Nonperturbative pair production and complex instantons

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- With Greger Torgrimsson and Jonatan Wårdh.
- PRD **92** (2015) 025009 & arXiv:1506.09186

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Outline

Nonperturbative pair production in strong external fields [1]

Outline

1. Pair production and wordline instantons.
2. Temporal vs. spatial inhomogeneities.
3. Branch cuts and poles.

[1] . . . S.P. Kim & D. Page, PRD 73 (2006) 065020, G. Dunne et al PRD 80 (2009) 111301, R. Schützhold et al PRL 101 (2008) 130404, F. Hebenstreit et al., PRL 102 (2009) 150404 A. Di Piazza et al PRL 103 (2009) 170403, S.S. Bulanov et al PRL 104 (2010) 220404, A. Gonoskov et al PRL 111 (2013) 060404, A. Otto et al., PRD 91 (2015) 105018 . . .

The effective action

- Probability of pair production:

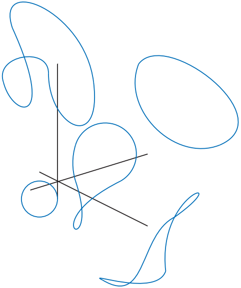
$$\mathbb{P}_{\text{pairs}} = 1 - e^{-2\text{Im}\,\Gamma} \simeq 2\text{Im}\,\Gamma\,.$$

- One-loop worldline representation[2]:

$$\Gamma = \int_0^\infty \frac{\mathrm{d}T}{T} \oint \mathcal{D}^4x\ e^{-iS}$$

- Classical action:

$$S = \frac{m^2 T}{2} + \int_0^1 \mathrm{d}\tau \left[\frac{\dot{x}^\mu \dot{x}_\mu}{2T} + eA_\mu(x)\dot{x}^\mu \right].$$



[2] Feynman PR 80 (1950) 440, Schwinger PR 82 (1951) 664, Affleck et al., NPB 197 (1982) 509, Bern & Kosower NPB 379 (1992) 451, Strassler NPB 385 (1992) 145, Schubert APPB 27 (1996) 3965

Semiclassical approximation

$$\oint \mathcal{D}x\; e^{-iS} \simeq \frac{e^{-iS[x_{\text{cl}}]}}{\sqrt{\text{Det}\; S''[x_{\text{cl}}]}}$$

1. Instanton: periodic classical path. $\ddot{x}^{\mu}_{\text{cl}} = T\dfrac{e}{m}F^{\mu}{}_{\nu}(x_{\text{cl}})\dot{x}^{\nu}_{\text{cl}}$

- Complex-valued loop.

Lavrelashvili et al., NPB 329 (1990) 98, Kim & Page PRD 75 (2007) 045013

Bender et al., PRL 104 (2010) 061601, Dumlu & Dunne PRD 84 (2011) 125023

2. Fluctuations around x_{cl} → prefactor determinant.

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2. Fluctuations around $x_{\text{cl}} \rightarrow$ prefactor determinant.

? Instantons not needed explicitly... why?

Dunne, Wang, Gies and Schubert PRD 73 (2006) 065028

Temproal vs. spatial structure

- From considering $E(t)$ and $E(z)$:

  "...temporal inhomogeneities tending to enhance local pair production, with spatial inhomogeneities tending to suppress local pair production."

  Dunne & Schubert, PRD 72 (2005) 105004

- $E(t+z)$: LFCA gives exact result. Tsamis et al, PRD62 (2000) 125005

Temproal vs. spatial structure

- $\boxed{E^z = E_0 f(\omega q)}$ depending on interpolating coordinate q

Hornbostel, PRD **45** (1992) 3781

$$q := t\cos\frac{\theta}{2} + z\sin\frac{\theta}{2}\,, \qquad \theta \in [0, \pi]$$

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$$q := t\cos\frac{\theta}{2} + z\sin\frac{\theta}{2}\,, \qquad \theta\in[0,\pi]$$

$$= t \to \frac{1}{\sqrt{2}}(t+z) \to z\;.$$

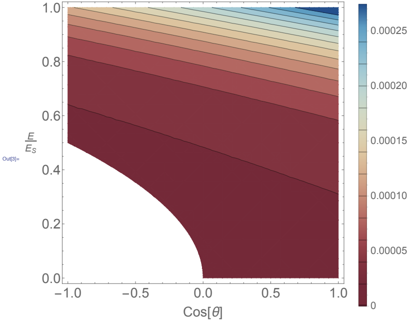
- Same strength, frequency, direction. Not a boost.

