Quo vadis, neutrino flavor models

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The flavor problem

Mass hierarchy?
- Why 3 generations?

Mixing hierarchy?
- Origin of neutrino mass?
The flavor problem

Mass hierarchy?

Mixing hierarchy?

why 3 generations?

origin of neutrino mass?
The flavor problem

Mass hierarchy?  Mixing hierarchy?

Is the flavor problem the evidence of some underlying symmetry?

why 3 generations?  origin of neutrino mass?
Flavor symmetry

SU(2) gauge doublet

triplet irreducible representation of a flavor group?
Lepton mixing: status

Is the atmospheric angle maximal?

Reactor angle not zero!

large solar  large atmospheric  small reactor
Is the atmospheric angle maximal?

Yes

With abelian flavor symmetry difficult to get maximal mixings

Strong indications in favor of non abelian flavor symmetry

No

everything is possible: there are no particular indication on the flavor symmetry...
maximal mixing & mu-tau symmetry

\[
\sin \theta_{23} = \frac{1}{\sqrt{2}} \\
\sin \theta_{13} = 0
\]

\[\nu_\mu \leftrightarrow \nu_\tau\]
exchange symmetry
In the neutrino sector

\[\mu \leftrightarrow \tau\]
exchange symmetry
broken in the charged sector

Babu, Ma, Valle 02
Grimus, Lavoura, 04
Altarelli, Feruglio 05
Caravaglios, M, 05
......
maximal mixing & mu-tau symmetry

Difficult to get because

\[
\begin{pmatrix}
\mu \\
\nu^\mu
\end{pmatrix}
\]

exchange symmetry
In the neutrino sector

\[
\nu^\mu \leftrightarrow \nu^\tau
\]

exchange symmetry
broken in the charged sector

\[
\mu \leftrightarrow \tau
\]
maximal atmospheric mixing

origin of mu-tau neutrino symmetry

Non-abelian symmetry
Symmetry of tetrahedron
Triplet representation
Finite subgroup of SU(3)

$\mu \leftrightarrow \tau$

Broken in the charged lepton sector: but $\mathbb{Z}_3$ preserved

$\nu_\mu \leftrightarrow \nu_\tau$

In the neutrino sector
maximal atmospheric mixing
and reactor angle

\[ \sin \theta_{13} \sim \lambda_c \neq 0 \]

M, Patel, Peinado 11
Boucenna, M, Tortola, Valle 11
King, 12
King, Luhn 12

We expect deviations of the
\[ \nu_\mu \leftrightarrow \nu_\tau \]
exchange symmetry in the neutrino sector
What if the atmospheric angle is not maximal?
Anarchy: neutrino mass matrix has random entries

$M = m \begin{pmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{pmatrix}$

implies mixing and ratio of masses of order one in rough agreement with data

Hall, Murayama, Weiner 99
discrete non abelian symmetries so far are not excluded

connection between atmospheric and reactor angles, i.e.

$\sin \theta_{23} = \frac{1}{\sqrt{2}} + \sin \theta_{13}\cos \delta$

mixing sum rule

Ballet, King, Luhn, Pascoli, Schmidt 13
Discrete symmetries & Mass sum rules

\[ \alpha m_1 + \beta m_2 = m_3 \]

\[ \frac{\alpha}{m_1} + \frac{\beta}{m_2} = \frac{1}{m_3} \]

\[ \alpha \sqrt{m_1} + \beta \sqrt{m_2} = \sqrt{m_3} \]

\[ \frac{\alpha}{\sqrt{m_1}} + \frac{\beta}{\sqrt{m_2}} = \frac{1}{\sqrt{m_3}} \]

Altarelli, Feruglio, Hagedorn 08
Hirsch, M, Valle 08
Bazzocchi, M, Merlo 09
Altarelli, Meloni 09
Barry, Rodejohann 10/11
Dorame, Meloni, M, Peinado, Valle 11

