Linking the leptonic CP violation to the quark unitarity triangle

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3 Summary

1 Introduction

Neutrino Flavor Mixing is different from the quark one !
However, theory could not predict two large mixing angles.

$$U_{PMNS} \sim \begin{pmatrix} 0.8 & 0.55 & 0.15 \\ -0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix} \qquad U_{CKM} \sim \begin{pmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.04 \\ 0.008 & 0.04 & \sim 1 \end{pmatrix}$$

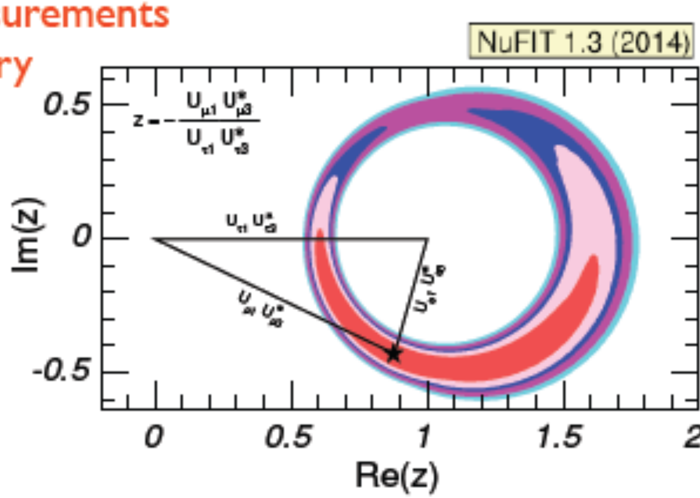
$\delta \sim$ unknown

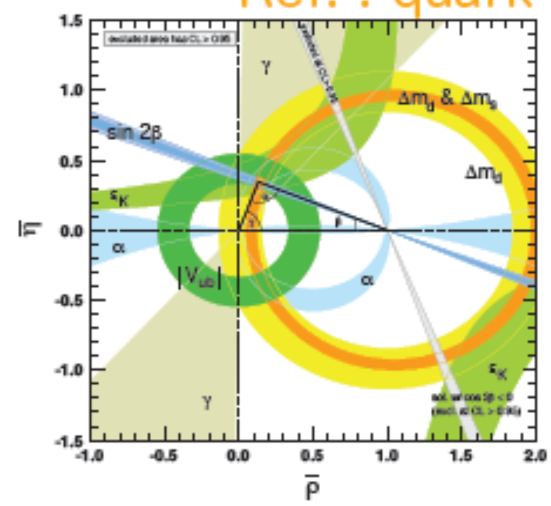
$\delta = 70°$

Leptonic unitarity triangle

Ref. : quark

- combining all measurements
- assuming the unitary





Is the CP violation of neutrinos is predictable ?

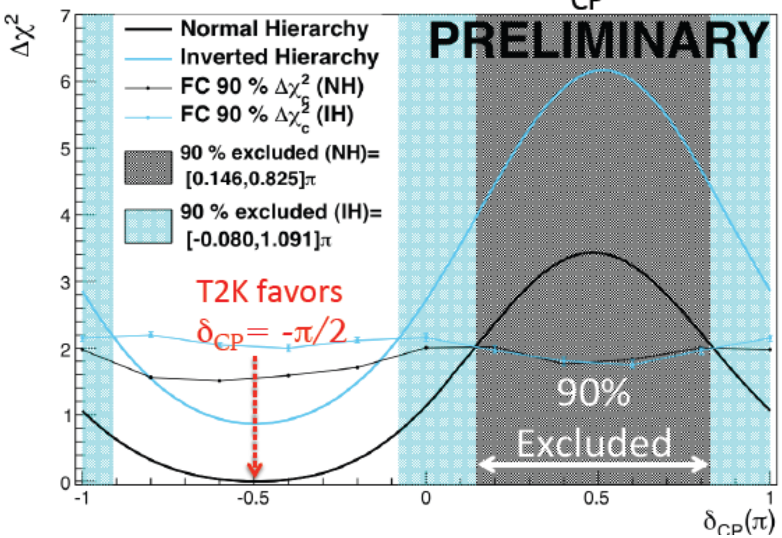
Talks of Mu-Chun Chen and Arsenii Titov

Another approch: Linking to Quark CP violation

T2K Joint $\nu_e + \nu_\mu$ analysis (6.57×10^{20} POT)

