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Impact of quantum effects on relativistic electron motion in a standing wave

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Heidelberg
24.07.2015

Quantum effects of electron motion in relativistically strong electromagnetic field

1) Quantization of motion

$$\delta r \, \delta p > h$$

$$\delta r \sim mc^2 \gamma / eF_{eff}$$ spatial scale of motion

$$\delta p \sim mc\gamma \delta\gamma / \gamma$$ momentum change

$m, -e, \vec{v}, \gamma$ mass, charge, velocity and Lorentz-factor of electron

c speed of light

$$\vec{F}_{eff} = \vec{E} + \frac{\vec{v}}{c} \times \vec{B}$$

$\vec{E}$ electric field

$\vec{B}$ magnetic field

$$\delta\gamma / \gamma > F_{eff} / \gamma^2 F_{cr}$$

$$F_{cr} = m^2 c^3 / e\hbar \sim 10^{18} Vm^{-1}$$

Electron motion can be treated as classical

V.N. Bayer, V.M. Katkov, V.S. Fadin, Radiation of relativistic electrons, (Atomizdat, Moscow, 1973)

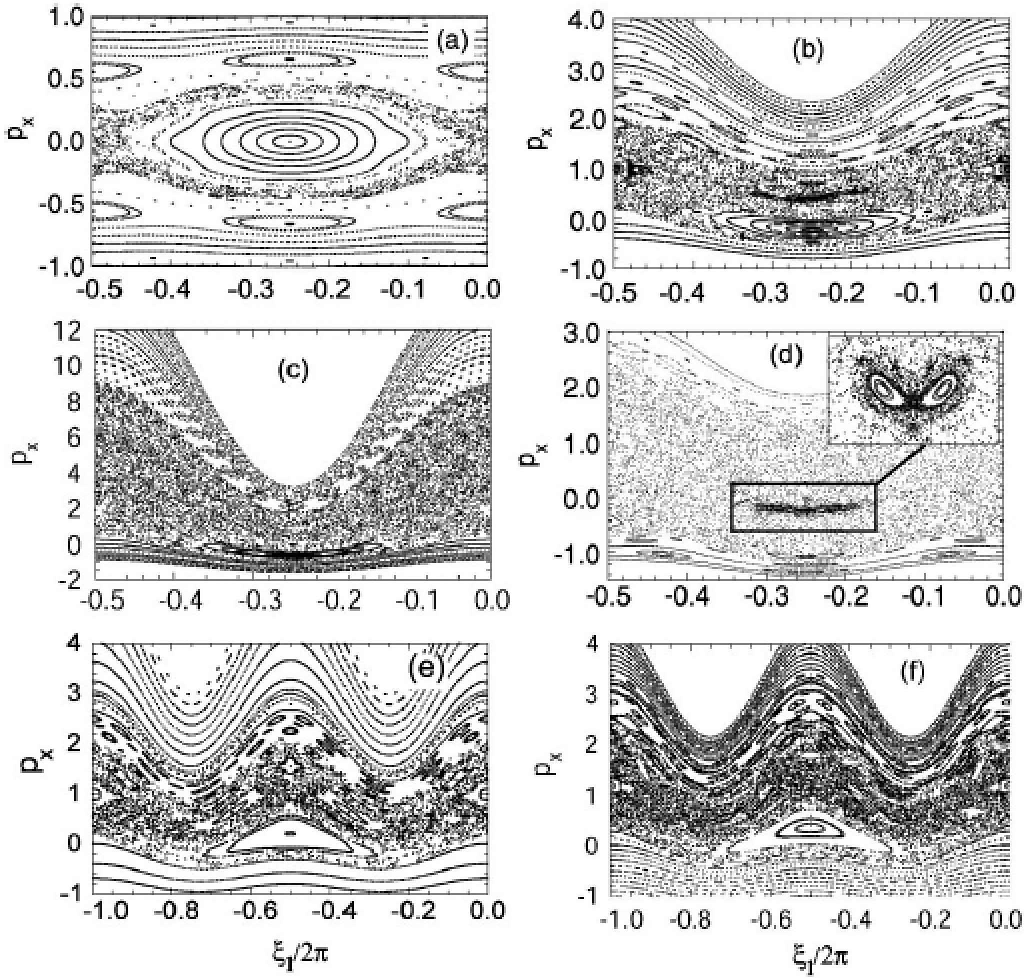
2) Quantization of radiation

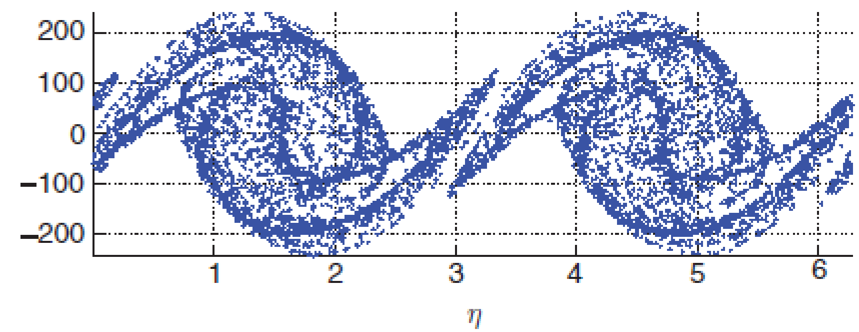
Outline

- Introduction
- Electron motion in field of plane standing wave
    - Electron trajectories
    - Lyapunov characteristic exponents
    - Quasiclassical approach
    - Electron bunch motion
    - Markov chain
    - Radiation pattern
    - Entropy
    - Ultrarelativistic cases
- Conclusion

Introduction

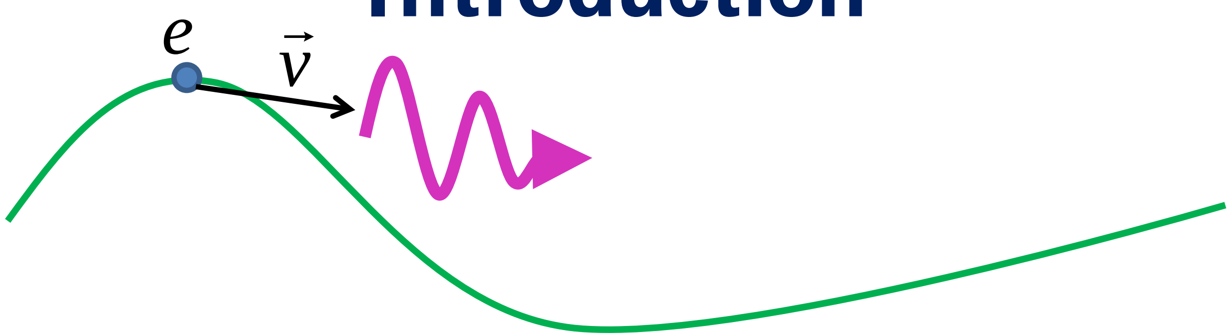
1. Curiosity
2. Beauty





J. T. Mendon¸ca and F. Doveil, Journal of Plasma Physics 28, 485 (1982)
Y. Sentoku et al., Applied Physics B 74, 207 (2002)
Z. M. Sheng et al., Phys. Rev. Lett. 88, 055004 (2002)
G. Lehmann and K. H. Spatschek, Phys. Rev. E 85, 056412 (2012)

Introduction



•Lorentz-Abraham-Dirac force

Paul A.M. Dirac, Proc. Roy. Soc. of London. A929:0148-0169 (1938)

unphysical solution (self-acceleration without external field)

