Critical Schwinger Pair Production

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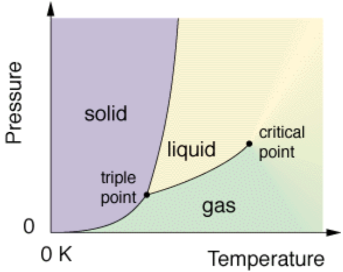
Universality

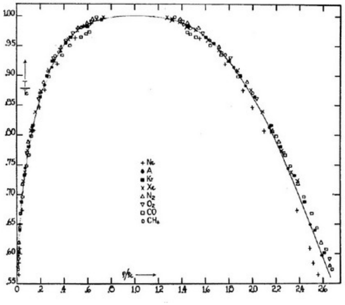
Universality

(van der Waals'1873; Gibbs; Ehrenfest; Landau; ...)

▷ liquid-gas phase diagram

density-temperature phase diagram





(Guggenheim'45)

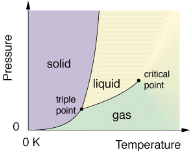
▷ universal scaling law near critical point

$$\rho \sim (T_c - T)^{\beta}, \quad \beta \simeq \frac{1}{3}$$

Universality classes

(60s-early 70s: Widom; Pokrovskii,Patashinskii; Kadanoff; …; Wilson'71)

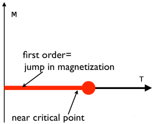
▷ liquid-gas:



▷ order parameter Φ

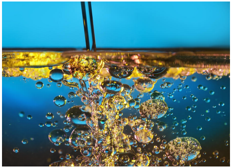
$$\Phi = \rho \quad \text{density}$$

▷ ferromagnet:



$$\Phi = M \quad \text{magnetization}$$

▷ fluid mixtures:



$$\Phi = C \quad \text{composition}$$

Universality classes

(60s-early 70s: Widom; Pokrovskii,Patashinskii; Kadanoff; ...; Wilson'71)

▷ Universal scaling law:

$$\Phi \sim (T_c - T)^{\beta}$$

▷ Critical exponent (3D Ising universality class): (review: Vicari'07)

$$\beta \simeq 0.326(2) \qquad \simeq \frac{1}{3}$$

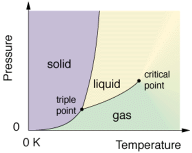
▷ Quantitative critical behavior depends only on:

- dimension
- symmetries
- long-range degrees of freedom

$\Longrightarrow$ microscopic details become irrelevant

Critical phenomena, scaling laws and universality

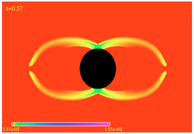
▷ statistical physics:



▷ order parameter Φ

$$\Phi =\sim (T_c - T)^{\beta}$$

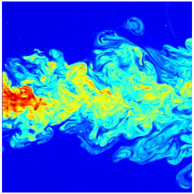
▷ gravitational collapse:



▷ black-hole mass (Choptuik'93)

$$\Phi \equiv M \sim (p - p_c)^{\beta}, \quad \beta \simeq 0.37$$

▷ turbulence:

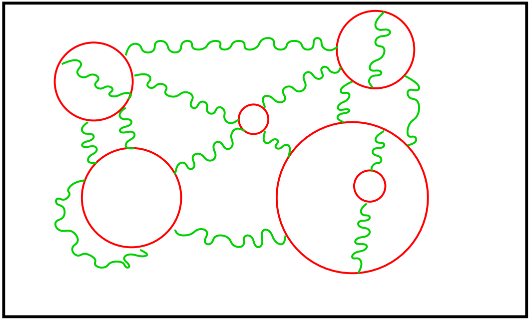


▷ energy spectrum (Kolmogorov'41)

$$E(k) \sim k^{-\beta}, \quad \beta \simeq \frac{5}{3}$$

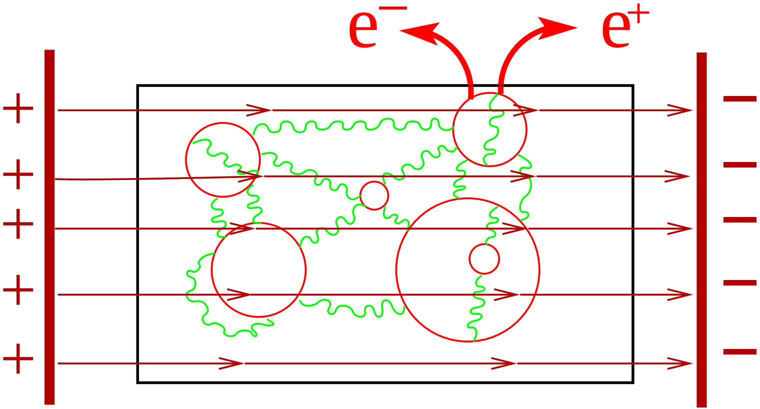
Critical Schwinger Pair Production

Schwinger Pair Production



(Sauter'31; Heisenberg,Euler'36; Schwinger'51)

