Conceptual problems in QED

Polarization of the vacuum:

Challenges and open questions

Rainer Grobe
Intense Laser Physics Theory
Illinois State University

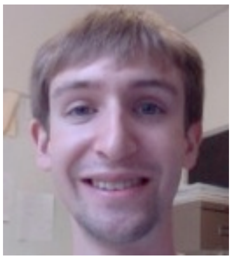
Prof. Charles Su

Prof. Neil Christensen

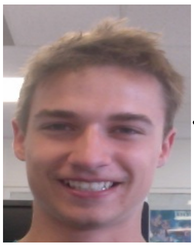
Coworkers

Dr. QingZheng Lv

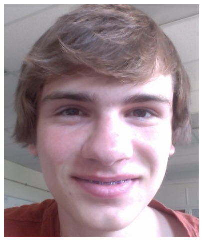
Dr. Sven Ahrens



Andrew Steinacher

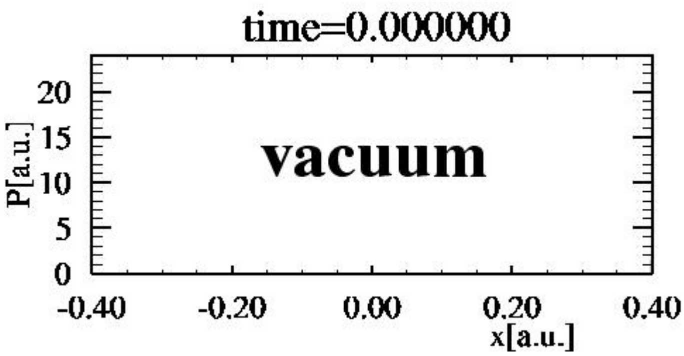


Jarrett Betke



Will Bauer

Pair creation: Space-time simulations



1998: Dirac equation on 1d numerical grid
2002: Generalization to quantum field theory

outcomes:

- Klein paradox resolved
- Zitterbewegung
- Localization problem
- Creation dynamics inside interaction zone
- ....

Does the $e^- - e^+$ attraction increase the pair creation?

(1) No effect

Coulomb energy for $e^- - e^+$ apart by $\lambda_{Compton}$

=> $E = k\, e^2/\lambda^2$ $(= mc^2/ 860)$

=> negligible

but

- Coulomb's law questionable
  for distances 10^{-13} m and times 10^{-21} s
- there are no forces
- how do two particles interact with each other?

PRL 106, 023601 (2011)

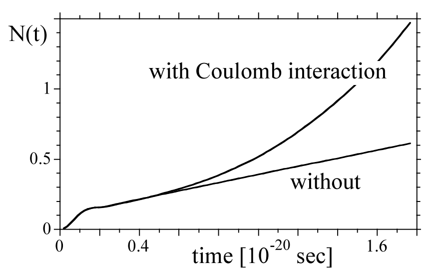
Does the $e^- - e^+$ attraction increase the pair creation?

(2) No

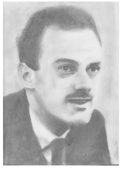
attractive particles harder to separate
(Dr. F. Hebenstreit 16:40 today)

(3) Yes

one-photon exchange in two-loop
correction to the E-H Lagrangian
(Prof. C. Schubert, 15:50 today)



How to incorporate $e^- - e^+$ attraction into a simulation ?



$$i\,\partial_t\,\psi(z,t) = [c\,\sigma_1\,[p - A(z,t)/c] + \sigma_3\,c^2 + V(z,t)]\;\psi(z,t)$$

e^--e^+ field operator

permit particles to generate their own fields
via
their **charge density ρ**

$$(\partial_{ct}^{\,2} - \partial_z^{\,2})\,V(z,t) = 4\pi\,\rho(z,t)$$
$$(\partial_{ct}^{\,2} - \partial_z^{\,2})\,A(z,t) = 4\pi c^{-1}\,j(z,t)$$