•Landau-Lifshitz force

L. D. Landau and E. M. Lifshitz, The classical theory of fields, (Elsevier, Oxford, 1975)

breaking of conservation law

•Sokolov force, Esirkepov force

I.V. Sokolov et al., Phys. Plasmas 16, 093115 (2009)
T. Zh. Esirkepov et al., arXiv:1412.6028 (2014)

continuous action

▪Probabilistic approach allows for energy losses and discreetness of radiation

V. Anguelov and H. Vankov, J. Phys. G **25**, 1755 (1999)

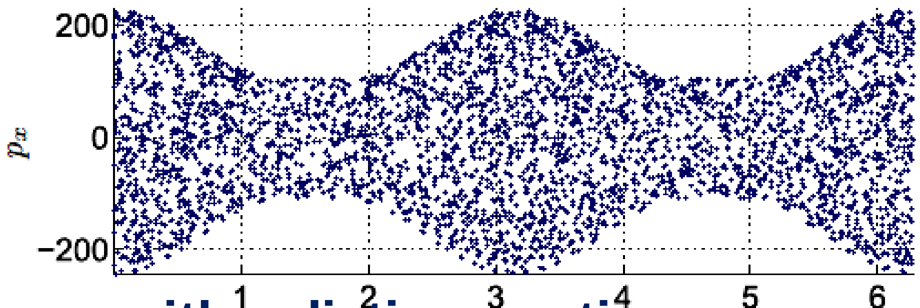
Introduction

1)Phase space contraction

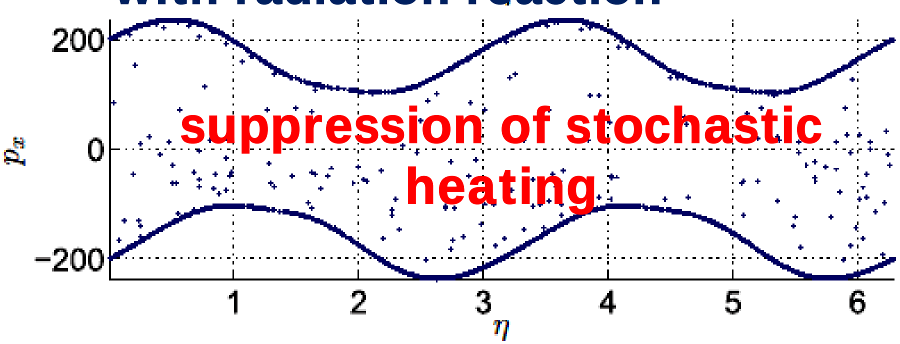
M. Tamburini et al., Nucl. Instrum. Methods Phys. Res., Sect. A **653**, 181 (2011)

Electron motion in field of linearly polarized plane standing wave with amplitude $a = eE_{max} / m\omega c = 136$

without radiation reaction



with radiation reaction



suppression of stochastic heating

contraction time $t_{sq} \sim 400T_L$

G. Lehmann and K. Spatschek, Phys. Rev. E 85, 056412 (2012)

2)Stochasticity

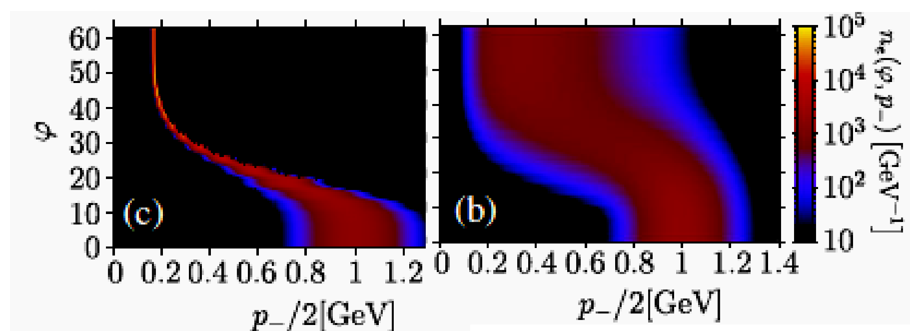
C. S. Shen and D. White, Phys. Rev. Lett. 28, 455 (1972).

Interaction of laser pulse with counter-propagating electron beam

$\varepsilon_{el} = 1 \pm 0.1 GeV$ electron energy

$I = 10^{22} W / cm^2$ laser pulse intensity

$\tau = 30\ fs$ pulse duration



Landau-Lifshitz force

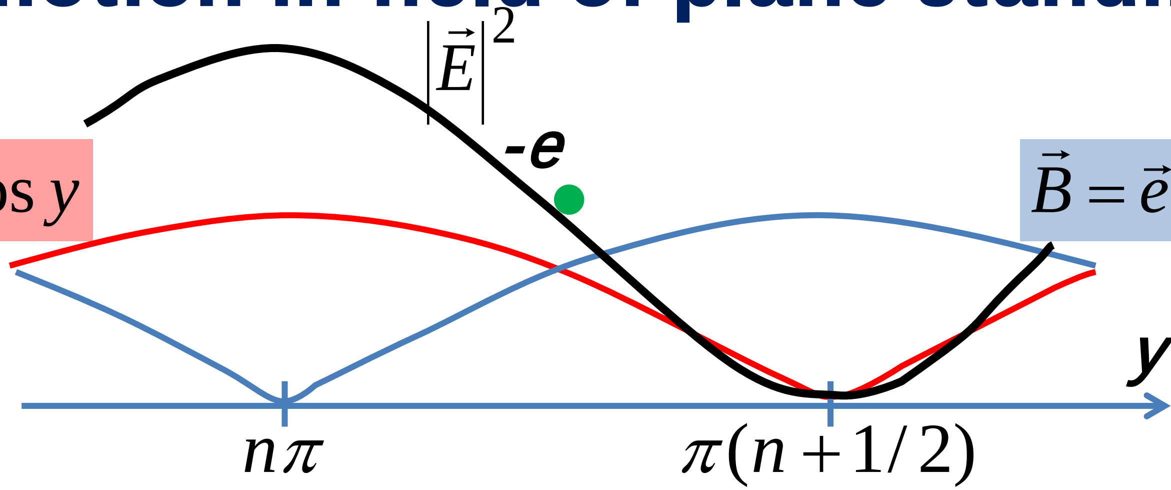
Probabilistic approach

Stochasticity leads to broadening of electron distribution

N. Neitz and A. Di Piazza, Phys. Rev. Lett. **111**, 054802 (2013)

Electron motion in field of plane standing wave

$\vec{E} = \vec{e}_x a \sin\tau \cos y$



$\vec{B} = \vec{e}_z a \cos\tau \sin y$

$E_0 \to eE/mwc = a$

$p_i \to p_i/mc$

$t \to \tau = wt$

$y \to yw/c$

$\delta = \dfrac{4\pi r_e}{3\lambda}$

$$\begin{cases} \dfrac{dy}{d\tau} = p_y/\gamma \\ \dfrac{dp_y}{d\tau} = \dfrac{p_x a \cos\tau \sin y}{\gamma} - \delta p_y \gamma a^2 \\ \times\left\{2\sin^2\tau\cos^2 y + \cos^2\tau - \cos^2 y - \dfrac{1}{\gamma}\left(\dfrac{p_y}{2}\sin 2\tau \sin 2y + \dfrac{1}{\gamma}\cos^2\tau\sin^2 y - \dfrac{p_x{}^2}{\gamma}\sin^2\tau\cos^2 y\right)\right\} \\ \dfrac{dp_x}{d\tau} = -a\sin\tau\cos y - \dfrac{p_y a \cos\tau \sin y}{\gamma} - \delta p_x \gamma a^2 \\ \times\left\{2\sin^2\tau\cos^2 y + \cos^2\tau - \cos^2 y - \dfrac{1}{\gamma}\left(\dfrac{p_y}{2}\sin 2\tau \sin 2y + \dfrac{1}{\gamma}\cos^2\tau\sin^2 y - \dfrac{p_x{}^2}{\gamma}\sin^2\tau\cos^2 y\right)\right\} \end{cases}$$

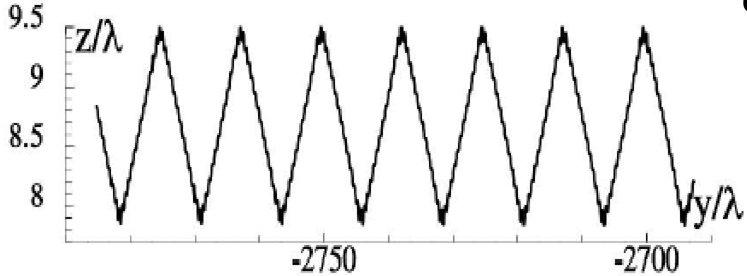
parameters: a, δ

$\delta a^3, \gamma >> 1$

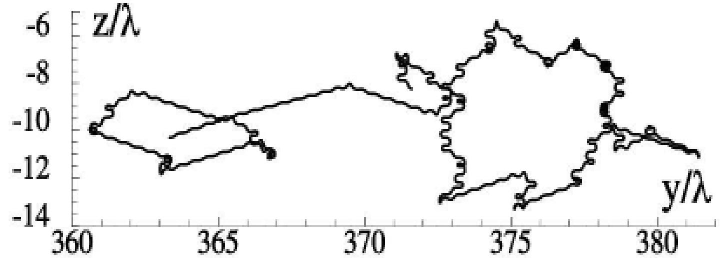
Electron motion in field of plane standing wave
Trajectories