- Simultaneous fit to ν_e and ν_μ data
  - 4 oscillation parameters Δm^2_{23}, θ_{23}, θ_{13}, δ_{CP}
- Reactor constraints (PDG2013): $\sin^2 2\theta_{13}=0.095\pm0.010$

Constraint on δ_{CP}



2 Linking Leptonic CP violation to quark unitarity triangle

2.1 Quark mixing sum rules

Assume Texture zero

$$\theta_{13}^d = \theta_{13}^u = 0$$

$$M_{u(d)} = \begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$$

Antusch, King, Malinsky, Spinrath, Phys. Rev. D81 (2010) 033008

$$\delta_{\rm CKM} = {\rm Arg}\left[1 - \frac{\theta_{12}^d}{\theta_{12}^u} e^{-i(\delta_{12}^d - \delta_{12}^u)}\right]$$

$$\phi_2(\alpha) = {\rm Arg}\left[-\frac{U_{td}U_{tb}^*}{U_{ud}U_{ub}^*}\right] = \delta_{12}^d - \delta_{12}^u$$

Definitions

$$V_{uL} M_u V_{uR}^\dagger = \mathrm{diag}(m_u, m_c, m_t)\ , \qquad V_{dL} M_d V_{dR}^\dagger = \mathrm{diag}(m_d, m_s, m_b)$$

$$V_{qL}^\dagger = U_{23}^{qL} U_{13}^{qL} U_{12}^{qL} \qquad U'_{\mathrm{CKM}} = V_{uL} V_{dL}^\dagger$$

$$U_{12}^{qL} = \begin{pmatrix} c_{12}^q & s_{12}^q e^{-i\delta_{12}^q} & 0 \\ -s_{12}^q e^{i\delta_{12}^q} & c_{12}^q & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U_{23}^{qL} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23}^q & s_{23}^q e^{-i\delta_{23}^q} \\ 0 & -s_{23}^q e^{i\delta_{23}^q} & c_{12}^q \end{pmatrix}$$

$$U'_{\mathrm{CKM}} = U_{12}^{uL\dagger} U_{13}^{uL\dagger} U_{23}^{uL\dagger} U_{23}^{dL} U_{13}^{dL} U_{12}^{dL}$$

$$U_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CKM}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\rm CKM}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\rm CKM}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\rm CKM}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\rm CKM}} & c_{23}c_{13} \end{pmatrix},$$

$$U'_{\rm CKM} = U_{23}U_{13}U_{12} \qquad U_{12} = \begin{pmatrix} c_{12} & s_{12}e^{-i\delta_{12}} & 0 \\ -s_{12}e^{i\delta_{12}} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

δ_{ij}

Three phases appear, but we can remove two phase by multiplying the phase matrix in right-hand side.

$$\mathrm{diag}(e^{i\gamma_1}, e^{i\gamma_2}, e^{i\gamma_3})$$

$$\delta_{\rm CKM} = \delta_{13} - \delta_{23} - \delta_{12}$$

CKM mixing is expressed in terms of the left-handed mixing angles and phases of up- and down-quarks.