See also Ji & Mitchell, PRD **64** (2001) 085013, Heinzl & Ilderton JPA **40** (2007) 9097

The effective action: example

$$E^z = E_0 \text{sech}^2(\omega q) \qquad q = t\cos\frac{\theta}{2} + z\sin\frac{\theta}{2}$$

- Imaginary part of Γ



- $\omega = m/2$
- Smooth function of $c \equiv \cos\theta$.
- $\mathbb{P}$ greater for q timelike

Ilderton, Torgrimsson, Wårdh 1506.09186 [hep-th]

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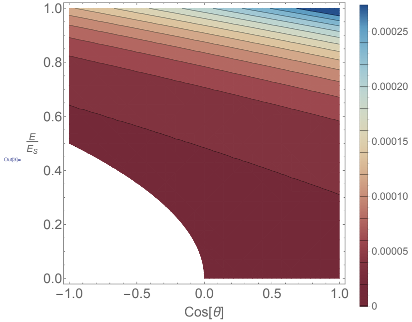


- Adiabacity: $\gamma = \dfrac{m\omega}{eE_0}$
- $c \equiv \cos\theta$
- Critical curve $c\gamma^2 = -1$

Nikishov, NPB 21 (1970) 346

Gies & Klingmuller, PRD 72 (2005) 065001

Ilderton, Torgrimsson, Wårdh 1506.09186

- Behaviour near critical points: talk by H. Gies.

Gies and Torgrimsson, to appear.

Instantons

Example: sech^2.

- Equation of motion: $$\dot{q}^2 = T^2\left(c + \frac{1}{\gamma^2}\tanh^2(\omega q)\right)$$
- Solution: c.f. Dunne & Schubert, PRD 72 (2005) 105004

$$q(\tau) = \frac{1}{\omega}\sinh^{-1}\left[\frac{i\sqrt{c\gamma^2}}{\sqrt{1+c\gamma^2}}\sin 2n\pi(\tau - \tau_0)\right]$$

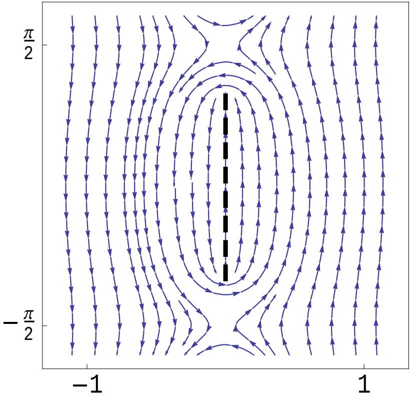
- Interpolation dependence: $c\gamma^2$. Integration constant τ_0.
- Square root $\implies$ branch cut.

$\rightarrow$ Streamplots, with $\dot{q}$ as a velocity field.

Interpolating instantons

Complex $q(\tau)$ plane

$$c = 1 \qquad E \equiv E(t)$$



- Various complex τ_0.
- $q(\tau)$ circles a branch.
- Reflected from poles in $\mathbf{E}$ at $\pm\frac{i\pi}{2\omega}$.

  Schützhold et al. PRL 101 (2008) 130404

  Schneider & Schützhold 1407.3584 [hep-th]

  Linder et al., 1505.05685 [hep-th]

