Where does CP violation actually come from?

Mu-Chun Chen, University of California, Irvine

WIN 2015, Heidelberg, Germany, June 8 - 13, 2015

Where can CP violation possibly come from?

Mu-Chun Chen, University of California, Irvine

Based on work in collaboration with
Maximillian Fallbacher, K.T. Mahanthappa, Michael Ratz, Andreas Trautner

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CP Violation in Nature

- CP violation: required to explain matter-antimatter asymmetry
- So far observed only in flavor sector
  - SM: CKM matrix for the quark sector
    - experimentally established δ_{CKM} as major source of CP violation
- Search for new source of CP violation:
  - CP violation in neutrino sector
  - if found $\Rightarrow$ phase in PMNS matrix
- Discrete family symmetries:
  - suggested by large neutrino mixing angles
  - neutrino mixing angles from group theoretical CG coefficients

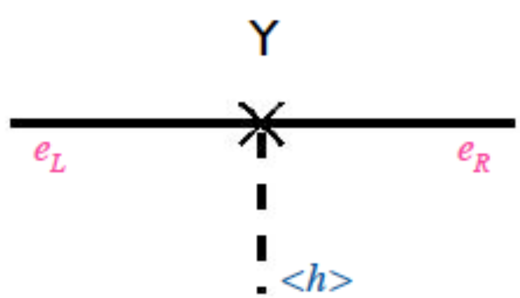
Discrete (family) symmetries $\Leftrightarrow$ Physical CP violation

Origin of CP Violation

- CP violation ⇔ complex mass matrices

$$\overline{U}_{R,i}(M_u)_{ij}Q_{L,j} + \overline{Q}_{L,j}(M_u^\dagger)_{ji}U_{R,i} \xrightarrow{\mathcal{CP}} \overline{Q}_{L,j}(M_u)_{ij}U_{R,i} + \overline{U}_{R,i}(M_u)^*_{ij}Q_{L,j}$$

- Conventionally, CPV arises in two ways:
  - Explicit CP violation: complex Yukawa coupling constants Y
  - Spontaneous CP violation: complex scalar VEVs <h>



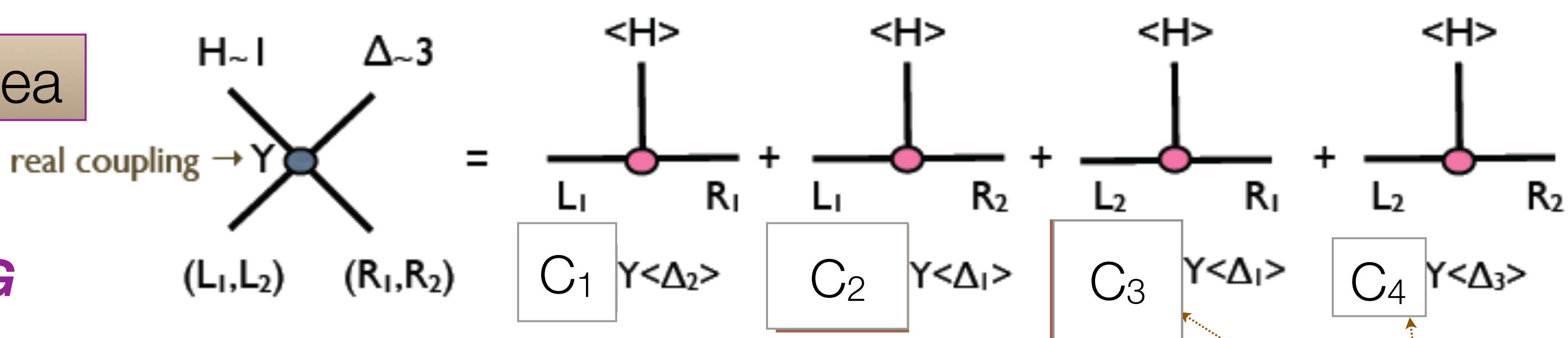
A Novel Origin of CP Violation

M.-C.C., K.T. Mahanthappa
Phys. Lett. B681, 444 (2009)

- Complex CG coefficients in certain discrete groups ⇒ explicit CP violation
  - Real Yukawa couplings, real scalar VEVs
  - CPV in quark and lepton sectors purely from complex CG coefficients
  - No additional parameters needed ⇒ extremely predictive model!