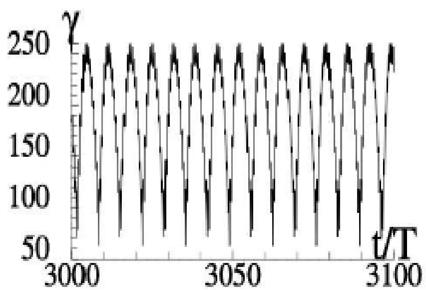
$a = 136$

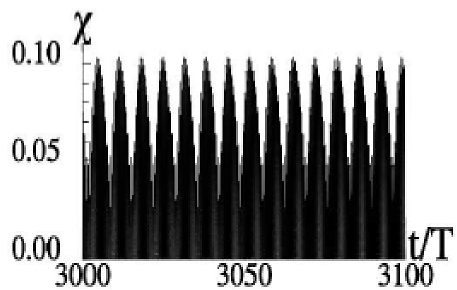
Landau-Lifshitz force

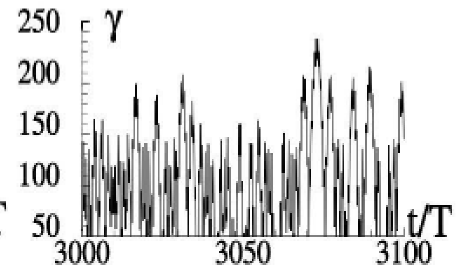


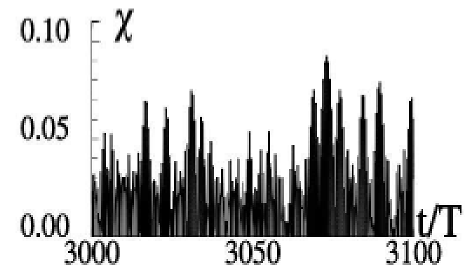
Quasiclassical approach











Quantum parameter $\chi \sim \frac{\hbar\omega_L}{mc^2} a^2 \sim 0.05$

Average photon energy $\varepsilon_\gamma \sim \hbar\omega_L a^3 \sim 3.3$

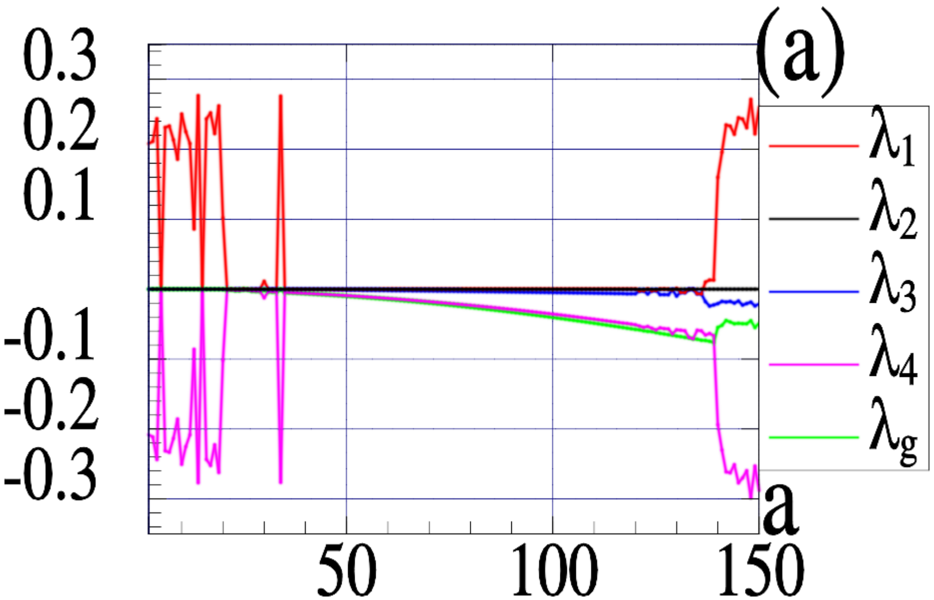
Average number of photons over wave period $N_\gamma \sim \frac{2\pi e^2}{\hbar c} a \sim 6$

Power of radiation averaged over wave period $P_{rad} \sim \frac{2e^2}{3\hbar c} a^4 \hbar\omega_L$

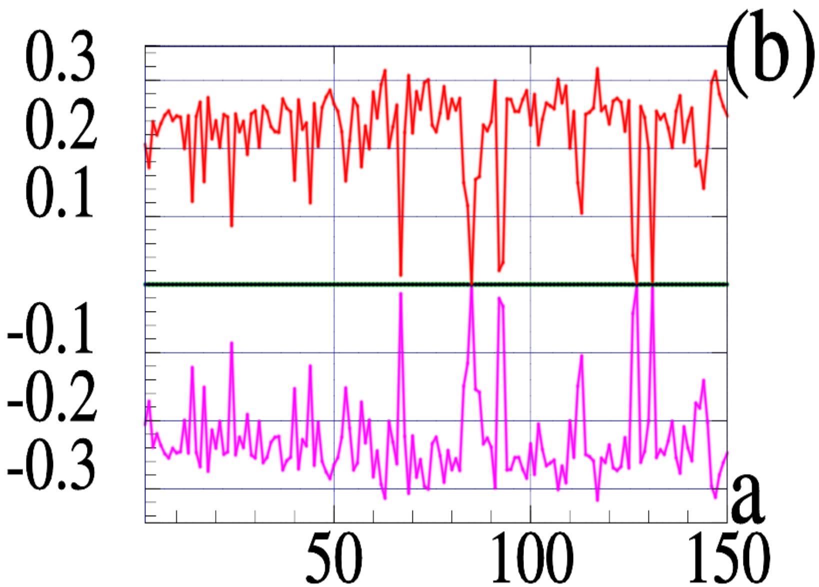
Classical approach to description of radiation losses should work well ?!

Electron motion in field of plane standing wave
Lyapunov characteristic exponents

Landau-Lifshitz force



without radiation reaction



$\lambda_1 = 0; \lambda_2 < 0 \ if \ \ 35 < a < 137$

Regular attractor, reduced stochastic heating

$\lambda_1 > 0 \ if \ \ 35 < a < 137$

chaotic motion, stochastic heating

$\sum \lambda_i < 0, |\lambda_2| < 0.007$

the same perturbation vector

$\lambda_1 \sim 0.2$

Electron motion in field of plane standing wave
Quasiclassical approach

Particle motion is classical but without radiation reaction
Radiation is according to quantum theory

V.N. Bayer, V.M. Katkov, V.S. Fadin, Radiation of relativistic electrons, (Atomizdat, Moscow, 1973)

Optical path

$$L_{opt} = \log \frac{1}{1-r_1} \Rightarrow \int W\,dt = L_{opt}$$

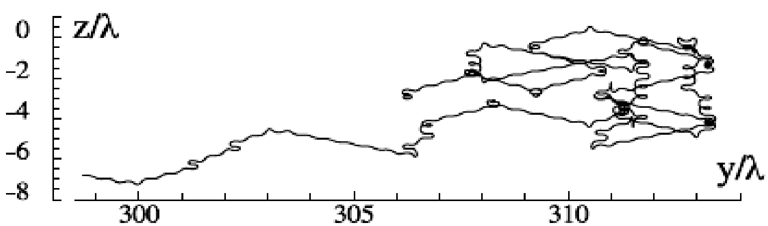
Probability of radiation per unit time

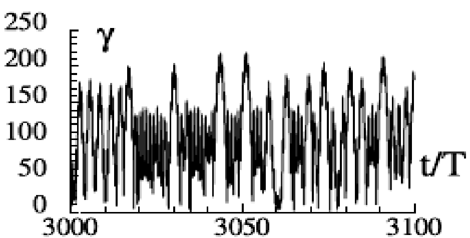
$$W = \frac{1}{3\sqrt{3}\pi}\frac{e^2}{\hbar c}\frac{mc^2}{\hbar\gamma}\int_0^\infty \frac{5u^2+7u+5}{(1+u)^3} K_{2/3}(2u/3\chi)\,du$$