Schwinger Pair Production

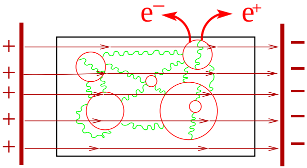


▷ electric fields: Schwinger pair production "vacuum decay"

(Sauter'31; Heisenberg,Euler'36; Schwinger'51)

Schwinger pair production

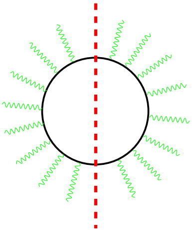
(SAUTER'31; HEISENBERG,EULER'36; SCHWINGER'51)



(ExHILP: ALKOFER, ILDERTON, GROBE, SCHUBERT, HEBENSTREIT, MEUREN, D'HUMIERES, KÄMPFER, KIM, MACKENROTH, MÜLLER, ELKINA, GRISMAYER, BELOV, BLINNE, EFIMENKO, KASPER, KHARIN, KOHLFÜRST, STROBEL, WÖLLERT)

▷ vacuum decay: $(E_{cr} = \frac{m^2}{e})$

$$\mathrm{Im}\,\Gamma = \mathrm{Vol}\,\frac{(eE)^2}{8\pi^3}\sum_{n=1}^{\infty}\frac{\exp\left(-\frac{m^2}{e}\frac{n\pi}{E}\right)}{n^2}$$

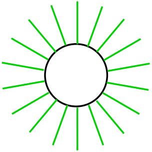


"nonperturbative" phenomenon

$\Longrightarrow$ $E = \text{const.}$, no critical behavior

Effective action in worldline representation

$$\Gamma[A] \quad = \quad \sum$$



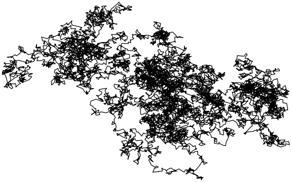
$$= -\frac{1}{2}\int_0^\infty \frac{dT}{T}\, e^{-m^2 T}\, \mathcal{N}\int_{x(T)=x(0)} \mathcal{D}x(\tau)\, e^{-\int_0^T d\tau \left(\frac{\dot{x}^2}{4} + i\dot{x}\cdot A(x(\tau))\right)}$$

(Feynman'50)

(Bern&Kosower'92; Strassler'92)

(Schmidt&Schubert'93)

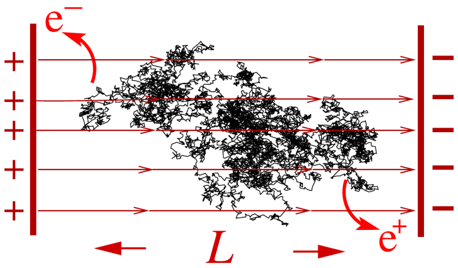
$$x(T) \quad =$$



▷ worldline ∼ "spacetime trajectories of quantum fluctuations"

e^+e^- pair production

▷ Pair production requires delocalization !



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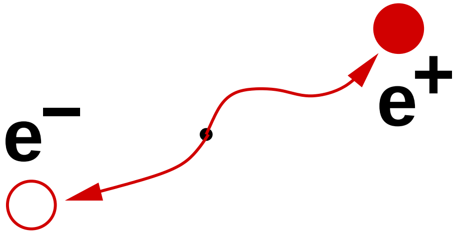
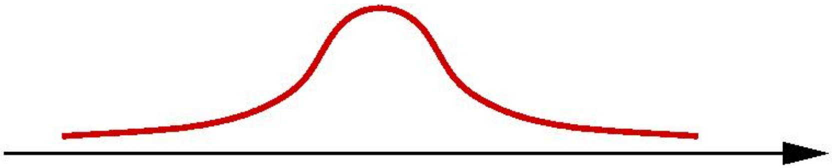


$$e \int ds \cdot \mathbf{E} > 2m$$

e^+e^- pair production

▷ e.g., a localized field $E \sim \mathrm{sech}^2 kx$:



