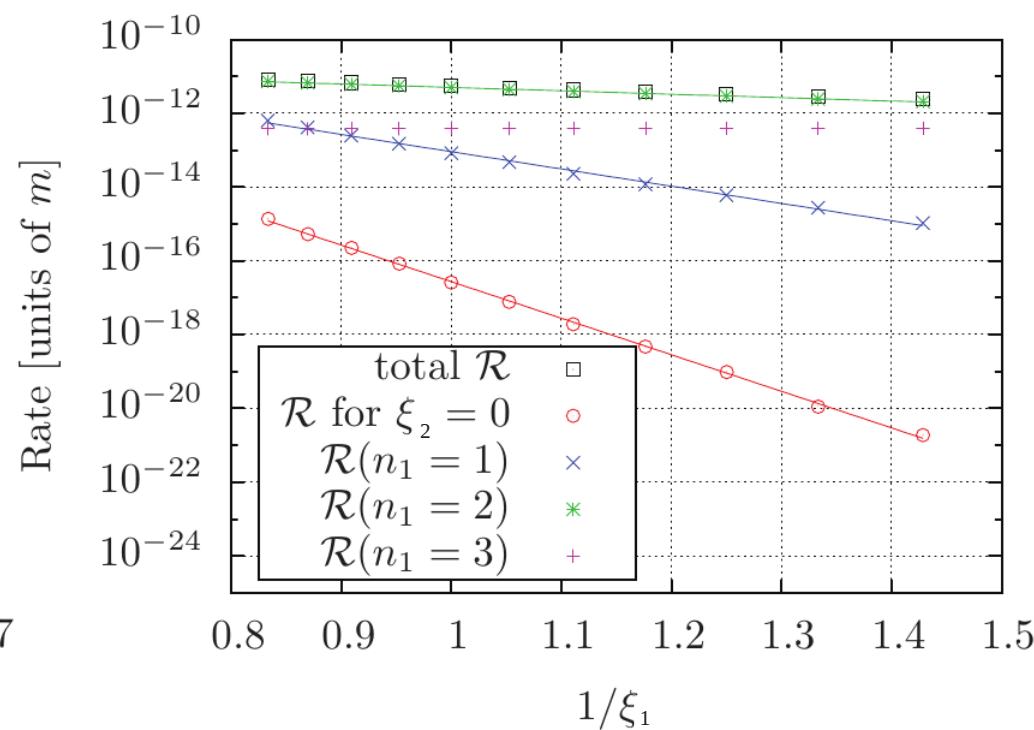
# Dynamically assisted Breit-Wheeler pair creation