8

Charge density ρ(z,t)

$$\rho(z,t) = \langle\,\Phi(t{=}0)\,|\,\underbrace{-[\Psi^\dagger(z,t),\Psi(z,t)]/2}_{\text{charge operator}}\,|\,\Phi(t{=}0)\rangle$$

Example: single electron $\quad|\,\Phi(t{=}0)\,\rangle = b_P^\dagger\,|\,\mathrm{vac}\,\rangle$

$$\rho(z,t) = -\underbrace{|\phi_P(+;z,t)|^2}_{\text{electron's wave function (problematic)}} + \underbrace{\left(\Sigma_{E(+)}\,|\phi_E(+;z,t)|^2 - \Sigma_{E(-)}\,|\phi_E(-;z,t)|^2\right)/2}_{\text{vacuum's polarization density (not understood)}}$$

electron's wave function

(problematic)

vacuum's polarization density

(not understood)

Overview:

Problems with the charge density

Part I: Single particle: $\rho(z,t) = -\ |\phi_P(+;z,t)|^2$

self-repulsion

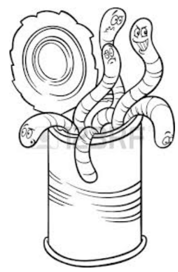
Part II: Vacuum: $\rho(z,t) = (\Sigma_{E(+)} |\phi_E(+;z,t)|^2 - \Sigma_{E(-)} |\phi_E(-;z,t)|^2)/2$

- Feynman propagator approach

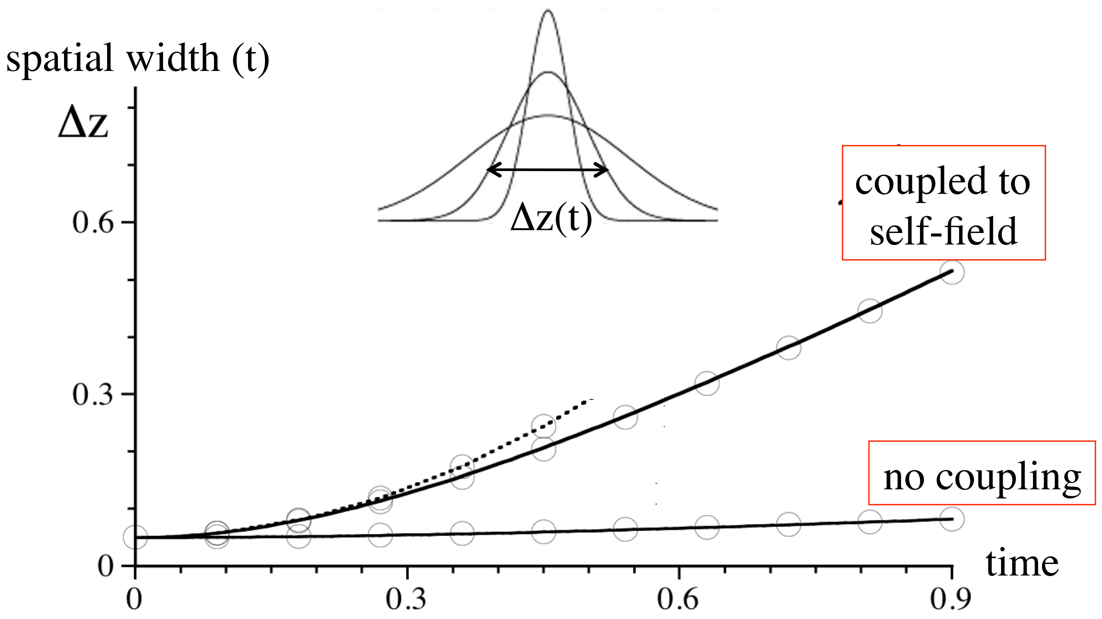
  no dynamics

- External field approximation

  unphysical solutions



Spatial width of a Gaussian wave packet Δz(t)



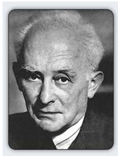
no coupling	=>	Δz(t)	increases due to $\Delta p \neq 0$
with coupling	=>	Δz(t)	explodes due to self-repulsion