Basic idea

Discrete symmetry ***G***

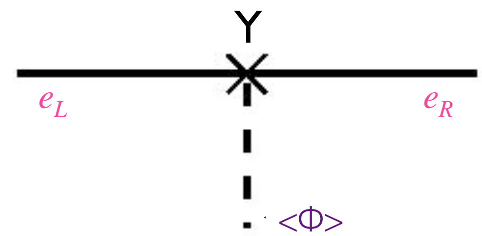


- Scalar potential: if Z_3 symmetric ⇒ $\langle\Delta_1\rangle = \langle\Delta_2\rangle = \langle\Delta_3\rangle \equiv \langle\Delta\rangle$ real
- Complex effective mass matrix: **phases determined by group theory**

$$\begin{array}{ccc} & (\,L_1 \quad L_2\,) & \\ M = & \begin{pmatrix} C_1 & C_3 \\ C_2 & C_4 \end{pmatrix} Y \langle\Delta\rangle & \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} \end{array}$$

$C_{1,2,3,4}$: complex CG coefficients of ***G***

A Novel Origin of CP Violation



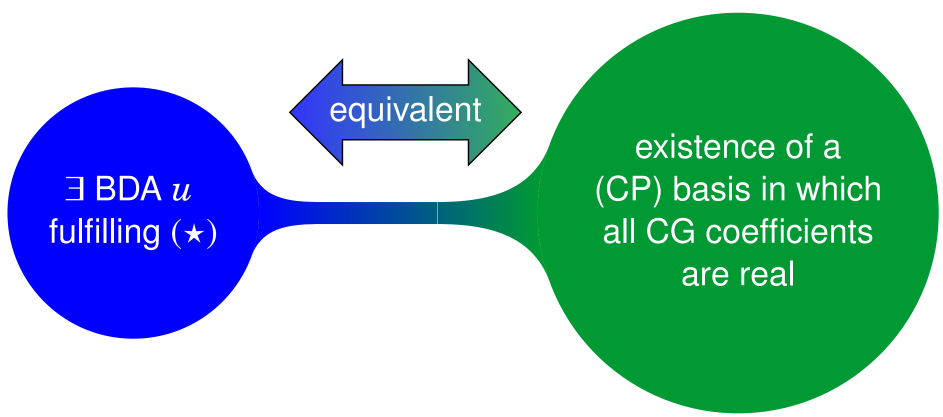
- Conventionally:
  - Explicit CP violation: complex Yukawa couplings
  - Spontaneous CP violation: complex Higgs VEVs
- Complex CG coefficients in discrete groups ⇒ explicit CP violation in quark and lepton sectors (e.g. $\delta \neq 0$)

M.-C.C, K.T. Mahanthappa, Phys. Lett. B681, 444 (2009);
M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, Nucl. Phys. B (2014)

- Conditions for a discrete group to admit real CG's

Bickerstaff, Damhus, 1985

$$\boxed{\exists \text{ automorphism } u \text{, such that } \lambda_k(R) = \lambda_k(u(R))^* \text{ for all } R \in G}$$



A Novel Origin of CP Violation

M.-C.C, M. Fallbacher, K.T. Mahanthappa,
M. Ratz, A. Trautner, NPB (2014)

- **More generally, for discrete groups that do not have class-inverting, involutory automorphism, CP is generically broken by complex CG coefficients (Type I Group)**
- **Non-existence of such automorphism ⇔ Physical CP violation**

CP Violation from Group Theory!