Depencence on energy gap



$$(k_2 + k_\gamma)^2 = 4m^2(1 - \Delta)$$

Assistance by two high-frequency photons dominates



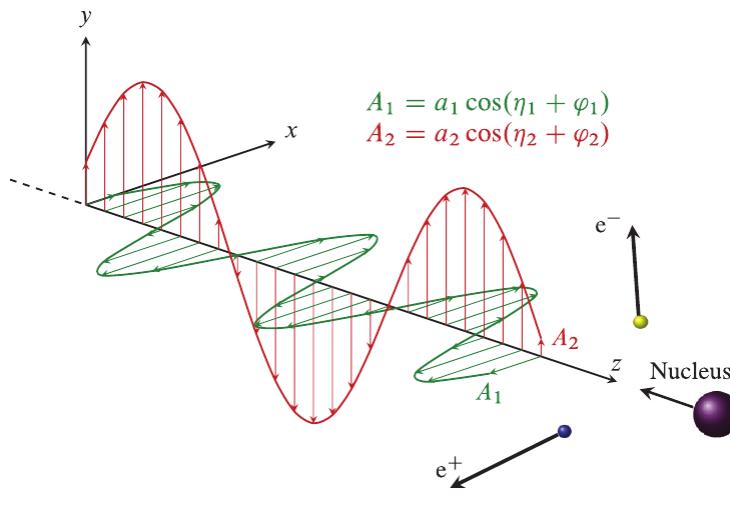
$$\omega_\gamma = 0.95 m$$

$$\omega_1 = 0.05 m$$

$$\omega_2 = \omega_\gamma / 2, \xi_2 = 0.045$$

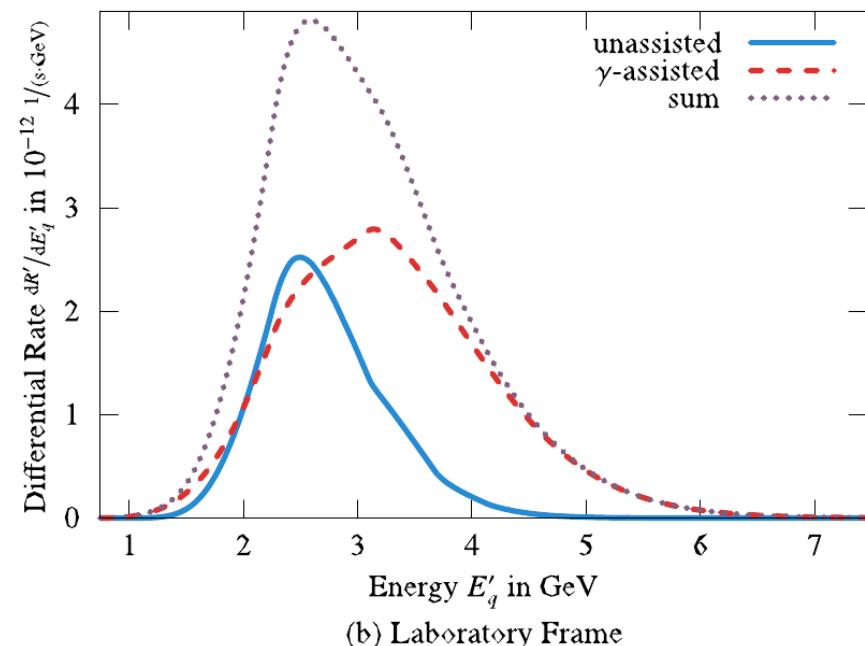
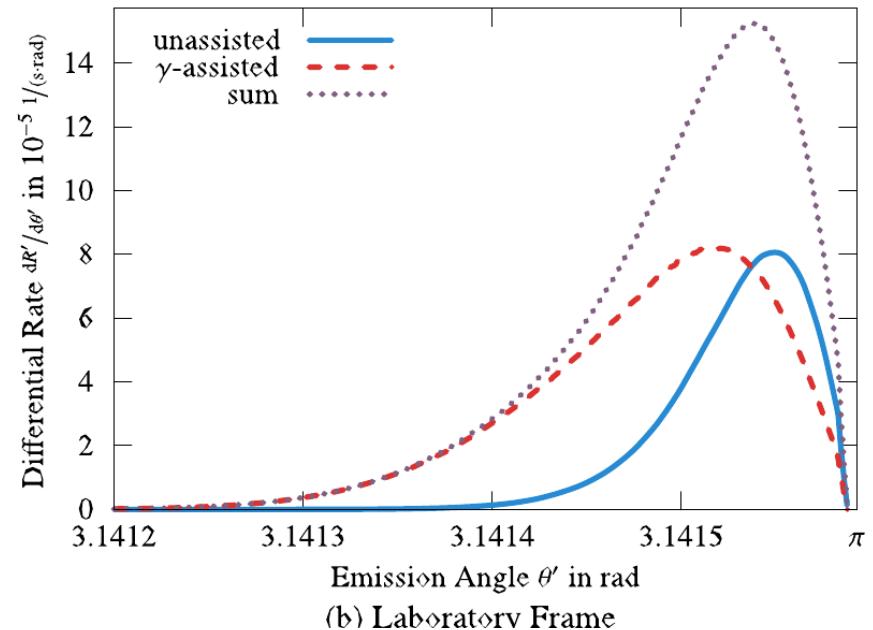
[see also Wu & Xue, PRD 90, 013009 (2014);  
 Narozhny & Fofanov, JETP 90, 415 (2000)]

# Dynamically assisted Bethe-Heitler pair creation

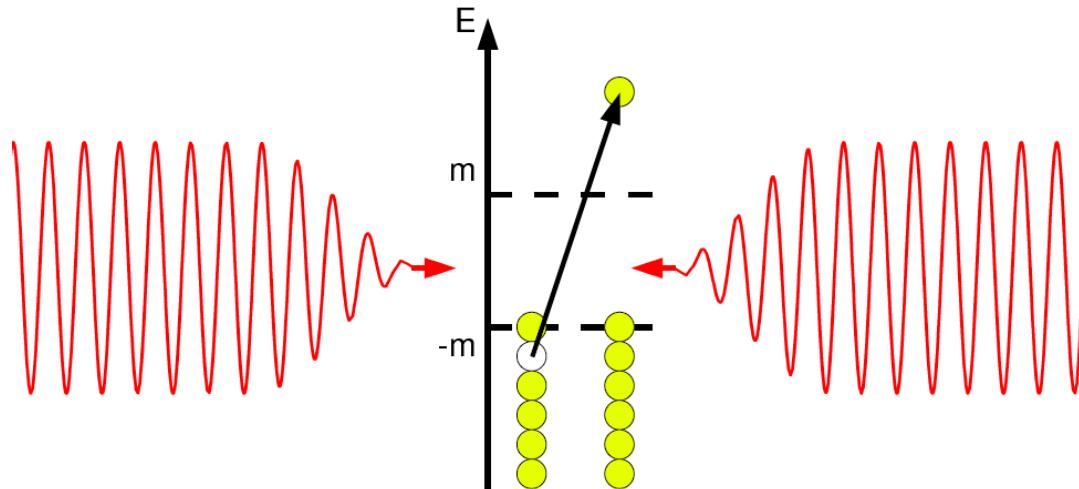


Angular & energy distributions  
of  $e^+$  may differ even when total  
contributions similar in size

$\omega_1 = 2.3 \text{ eV}, \xi_1 \approx 1$   
 $\omega_2 = 70 \text{ eV}, \xi_2 \approx 10^{-8}$   
 $\gamma = 7000$