PRA 90, 034101 (2014)

Origin of self-repulsion

$\phi(z,t)$				$\phi(z,t+\Delta t)$
e^-	=>	repulsive E-field	=>	self-repulsion
Dirac		Maxwell		Dirac

- $|\phi(z)|^2$
  = temporal average of consecutive measurements
  ≠ simultaneous occurrence of measured locations



- Maxwell: $-|\phi(z)|^2 = \rho(z)$ acts as a simultaneous charge density and generates repulsive E-field

- Dirac: simultaneous reaction of $\phi(z,t)$ to repulsive E-field

Self-repulsion: a classical effect?

quantum mechanical approach

$$i\,\partial_t\,\phi(z,t) \;=\; [c\,\sigma_1\,[p - A(z,t)/c] + \sigma_3\,c^2 + V(z,t)]\,\phi(z,t)$$

$$(\partial_{ct}^{\;2} - \partial_z^{\;2})\,V(z,t) \;=\; 4\pi\,\rho(z,t)$$

$$(\partial_{ct}^{\;2} - \partial_z^{\;2})\,A(z,t) \;=\; 4\pi c^{-1}\,j(z,t)$$

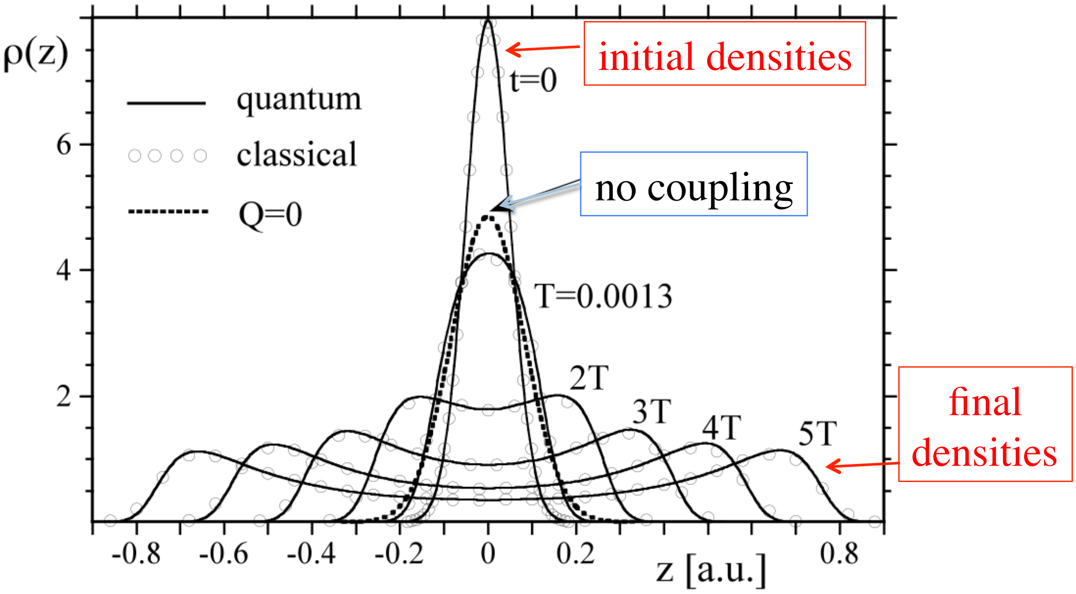
classical mechanical N-particle approach

$$H \;=\; \Sigma_{i=1}^{N}\,[\,(m_{eff}^{\;2}c^4 + c^2\,p_i^{\;2})^{1/2} \;+\; q_{eff}\,\Sigma_{j=1}^{N}\,V(z_i,z_j)]$$

$$\partial_t\,z_i \;=\; \partial H/\partial p_i$$

$$\partial_t\,p_i \;=\; -\partial H/\partial z_i$$

quantum self-repulsion = classical effect



How to remove self-repulsion
from a simulation ?