$$\theta_{23} e^{-i\delta_{23}} = (s^d_{23} e^{-i\delta^d_{23}} - s^u_{23} e^{-i\delta^u_{23}}) + s^u_{12} e^{i\delta^u_{12}} (s^d_{13} e^{-i\delta^d_{13}} - s^u_{13} e^{-i\delta^u_{13}}) ,$$

$$\theta_{13} e^{-i\delta_{13}} = -s^u_{12} e^{-i\delta^u_{12}} (s^d_{23} e^{-i\delta^d_{23}} - s^u_{23} e^{-i\delta^u_{23}}) + (s^d_{13} e^{-i\delta^d_{13}} - s^u_{13} e^{-i\delta^u_{13}}) ,$$

$$\theta_{12} e^{-i\delta_{12}} = (s^d_{12} e^{-i\delta^d_{12}} - s^u_{12} e^{-i\delta^u_{12}}) + s^u_{13} e^{-i\delta^u_{13}} (s^d_{23} e^{-i\delta^d_{23}} - s^u_{23} e^{-i\delta^u_{23}}) ,$$

CKM angles

up- and down-quark left-handed mixing

12 parameters

$$\delta_{\rm CKM} = \delta_{13} - \delta_{23} - \delta_{12}$$

Suppose vanishing left-handed (1-3) mixing in both up-quarks and down quarks.

$\theta_{13\,L}^{u,d} = 0$

$$M_{u(d)} = \begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$$

$$U'_{\mathrm{CKM}} = U_{12}^{uL\dagger} U_{23}^{uL\dagger} U_{23}^{dL} U_{12}^{dL}$$

U_{13}^{qL} is the unit matrix

$$\begin{cases} \theta_{23} e^{-i\delta_{23}} = \theta^d_{23} e^{-i\delta^d_{23}} - \theta^u_{23} e^{-i\delta^u_{23}} , \\ \theta_{13} e^{-i\delta_{13}} = -\theta^u_{12} e^{-i\delta^u_{12}} (\theta^d_{23} e^{-i\delta^d_{23}} - \theta^u_{23} e^{-i\delta^u_{23}}), \\ \theta_{12} e^{-i\delta_{12}} = \theta^d_{12} e^{-i\delta^d_{12}} - \theta^u_{12} e^{-i\delta^u_{12}} , \end{cases}$$

Mixing sum rules

- $\theta^u_{12} = \dfrac{\theta_{13}}{\theta_{23}}$
- $\theta^d_{12} = \left| \theta_{12} - \dfrac{\theta_{13}}{\theta_{23}} e^{-i\delta_{\rm CKM}} \right|$

Antusch, King, Malinsky, Spinrath, Phys. Rev. D81 (2010) 033008

- $\delta_{\rm CKM} = {\rm Arg} \left[1 - \dfrac{\theta^d_{12}}{\theta^u_{12}} e^{-i(\delta^d_{12} - \delta^u_{12})} \right]$

$$\delta_{\rm CKM} = \delta_{13} - \delta_{23} - \delta_{12}$$

CKM phase is expressed in terms of parameters of the up- down-quark mixing angles and phases.

$$\phi_1(\beta) = \mathrm{Arg}\left[-\frac{U_{cd}U_{cb}^*}{U_{td}U_{tb}^*}\right] = \mathrm{Arg}\left[1 - \frac{\theta_{12}^u}{\theta_{12}^d} e^{-i(\delta_{12}^d - \delta_{12}^u)}\right]$$

$$\phi_2(\alpha) = \mathrm{Arg}\left[-\frac{U_{td}U_{tb}^*}{U_{ud}U_{ub}^*}\right] = \delta_{12}^d - \delta_{12}^u$$

