



Scottish Universities  
Physics Alliance



# Radiation production from the LWFA and its relevance to high field physics

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# Laser-plasma accelerators

## Parameters

- Undulator radiation

- Betatron radiation

- Radiation reaction

- SCAPA

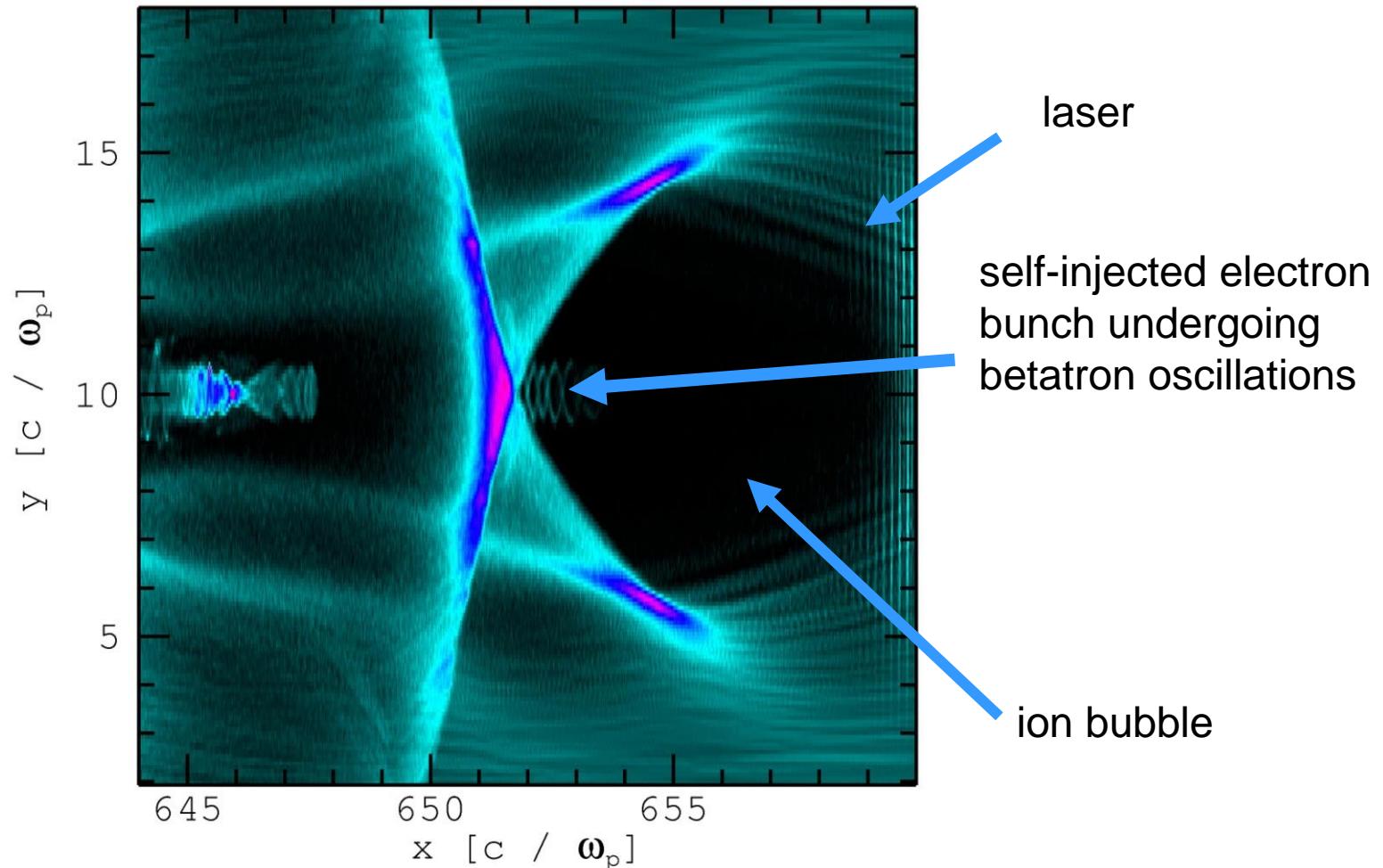
# Bubble structure – relativistic regime



First experiments on controlled acceleration:  
2004

- ALPHA-X (UK: IC, Strathclyde, RAL)
- LBNL (US)
- LOA (France)

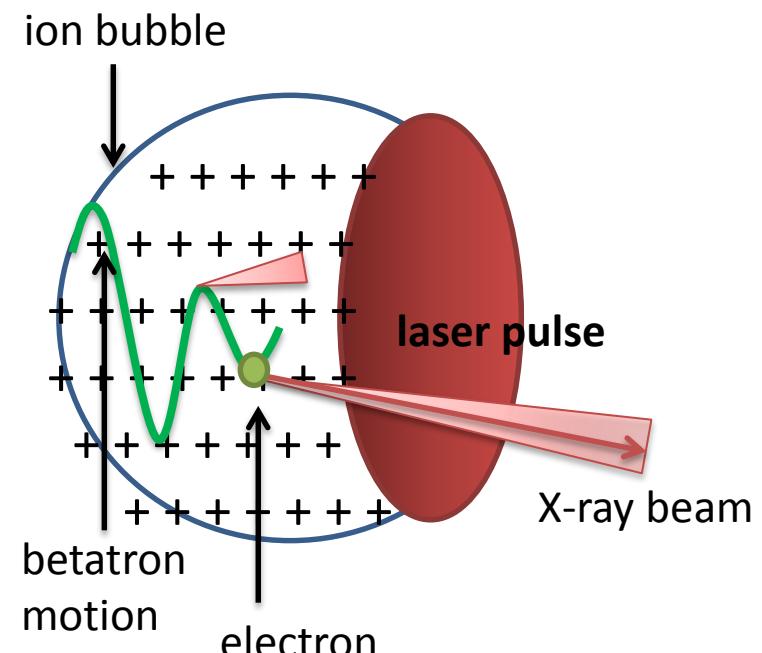
$$\text{ion bubble radius } R \approx \frac{\sqrt{a_0} \lambda_p}{2}$$



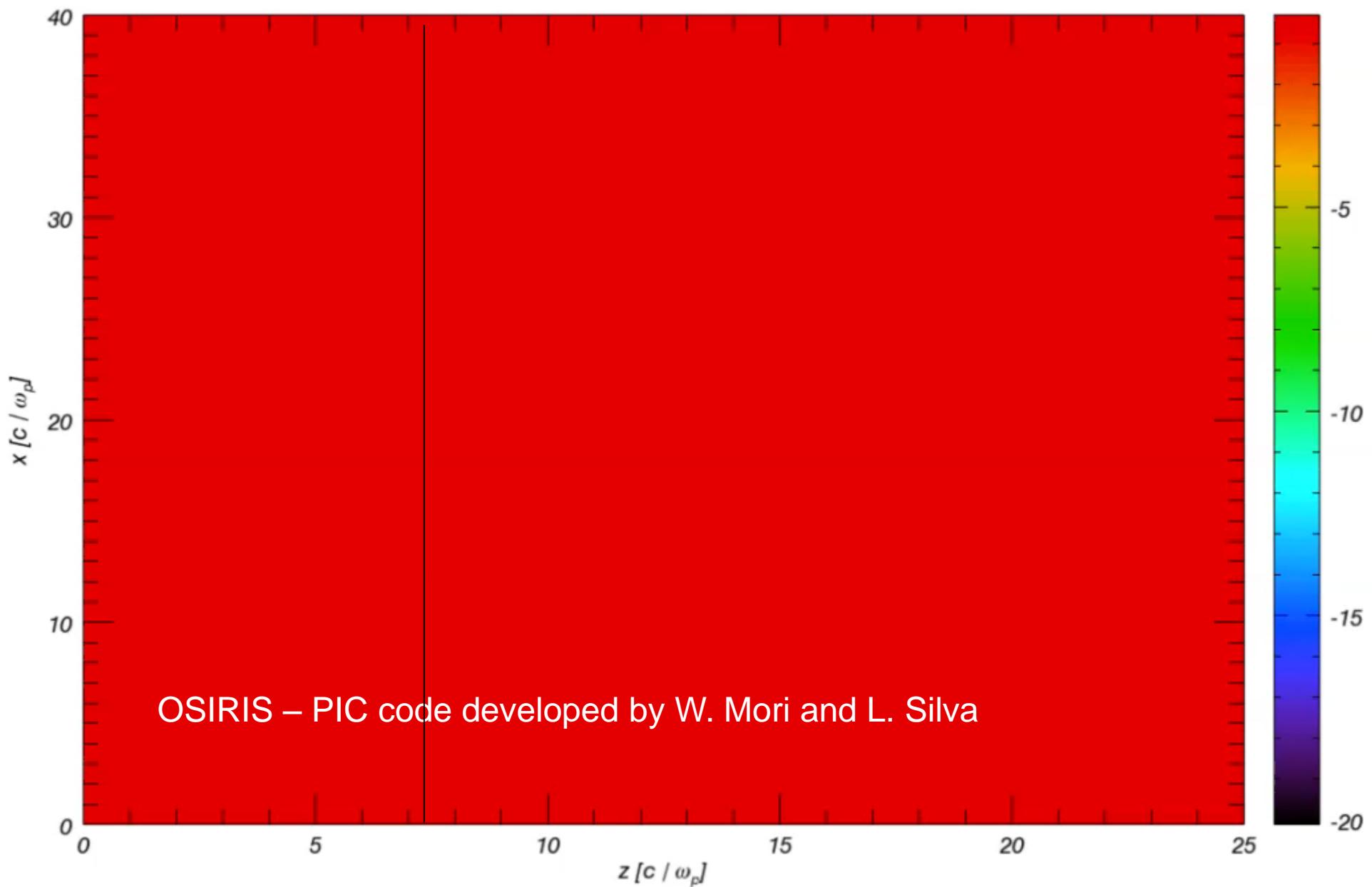
# LWFA

## BUBBLE REGIME

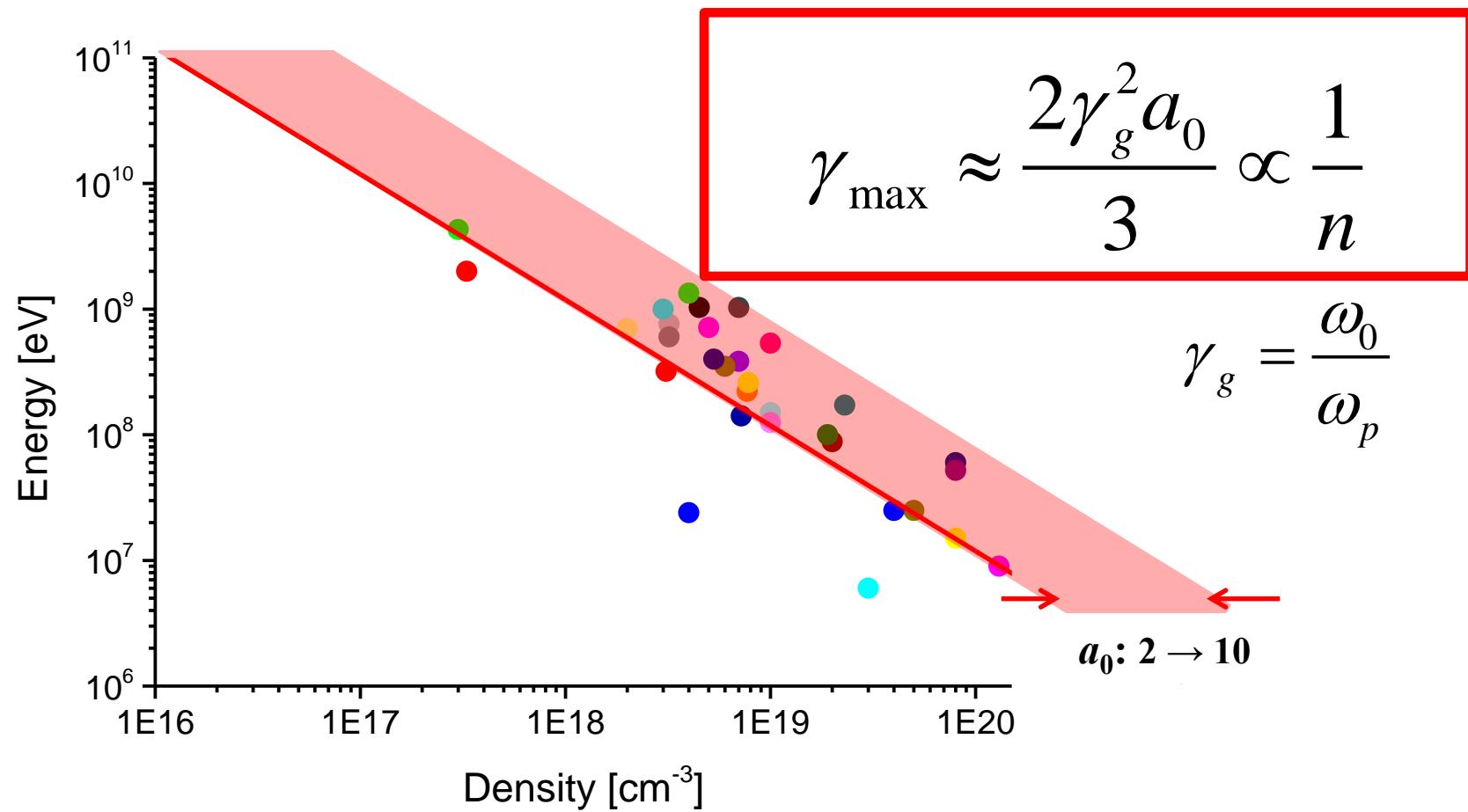
- Laser ponderomotive force creates stable evacuated spherical structures trailing the laser pulse
- Trapped electrons undergo transverse oscillations while accelerating
- Synchrotron like radiation is emitted in a narrow cone



Time = 0.00 [1 /  $\omega_p$ ]



# Energy Scaling



Need a combination of bubble regime (must be above critical power for relativistic self-focussing) and linear regime to get to very high energies

# ALPHA-X: Advanced Laser Plasma High-energy Accelerators towards X-rays – Template for SCAPA

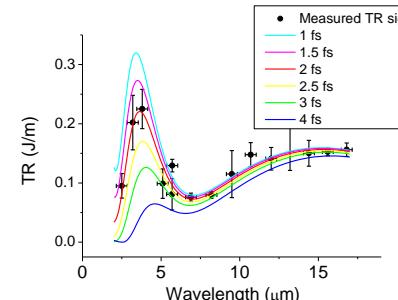


Compact R&D facility to develop and apply femtosecond duration particle, synchrotron, free-electron laser and gamma ray sources