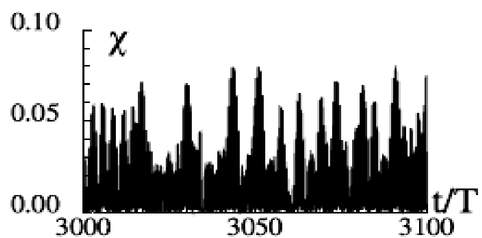
Photon radiation is consistent with spectral probability density

$$\frac{dW(\chi,\eta)}{d\eta} = \frac{1}{\pi\sqrt{3}}\frac{e^2}{\hbar c}\frac{mc^2}{\hbar\omega\gamma}\left[\int_{\frac{2\eta}{3(1-\eta)\chi}}^{\infty} K_{5/3}(y)dy + \frac{\eta^2}{1-\eta}K_{2/3}\left(\frac{2\eta}{3(1-\eta)\chi}\right)\right]$$

R. Duclous, J. G. Kirk, and A. R. Bell, Plasma Physics and Controlled Fusion 53, 015009 (2011)



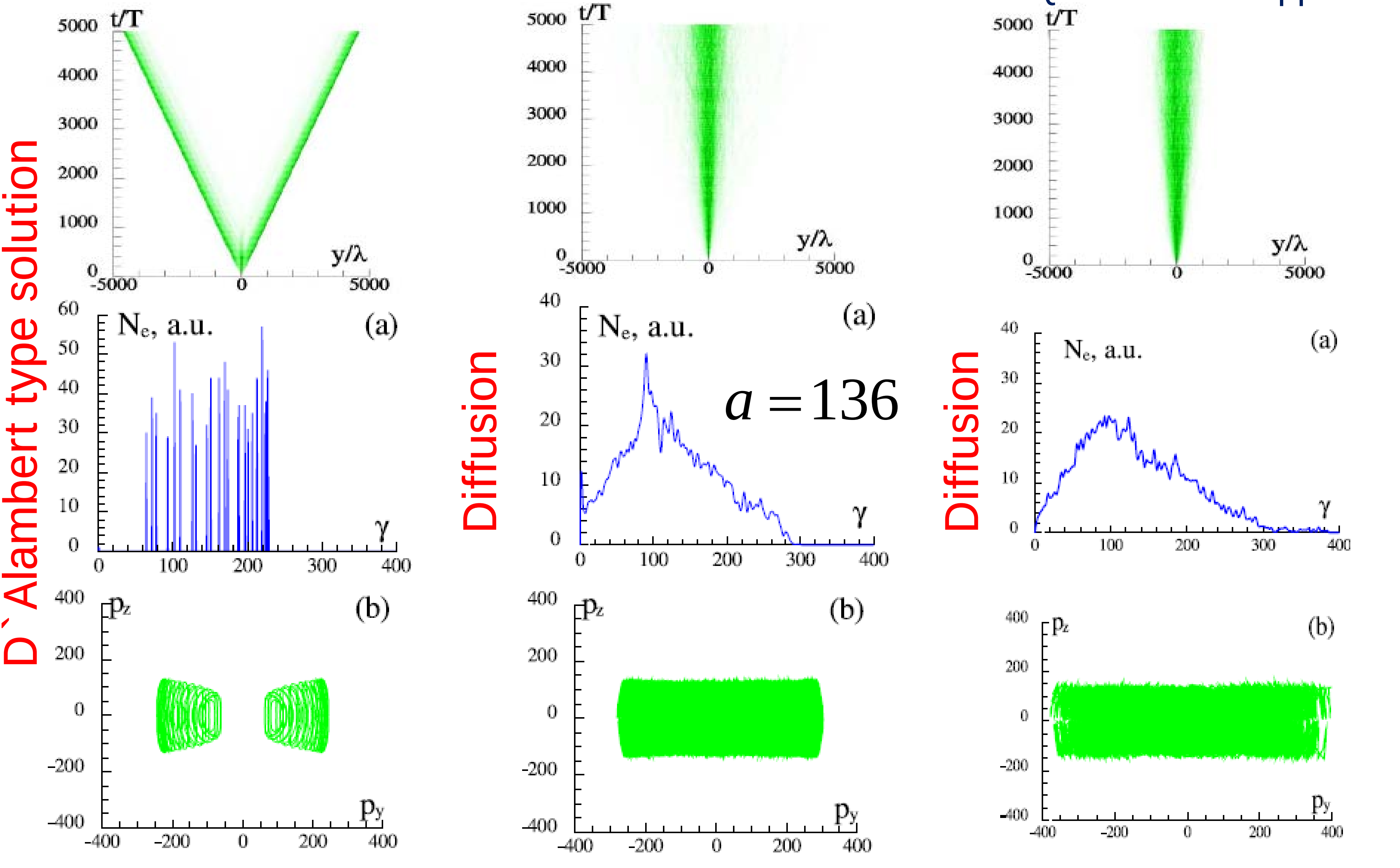




Electron motion like in classical case without radiation reaction at 35<a<137

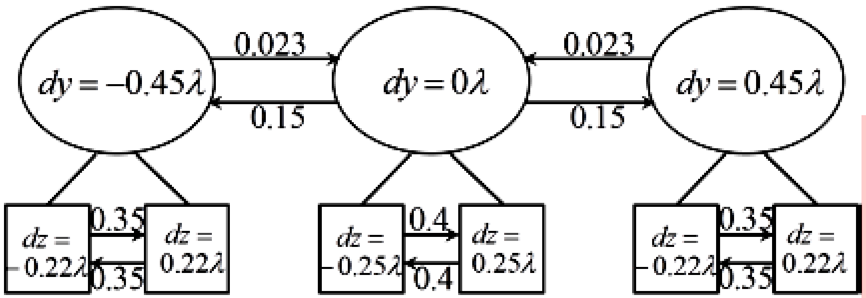
Electron motion in field of plane standing wave
Electron bunch motion

Landau-Lifshitz force | Without radiation reaction | Quasiclassical approach



Electron motion in field of plane standing wave
Markov chain

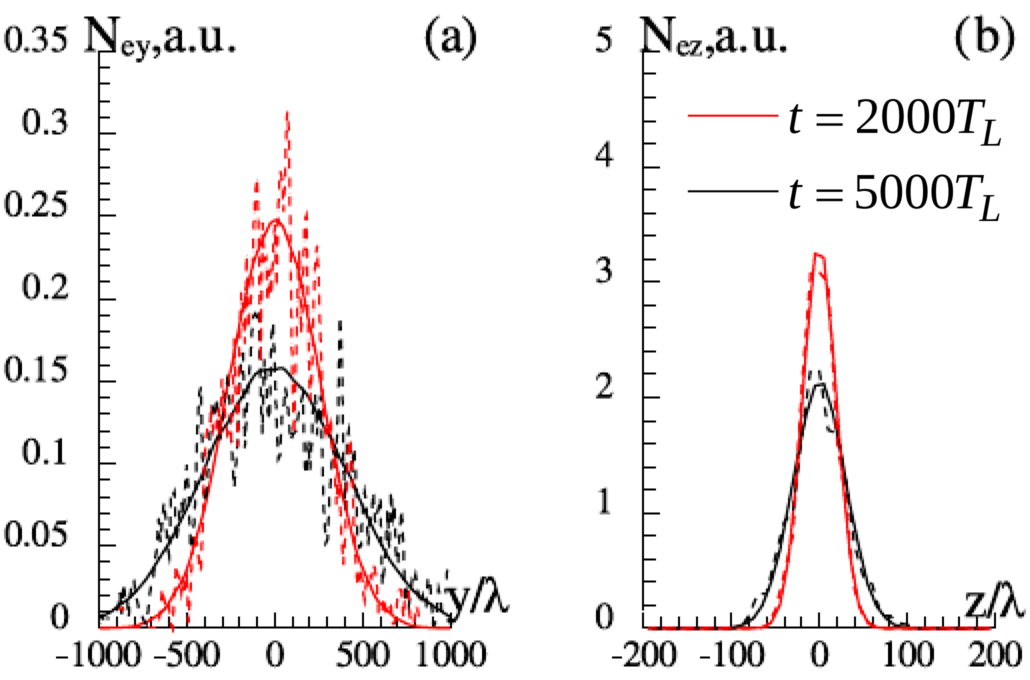
Scheme of motion



Transition matrix

$$P_{ij} = \begin{pmatrix} 0.977 & 0.023 & 0 \\ 0.15 & 0.7 & 0.15 \\ 0 & 0.023 & 0.977, \end{pmatrix}$$