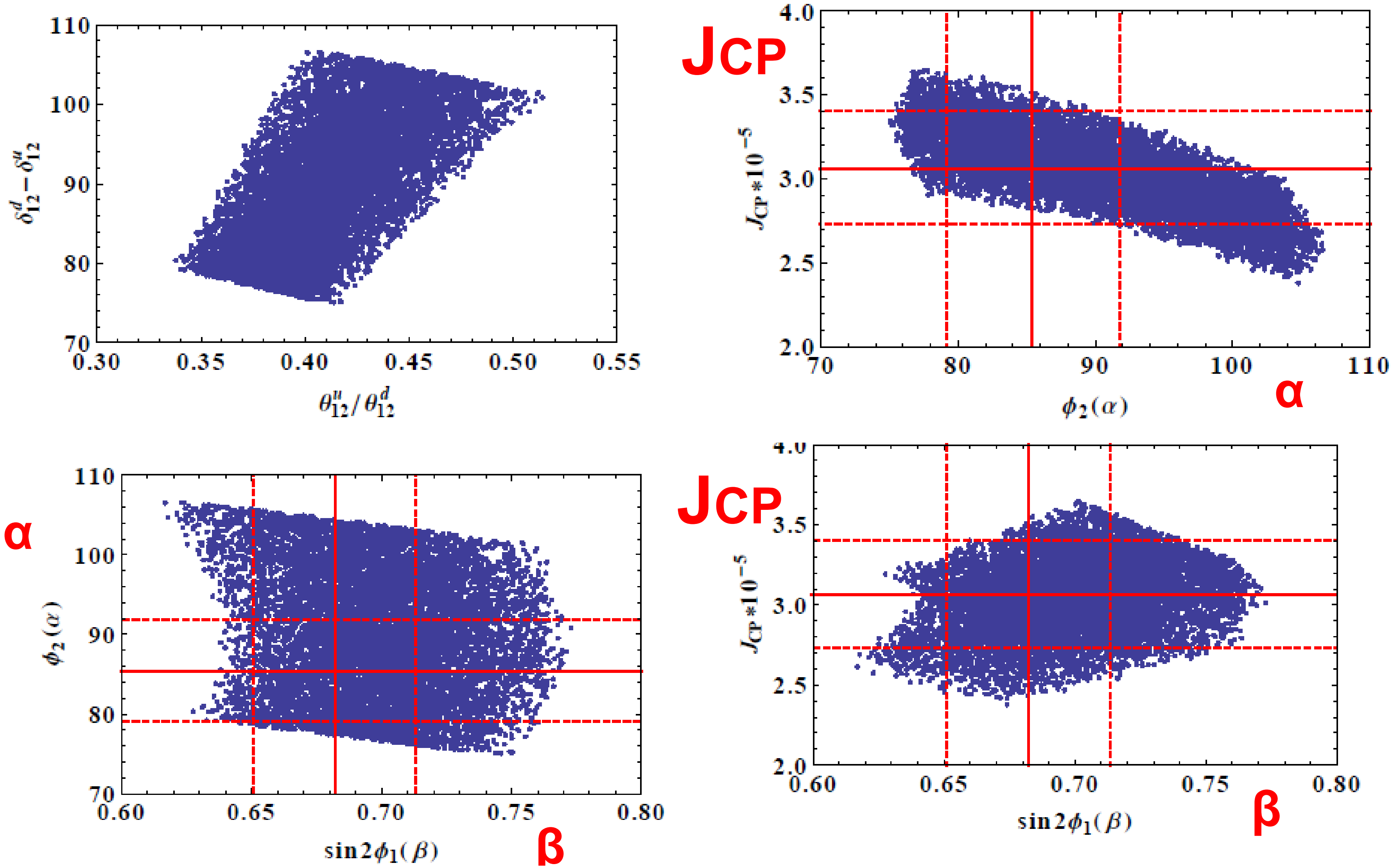
$$\phi_3(\gamma) = \mathrm{Arg}\left[-\frac{U_{ud}U_{ub}^*}{U_{cd}U_{cb}^*}\right] = \mathrm{Arg}\left[1 - \frac{\theta_{12}^d}{\theta_{12}^u} e^{-i(\delta_{12}^d - \delta_{12}^u)}\right]$$

δ_{CKM}

$$J_{CP} = \mathrm{Im}\left[U_{us}U_{cb}U_{ub}^*U_{cs}^*\right] = |U_{cb}|^2 \theta_{12}^u \theta_{12}^d \sin(\delta_{12}^d - \delta_{12}^u)$$