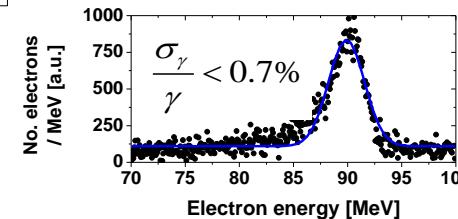


Jaroszynski et al., (Royal Society Transactions, 2006)

CTR: electron bunch duration:  
1-3 fs



electron beam spectrum

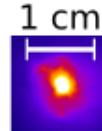


phase contrast imaging with 50 keV photons



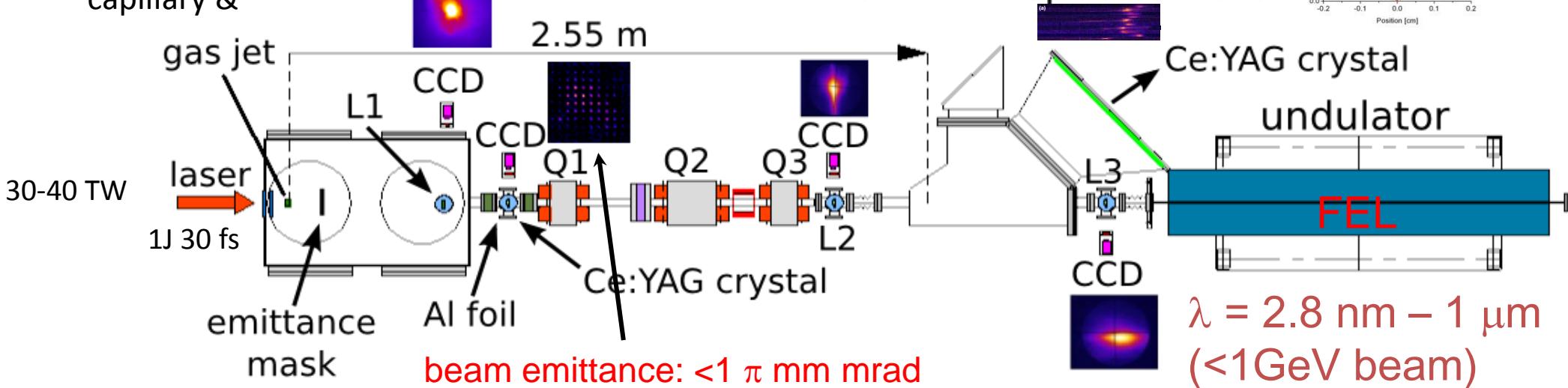
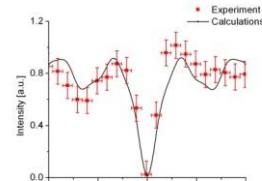
capillary &

gas jet



2.55 m

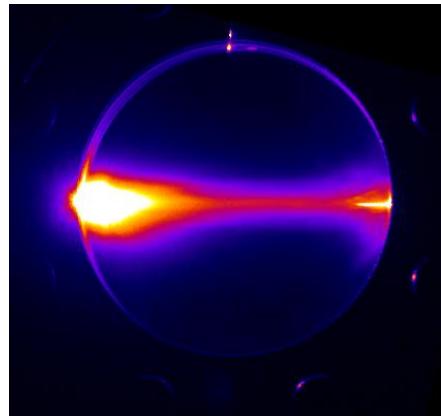
electron spectrometer



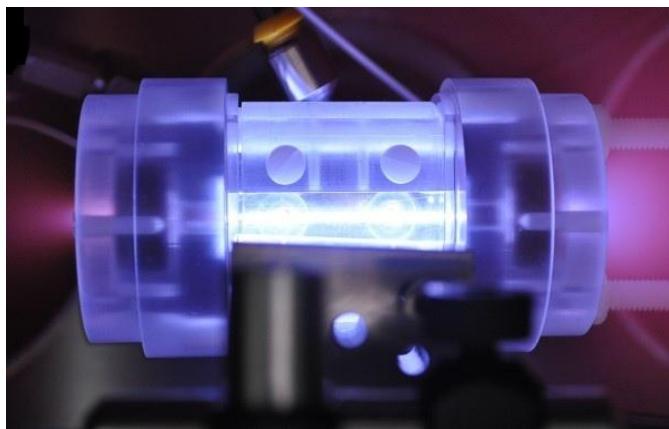
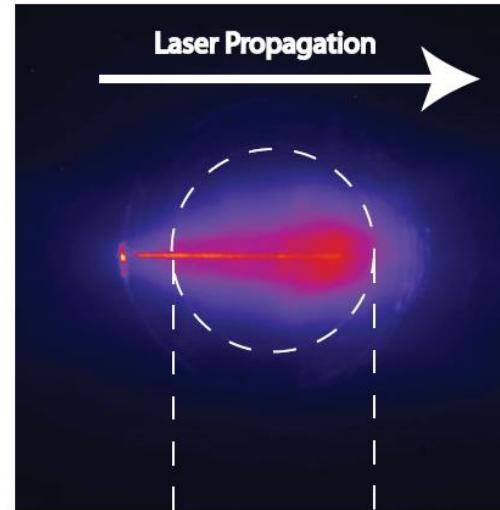
Brilliant particle source: 10 MeV → GeV, kA peak current, fs duration

# Plasma media: capillary, gas jet and plasma cells

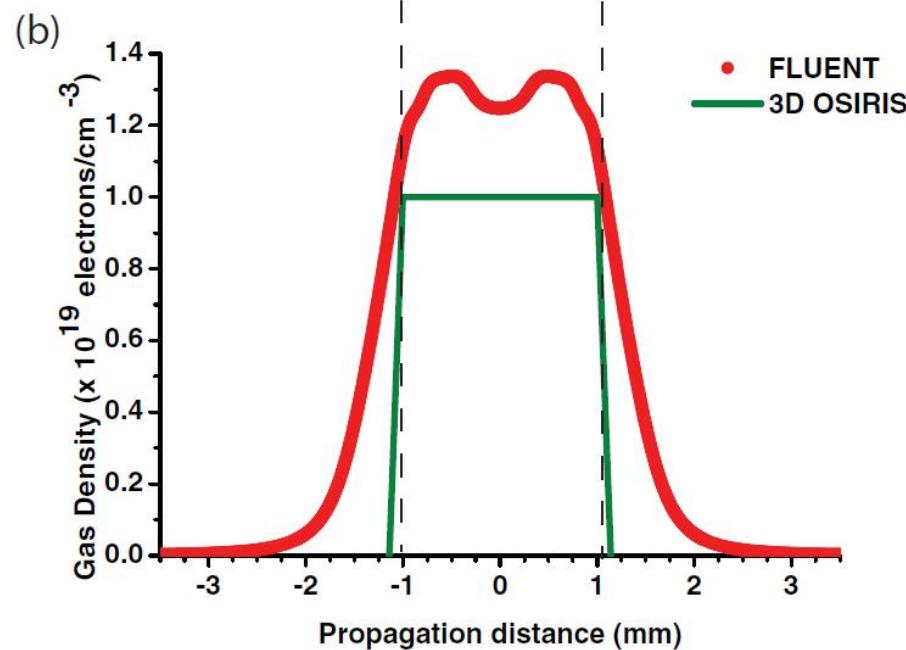
4 cm long gas  
Cell  
10 J, 50 fs  
= 850 MeV  
(RAL)



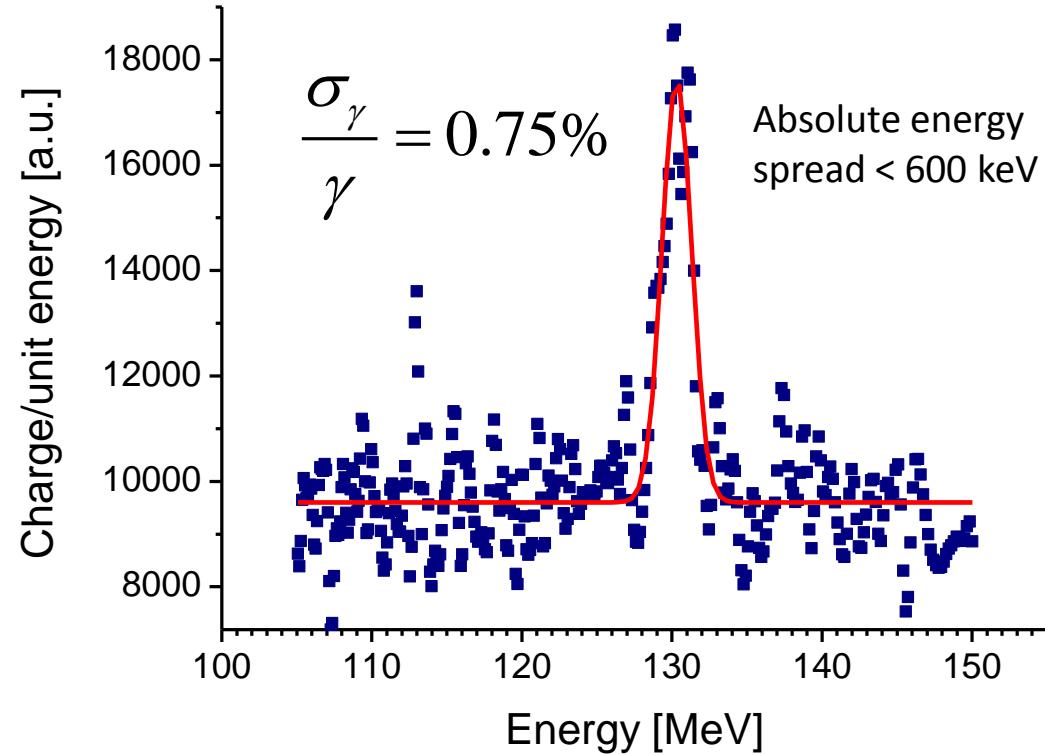
2 mm  
Gas jet  
1 J, 40 fs  
= 100 – 300 MeV



4 cm  
Plasma capillary  
10 J, 50 fs  
~ 1 GeV (RAL)

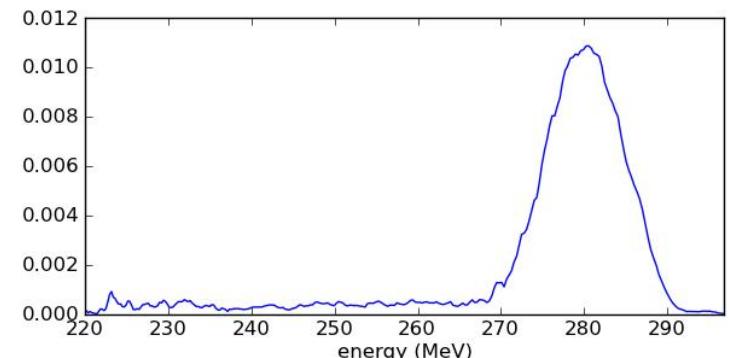
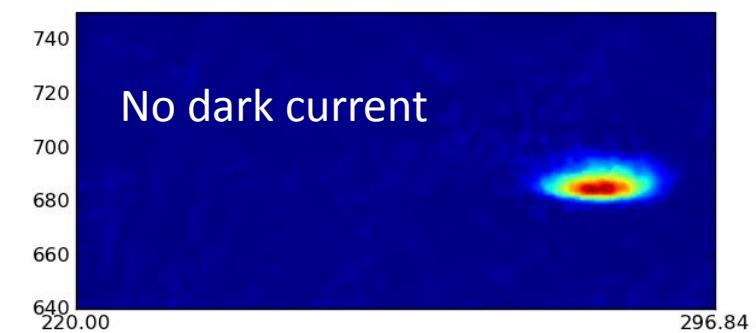
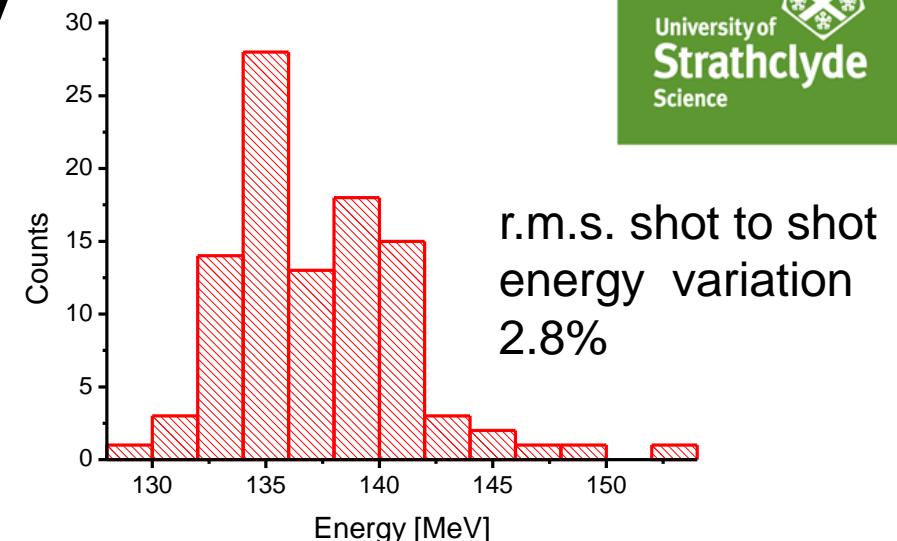