Each half of a wave period an electron has a certain character of motion. Motion during next half-period is determined in probabilistic way by present character of motion



Diffusion

$$N_{er} = \frac{N_0}{\sqrt{4\pi D_{\parallel,\perp} t}} \exp\left(-\frac{r^2}{4D_{\parallel,\perp} t}\right)$$

Numerical simulation (dashed line)

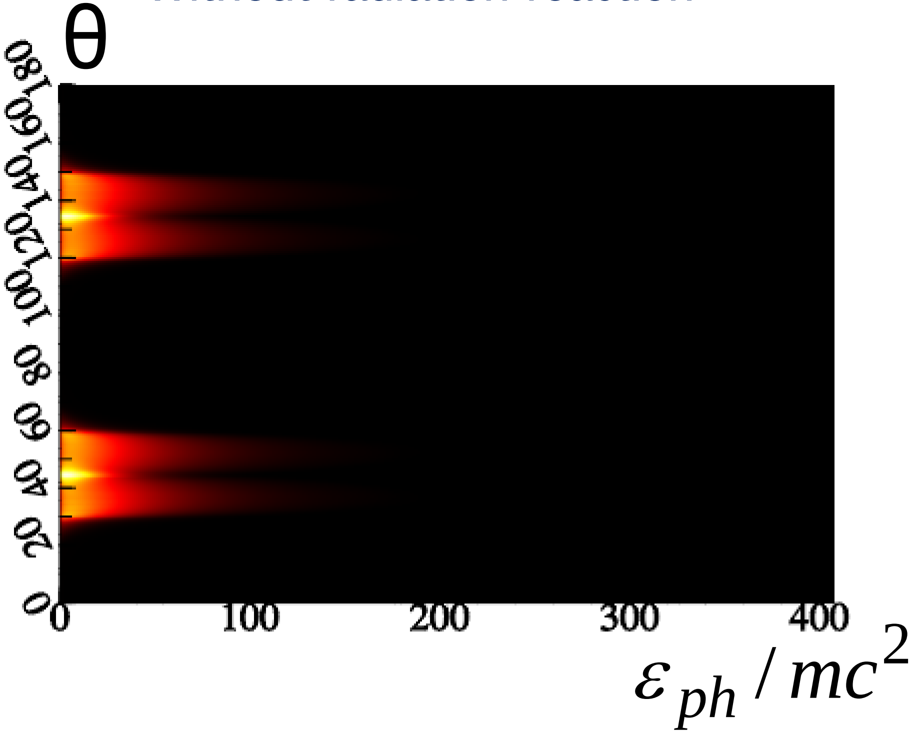
$D_y \approx 100 \qquad D_z \approx 0.6$

Analytical treatment (solid line)

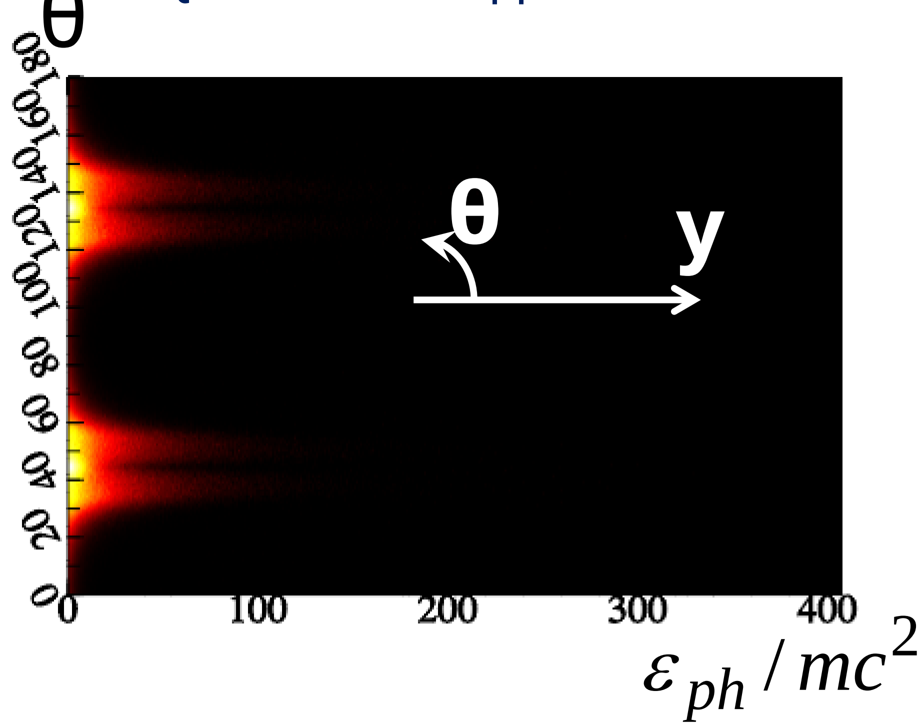
$D_y \approx 4^2 2\pi \qquad D_z \approx 0.3^2 2\pi$

Electron motion in field of plane standing wave
Radiation pattern

Without radiation reaction



Quasiclassical approach



Attainable energies are approximately the same, but in quasiclassical case aperture angle is larger and radiation pattern loses its fine structure because of stochasticity

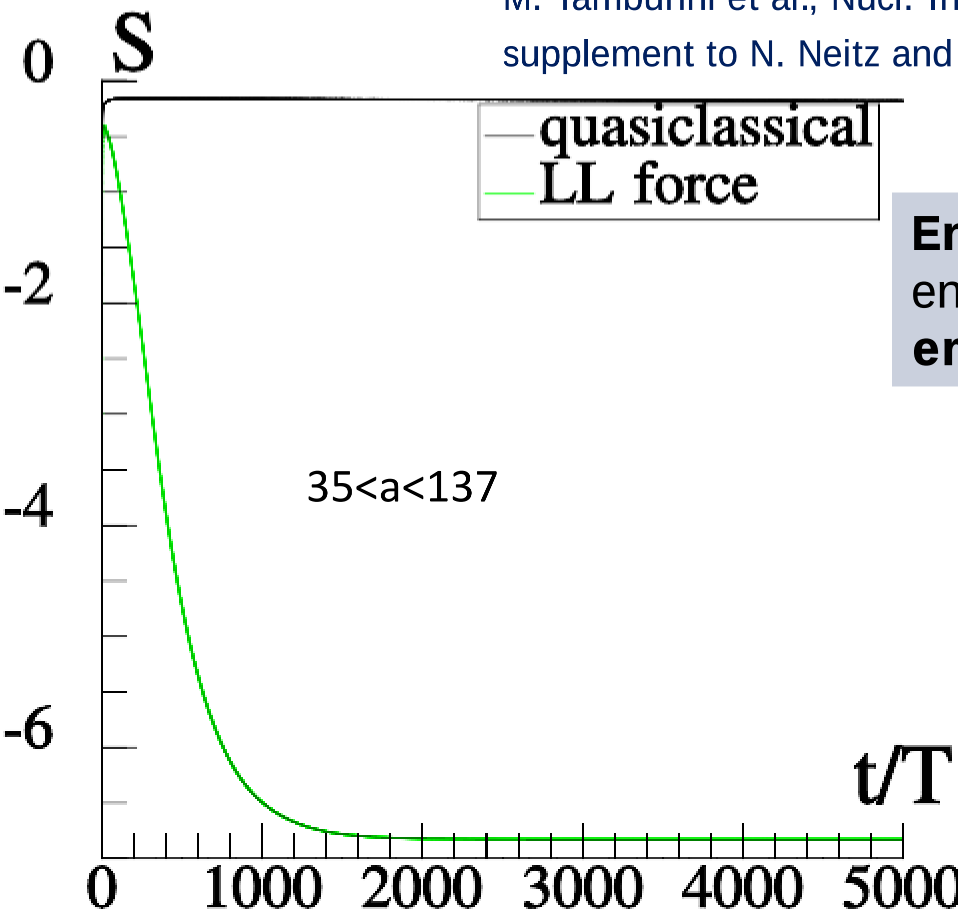
Electron motion in field of plane standing wave
Entropy

$$S = -\int f \ln f d\vec{r} d\vec{p}$$

f – distribution function

M. Tamburini et al., Nucl. Instrum. Methods Phys. Res., Sect. A **653**, 181 (2011)

supplement to N. Neitz and A. Di Piazza, Phys. Rev. Lett. **111**, 054802 (2013)