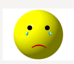
Problems with the charge density

Part I: Single particle: $\rho(z,t) = -\,|\phi_P(+;z,t)|^2$

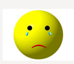
 self-repulsion

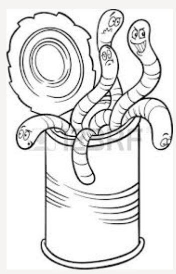
Part II: Vacuum: $\rho(z,t) = (\Sigma_{E(+)}\,|\phi_E(+;z,t)|^2 - \Sigma_{E(-)}\,|\phi_E(-;z,t)|^2)/2$

- Feynman propagator approach

 no dynamics

- External field approximation

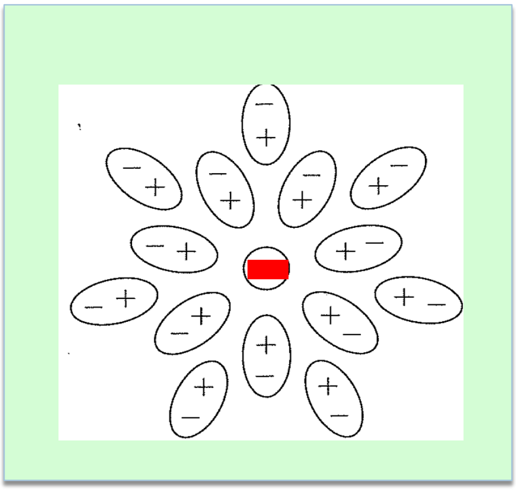
 unphysical solutions



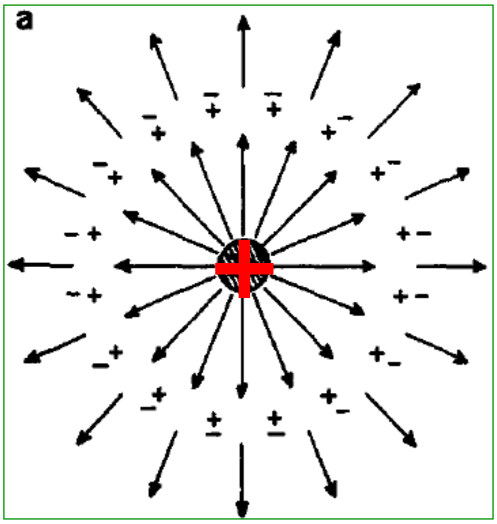
1. conceptual question ...

[redacted]

Peskin Schroeder Fig. 7.8



Greiner Fig. 1.3a



Which sketch is actually correct ?

2. conceptual question ...

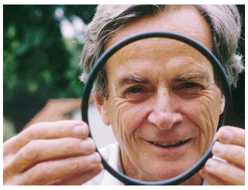
Are vacuum polarization charges real ?

Yes,

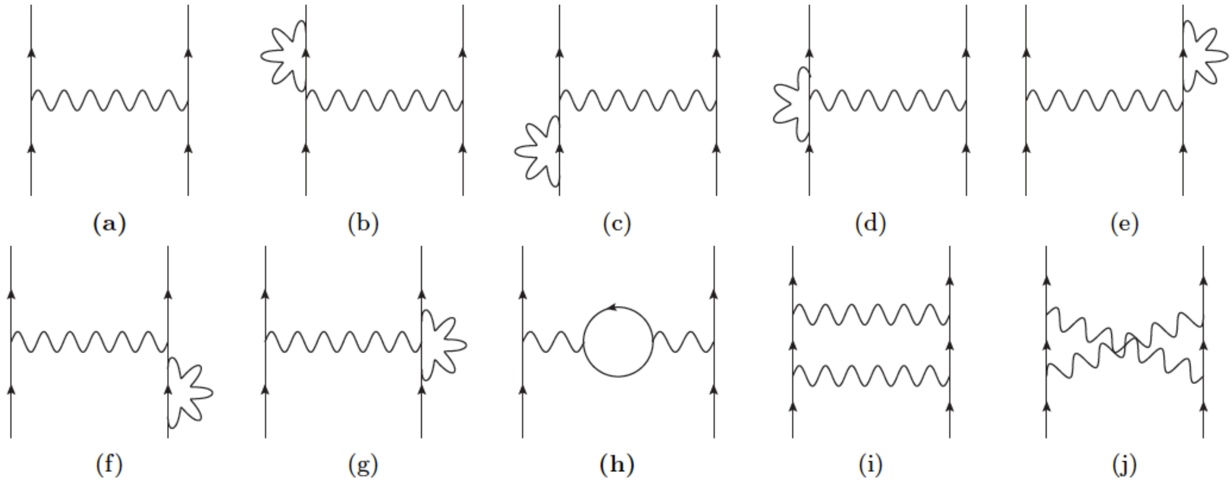
- charge density $\langle\, [\Psi^{\dagger}(z), \Psi(z)]/2 \,\rangle$ is nonzero