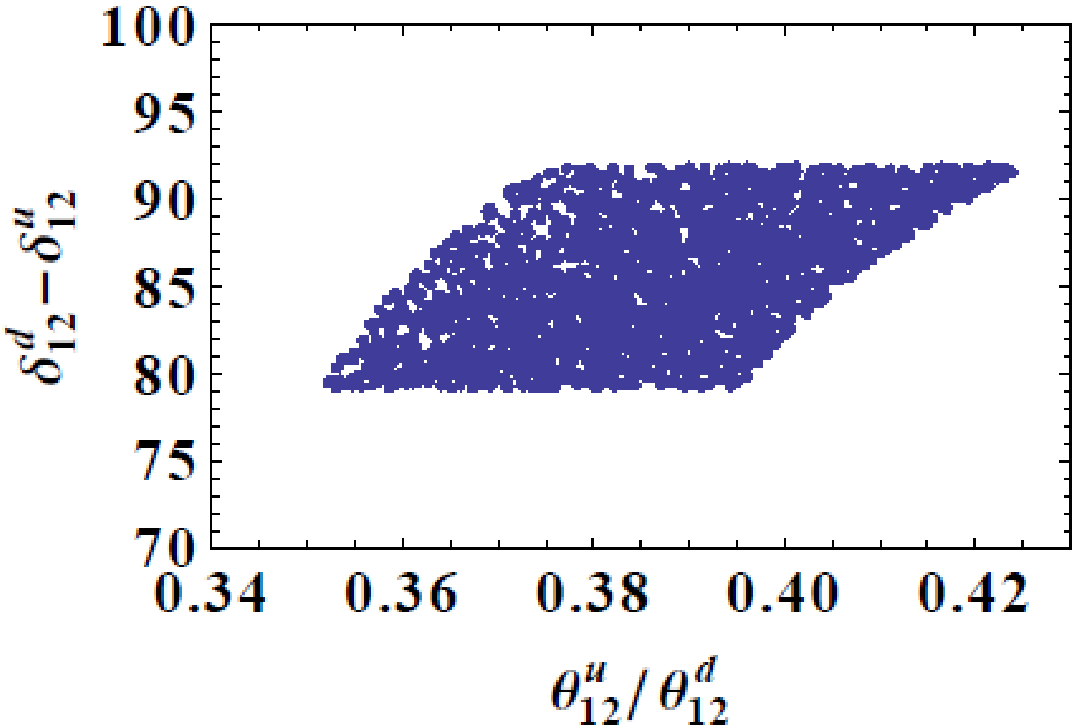
Unitarity Triangle

Input: V_{us}, V_{ub}, V_{cb}, γ



$$\sin 2\phi_1(\beta) = 0.682 \pm 0.019, \quad \phi_2(\alpha) = 85.4^{\circ}{}^{+3.9^\circ}_{-3.8^\circ}\ , \quad J_{CP} = 3.06^{+0.21}_{-0.20} \times 10^{-5}\ .$$

Impose constraints of α, β, J_{cp}



2.2 Lepton mixing sum rules

$$U'_{\mathrm{PMNS}} = \mathrm{diag}(e^{i\delta_e}, e^{i\delta_\mu}, e^{i\delta_\tau}) \cdot U_{\mathrm{PMNS}} \cdot \mathrm{diag}(e^{-i\varphi_1/2}, e^{-i\varphi_2/2}, 1)$$