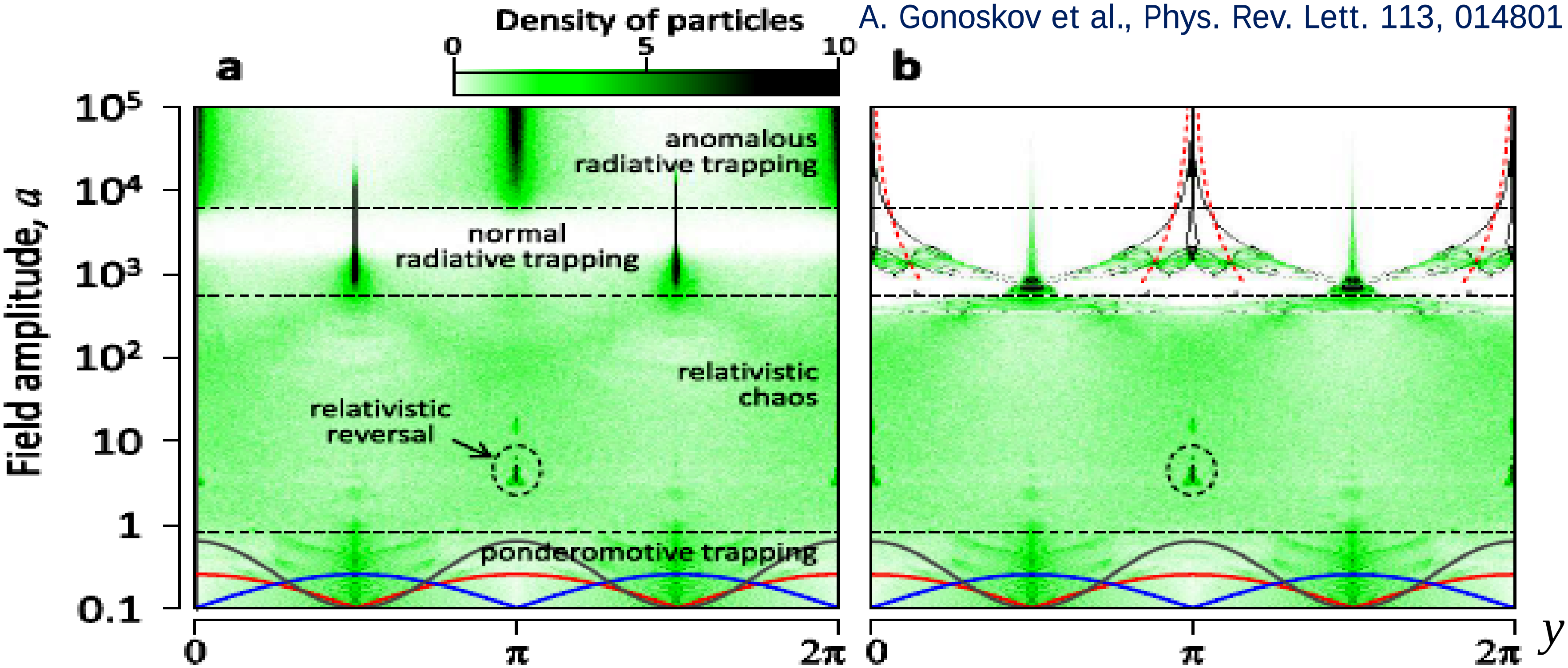
Energy losses tend to **decrease** entropy while **discreteness of photon emission increases** entropy

At 35<a<137 electron motion is stronger influenced by stochasticity than by radiation losses

Electron motion in field of plane standing wave
Ultrarelativistic case

Quasiclassical approach — Landau-Lifshitz force

A. Gonoskov et al., Phys. Rev. Lett. 113, 014801 (2014)



$$I_{th}^{NRT} \approx 5\times 10^{23}\frac{W}{cm^2}\times\left(\frac{0.81\mu m}{\lambda}\right)^{\frac{4}{3}}$$

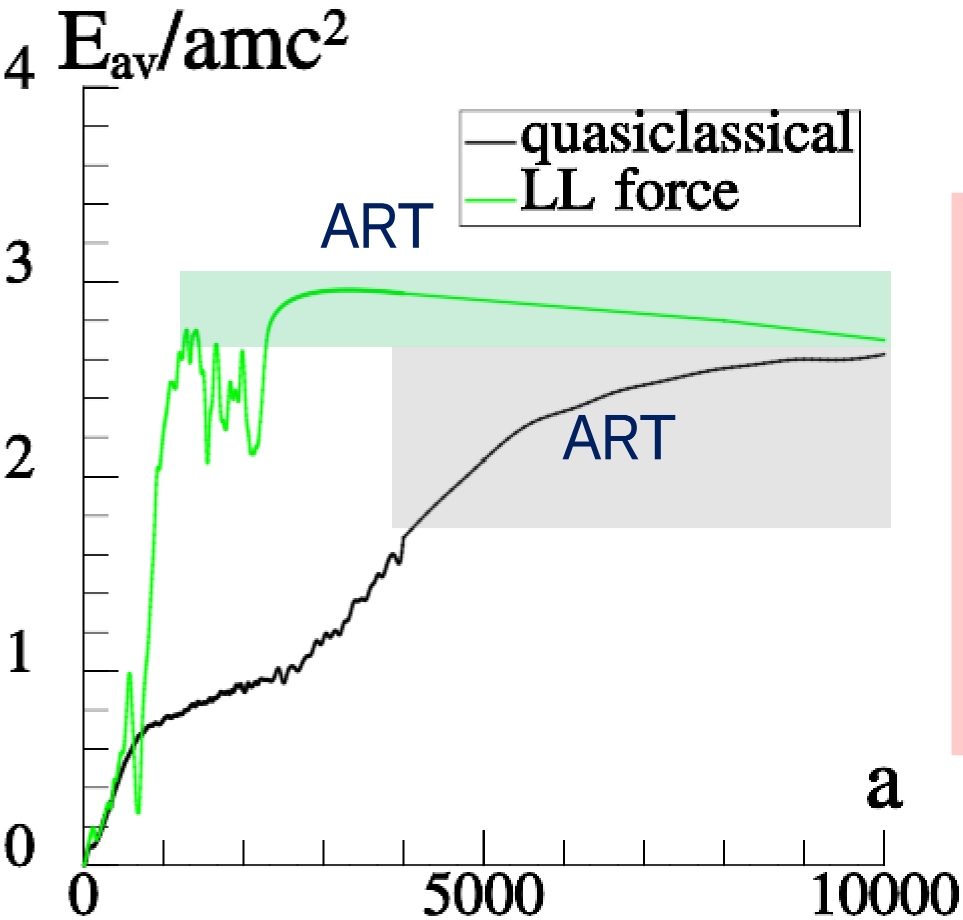
$$I_{th}^{ART} \approx 6\times 10^{25}\frac{W}{cm^2}\times\left(\frac{0.81\mu m}{\lambda}\right)^{\frac{4}{3}}$$

$$I_{th}^{NRT} \approx 5\times 10^{23}\frac{W}{cm^2}\times\left(\frac{0.81\mu m}{\lambda}\right)^{\frac{4}{3}}$$

$$I_{th}^{ART} \approx 8\times 10^{24}\frac{W}{cm^2}\times\left(\frac{0.81\mu m}{\lambda}\right)^{\frac{4}{3}}$$