# Energy spread, beam loading and stability

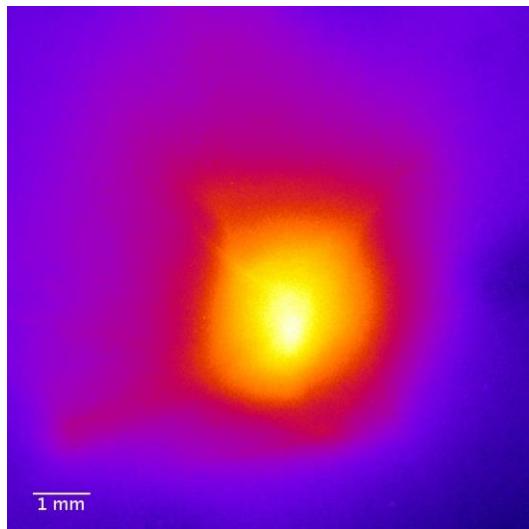


Maximum energy obtained in 2 mm  $\approx 300$  MeV

With stable laser and gas jet



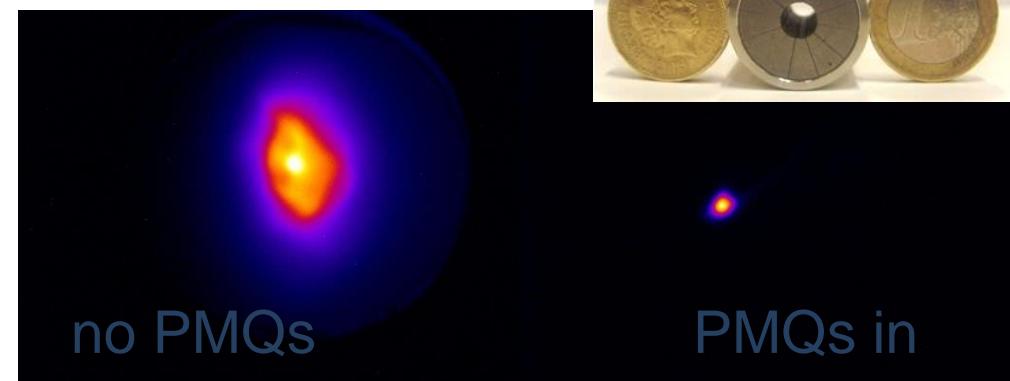
# Electron Beam- Pointing Stability



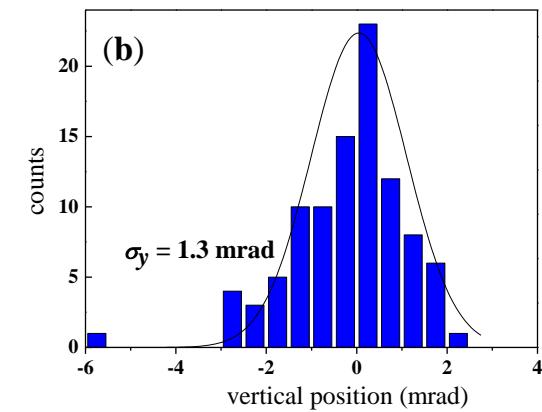
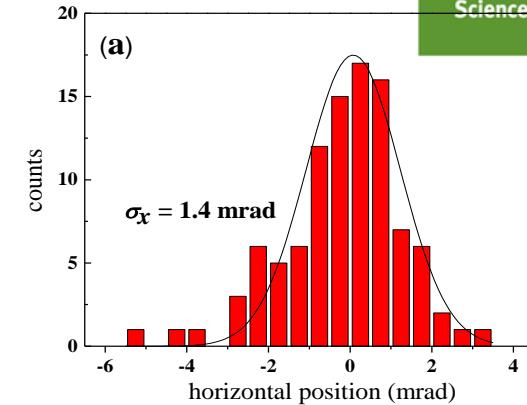
Electron  
beam  
recorded on  
YAG screen

High  
resolution

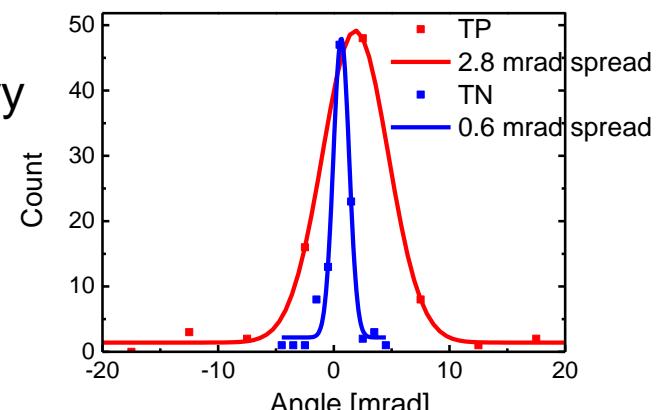
Electron beam pointing deviation is less  
than one spot size



Gas jet

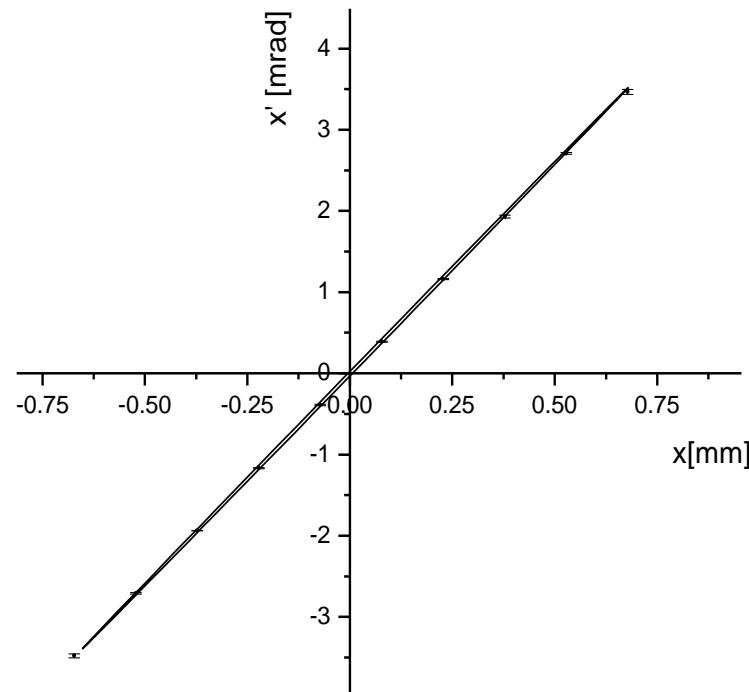
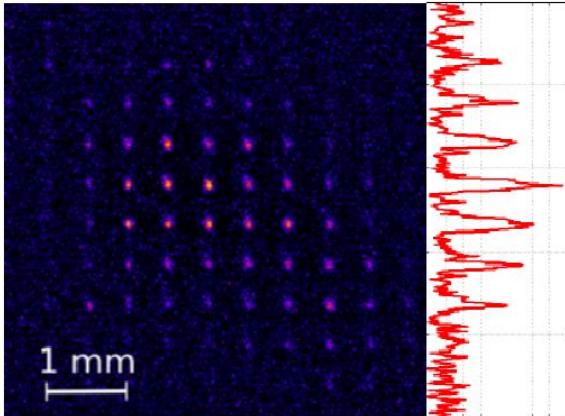


Capillary

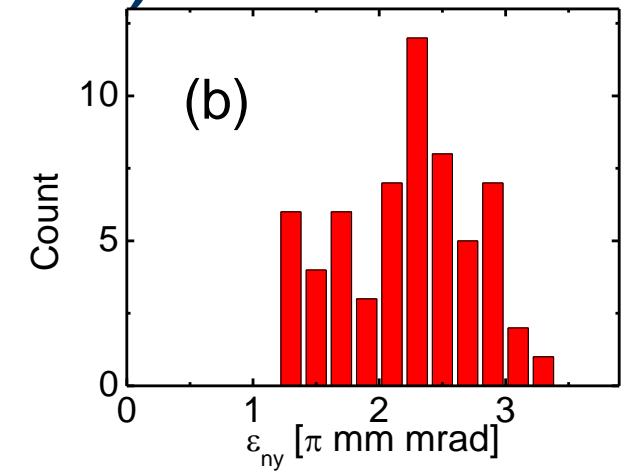
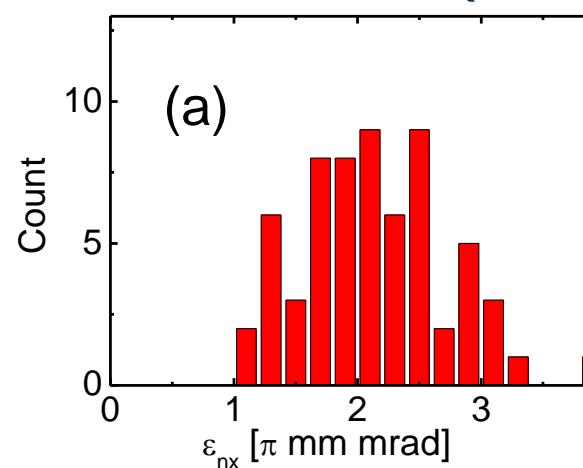


# Emittance

- Thin tungsten mask with holes  $\phi \sim 25 \mu\text{m}$

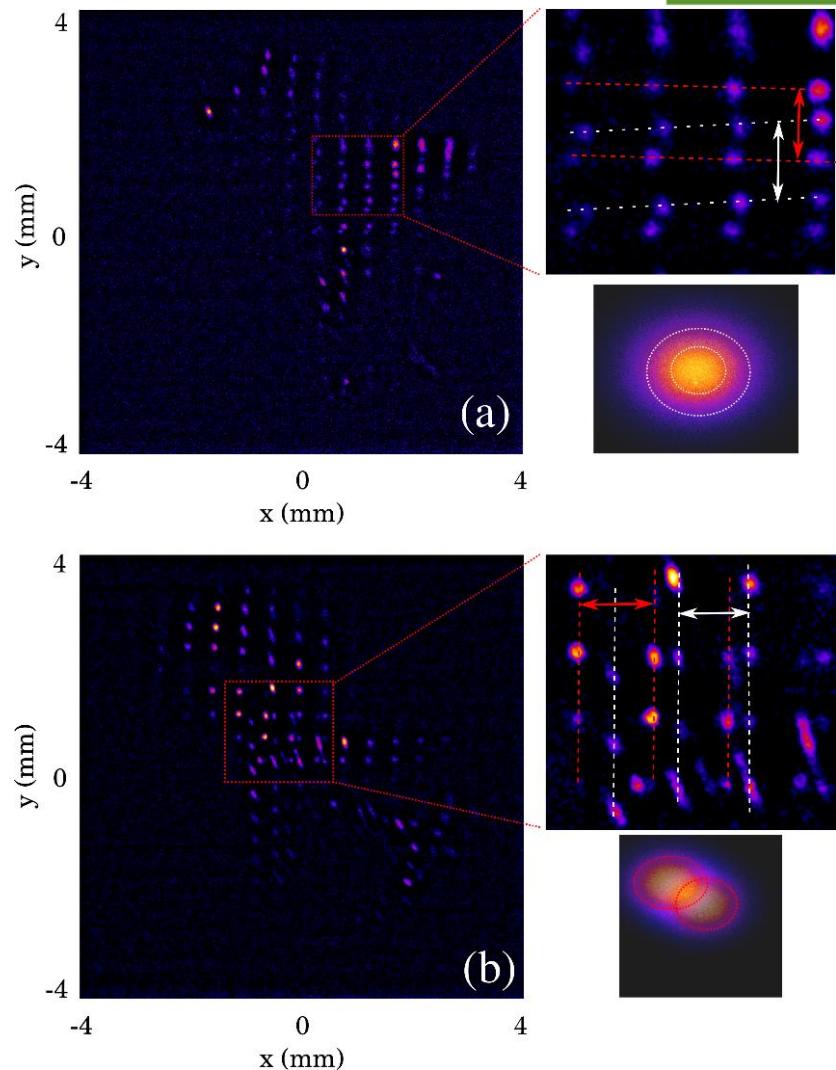
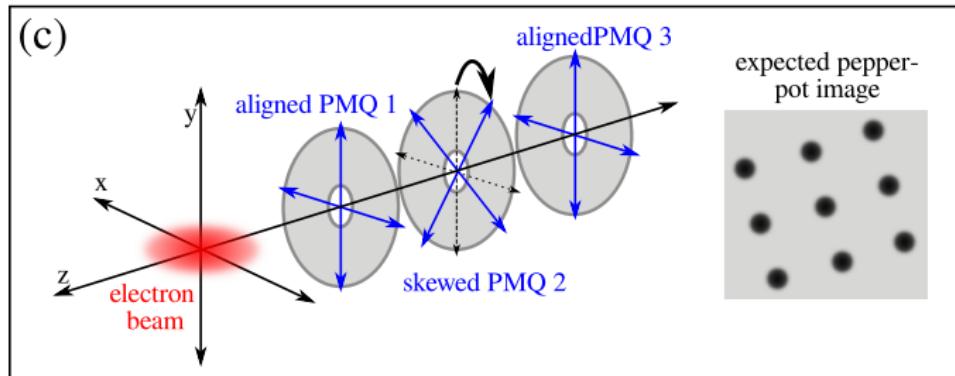
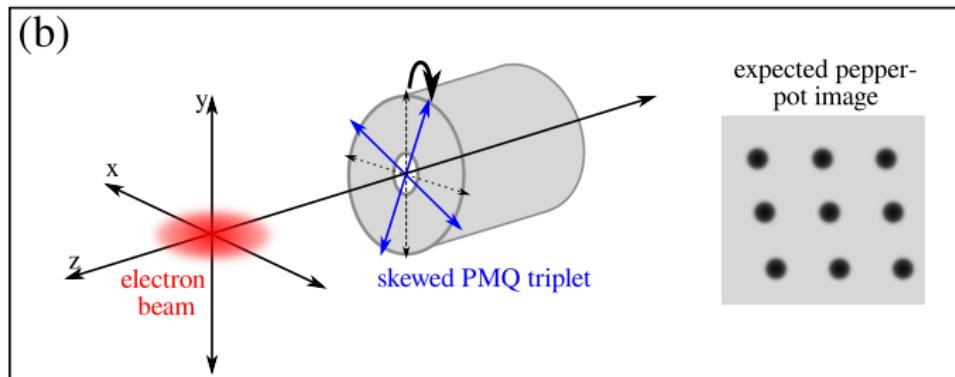
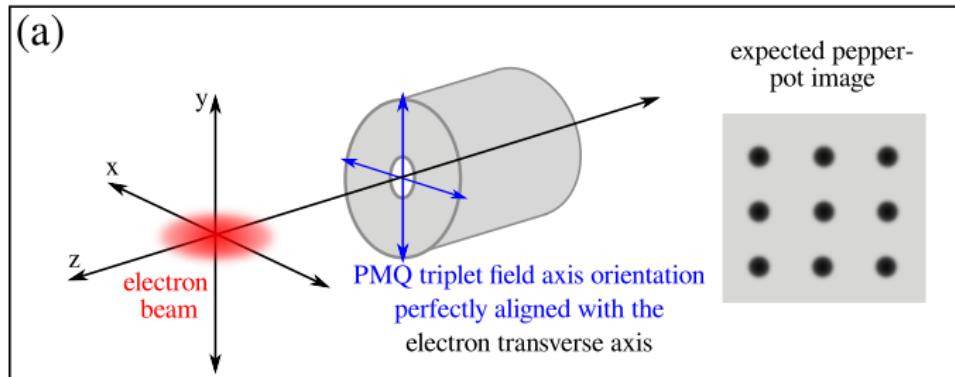