Discrete (flavor) symmetry G

Type II: one can impose a physical CP transformation

Type I groups G_{I}: generic settings based on G_{I} do not allow for a physical CP transformation

Type II A groups $G_{\mathrm{II\ A}}$: there is a CP basis in which all CG's are real

Type II B groups $G_{\mathrm{II\ B}}$: there is no basis in which all CG's are real

For further insights, see, M. Fallbacher, A. Trautner, NPB (2015)

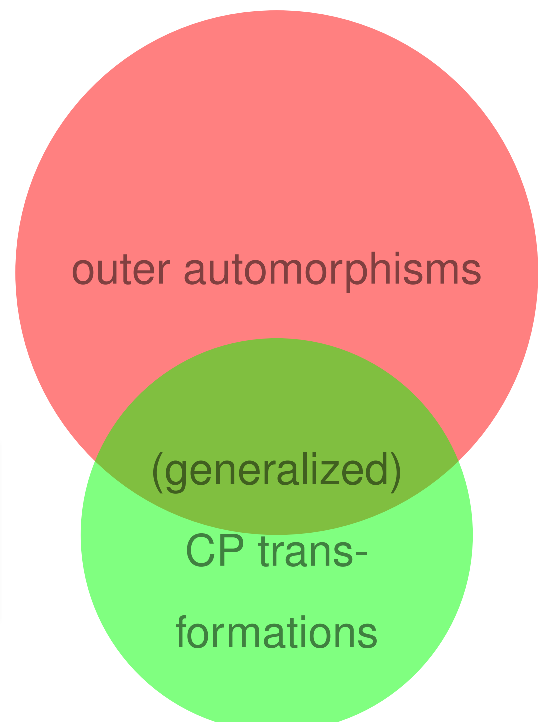
Physical CP Transformation

- **NOT all outer automorphisms correspond to physical CP transformations**

- **Condition on automorphism for *physical* CP transformation**

$$\rho_{\boldsymbol{r}_i}\big(u(g)\big) \;=\; U_{\boldsymbol{r}_i}\, \rho_{\boldsymbol{r}_i}(g)^*\, U_{\boldsymbol{r}_i}^{\dagger} \quad \forall\, g \in G \text{ and } \forall\, i$$

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)



class inverting, involutory automorphisms $\Leftrightarrow$ **physical CP transformations**

Examples

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Type I: all odd order non-Abelian groups

group	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\Delta(27)$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$
SG	(20,3)	(21,1)	(27,3)	(27,4)

- Type IIA: dihedral and all Abelian groups

group	S_3	Q_8	A_4	$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	T′	S_4	A_5
SG	(6,1)	(8,4)	(12,3)	(24,1)	(24,3)	(24,12)	(60,5)

- Type IIB

group	$\Sigma(72)$	$((\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4) \rtimes \mathbb{Z}_4$
SG	(72,41)	(144,120)

Example for a type I group:

$$\mathbf{\Delta(27)}$$

- decay asymmetry in a toy model
- prediction of CP violating phase from group theory

Toy Model based on Δ(27)

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Field content

fermions

field	S	X	Y	Ψ	Σ
$\Delta(27)$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{3}$
U(1)	$q_\Psi - q_\Sigma$	$q_\Psi - q_\Sigma$	0	q_Ψ	q_Σ

$q_\Psi - q_\Sigma \neq 0$

- Interactions

$$\mathscr{L}_{\text{toy}} = F^{ij}\ S\,\overline{\Psi}_i \Sigma_j + G^{ij}\ X\,\overline{\Psi}_i \Sigma_j + H^{ij}_{\Psi}\ Y\,\overline{\Psi}_i \Psi_j + H^{ij}_{\Sigma}\ Y\,\overline{\Sigma}_i \Sigma_j + \text{h.c.}$$

$$F = f\,\mathbb{1}_3$$

$$G = g\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$H_{\Psi/\Sigma} = h_{\Psi/\Sigma}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