Interpolating instantons

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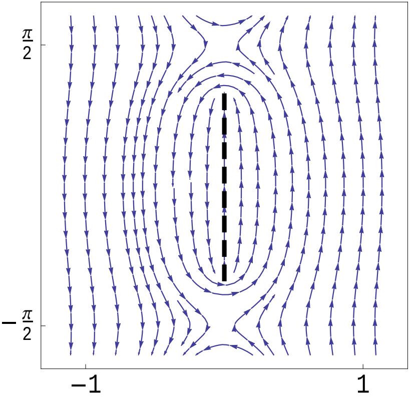
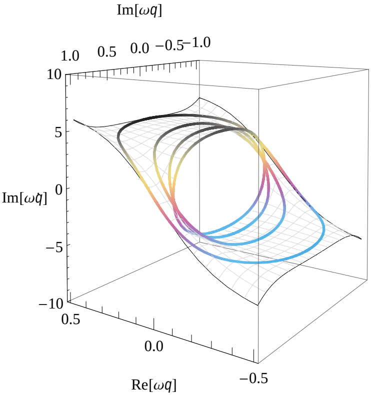
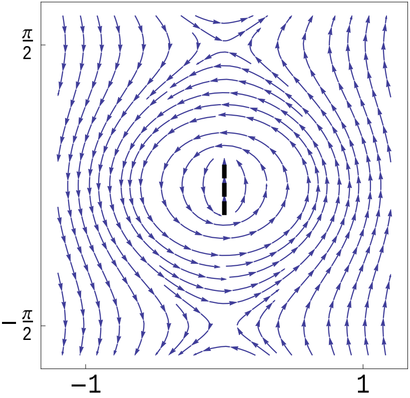




Interpolating instantons

Complex $q(\tau)$ plane

$c = 1/2 \qquad E \equiv E(t\sqrt{3} + z)$

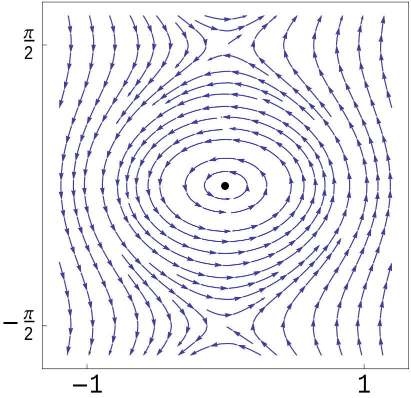


- $1 > c > 0$.
- Branch points closer.
- (Poles in same place.)

Interpolating instantons

Complex $q(\tau)$ plane

$$c = 0 \qquad E \equiv E(t+z)$$

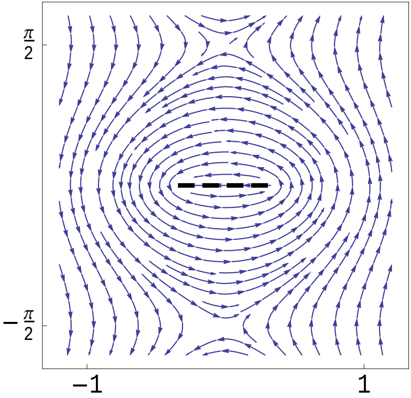


- $\mathbf{E}$: lightfront time.
- Branch points coalesce.
- Become a pole.

Interpolating instantons

Complex $q(\tau)$ plane

$$c = -1/2 \qquad E \equiv E(t + z\sqrt{3})$$

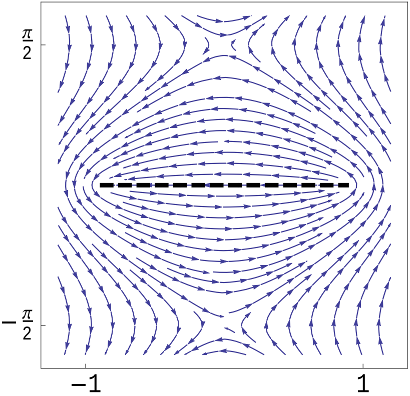


- Spacelike dependence.
- Branch grows again.
- Along real axis.

Interpolating instantons

Complex $q(\tau)$ plane

$c = -1 \qquad E \equiv E(z)$



- If $c\gamma^2 < -1$, branch is ∞ long.
- No periodic solutions.
- No pair production.

Dunne & Schubert, PRD 72 (2005) 105004

Dietrich & Dunne JPA 40 (2007) F825

Cauchy's integral theorem

- Instantons:

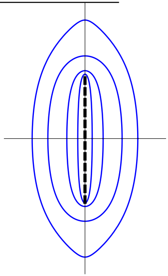
$$q(\tau) = \frac{1}{\omega}\sinh^{-1}\left[\frac{i\sqrt{c\gamma^2}}{\sqrt{1+c\gamma^2}}\sin 2n\pi(\tau-\tau_0)\right]$$

- Instanton contribution: an integral over the instanton itself.

$$-iS[x_{\text{cl}}] \sim \oint \mathrm{d}q\ \dot{q} = -\frac{\pi E_S}{E_0}\frac{2}{1+\sqrt{1+c\gamma^2}}$$

- S_{cl} independent of τ_0.
- Cauchy's theorem: can deform contour freely.
- Vary τ_0: curve remains an instanton solution.