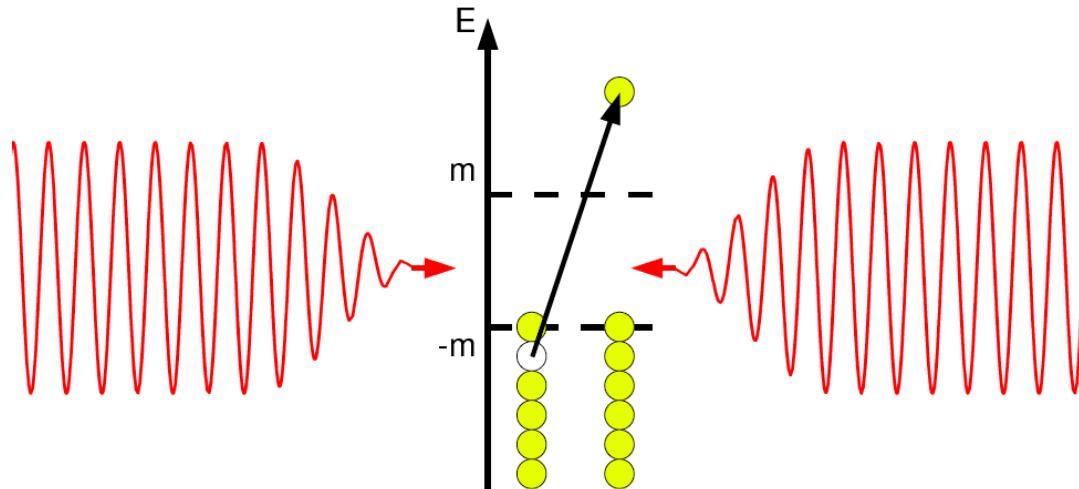


# Pair creation in oscillating $E$ -field

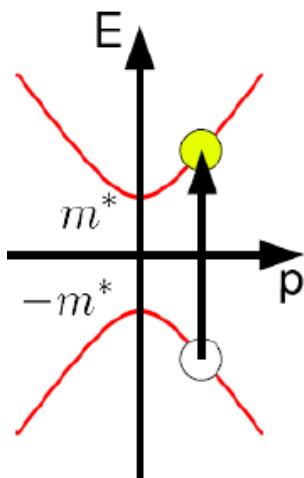


$$A = A_0 [ \sin(\omega t - kz) + \sin(\omega t + kz) ] = 2A_0 \sin(\omega t) \cos(kz) \approx 2A_0 \sin(\omega t)$$

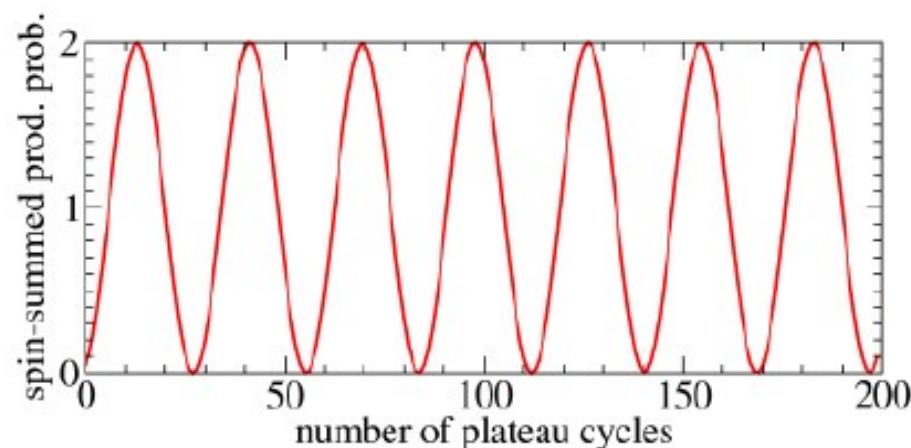
# Pair creation in oscillating $E$ -field



$$A = A_0 [ \sin(\omega t - kz) + \sin(\omega t + kz) ] = 2A_0 \sin(\omega t) \cos(kz) \approx 2A_0 \sin(\omega t)$$



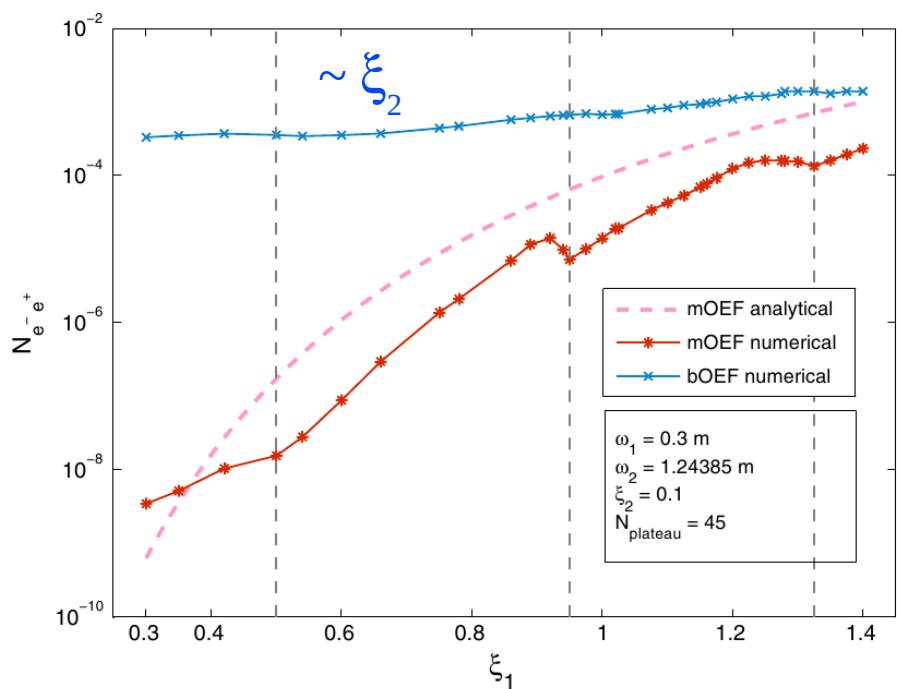
Momentum conservation reduces the problem to a two-level system, which undergoes Rabi oscillations at certain frequencies.