See talk of Merle
Mass sum rules & Onubb

King, Merle, M, Shimizu, Tanimoto 14

See talk of Merle

\[\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3\]
\[2\tilde{m}_2 + \tilde{m}_3 = \tilde{m}_1\]
\[\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3\]
\[\tilde{m}_1 + \frac{\sqrt{3}+1}{2}\tilde{m}_3 = \frac{\sqrt{3}-1}{2}\tilde{m}_2\]
\[\tilde{m}_1^{-1} + \tilde{m}_2^{-1} = \tilde{m}_3^{-1}\]
\[2\tilde{m}_2^{-1} + \tilde{m}_3^{-1} = \tilde{m}_1^{-1}\]
\[\tilde{m}_1^{-1} + \tilde{m}_3^{-1} = 2\tilde{m}_2^{-1}\]
\[\tilde{m}_3^{-1} \pm 2i\tilde{m}_2^{-1} = \tilde{m}_1^{-1}\]
\[\sqrt{\tilde{m}_1} - \sqrt{\tilde{m}_3} = 2\sqrt{\tilde{m}_2}\]
\[\sqrt{\tilde{m}_1} + \sqrt{\tilde{m}_3} = 2\sqrt{\tilde{m}_2}\]
\[\tilde{m}_1^{-1/2} + \tilde{m}_2^{-1/2} = 2\tilde{m}_3^{-1/2}\]
Flavor symmetries: multi-Higgs vs flavon

\[ y_{i j k} \bar{L}_i l_{Rj} H_k \]

\[ \frac{1}{\Lambda} y_{i j k} \bar{L}_i l_{Rj} H \phi_k \]
Flavor symmetries: multi-Higgs vs flavon

\[ y_{ijk} \bar{L}_i l_{R_j} H_k \]

\[ \frac{1}{\Lambda} y_{ijk} \bar{L}_i l_{R_j} H \varphi_k \]

No signal at LHC,...
Flavor symmetries: multi-Higgs vs flavon

\[ y_{ijk} \overline{L}_i l_{Rj} H_k \]

\[ \frac{1}{\Lambda} y_{ijk} \overline{L}_i l_{Rj} H \varphi_k \]

New Higgs at the weak scale FCNC, LFV,...

\[ H \rightarrow ee, H \rightarrow \mu \mu, \ldots \]

Flavor symmetry gives relations between branching ratios

See talk of Vicente
Higgs decay and flavor symmetry, i.e.

Based on A4: it is distinguishable and falsifiable

See also: triality, Ma PRD (10)
Higgs decay in S3, Bhattacharyya et al PRD (12)
Lepton non-universality and flavor symmetry

\[ R_K = \frac{\text{BR}(B \rightarrow K\mu^+\mu^-)}{\text{BR}(B \rightarrow Ke^+e^-)} = 0.745^{+0.090}_{-0.074} \pm 0.036 \]

Only 2.6 sigma

LHCb collaboration PRL (14)

\( R = 1 \) in the SM
Lepton non-universality and flavor symmetry

\[ R_K = \frac{\text{BR}(B \rightarrow K\mu^+\mu^-)}{\text{BR}(B \rightarrow Ke^+e^-)} = \frac{0.745^{+0.090}_{-0.074}}{0.036} \]

Only 2.6 sigma

LHCb collaboration PRL (14)

- Minimal Flavor Violation: Alonso et al 1505.05164
  Lee, Tandeon 1505.04692

- Z'
  Celis et al 1505.03079
  Crivellin et al 1504.07928

- Neutrino oscillation
  Boucenna et al 1503.07099
Lepton non-universality and flavor symmetry

Leptoquark

Hiller, Schmaltz JHEP (15)

\[ \mathcal{L} = -\lambda_{q\ell} \Delta (\bar{q}P_L \ell) \] for SU(2) doublet

\[ \mathcal{L} = -\lambda_{q\ell} \Delta^* (\bar{q}\ell) \] for SU(2) triplet
Lepton non-universality and flavor symmetry