Electron motion in field of plane standing wave
Ultrarelativistic case

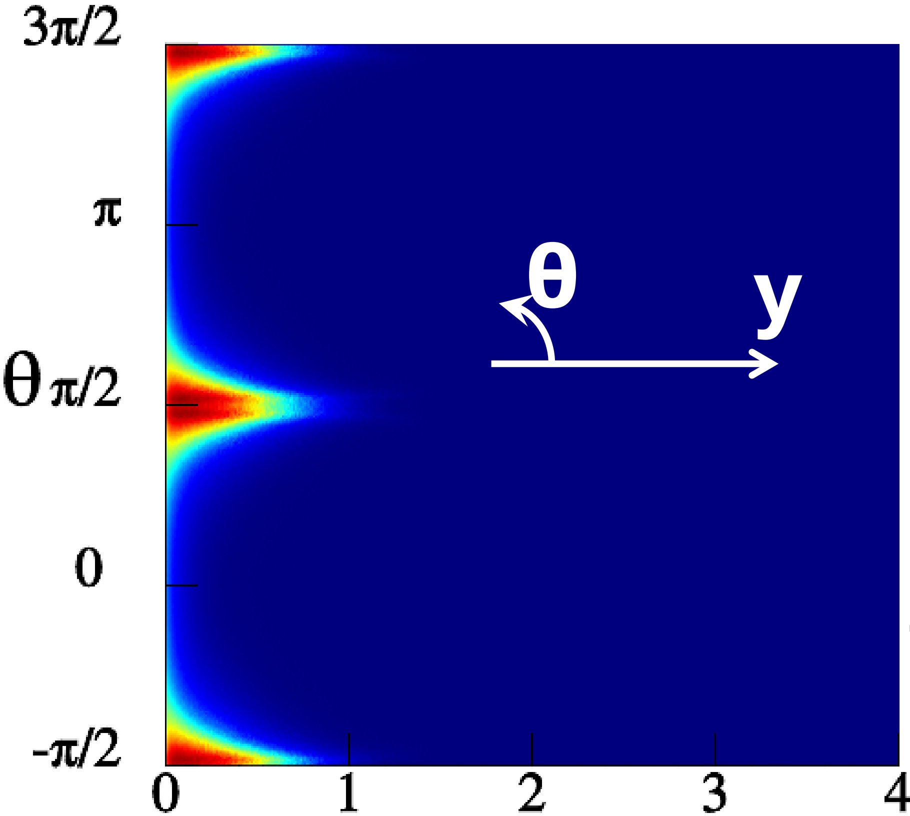
E_{av} is energy radiated over the wave period



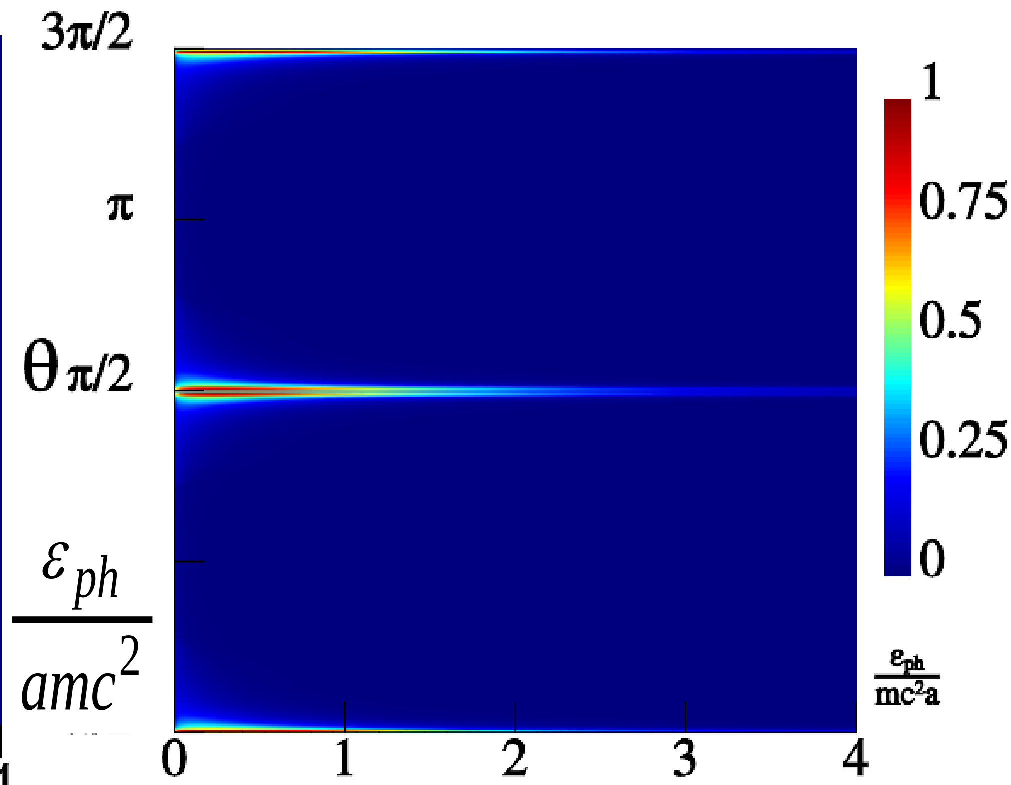
- At 500<a<10000 radiation losses are overestimated by classical theory
- E_{av} values are approximately the same for ART amplitudes.
- ART trajectories are similar in both approaches.

Electron motion in field of plane standing wave
Ultrarelativistic case

Quasiclassical approach



Landau-Lifshitz force



In classical case radiation losses occurs at a certain time while in quasiclassical case they are distributed over a larger part of trajectory, but total radiated energy is approximately the same in these cases.