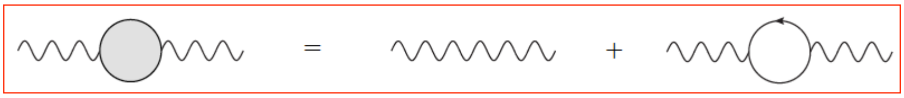
No,

- they are just displaced virtual charges
- just a mental picture to visualize
  non-Coulombic behavior of the true force



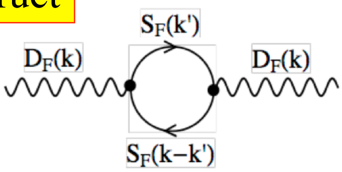
(1) Propagator based approach





true force = $1/r^2$ + correction

construct



$$-\int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[V_\sigma S_F(p) V_\lambda S_F(p+k)\right]$$

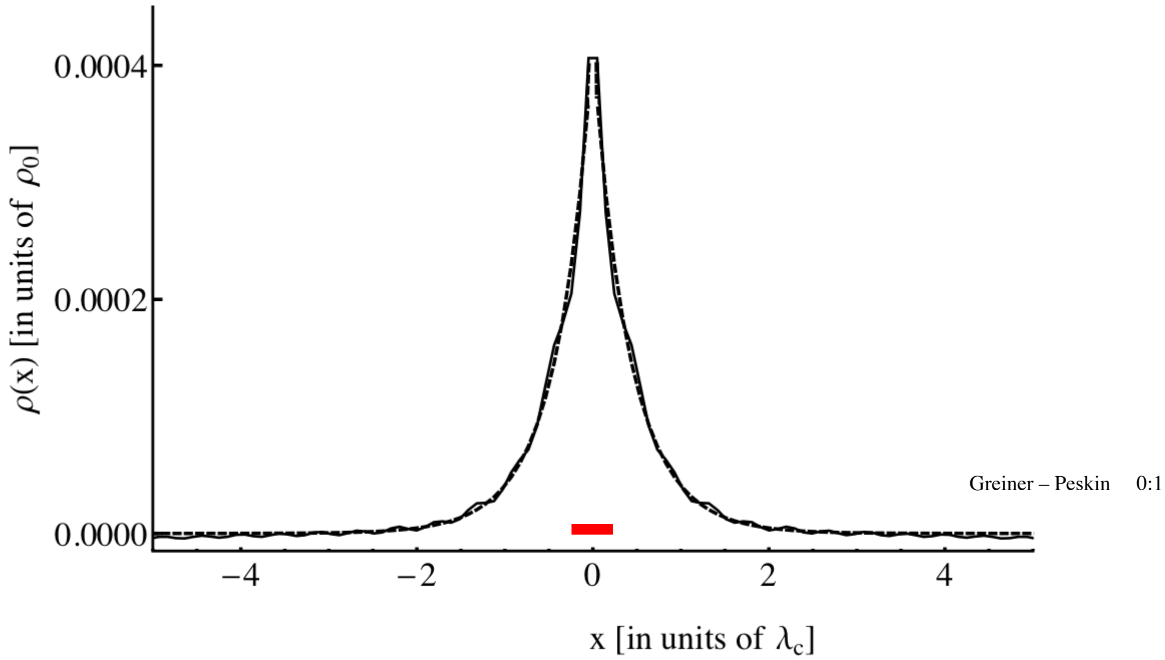
$$D_F'^{\mu\nu}(k) = D_F^{\mu\nu}(k) + D_F^{\mu\sigma}(k)\left[i\Pi_{\sigma\lambda}(k)\right] D_F^{\lambda\nu}(k)$$