- divergence  $1 - 2 \text{ mrad}$  with  $125 \text{ MeV}$  electrons
- average  $\varepsilon_N = (2.2 \pm 0.7)\pi \text{ mm mrad}$
- **best  $\varepsilon_N = (1.0 \pm 0.1)\pi \text{ mm mrad}$**
- Elliptical beam:  $\varepsilon_{N,X} > \varepsilon_{N,Y}$
- Upper limit because of resolution
- **With PMQs emittance grows by factor of 5 (measured)**



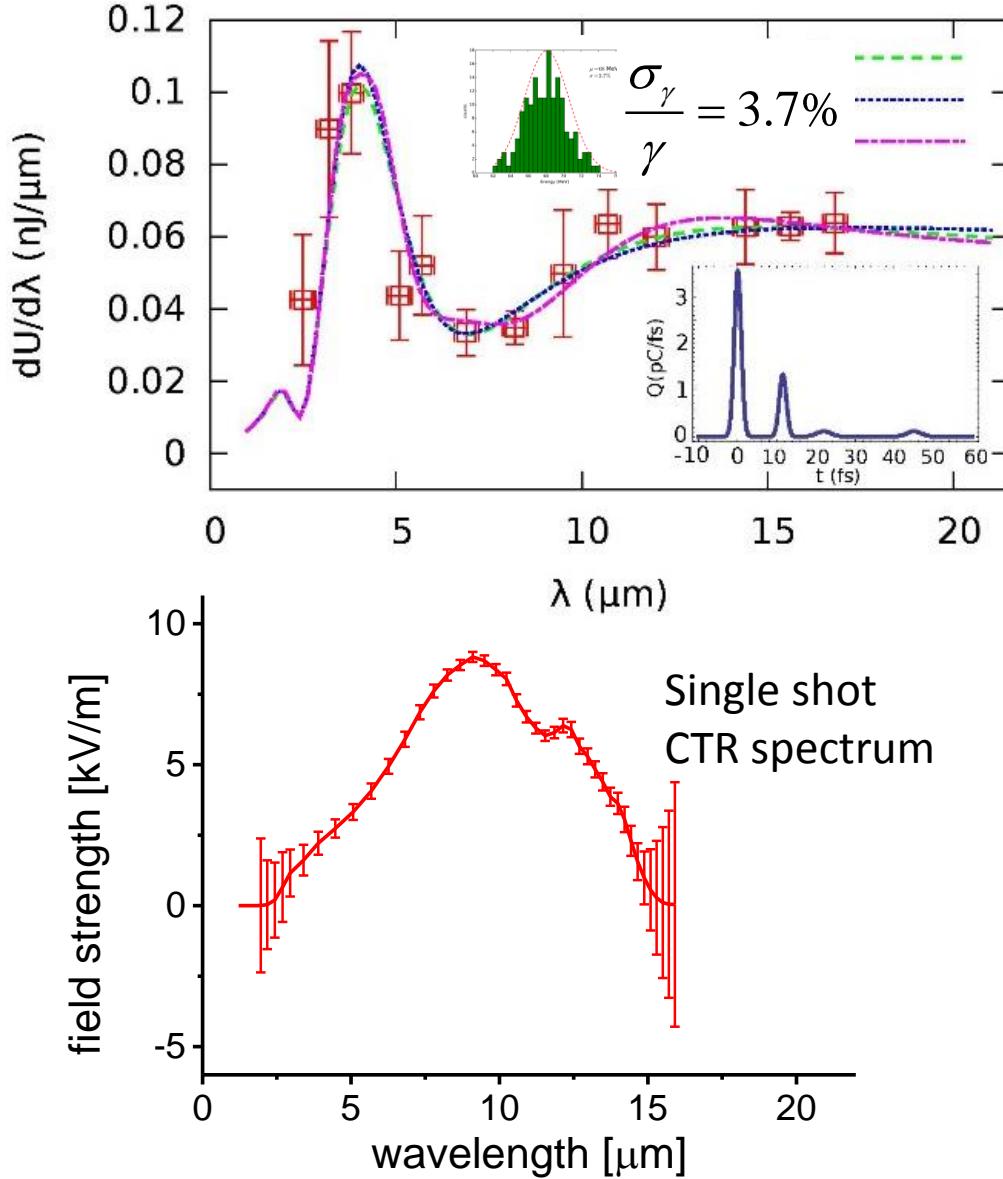
See Brunetti et al., PRL (2010)

# Evidence of multiple beams



Manahan et al., New J. Phys. (2014).

# Bunch length measurements: Coherent Transition Radiation



Coherent transition radiation spectrum gives bunch length

Chirp:

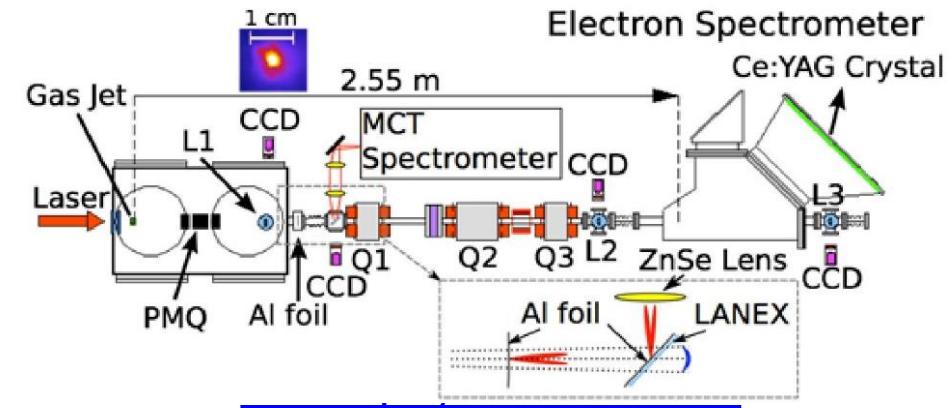
$$\frac{\delta\gamma}{\delta z} \approx \frac{2(z - R)\gamma_{\max}}{R^2}$$

2 fs bunch measured at 1 m from source

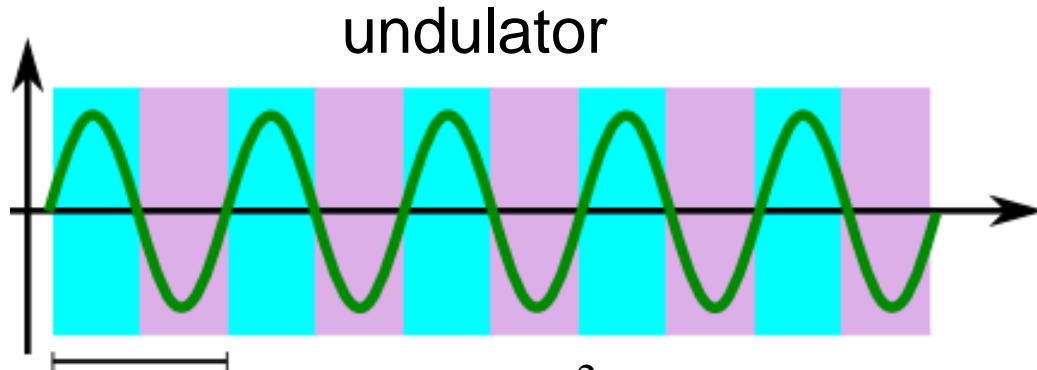
Peak current several kiloAmperes

$$\delta_{foil} = \delta_{acc} + \frac{D\delta_\gamma}{\gamma_0^3}$$

Ultra-short bunches: ~ 1 fs at source – Peak current several kA



# Radiation from relativistic electrons



$$\omega_u = \frac{4\pi c \gamma^2}{\lambda_u (1 + a_u^2 / 2 + \gamma^2 \theta^2)}$$

Photons per electron  
**per undulator period**

$$N_{phot} = \frac{2\pi}{3} \alpha a_u^2, \quad a_u < 1$$

$$N_{phot} = \frac{5\sqrt{3}\pi}{6} \alpha a_u, \quad a_u > 1$$

$$z(t) = \left(1 + \frac{a_u^2}{4\gamma^2}\right)vt - \frac{a_u^2}{8\gamma^2 k_u} \cos(2\omega_u t)$$

$$x(t) = \frac{a_u}{k_u \gamma} \cos(\omega_u t)$$

$$\omega_u = k_u \left(1 + \frac{a_u^2}{4\gamma^2}\right) v$$

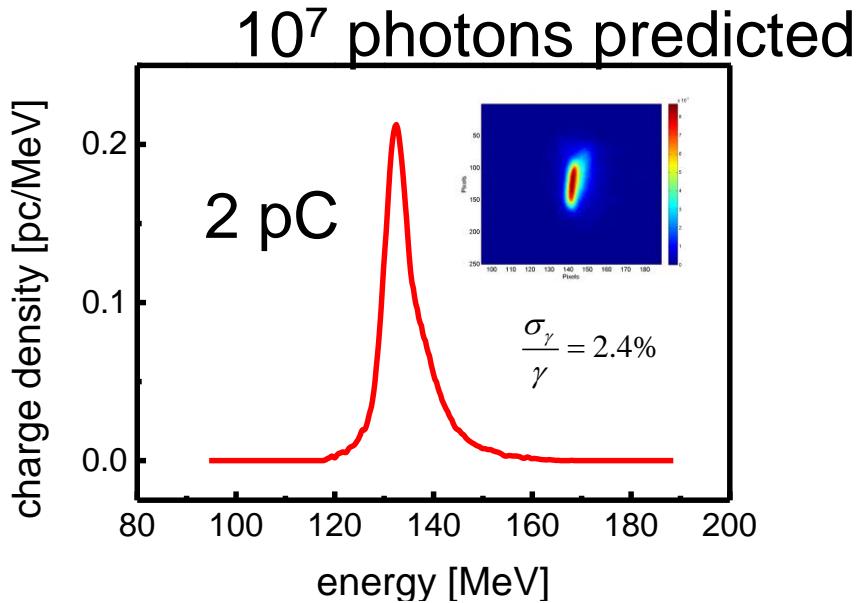
$$k_u = 2\pi / \lambda_u$$

$a_u < 1$ : undulator radiation

$a_u > 1$ : wiggler radiation

$\alpha$ : fine structure constant

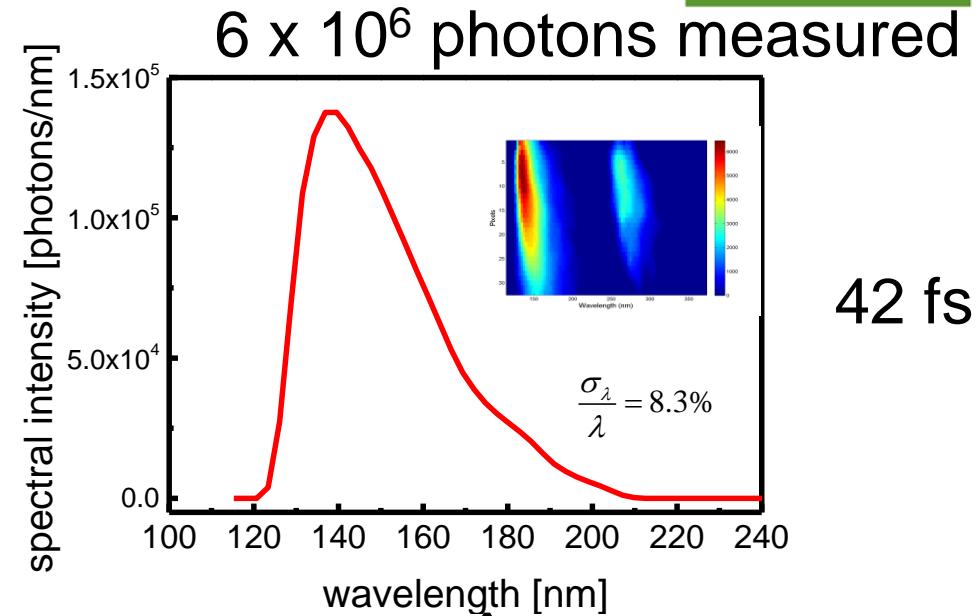
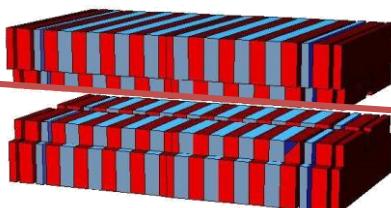
# Strathclyde UV undulator measurements



$$\lambda = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{a_u^2}{2} + \gamma^2 \theta^2\right);$$

MP Anania, et al.,  
Applied Physics Letters  
104, 264102 (2014)