Non-Abelian flavor symmetry

\[
\lambda \equiv \begin{pmatrix}
\lambda_{de} & \lambda_{d\mu} & \lambda_{d\tau} \\
\lambda_{se} & \lambda_{s\mu} & \lambda_{s\tau} \\
\lambda_{be} & \lambda_{b\mu} & \lambda_{b\tau}
\end{pmatrix} \sim \lambda_0 \begin{pmatrix}
\rho_d \kappa & \rho_d & \rho_d \\
\rho \kappa & \rho & \rho \\
\kappa & 1 & 1
\end{pmatrix}
\]

De Medeiros, Hiller 1503.01084

\[
\mathcal{B}(B \to K\mu^±e^±) \simeq 3 \cdot 10^{-8} \kappa^2 \left( \frac{1 - R_K}{0.23} \right)^2,
\]

\[
\mathcal{B}(B \to Ke^±\tau^±) \simeq 2 \cdot 10^{-8} \kappa^2 \left( \frac{1 - R_K}{0.23} \right)^2,
\]

\[
\mathcal{B}(B \to K\mu^±\tau^±) \simeq 2 \cdot 10^{-8} \left( \frac{1 - R_K}{0.23} \right)^2,
\]

\[
\mathcal{B}(\mu \to e\gamma) \simeq 2 \cdot 10^{-12} \frac{\kappa^2}{\rho^2} \left( \frac{1 - R_K}{0.23} \right)^2,
\]

\[
\mathcal{B}(\tau \to e\gamma) \simeq 4 \cdot 10^{-14} \frac{\kappa^2}{\rho^2} \left( \frac{1 - R_K}{0.23} \right)^2,
\]

\[
\mathcal{B}(\tau \to \mu\gamma) \simeq 3 \cdot 10^{-14} \frac{1}{\rho^2} \left( \frac{1 - R_K}{0.23} \right)^2,
\]

\[
\mathcal{B}(\tau \to \mu\eta) \simeq 4 \cdot 10^{-11} \rho^2 \left( \frac{1 - R_K}{0.23} \right)^2.
\]
While large neutrino mixing seems to give indication of non-abelian family symmetry.

Typically the neutrino flavor symmetries do not naturally fit the quark sector.

Non abelian symmetries must be introduced to fit also the quark sector.

\[ G_f \equiv A_4 \otimes Z_n \otimes U(1)_{f_n} \otimes \ldots \]
Flavor symmetries: from neutrino to quarks

While large neutrino mixing seems to be an indication of non-abelian family symmetries, typically the neutrino flavor symmetries do not naturally fit the quark sector. Non-abelian symmetries must be introduced to fit also the quark sector. The problem is to find a simple common framework.

\[ G_f \equiv A_4 \otimes Z_n \otimes U(1)_f \otimes \ldots \]
GUT

LHC

Quarks
B-physics

Gf

DM

Neutrino
0nubb, CP
Sterile, LFV

Inflation

BAU
Summary

The future direction of flavor model building will strongly depend on experimental results:

- atmospheric mixing
- leptonic CP violation
- mass hierarchy

If the atmospheric angle is VERY close to be maximal, evidence of a neutrino mu-tau symmetry

If the atmospheric angle is NOT maximal, even anarchy is possible

BUT

flavor symmetries imply testable mixing & mass sum rules
Summary

In the multi-Higgs case, hopefully LHC will give us some indication.

We can also have indications from Flavor Physics like B decays.

From the theoretical point of view, a possible criteria to proceed is to link very different topics and problems like dark matter and neutrino and so on....
Tri-bi-maximal mixing: theoretical ansatz

\[
\sin \theta_{12} = \frac{1}{\sqrt{3}}
\]

**Tri-maximal solar angle**

\[
U_{HPS} = \begin{pmatrix}
\sqrt{2/3} & 1/\sqrt{3} & 0 \\
-1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\
-1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2}
\end{pmatrix}
\]

**Reactor angle zero**

\[
\sin \theta_{23} = \frac{1}{\sqrt{2}}
\]

**Maximal atmospheric angle**