# Pair creation in bifrequent oscillating $E$ -field

$$\vec{A}(t) \sim [\xi_1 \sin(\omega_1 t) + \xi_2 \sin(\omega_2 t)] \vec{e}_y$$

Number density of created pairs

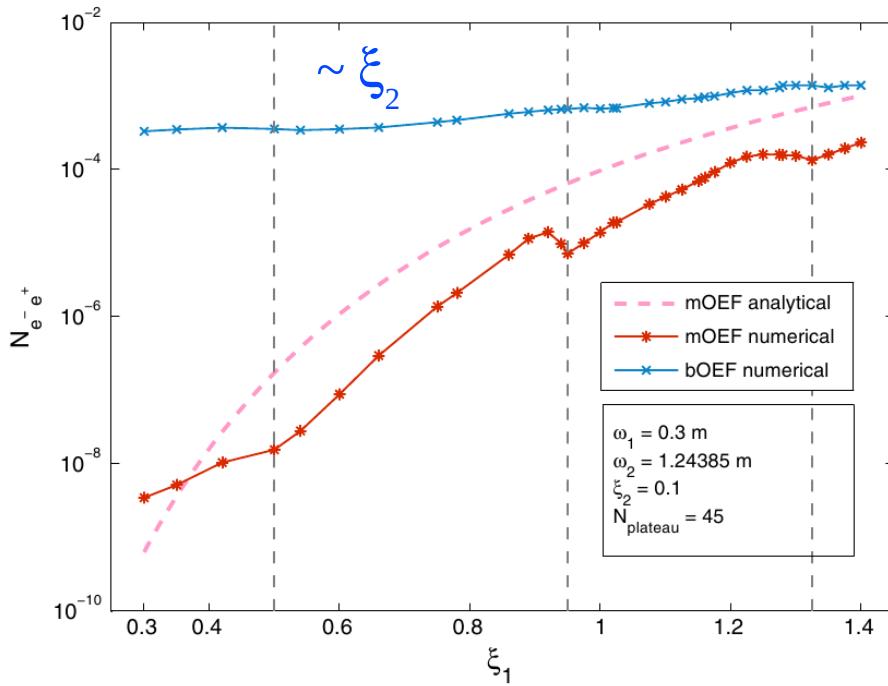


Resonances:  $n_1 \omega_1 + n_2 \omega_2 = 2\varepsilon(p)$

# Pair creation in bifrequent oscillating $E$ -field

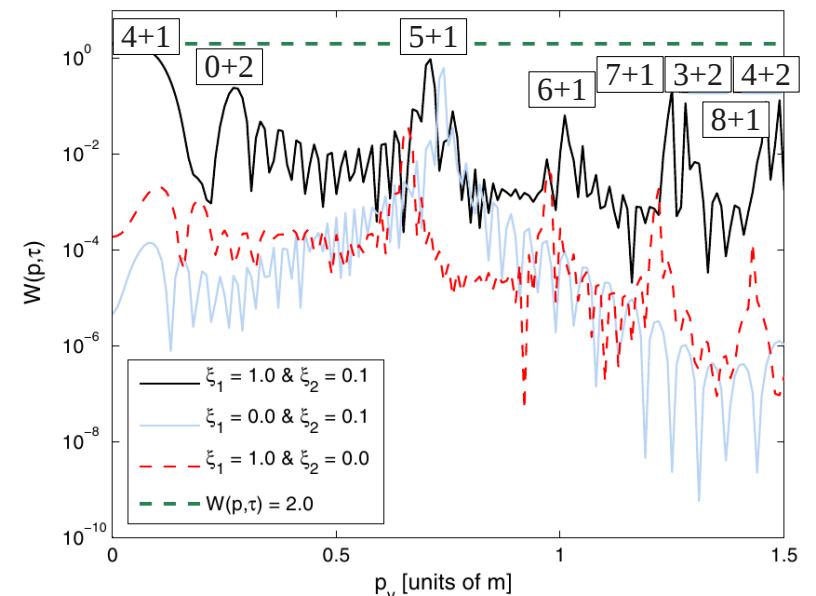
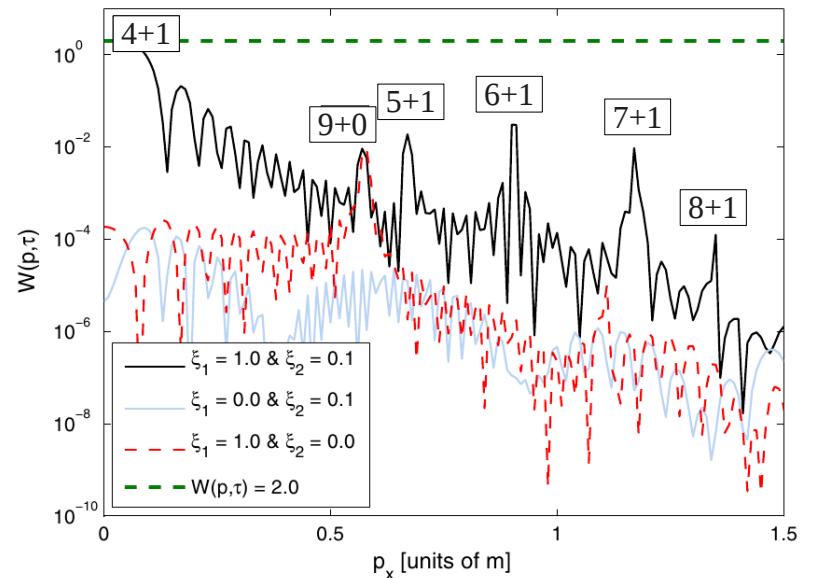
$$\vec{A}(t) \sim [\xi_1 \sin(\omega_1 t) + \xi_2 \sin(\omega_2 t)] \vec{e}_y$$