$N_u=100, \lambda_u=1.5$  cm



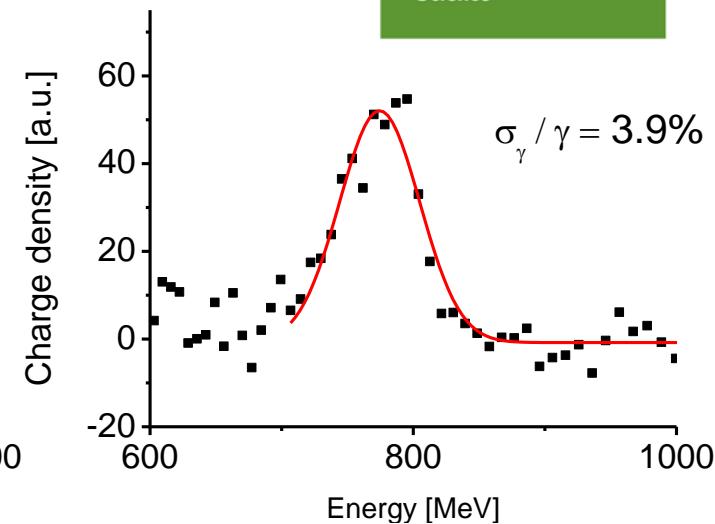
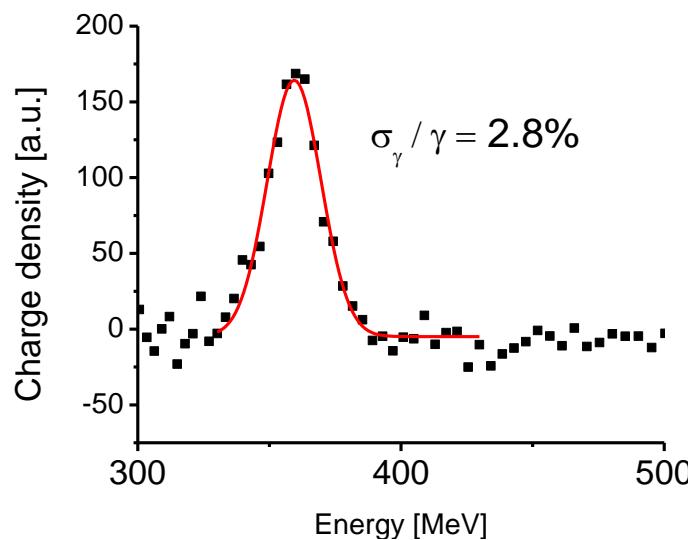
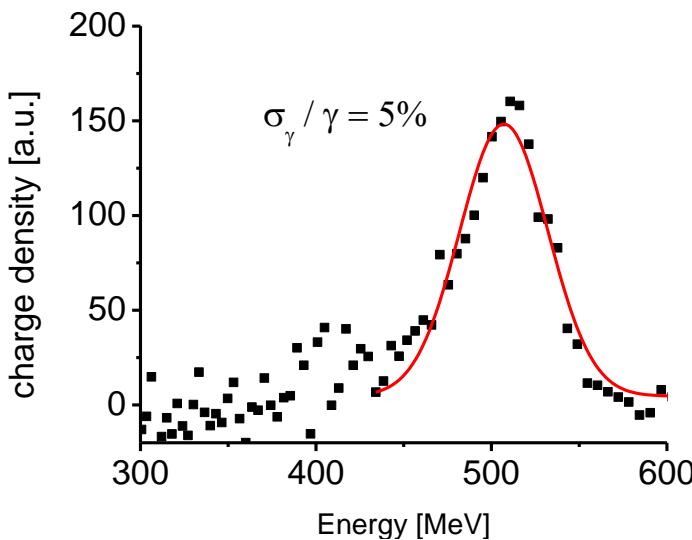
$$\frac{\Delta\lambda}{\lambda} \approx \frac{1}{2N_u} \quad a_u = \frac{e\hat{B}\lambda_u}{2\pi mc}$$

$$2\sigma_\gamma / \gamma \approx \left[ (\sigma_\lambda / \lambda)_{measured}^2 - (\theta^2 \gamma^2)^2 - 1/N_u^2 \right]^{1/2}$$

**First experiments in visible:**  
HP Schlenvoigt, et al.,  
Nature Physics 2008

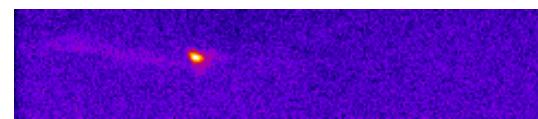
$$\frac{1}{\sqrt{N_u \gamma}}$$

# 5 J laser at RAL: Electron Beams from Capillary

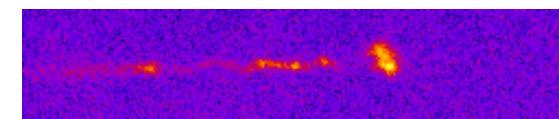


RAL GEMINI: limited by spectrometer resolution – maximum energy measured 850 MeV from 4 cm capillary.

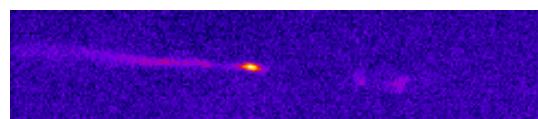
$$\gamma_d \approx 2\gamma_g^2 a_0 / 3$$



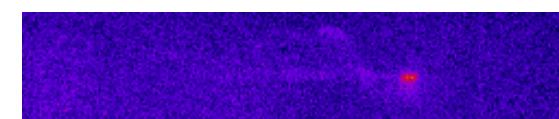
$E_0 = 260$  MeV,  $\sigma_\gamma / \gamma_{\text{MEAS}} \sim 2.5\%$



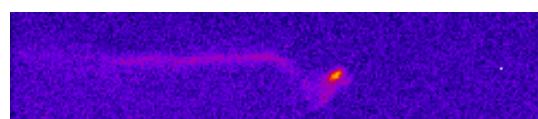
$E_0 = 610$  MeV,  $\sigma_\gamma / \gamma_{\text{MEAS}} \sim 4.5\%$



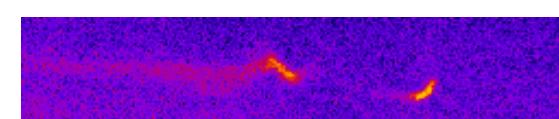
$E_0 = 340$  MeV,  $\sigma_\gamma / \gamma_{\text{MEAS}} \sim 2.5\%$



$E_0 = 690$  MeV,  $\sigma_\gamma / \gamma_{\text{MEAS}} \sim 4\%$

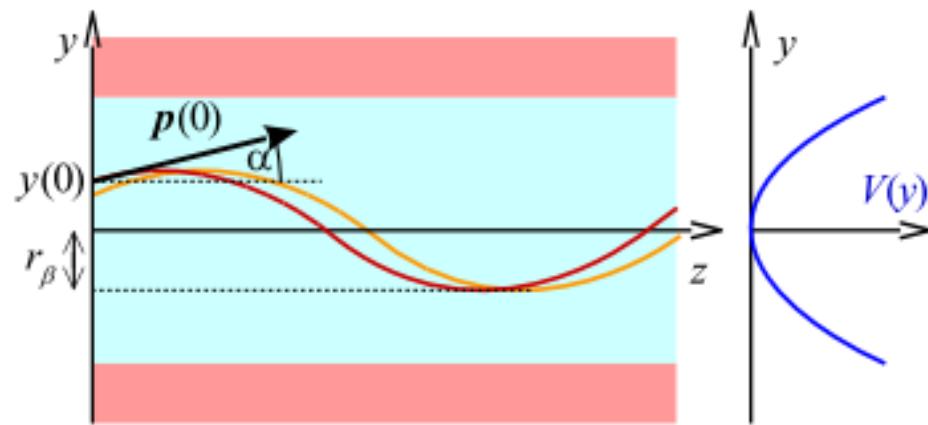


$E_0 = 510$  MeV,  $\sigma_\gamma / \gamma_{\text{MEAS}} \sim 3\%$



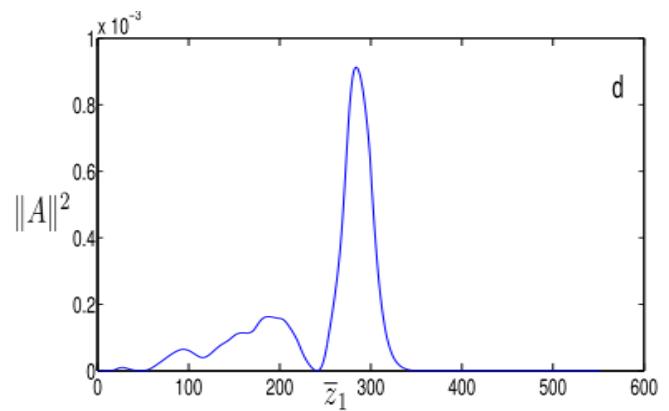
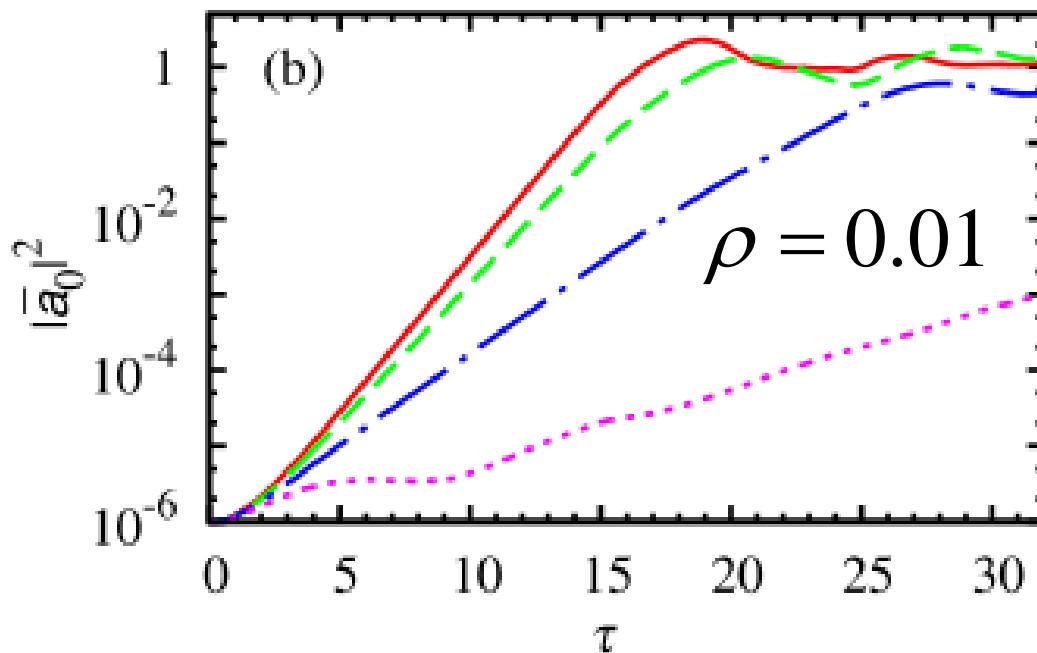
$E_0 = 770$  MeV,  $\sigma_\gamma / \gamma_{\text{MEAS}} \sim 4\%$

# Ion Channel Laser



Similar to FEL but with variable wiggler parameter for each electron

$$\rho = \left[ \eta_h^2 \eta_m \bar{\eta}_f \omega_b^2 R_\beta^2 / (8 \tilde{\gamma}_0 c^2) \right]^{1/2} \approx 3.3 \times 10^{-3} \left[ (n_b / 10^{18} \text{ cm}^{-3}) / \tilde{\gamma}_0 \right]^{1/2} (R_\beta / \mu\text{m}).$$



# Betatron radiation emission during LWFA

## SCALING LAWS

### Acceleration:

- Bubble radius:  $R = 2\sqrt{a_0}/k_p$
- Dephasing Length:  $L_d \approx 2/3\omega_0^2/\omega_p^2$ ,  $\omega_p = \sqrt{4\pi n_e e^2 / m_e}$
- Max Energy:  $\gamma_d \approx 2\gamma_g^2 a_0 / 3$

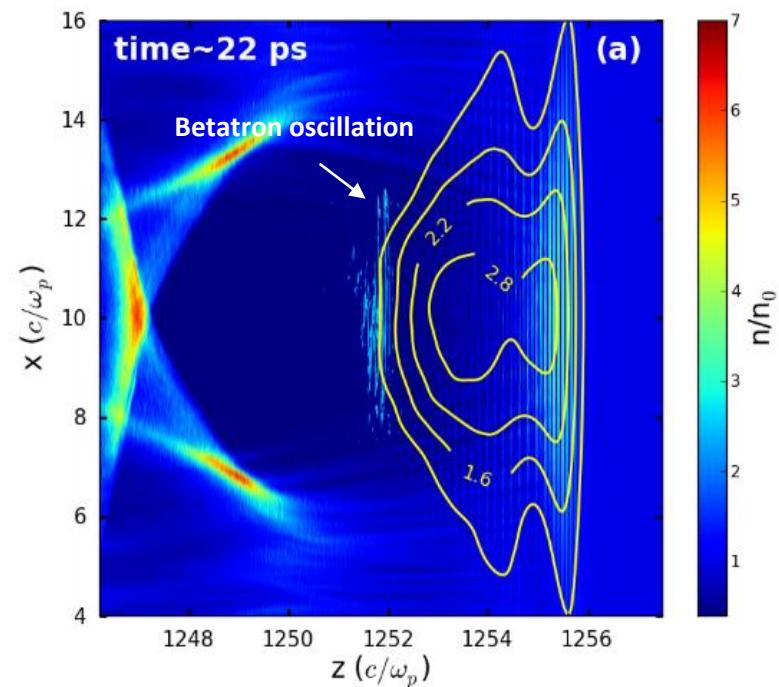
### Betatron Radiation:

- Betatron Frequency:  $\omega_\beta = \omega_p / \sqrt{2\gamma}$
- Transverse momentum:  $a_\beta \propto \sqrt{\gamma n_e} r_\beta$
- Divergence:  $\vartheta = a_\beta / \gamma$
- Critical photon Energy:  $E_c \propto \gamma^2 n_e r_\beta$        $h_c \approx 3a_\beta^3 / 8$
- Efficiency:  $N_{phot/cycle} = \alpha a_\beta$

# Betatron radiation emission during LWFA

## BETATRON RESONANCE

- The bubble partially filled by laser pulse
- Electrons enter into resonance with the laser field
- Laser drives larger amplitude betatron oscillations:  
**Increase in  $r_\beta$ ,  $a_\beta$ ,  $E_c$**