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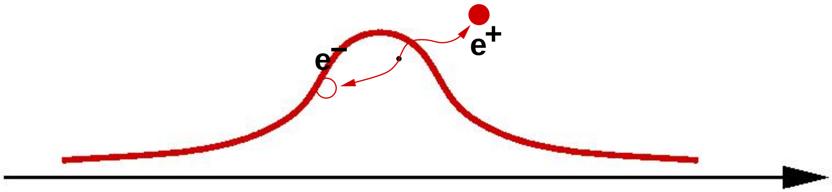

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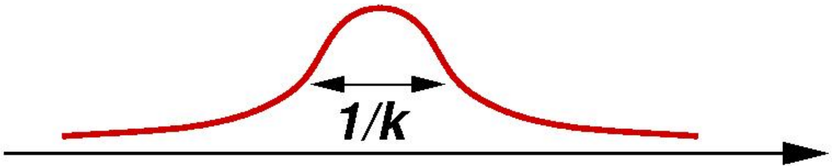
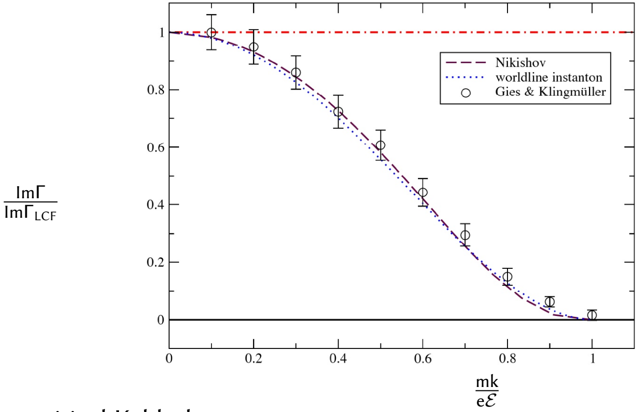


Exactly soluble example

▷ e.g., a localized field $E(x) = \mathcal{E}\mathrm{sech}^2 kx$: (Nikishov'70; HG,Klingmüller'05; Dunne,HG,Wang,Schubert'06)



▷ critical Keldysh parameter:

$$\gamma = \frac{mk}{e\mathcal{E}} \equiv \frac{2m}{E_{el}}, \qquad \gamma_{cr} = 1$$

Universality?

- critical scaling?

$$\text{“order parameter”} \quad \Phi \;=\; \mathrm{Im}\,\Gamma \sim (1-\gamma^2)^\beta \qquad ?$$

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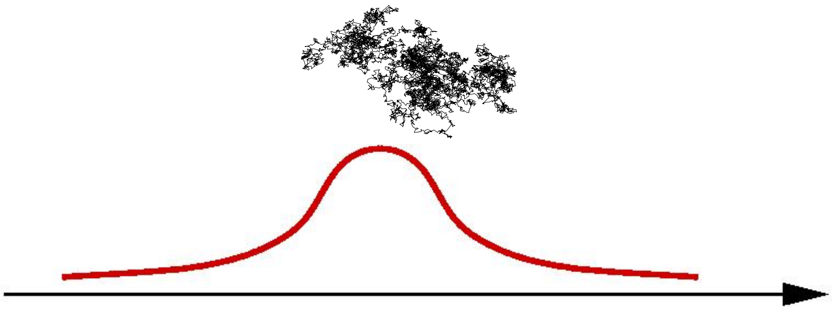
▷ onset of pair production dominated by long-range fluctuations



Universality?

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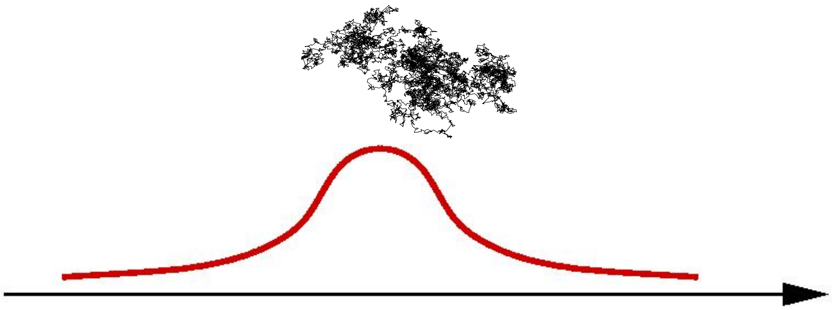


BUT: explicit mass scale m!