infinities, Feynman parameterization, regularization

$$D'_{F\mu\nu}(k) = D_{F\mu\nu}(k)\left[1 + \frac{e_b^2 k_e}{\hbar c} P(k^2)\right]$$

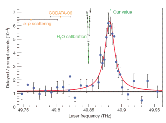
charge renormalization: find $e_{bare} = f(e_{physical})$

$$\rho(x) = 2 e \alpha \hbar m^{-1} c^{-1} \int_1^\infty dt\, \tau^{-3} \sqrt{(\tau^2-1)}^{-1} \exp[-2mc\tau|x|/\hbar]$$

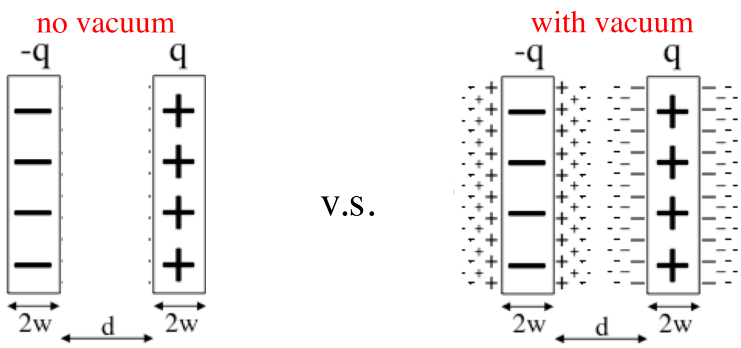


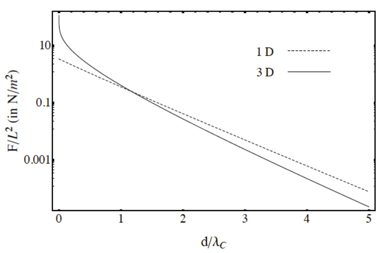
E.A. Uehling, Phys. Rev. 48, 55 (1935)
E.H. Wichmann and N.H. Kroll, Phys. Rev. 101, 843 (1956)

3. conceptual question ...



Effect of vacuum polarization on mechanical forces F ?





Lv et al. PRA (submitted)

effect of the vacuum:

- F increases
- F depends on spacing d

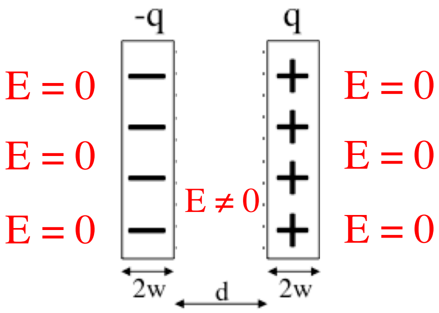
4. conceptual question ...

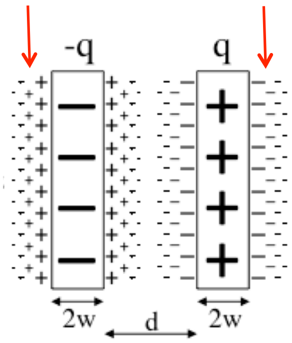
Is an electric field E necessary to polarize the vacuum ?