# Betatron radiation emission during LWFA

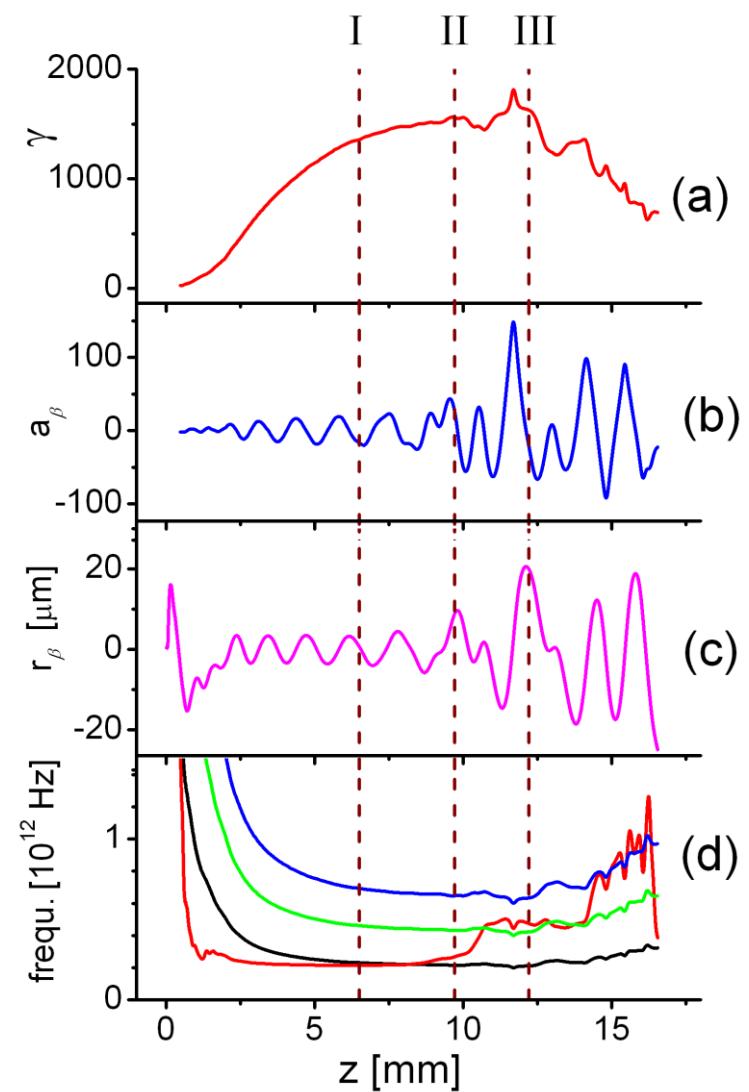
Cipiccia et al., Nature Physics, 2011

- Three different regimes:
  - Non resonant (I)
  - Weakly resonant (II)
  - Strongly resonant (III)

$$\ddot{y} + \Gamma \dot{y} + \omega_\beta^2 y = F_{Ly} / (m\gamma)$$

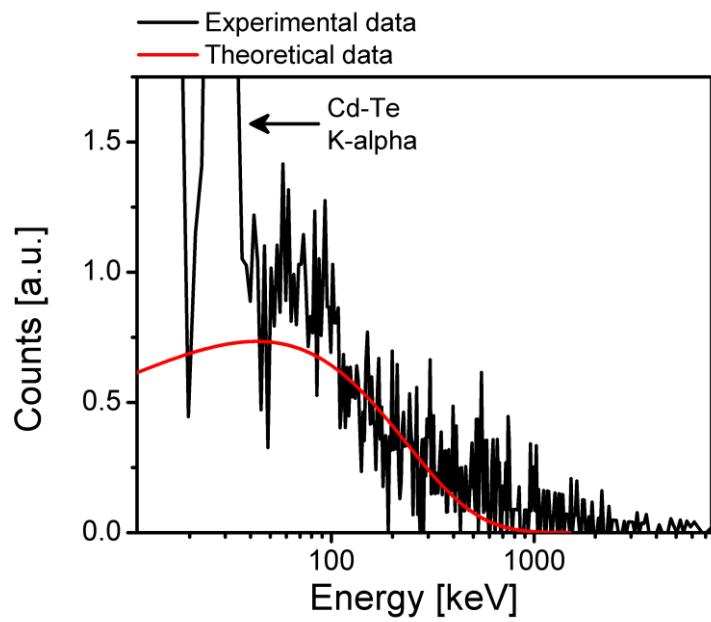
$$\Gamma = \dot{\gamma} / \gamma \quad F_{Ly} = e(dA/dt - \dot{y} \partial A / \partial y)$$

$$\tilde{\omega} = \omega_0' \left( \frac{1}{2\bar{\gamma}_z^2} + \frac{1}{2\gamma_g'^2} \right) \bar{\gamma}_z = \gamma (1 + a_\beta^2 / 2)^{-1/2}$$



# ASTRA Gemini: Experimental Results

Case II Weakly Resonant

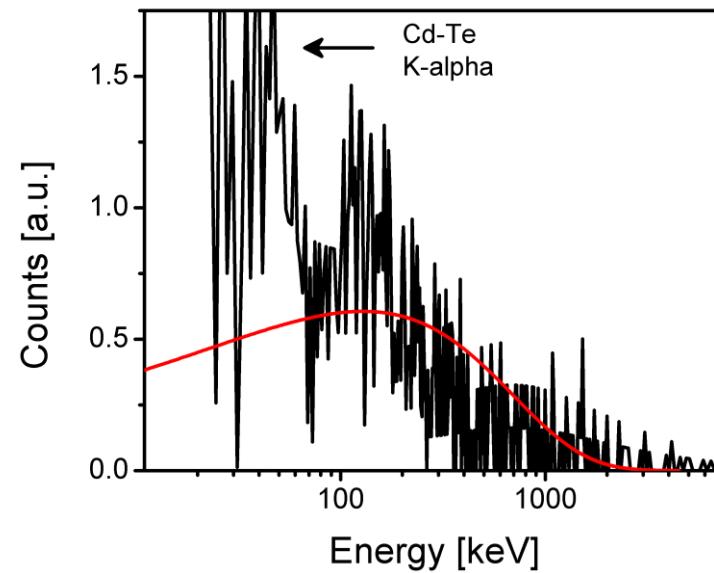


$$E_c = 150 \text{ keV}$$

$$r_\beta = 7 \text{ mm}$$

$$a_\beta = 50$$

Case III: Strongly Resonant:

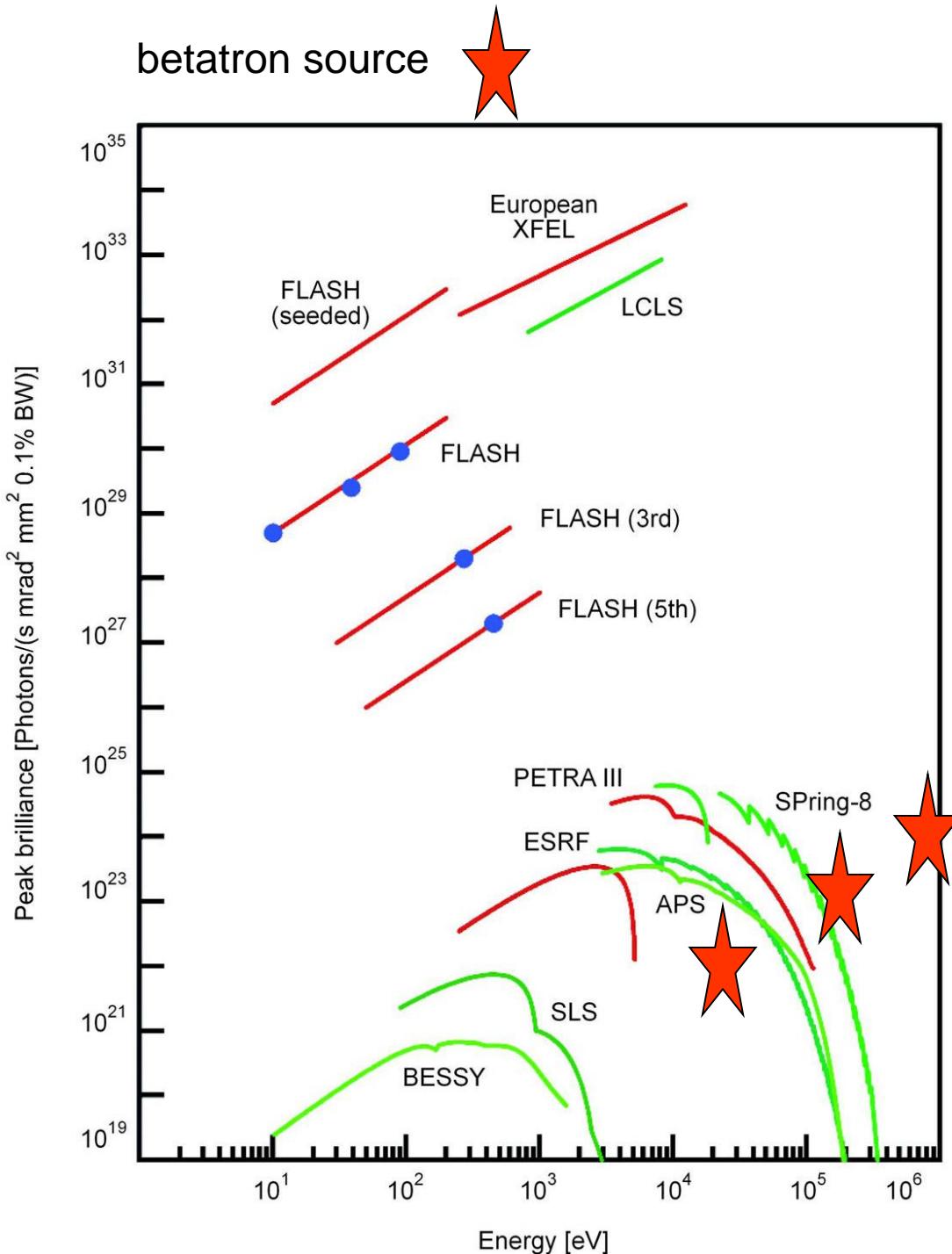


$$E_c = 450 \text{ keV}$$

$$r_\beta = 20 \mu\text{m}$$

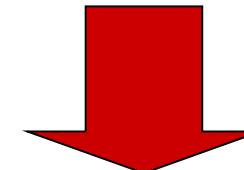
$$a_\beta = 150$$

# Brilliance



## Betatron peak brilliance

10<sup>22</sup> - 10<sup>23</sup> photons/s mrad<sup>2</sup>  
mm<sup>2</sup> 0.1% B.W



Laser: 18 J,  $a_0 = 4$   
10<sup>24</sup> photons/s mrad<sup>2</sup> mm<sup>2</sup>  
0.1% B.W

# Scaling up photon energy

## Current:

- Laser Energy: 4-5 J
- Pulse length: 50-60 fs
- Initial  $a_0$ : 2
- $E_c = 10 - 150$  keV
- No. photons/shot:  $>3 \times 10^{10}$

## Bigger laser:

- Laser: 18 J,  $a_0 = 4$
- Max electron energy : 1.5 GeV
- Electron beam charge: 300 pC
- N photon/shot 5-25 MeV =  $6 \times 10^8$
- Max repetition rate: 10 Hz
- $10^{10}$  10 MeV photons per second:
- 10 mW of 10 MeV photons

# Radiation Reaction studies

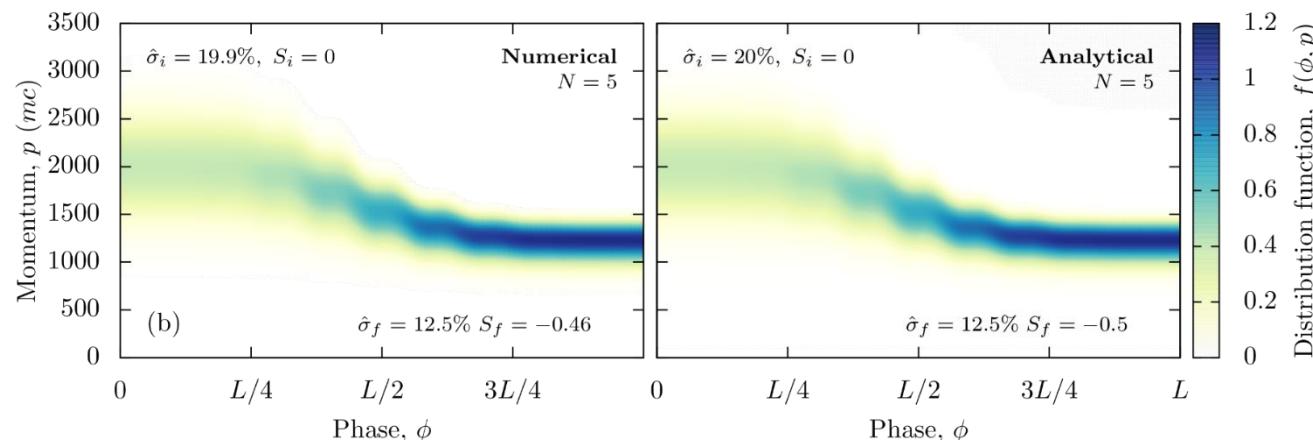
- Semi-classical:
  - Radiation reaction in a Coulomb field
- Stochastic:
  - Interaction with two-colour pulses
  - Chirped pulses
  - Longitudinal vs. transverse beam cooling

# Radiation reaction

- **Lorentz-Abraham-Dirac:** relativistic equation of motion including radiation reaction
- **Landau-Lifshitz:** treat radiation reaction as a small perturbation to external forces
- Often claimed LL is valid, provided quantum effects can be ignored [Spohn 2000; Kravets *et al.* 2013]
- Quantum effects can typically be ignored provided the observed electric field  $\hat{E} \ll E_S$ , where  $E_S = 1.3 \times 10^{18}$  V/m is the Schwinger critical field
- **Semi-classical extension:**
  - Classical models overestimate radiation as quantum effects become important
  - Following Kirk *et al.* (2009) and Thomas *et al.* (2012), scale the radiation reaction force by a function of the quantum nonlinearity parameter,  $\chi = \frac{\hat{E}}{E_S}$   
$$g(\chi) = (1 + 12\chi + 31\chi^2 + 3.7\chi^3)^{-4/9}$$

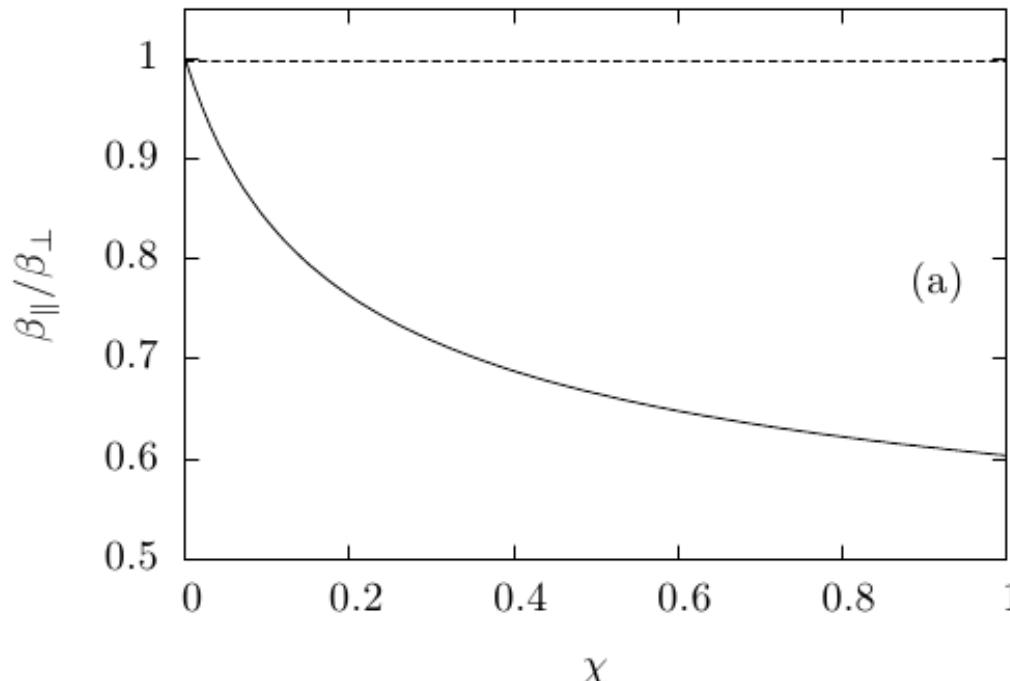
# Analytical solution of Vlasov equation with LL