Critical Schwinger pair production

(HG,Torgrimsson'15)

▷ semiclassical critical regime:

$$\left(\frac{e\mathcal{E}}{m^2}\right)^2 \ll 1-\gamma^2 \ll 1$$

$\Longrightarrow$ dominance of Euclidean worldline instantons

(Affleck,Alvarez,Manton'81)
(Dunne,Schubert'05)
(Dunne,Wang,HG,Schubert'06)

$$\Gamma[A] = -\int_0^\infty \frac{ds}{s} e^{-im^2 s} \int_{x(s)=x(0)} \mathcal{D}x e^{i\int_0^s d\sigma \left(\frac{\dot{x}^2}{4} - eA\cdot\dot{x}\right)}$$

$$\text{Im } \Gamma[E] \sim \sum_{\text{inst}} \det S^{(2)}_{\text{inst}} \; e^{-S_{\text{inst}}}$$

▷ near critical regime: dominance of "largest" instanton

Critical scaling

▷ universality classes: (HG,Torgrimsson'15)

- e.g., unidirectional fields with monotonic potentials

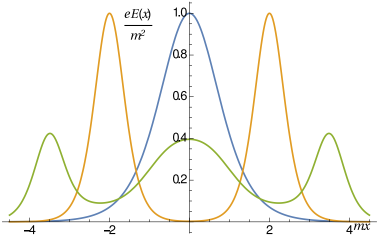
$$E \sim \mathcal{E} \times \frac{1}{|x|^p}, \quad \text{for} |x| \to \infty$$

▷ only asymptotic behavior matters:

$$\operatorname{Im}\Gamma[E] \sim (1-\gamma^2)^\beta, \quad \beta = \frac{5p+1}{4(p-1)} \quad \text{for } p > 3$$

▷ exponential asymptotics

$$p \to \infty, \quad \beta = \frac{5}{4}$$



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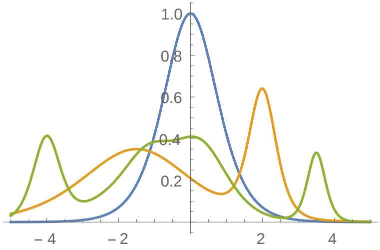
only asymptotic symmetry required



Critical scaling

▷ special universality classes: $1 < p < 3$ (HG,Torgrimsson'15)

$$\operatorname{Im}\Gamma[E] \sim (1-\gamma^2)^\beta \exp\left(-\frac{\pi m^2}{e\mathcal{E}} \frac{C}{(1-\gamma^2)^\lambda}\right), \quad \lambda = \frac{3-p}{2(p-1)}$$

$\Longrightarrow$ essential scaling

- $\sim$ BKT phase transition of XY model (Beresinski'71; Kosterlitz, Thouless'73)

  ...vortex unbinding, thin liquid Helium films

- $\sim$ (beyond) Miranski scaling ("conformal phase transition")

  (Miransky,Yamawaki'89)
  (Braun,Fischer,HG'11)

  ...many-flavor-QCD, walking technicolor

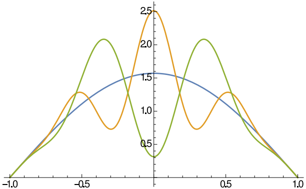
Critical scaling

▷ fields with compact support, special case:

$$E \sim \mathcal{E}|x_0 - x|, \quad \text{for}|x| < |x_0|$$

▷ scaling with log-corrections:

$$\operatorname{Im}\Gamma[E] \sim \frac{(1-\gamma^2)^{\frac{1}{2}}}{-\ln(1-\gamma^2)}$$



- $\sim$ phase transition with contribution from RG marginal operators

(Wegner'72)

2d 4-state Potts model, 2d Ising model with impurities
tricritical points, random graph systems

Conclusion

- Onset of Schwinger pair production

  = critical point dominated by long-range fluctuations

- Universality and independence of local details of field profile

  ...only field asymptotics is relevant

- Field profiles form universality classes

  ...critical Schwinger effect features a variety of scaling laws

Outlook

- generalization to other dimensions

  …partly straightforward

- inclusion of time-dependence

  …enhancement from multi-photon effects

  …critical point $\gamma_{\rm cr} \to \infty$?

  (Ilderton,Torgrimsson,Wardh'15)

- deeply-critical regime $1 - \gamma^2 \ll \left(\frac{e\mathcal{E}}{m^2}\right)^2 \ll 1$

  …unhanced universality?

- RG description?