Number density of created pairs



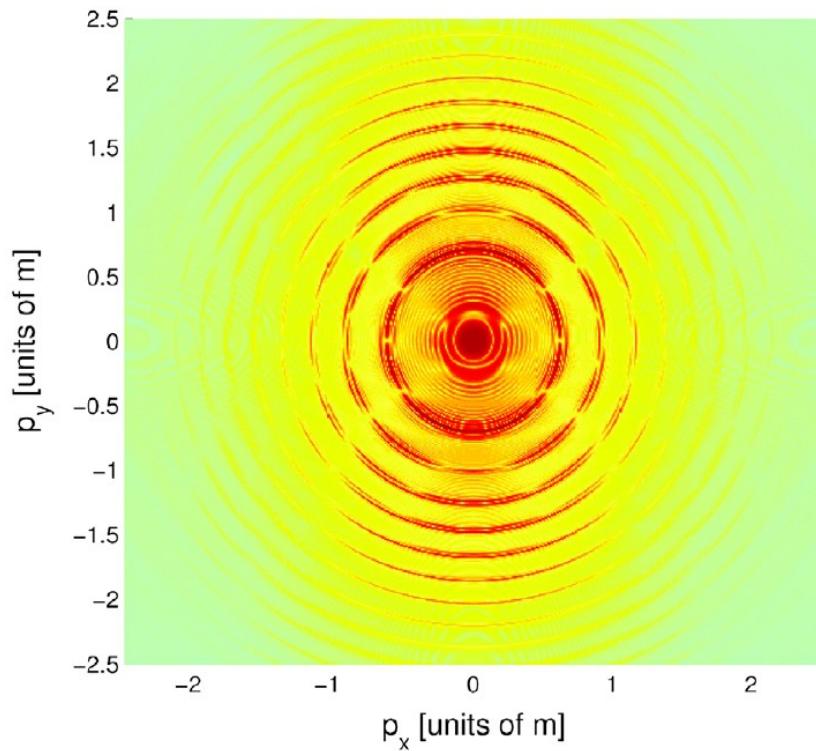
$$\text{Resonances: } n_1 \omega_1 + n_2 \omega_2 = 2\varepsilon(p)$$

Momentum spectra



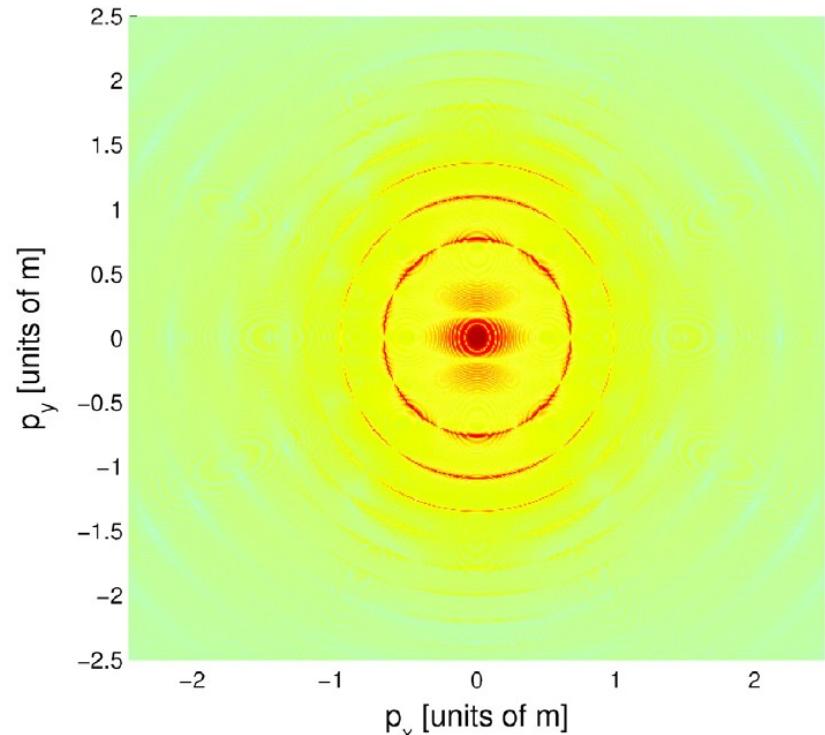
# Pair creation in bifrequent oscillating $E$ -field

[4+1] – resonance



$$\begin{aligned}\omega_1 &= 0.3m, \xi_1 = 1.0 \\ \omega_2 &= 1.24m, \xi_2 = 0.1 \\ N &= 16\end{aligned}$$