- Analytical solution to the Vlasov equation with radiation reaction according to the classical LL model, for an electron bunch in a plane wave [Yoffe *et al.* 2015]
- Distribution  $F$  satisfies  $\frac{dF}{d\phi} = -\beta F$ , where  $\phi = \omega t - \mathbf{k} \cdot \mathbf{x}$  is the phase and  $\beta < 0$  results in phase space contraction (beam cooling) and leads to solution of the form  $F(\phi, \mathbf{v}) = \frac{F(\phi^0, \mathbf{v}^0)}{G(\phi)^4}$  with  $\mathbf{v}_0$  replaced by  $\mathbf{v}$  using the LL equations of motion
- Reduced (longitudinal) distribution  $f$  satisfies  $f(\phi, v_\phi) = \frac{f(\phi^0, \frac{v_\phi}{G(\phi)})}{G(\phi)^2}$
- Comparison of analytical and numerical results (5-cycles,  $8 \times 10^{21}$  W/cm<sup>2</sup>, 7 fs FWHM; 1 GeV electrons with 20% initial spread)



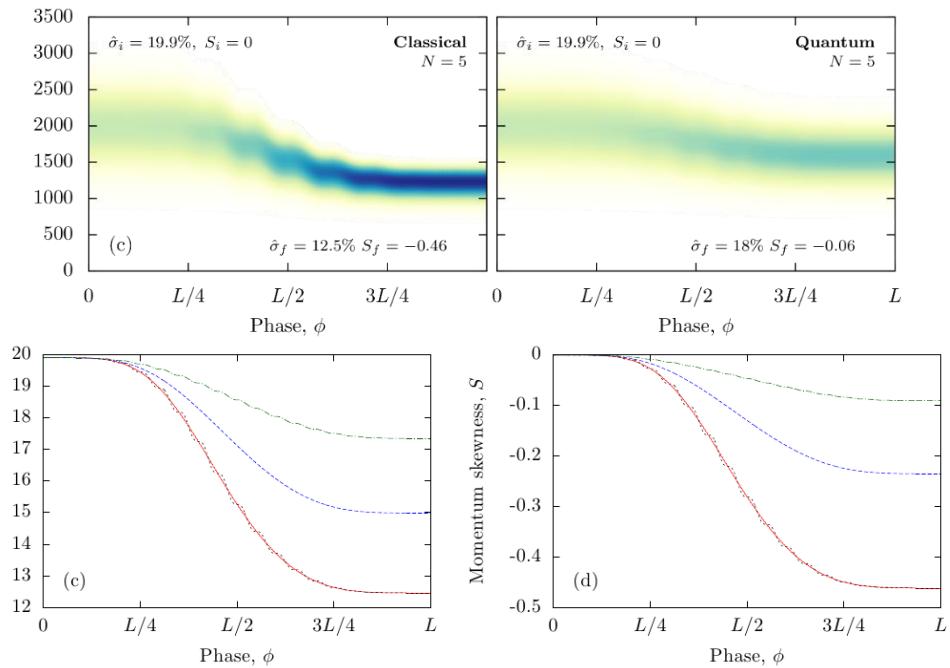
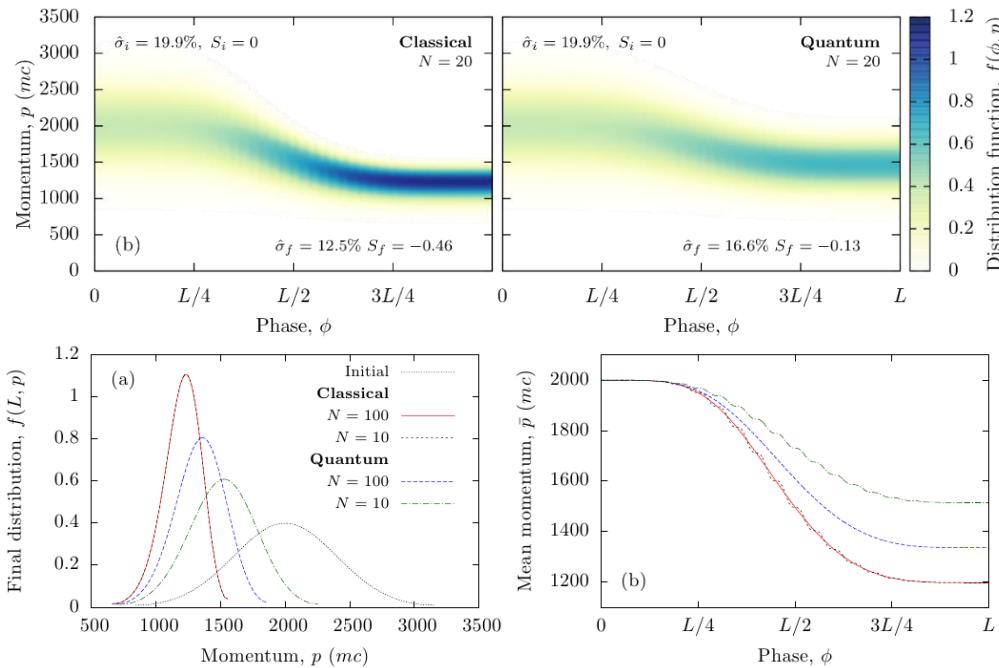
# Longitudinal vs. transverse cooling

- Analytical solution to the Vlasov equation - two contributions to cooling:  $\beta = \beta_{\parallel} + \beta_{\perp}$
- **Classical:**  $\hat{\beta}_{\parallel} = \hat{\beta}_{\perp}$  - equal cooling in longitudinal and transverse directions
- **Semi-classical:** reduction in beam cooling by  $g(\chi)$ :  $\beta_{\perp} = g(\chi)\hat{\beta}_{\perp}$   
Dependence of  $\chi$  on  $v_{\phi}$  produces an additional heating term,  
$$\frac{\beta_{\parallel}}{\beta_{\perp}} = 1 - \frac{2}{9}\chi g^{9/4}(\chi)(12 + 62\chi + 11.1\chi^2) \leq 1$$
- Reduced longitudinal beam cooling compared to transverse



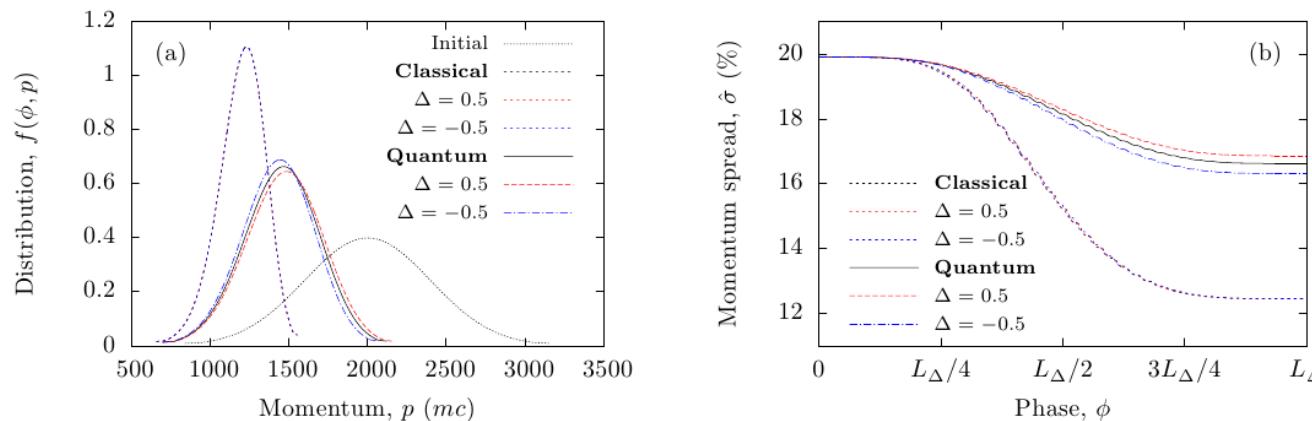
# Classical vs. semi-classical beam cooling

- Comparison of classical and semi-classical longitudinal beam cooling
  - Top:  $N = 5$  ( $8 \times 10^{21}$  W/cm $^2$ , 7 fs FWHM) and  $N = 20$  ( $2 \times 10^{21}$  W/cm $^2$ , 27 fs FWHM) plane-wave pulses;
  - Bottom:  $N = 10$  ( $4 \times 10^{21}$  W/cm $^2$ , 13 fs FWHM) and  $N = 100$  ( $4 \times 10^{20}$  W/cm $^2$ , 135 fs FWHM) plane-wave pulses;
  - 1 GeV electron bunches, with initial spread 20% and zero skewness
- Final spread increased from 12.5% to 16.6% ( $N = 20$ ) and 18% ( $N = 5$ ) – **REDUCED BEAM COOLING**
- Skewness reduced - semi-classical final distribution more Gaussian than the classical



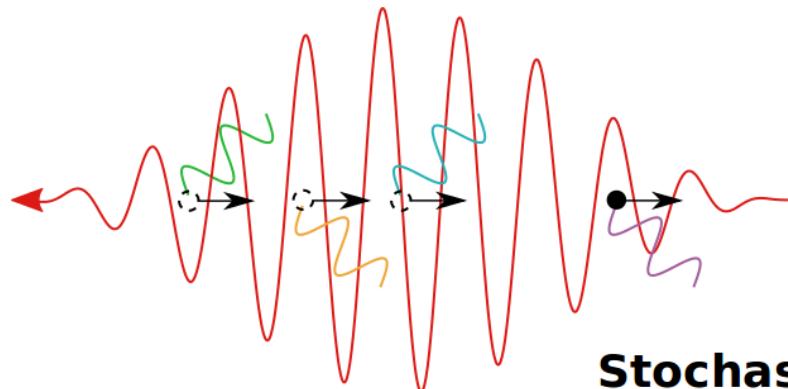
# Beam cooling with chirped pulses

- Semi-classical model: properties of the final state electron beam depend on the distribution of energy within the pulse - not only the total amount as in the classical case.
- The chirp modifies the distribution of energy within the pulse – how does this affect beam cooling?
- Number of cycles and total energy fixed to facilitate comparison



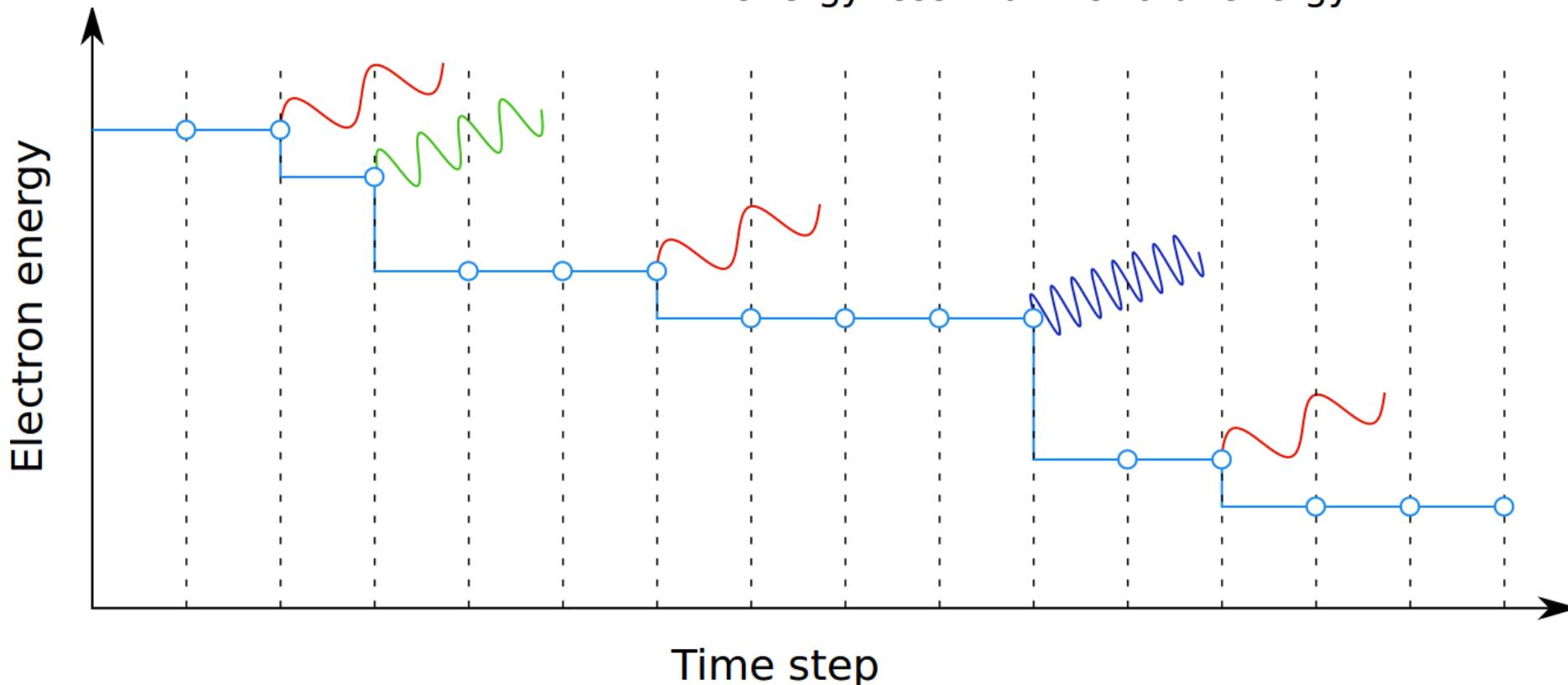
- Classical predictions insensitive to the chirp – as expected
- **Positive chirp:** pulse length reduced due to decreasing wavelength, so increase in peak intensity and increase in  $\chi$ . This suppresses radiation reaction effects, and hence **beam cooling is reduced**.
- **Negative chirp:** pulse length increases due to increasing wavelength, requiring a decrease in peak intensity to maintain total energy. Decrease in  $\chi$  leads to **further beam cooling**.