Electron motion in field of plane standing wave
Ultrarelativistic case

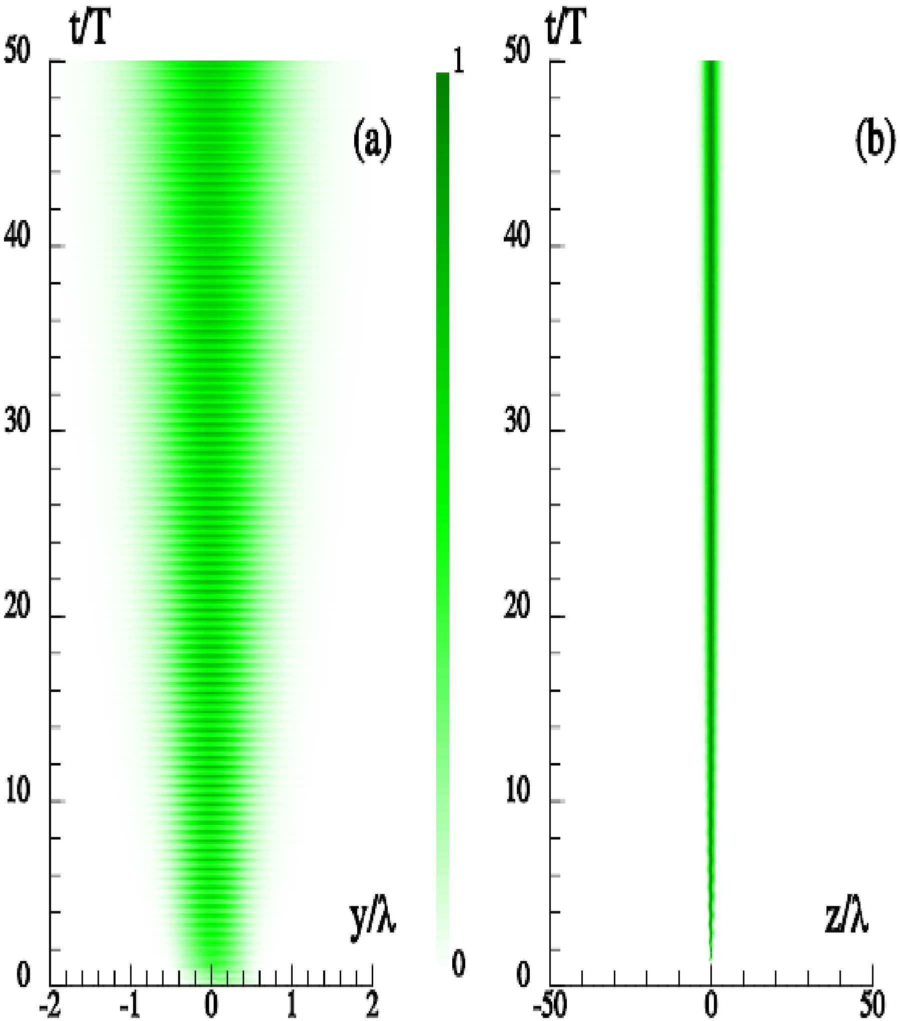
Quasiclassical approach

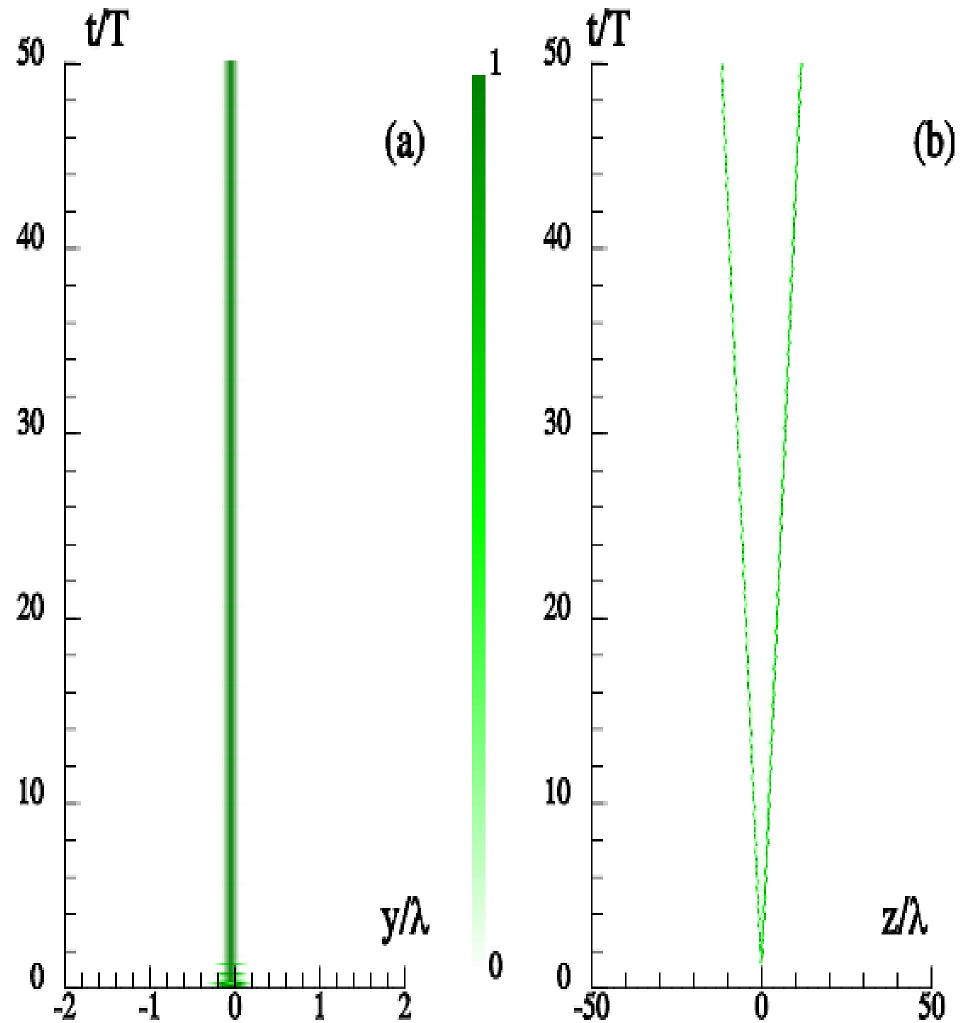
Diffusion

$a=4000$

Landau-Lifshitz force

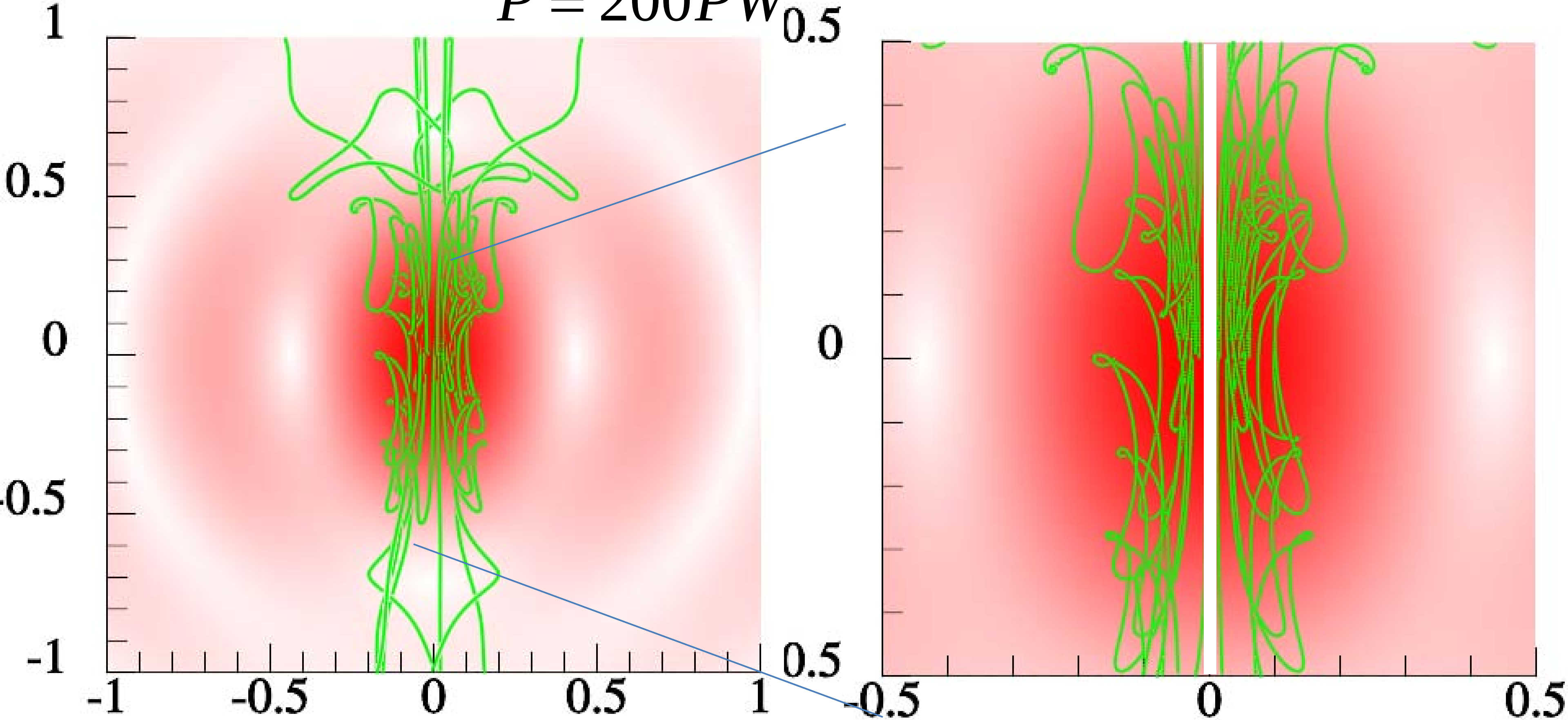
D'Alambert type solution





ART. Remark

$P = 200 PW$



If electron is born out of the tube with radius $r \approx 0.01\lambda$ than cascade is initiated.

Ultra-bright source of GeV photons from controlled electron-positron cascade in focused laser flelds of extreme intensity, A. Bashinov, A. Gonoskov, I. Gonoskov, E. Emenko, A. Ilderton, A. Kim, M. Marklund, A. Muraviev, and A. Sergeev, to be submitted.

Conclusion

1. Even if radiation losses are weak the discreteness of photon emission can play an important role. Despite predicted suppression of stochastic heating in classical description of radiation losses for $0.08\delta^{-1/3} < a < 0.33\delta^{-1/3}; \lambda >> 0.001\dot{A}$, the quasiclassical consideration doesn't confirm it.
2. It is shown that electron motion in the field of a linearly polarized plane standing wave can be described by Markov chain formalism, which is confirmed by numerical simulations.
3. At ultrarelativistic wave amplitudes both discreteness of photon emission and quantum corrections of radiation spectrum are important

A.V. Bashinov, A.V. Kim, A.M. Sergeev, arXiv:1502.02911 (2015)