Ilderton, Torgrimsson, Wårdh, PRD 92 (2015) 025009



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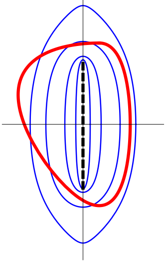
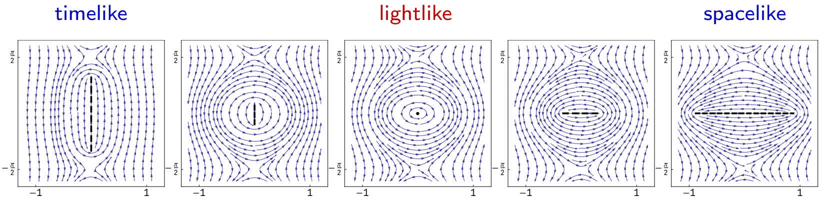
⟹ Only need structure circulated by the instantons.

- Hence details of instantons not needed.
- Distinguishes $c = 0$ from all other cases...
  (lightlike field dependence)



Branches vs. poles



- $\cos\theta + 0$ (timelike, spacelike): branch cut
  extended instantons

- $\cos\theta = 0$ (lightlike): pole
  instantons deformable to points

Ilderton, JHEP 1409 (2014) 166

The lightlike limit

$$S[x_{\text{cl}}] \sim \oint \mathrm{d}q \, \langle \text{ function with a simple pole } \rangle$$

- Poles $\implies$ residues. Ilderton, Torgrimsson, Wårdh, PRD 92 (2015) 025009

$\implies$ Local contributions to Γ in fields $E(t+z)$.

$\to$ Agrees with LCFA approximation. Tsamis et al, PRD62 (2000) 125005

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- Frequency ω multiplied by c in $S[x_{\mathsf{cl}}]$.

$$\exp\big(-iS[x_{\mathsf{cl}}]\big) = \exp\left(-\frac{\pi E_S}{E_0}\frac{2}{1+\sqrt{1+c\gamma^2}}\right)$$

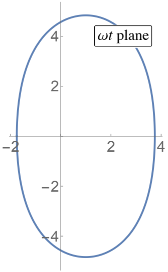
- Lose ω dependence when $c=0$.

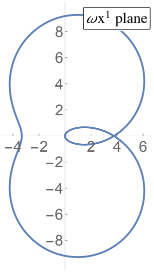
Pair production in ‘rotating’ fields

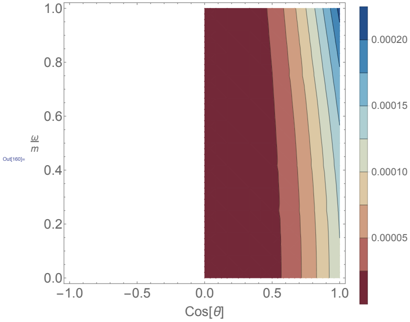
Marinov & Popov Yad.Fiz.18 (1972) 809, S.S. Bulanov PRE 69 (2004) 036408, Bai-Song et al, CPL29 (2012) 021102, Blinne & Gies PRD89 (2014) 085001, Strobel & Xue PRD91 (2015) 045016.

$$\mathbf{E} = \cos\frac{\theta}{2}\begin{pmatrix}\cos\omega q\\ \sin\omega q\end{pmatrix} \qquad \mathbf{B} = \sin\frac{\theta}{2}\begin{pmatrix}-\sin\omega q\\ \cos\omega q\end{pmatrix}$$

Rotating $\mathbf{E}(t)$ $\longrightarrow$ plane wave $\longrightarrow$ static $\mathbf{B}(z)$







Posters by A. Blinne and E. Strobel

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Conclusions

Instantons

- Complex loops $\implies$ only circulated structures matter.

Effective action

- Effect of field inhomogeneity encoded in complex plane.
- Probability always higher for timelike dependence.

Fields depending on lightfront time

- Branch $\rightarrow$ poles.
- Instantons deformable to points.
- Residue theorem $\implies$ local contributions. $\rightarrow$ LCFA.

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Thank you!