Majorana Phase

$$U'_{\mathrm{PMNS}} = U_{12}^{eL\dagger} U_{13}^{eL\dagger} U_{23}^{eL\dagger} U_{23}^{\nu L} U_{13}^{\nu L} U_{12}^{\nu L}$$

$$\begin{aligned}
c_{13}s_{23}e^{-i\delta_{23}} &= c^\nu_{13}s^\nu_{23}e^{-i\delta^\nu_{23}} - s^e_{23}c^\nu_{23}c^\nu_{13}e^{-i\delta^e_{23}} + s^e_{12}s^\nu_{13}e^{-i(\delta^e_{13}-\delta^e_{12})} \,, \\
s_{13}e^{-i\delta_{13}} &= -s^e_{12}c^\nu_{13}e^{-i\delta^e_{12}}(s^\nu_{23}e^{-i\delta^\nu_{23}} - s^e_{23}c^\nu_{23}e^{-i\delta^e_{23}}) + s^\nu_{13}e^{-i\delta^\nu_{13}} - s^e_{13}c^\nu_{13}c^\nu_{23}e^{-i\delta^e_{13}} \\
c_{13}s_{12}e^{-i\delta_{12}} &= c^\nu_{13}s^\nu_{12}e^{-i\delta^\nu_{12}} - s^e_{12}c^\nu_{12}c^\nu_{23}e^{-i\delta^e_{12}} + s^e_{13}s^\nu_{23}c^\nu_{12}e^{-i(\delta^e_{13}-\delta^\nu_{23})} \,.
\end{aligned}$$

Assume

$$\theta^e_{13} = \theta^\nu_{13} = 0$$ **with** $$\theta^e_{23} \ll 0.1$$

$$U_{\rm PMNS} = U^{eL\dagger}_{12} U^{eL\dagger}_{23} U^{\nu L}_{23} U^{\nu L}_{12}$$

$$c_{13} s_{23} e^{-i\delta_{23}} = s^\nu_{23} e^{-i\delta^\nu_{23}} - \theta^e_{23} c^\nu_{23} e^{-i\delta^e_{23}} \,,$$
$$\theta_{13} e^{-i\delta_{13}} = -\theta^e_{12} e^{-i\delta^e_{12}} \left(s^\nu_{23} e^{-i\delta^\nu_{23}} - \theta^e_{23} c^\nu_{23} e^{-i\delta^e_{23}} \right) ,$$
$$c_{13} s_{12} e^{-i\delta_{12}} = s^\nu_{12} e^{-i\delta^\nu_{12}} - \theta^e_{12} c^\nu_{23} c^\nu_{12} e^{-i\delta^e_{12}} \,,$$

Majorana Phases

$$\delta_{12} = \frac{1}{2}(\varphi_2 - \varphi_1) \ , \qquad \delta_{23} = -\frac{1}{2}\varphi_2 \ , \qquad \delta_{13} = \delta_{\rm PMNS} - \frac{1}{2}\varphi_1 \ .$$

$$\boxed{\delta_{\rm PMNS} = \delta_{13} - \delta_{23} - \delta_{12}}$$

Dirac phase is given by the same relation as in the quark sector

$\circ$ $\theta_{13} = \theta^e_{12} s_{23}$ $\quad s_{23} \simeq s^\nu_{23}$

Mixing sum rules

$\circ$ $s_{12} = s^\nu_{12} \left| 1 - \frac{\theta^e_{12} c^\nu_{23} c^\nu_{12}}{s^\nu_{12}} e^{-i(\delta^e_{12} - \delta^\nu_{12})} \right|$

$\circ$ $\delta_{\text{PMNS}} = \delta_{13} - \delta_{23} - \delta_{12} = \phi - 2\pi = \phi$

$V_e^\dagger V_\nu$

$$\phi = \text{Arg}\left[1 - \frac{s^\nu_{12}}{\theta^e_{12} c^\nu_{23} c^\nu_{12}} e^{-i(\delta^\nu_{12} - \delta^e_{12})}\right]$$