Yes: e^- and e^+ can only be separated by a force

No: for two parallel plates we have E=0 outside

but charges **ARE** induced





(2) External-field approximation based approach

$$\rho_{pol}(z) = \langle\, \mathbf{VAC} \,|\, -\mathbf{e}\, [\Psi^{\dagger}(z), \Psi(z)]/2 \,|\, \mathbf{VAC} \,\rangle$$

Dirac equation

$\rho_{ext}(z)$

$$[c\, \sigma_1\, p_z + mc^2\, \sigma_3 + \mathbf{e}\, \mathbf{V_{ext}(z)}]\, \Phi_E(z) = E\, \Phi_E(z)$$

$$\rho_{pol}(z) = \mathbf{e}\, \left(\Sigma_{E(+)} |\Phi_E(+;z)|^2 - \Sigma_{E(-)} |\Phi_E(-;z)|^2\right)/2$$

$\mathbf{e = e_{physical}}$ => no charge renormalization necessary

$$\langle e\,[\Psi^\dagger(z), \Psi(z)]/2 \rangle \neq \text{ correct charge density } \rho_{pol}(z)$$

- $\langle\, e\,[\Psi^\dagger(z), \Psi(z)]/2\, \rangle \;=\; \rho_{unphysical}(z) \;+\; \rho_{pol}(z)$

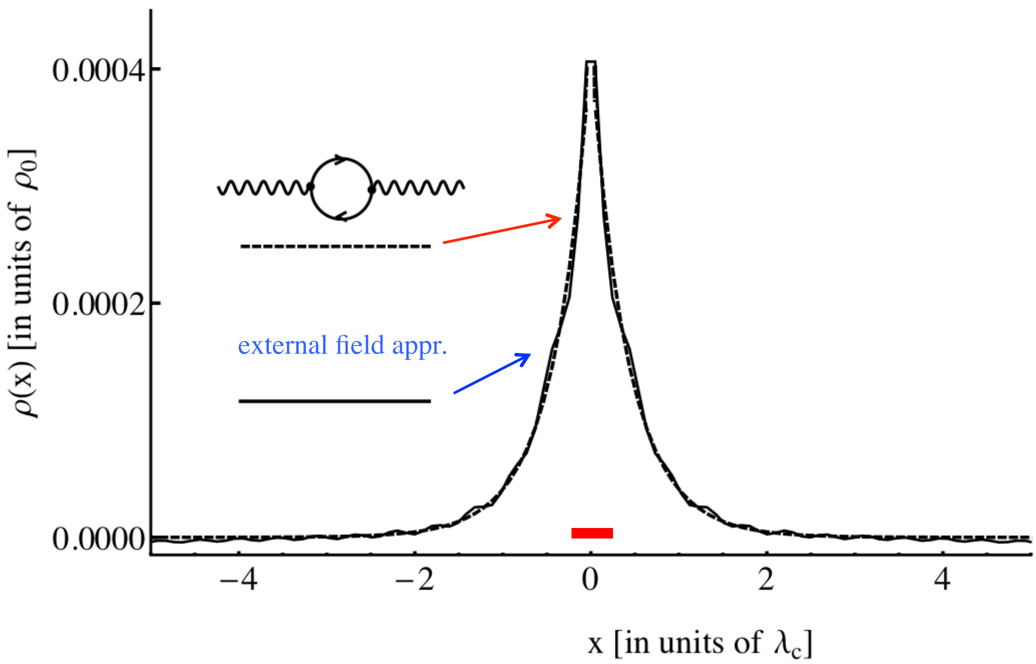
  Subtraction of unphysical contribution reveals $\rho_{pol}(z)$