# Stochastic effects



## Stochastic:

Electron emits (multiple) photons, each with energy less than its total energy



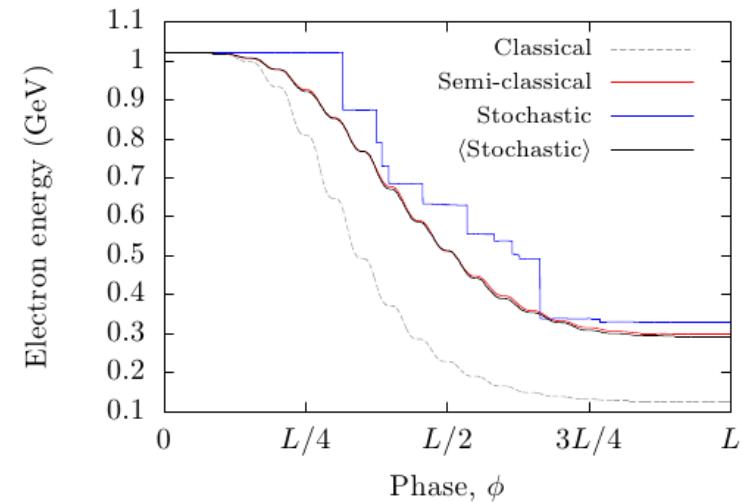
# Stochastic radiation emission

- As  $\chi$  increases, stochastic nature of photon emission becomes important.
- Stochastic model based on the differential probability  $dW = \Gamma d\phi$   
[Ritus 1985; Green and Harvey 2014]

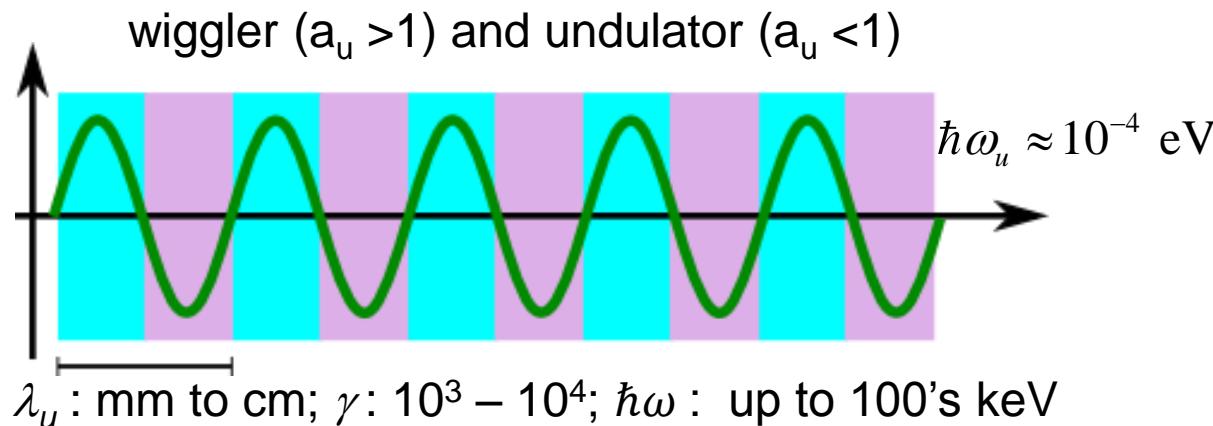
$$\Gamma = \int_0^\Omega d\tilde{\Omega} P(\Omega, \tilde{\Omega}) \quad \text{and} \quad P(\Omega, \tilde{\Omega}) = \frac{\alpha m}{\sqrt{3}\pi\Omega^2} \left[ \left( \frac{\Omega - \tilde{\Omega}}{\Omega} + \frac{\Omega}{\Omega - \tilde{\Omega}} \right) K_{2/3}(\tilde{\chi}) - \int_{\tilde{\chi}}^\infty dx K_{1/3}(x) \right]$$

The invariant parameters  $\Omega = \frac{k_a p^a}{m}$ ,  $\tilde{\Omega} = \frac{k_a \kappa^a}{m}$ , and  $\tilde{\chi} = \frac{2m\tilde{\Omega}}{[3\hbar a(\phi)\Omega(\Omega - \tilde{\Omega})]}$ .

- Electron propagates according to Lorentz force. At each step emits a photon if random number  $r \in [0,1) < dW$ .
- Determine random outgoing momentum by finding  $\tilde{\Omega}$  such that  $\int_0^{\tilde{\Omega}} dx P(\Omega, x) = \zeta \Gamma$  with  $\zeta \in [0,1)$ .
- Comparison of stochastic and semi-classical models:
  - $N = 10$  pulse with intensity  $I = 4.3 \times 10^{22} \text{ W/cm}^2$
  - Classical prediction shows significant beam cooling
  - Single stochastic prediction displays “steps”
- Ensemble of 1,000 stochastic runs in excellent agreement with semi-classical model, despite  $\langle \chi \rangle \simeq 0.8$



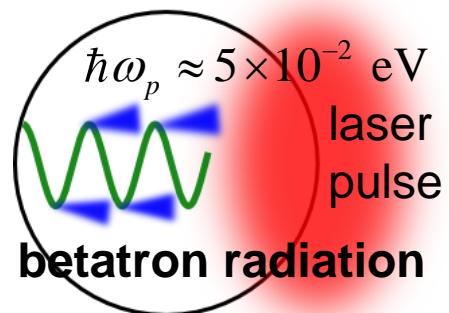
# Summary – radiation sources



$$\lambda = \frac{\lambda_w}{2\gamma^2} \left( 1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right)$$

$$N_{phot/cycle} \approx .015 a_{\beta,0}^2 \text{ for } a_{\beta,0} < 1$$

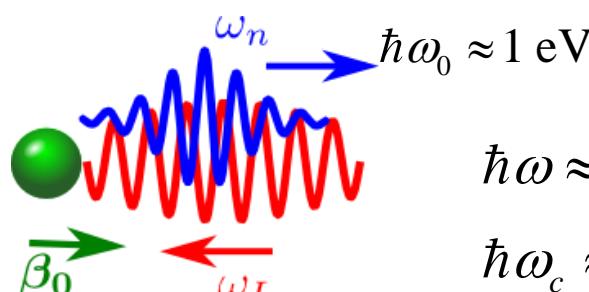
$$N_{phot/cycle} \approx .033 a_{\beta,0} \text{ for } a_{\beta,0} > 1$$



$$\lambda_\beta = \sqrt{2\gamma} \lambda_p$$

$$\hbar\omega \approx 1.4 \hbar\omega_p \gamma^{3/2} \text{ when } a_\beta < 1 \text{ (10 keV)}$$

$$\hbar\omega_c \approx \hbar\omega_p \gamma^{3/2} a_\beta \text{ when } a_\beta > 1 \text{ (10 MeV)}$$



$$\hbar\omega \approx 4 \hbar\omega_0 \gamma^2 \text{ when } a_0 < 1 \text{ (4 - 400 MeV)}$$

$$\hbar\omega_c \approx 3 \hbar\omega_0 \gamma^2 a_0 \text{ when } a_0 > 1 \text{ (< 1 GeV - nonlinear Compton)}$$

**Thomson/Compton back-scattering**

- Max:  $\gamma \approx 10^4$  (5 GeV)
- Undulator: <10 keV
- Wiggler: <100 keV
- Betatron: <20 MeV
- Thomson: <400 MeV
- Compton: Linear and nonlinear < 5 GeV

# Scottish Centre for the Application of Plasma-based Accelerators (SCAPA)

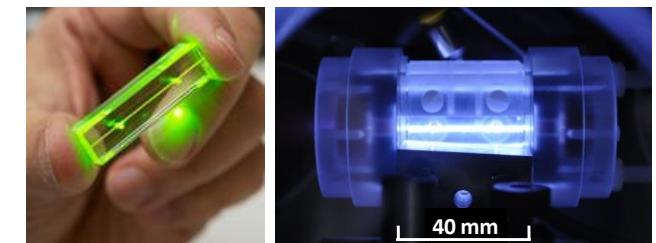


- Expansion of ALPHA-X laser-plasma accelerator facilities at Strathclyde with new laboratories.



- In-depth programme of Applications.
- Accelerator and source Research & Development.
- Knowledge Exchange & Commercialisation
- Engagement in European and other large projects.
- Training: Centre for Doctoral Training in the Application of Next Generation Accelerations
- 3 shielded areas with 7 accelerator beam lines.

- High-intensity femtosecond laser systems:
  - a) 350 TW (with provision for PW) @ 5 Hz,
  - b) 40 TW @ 10 Hz,
  - c) sub-TW @ 1 kHz.
- High-energy proton, ion and electron bunches.
- High-brightness fs duration X-ray & gamma-ray pulses.



Compact GeV electron accelerator and gamma-ray source

## APPLICATIONS

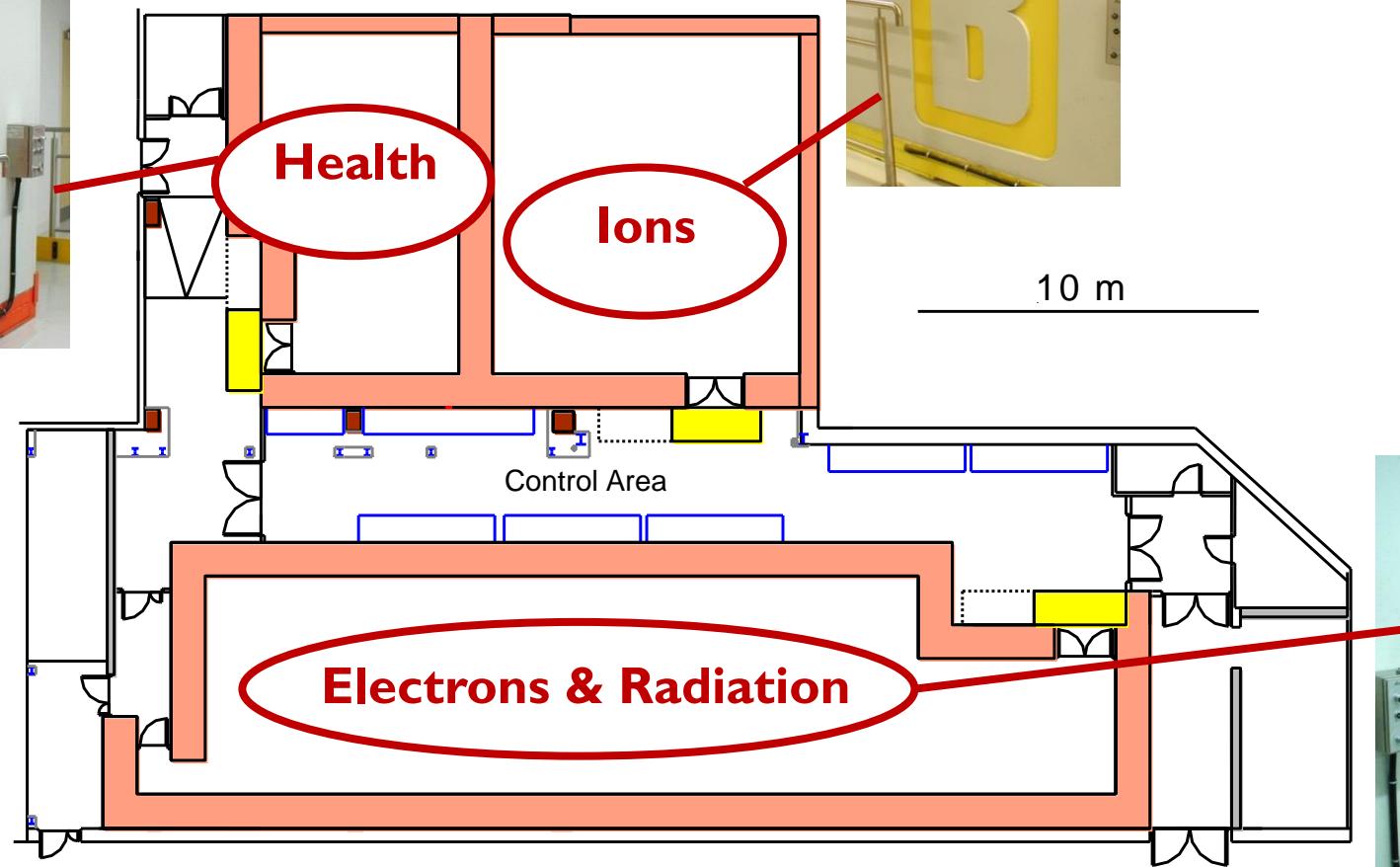
- Radiobiology
- Ultrafast Probing
- High-Resolution Imaging
- Radioisotope Production
- Detector Development
- Radiation Damage Testing

**SCAPA**

Concrete doors on air skates



10 m



**Strathclyde Team:** Dino Jaroszynski, Zheng Ming Sheng, Bernhard Hidding, Paul McKenna, Enrico Brunetti, Sijia Chen, Cristian Ciocarlan, Silvia Cipiccia, David Clark, Bernhard Ersfeld, Paul Farrell, David Grant, Peter Grant, Ranaul Islam, Karolina Kokourewicz, Panos Lepipas, Tom McCanny, Martin Mitchell, Graeme McKendrick, Grace Manahan, Adam Noble, Craig Picken, Guarav Raj, David Reboredo Gil, Phil Tooley, Gregory Vieux, Maria Weikum, Gregor Welsh, Mark Wiggins, Xue Yang, Sam Yoffe, Marie Boyd, Annette Sorensen, Gordon Rob, Brian McNeil, and Ken Ledingham



### **ALPHA-X: Current and past collaborators:**

Lancaster U., Cockcroft Institute / STFC - ASTeC, STFC - RAL CLF, U. St. Andrews, U. Dundee, U. Abertay-Dundee, U. Glasgow, Imperial College, IST Lisbon, U. Paris-Sud - LPGP, Pulsar Physics, UTA, CAS Beijing, U. Tsinghua, Shanghai Jiao Tong U., Beijing, Capital Normal U. Beijing, APRI, GIST Korea, UNIST Korea, LBNL, FSU Jena, U. Stellenbosch, U. Oxford, UCL, LAL, PSI, U. Twente, TUE, U. Bochum, IU Simon Cancer Center, Indianapolis, MGS Research, Inc., Madison, Royal Marsden, ELI-NP, ELI-ALPS, ELI-Beamlines ....

**Support:** Strathclyde University, EPSRC, CSO, EU-FP7 Laserlab & EUCARD, STFC

A close-up photograph of a salmon swimming upstream against a backdrop of turbulent, white-capped water. The fish is positioned on the right side of the frame, moving from left to right. Its body is dark on top, with a lighter belly and distinct pinkish-red lateral stripes. The water is filled with bubbles and spray, suggesting a strong current.

**FIN**

**Thank you**