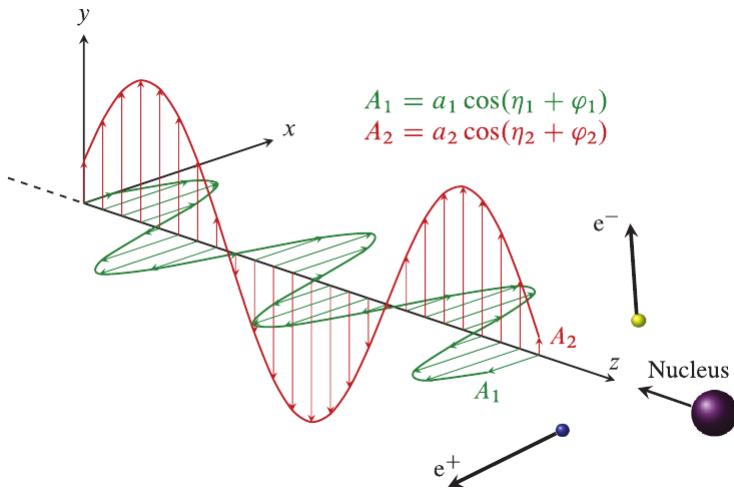
monochrom. [7] – resonance



$$\begin{aligned}\omega_1 &= 0.35m, \xi_1 = 1.0 \\ N &= 45\end{aligned}$$

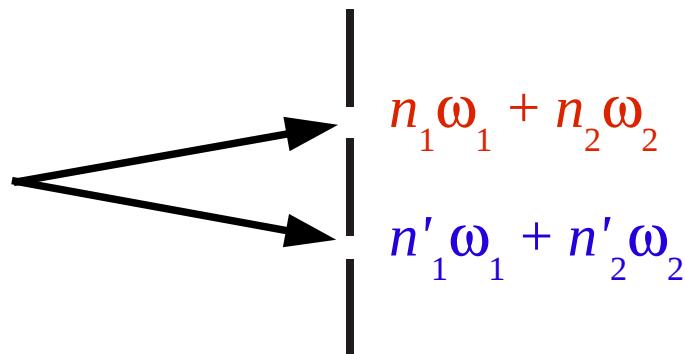
Akal, Villalba-Chavez, Müller, PRD 90, 113004 (2014)  
[see also Orthaber, Hebenstreit, Alkofer, PLB (2011)]

# Quantum interferences in bichromatic fields

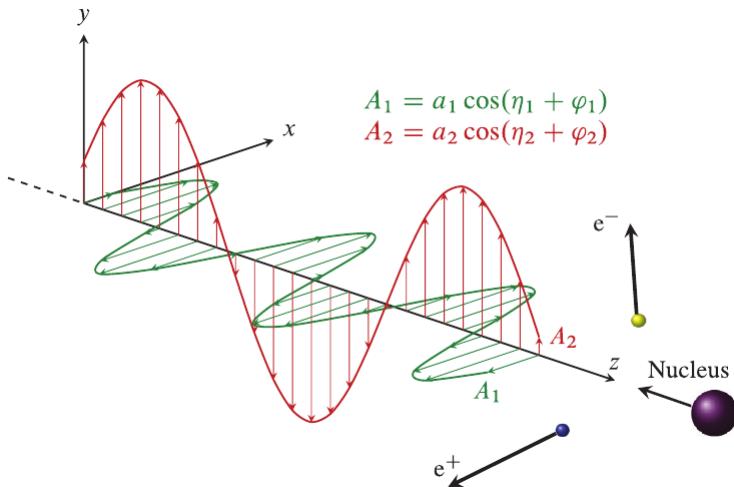


Suppose  $\omega_1 = 2\omega_2$  (commensurate)

**Two-pathway interference:**

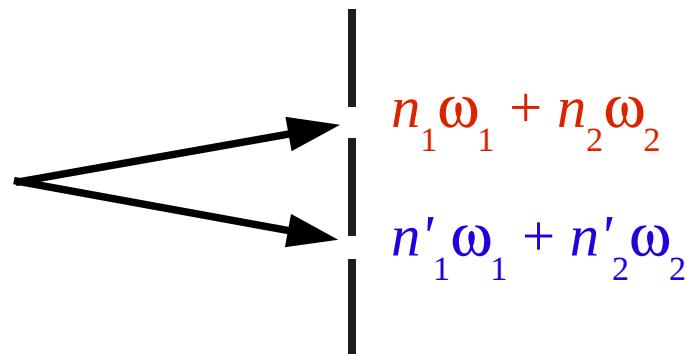


# Quantum interferences in bichromatic fields



Suppose  $\omega_1 = 2\omega_2$  (commensurate)