$V_u^\dagger V_d$

$$\delta_{\text{CKM}} = \text{Arg}\left[1 - \frac{\theta^d_{12}}{\theta^u_{12}} e^{-i(\delta^d_{12} - \delta^u_{12})}\right]$$

for quarks

3.3 Prediction of Leptonic CP violation

Let us consider the Pati-Salam symmetry $SU(4)_C \times SU(2)_L \times SU(2)_R$

$$F^{i\alpha a} = (\mathbf{4},\mathbf{2},\mathbf{1})^i = \begin{pmatrix} u_L^R & u_L^B & u_L^G & \nu_L \\ d_L^R & d_L^B & d_L^G & e_L^- \end{pmatrix}^i \quad \bar{F}^i_{\alpha x} = (\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}})^i = \begin{pmatrix} \bar{d}_R^R & \bar{d}_R^B & \bar{d}_R^G & e_R^+ \\ \bar{u}_R^R & \bar{u}_R^B & \bar{u}_R^G & \bar{\nu}_R \end{pmatrix}^i$$

$$\theta^e_{12} = \theta^d_{12}$$

$$\delta^d_{12} - \delta^u_{12} = \delta^e_{12} - \delta^\nu_{12} = \phi_2(\alpha)$$

Sum rules

$$\theta_{13} = \theta^e_{12} s_{23} \qquad \phi = \mathrm{Arg}\left[1 - \frac{s^\nu_{12}}{\theta^e_{12} c^\nu_{23} c^\nu_{12}} e^{-i(\delta^\nu_{12} - \delta^e_{12})}\right]$$

$$\theta_{13} = \theta_C/\sqrt{2}$$

$$\delta_{\mathrm{PMNS}} = \phi = \mathrm{Arg}\left[1 - \frac{s^\nu_{12}}{\theta^e_{12} c^\nu_{23} c^\nu_{12}} e^{i\phi_2(\alpha)}\right]$$

We can remove the phase from up-sector !

$$\theta_{23}e^{-i\delta_{23}} = \theta_{23}^{d}e^{-i\delta_{23}^{d}} - \theta_{23}^{u}e^{-i\delta_{23}^{u}}$$

$$\times \quad e^{i\delta_{23}^{u}}$$

$$\theta_{13}e^{-i\delta_{13}} = -\theta_{12}^{u}e^{-i\delta_{12}^{u}}(\theta_{23}^{d}e^{-i\delta_{23}^{d}} - \theta_{23}^{u}e^{-i\delta_{23}^{u}})$$

$$\times \quad e^{i(\delta_{12}^{u}+\delta_{23}^{u})}$$

$$\theta_{12}e^{-i\delta_{12}} = \theta_{12}^{d}e^{-i\delta_{12}^{d}} - \theta_{12}^{u}e^{-i\delta_{12}^{u}}$$

$$\times \quad e^{i\delta_{12}^{u}}$$

In this framework, we can take real mixing for up quarks and neutrinos.
Phases appear only for down quarks and the charged leptons
except for Majorana phase. Example: Tri-bimaximal mixing