- $\rho_{unphysical}(z) \;=\; \rho_{Foldy\text{-}Wouthuysen}(z)$

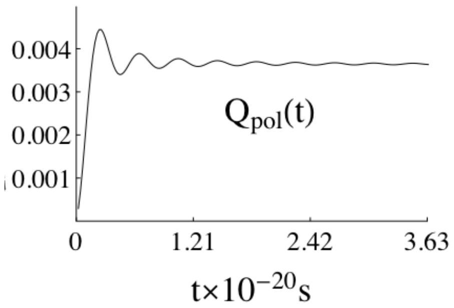
  $\rho_{unphysical}$ well understood

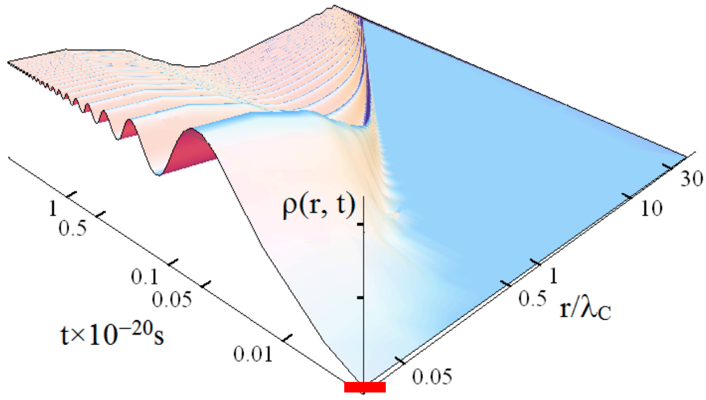
  true origin of $\rho_{unphysical}$ not understood

Comparison (after subtraction):
Feynman = external field approximation !



Dynamical polarization of the vacuum

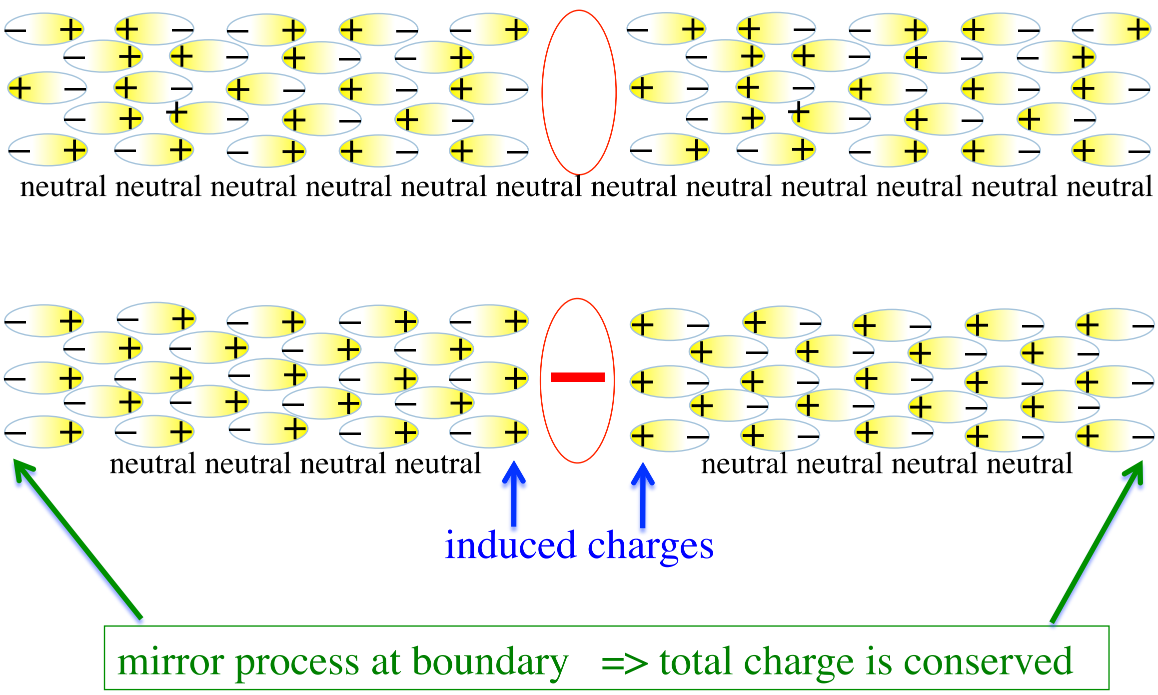




- local creation of positive charges (screening)
- approaches steady state ρ(z) (= Uehling)
- time-dependent charge renormalization

Q. Lv et al, PRA (submitted)

Charge conservation in the dynamical polarization



Q. Lv et al, PRA (submitted)

Summary

• Goal: impact of e^- - e^+ forces on pair creation

• Main tool: computational quantum field theory

• Early stage progress:

Feynman approach	=	External-field approximation
charge renormalization		unphysical solutions
static properties		space-time resolution