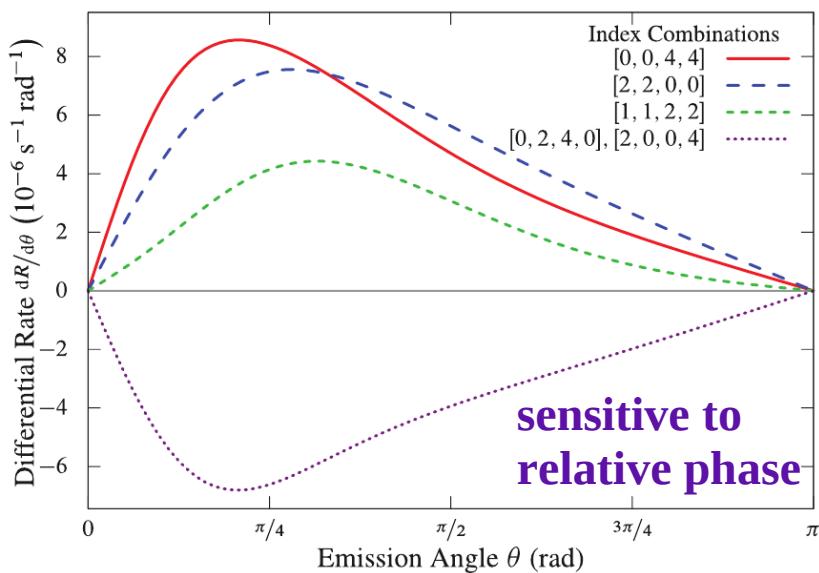
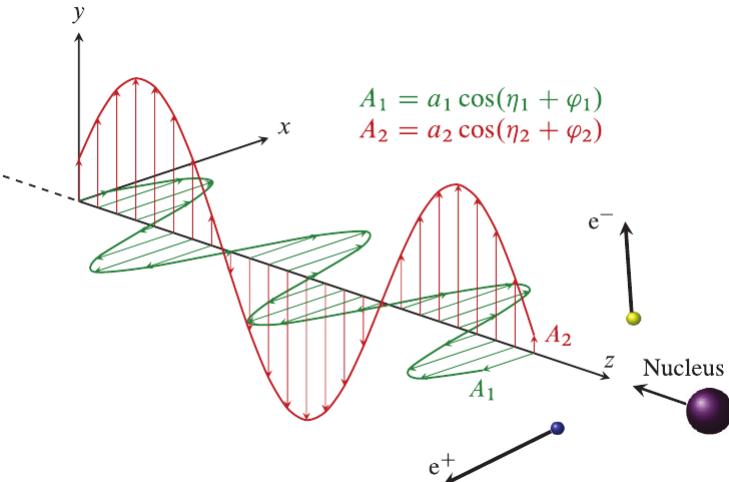
**Two-pathway interference:**



$$|\mathcal{S}|^2 = \sum_{n'_1, n'_2 \atop n_1, n_2} \mathcal{P}_{(n_1, n_2, n'_1, n'_2)}$$

**Maximize interference:**  $\xi_1^{-2} \sim \xi_2^{-4} \ll 1$

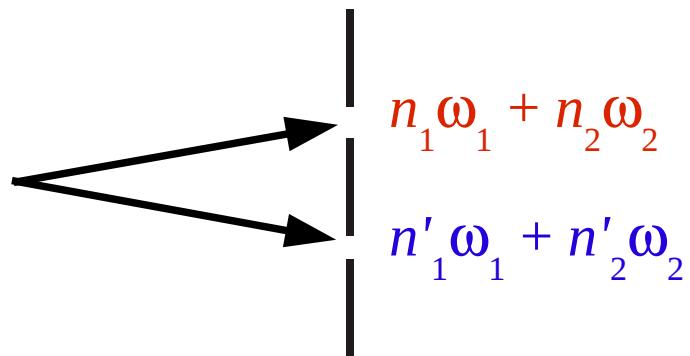
# Quantum interferences in bichromatic fields



$\omega_1 = 550 \text{ keV}$  (in nuclear frame)

Suppose  $\omega_1 = 2\omega_2$  (commensurate)

**Two-pathway interference:**



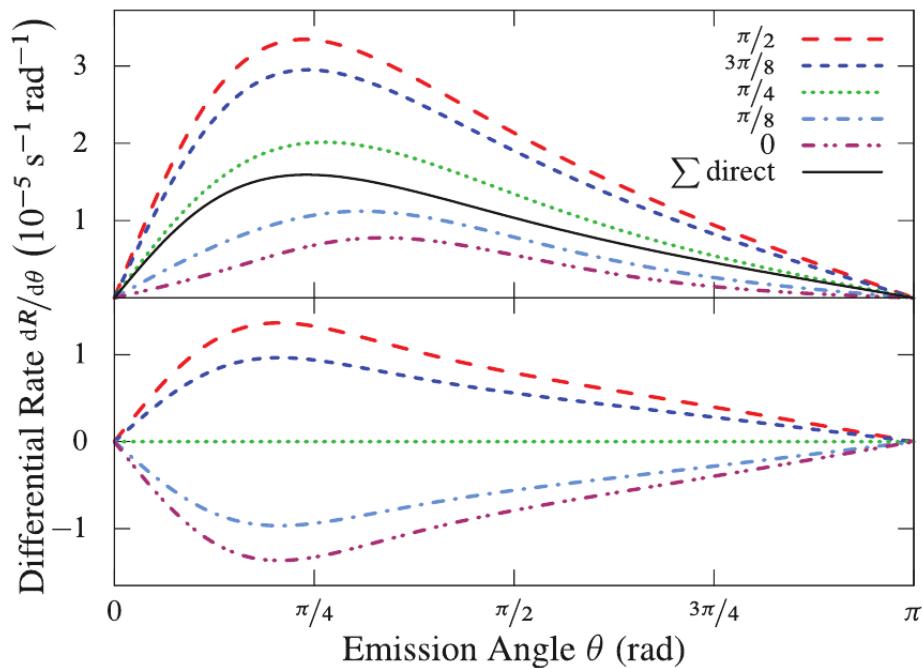
$$|\mathcal{S}|^2 = \sum_{n'_1, n'_2} \mathcal{P}_{(n_1, n_2, n'_1, n'_2)}$$