By using three conditions,

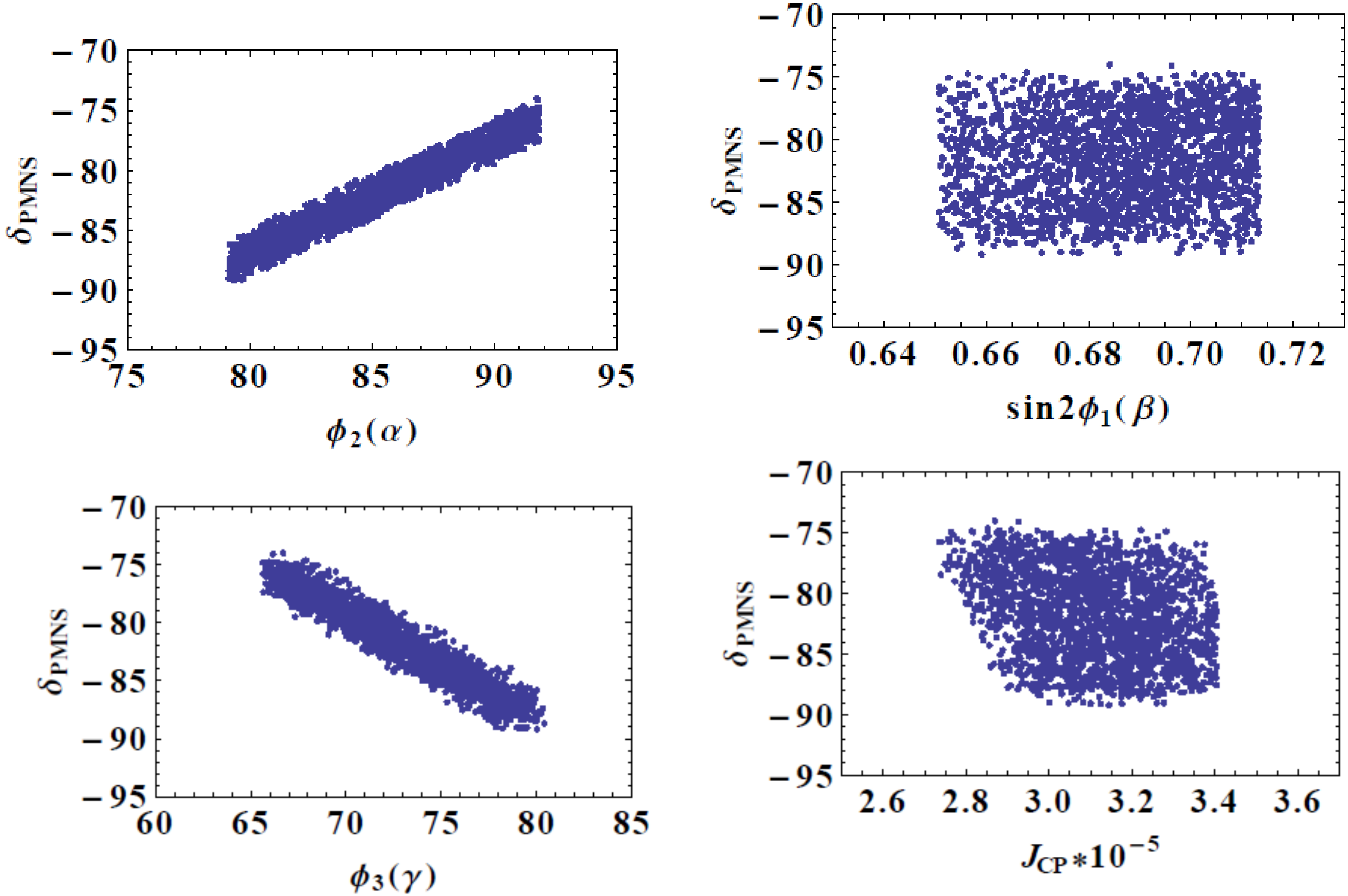
$$\theta^e_{12} = \theta^d_{12} \qquad s_{23} \simeq s^\nu_{23}$$

$$s_{12} = s^\nu_{12} \left| 1 - \frac{\theta^e_{12} c^\nu_{23} c^\nu_{12}}{s^\nu_{12}} e^{-i(\delta^e_{12} - \delta^\nu_{12})} \right|$$

we can calculate

$$\delta_{\rm PMNS} = \phi = {\rm Arg}\left[1 - \frac{s^\nu_{12}}{\theta^e_{12} c^\nu_{23} c^\nu_{12}} e^{i\phi_2(\alpha)} \right]$$

Prediction of δ_{PMNS} $-74^\circ \sim -89^\circ$



4 Summary

Leptonic CP violation can link to CKM phase
by the GUT relation: $-74^\circ \sim -89^\circ$

Predictions will be testable
by the precise data of
neutrino mixing angles, CP violating phase.

Is the CP violation detectable ?

Hyper-K !

Thank you !