**Maximize interference:**  $\xi_1^2 \sim \xi_2^4 \ll 1$

see also Krajewska & Kaminski, PRA 86, 021402 (2012)

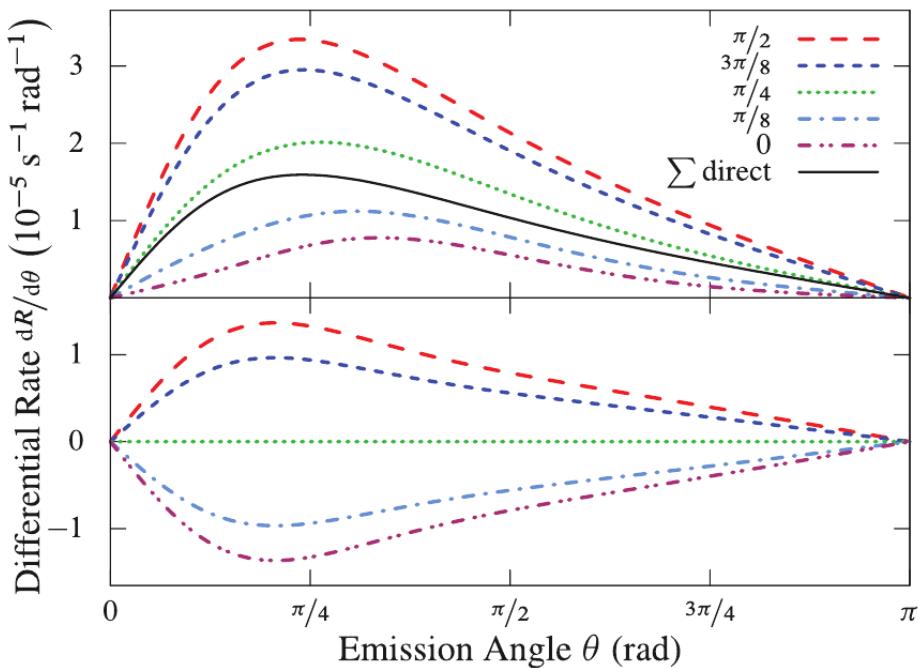
# Quantum interferences in bichromatic fields

Interference can affect total rate

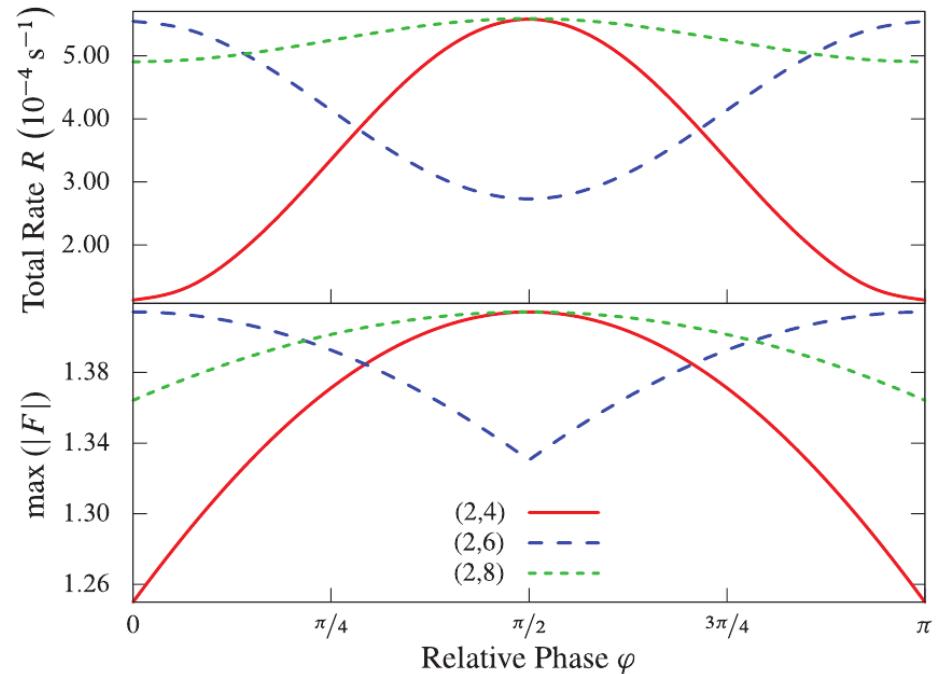


# Quantum interferences in bichromatic fields

Interference can affect total rate



Peak field strength relevant



$$F^2(ct - z) = \sum_{i=1}^2 \frac{c^2}{\mathbf{a}_i^2 \omega_i^2} E_i^2 = \sum_{i=1}^2 \sin^2(\eta_i + \varphi_i)$$

# Summary

- Theory of QED in strong laser fields relies on **Furry picture** and **Volkov states**
- Three standard types of laser-induced pair creation:  
with  $\gamma$ -photon, **Coulomb field** or **second laser beam**;  
various interaction regimes – tunable by laser parameters
- Recent developments: pair creation in **more complex field configurations** (e.g. two-colors or finite pulses)  
→ enriched creation dynamics, more realistic description  
and enhanced pair yields

**Thank you for your attention!**