with $\omega := \mathrm{e}^{2\pi\,\mathrm{i}/3}$

“flavor” structures determined by (complex) CG coefficients

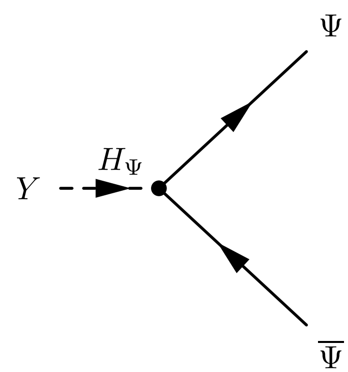
arbitrary coupling constants: f, g, h_Ψ, h_Σ

Toy Model based on Δ(27)

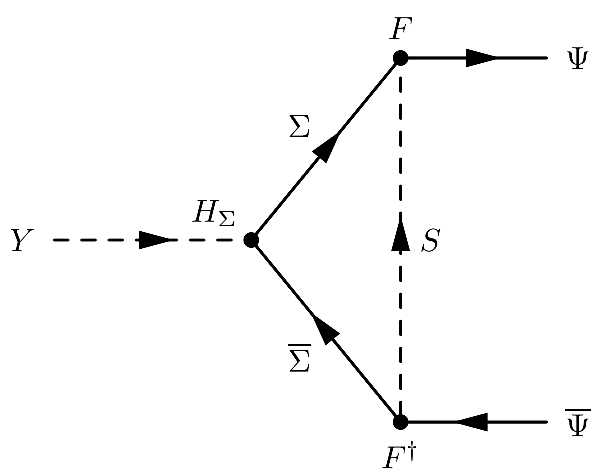
M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

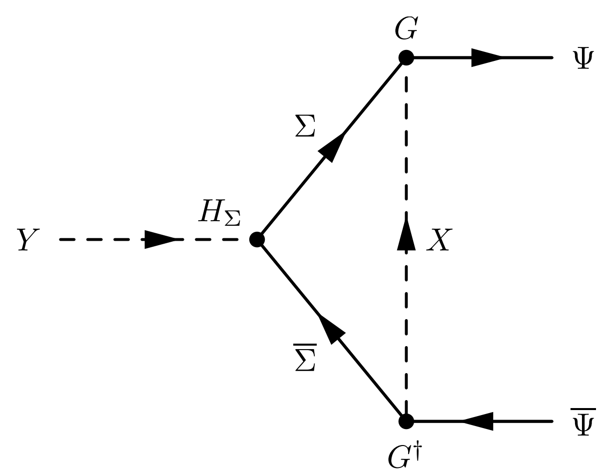
- Particle decay $Y \longrightarrow \overline{\Psi}\Psi$

interference of



with





Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Decay asymmetry

$$\varepsilon_{Y\to\overline{\Psi}\Psi} = \frac{\Gamma(Y\to\overline{\Psi}\Psi) - \Gamma(Y^*\to\overline{\Psi}\Psi)}{\Gamma(Y\to\overline{\Psi}\Psi) + \Gamma(Y^*\to\overline{\Psi}\Psi)}$$

$$\propto \ \mathrm{Im}\,[I_S]\ \mathrm{Im}\left[\mathrm{tr}\left(F^\dagger\, H_\Psi\, F\, H_\Sigma^\dagger\right)\right] + \mathrm{Im}\,[I_X]\ \mathrm{Im}\left[\mathrm{tr}\left(G^\dagger\, H_\Psi\, G\, H_\Sigma^\dagger\right)\right]$$

$$= \ |f|^2\ \mathrm{Im}\,[I_S]\ \mathrm{Im}\,[h_\Psi\, h_\Sigma^*] + |g|^2\ \mathrm{Im}\,[I_X]\ \mathrm{Im}\,[\omega\, h_\Psi\, h_\Sigma^*]\ .$$

one-loop integral $I_S = I(M_S, M_Y)$

one-loop integral $I_X = I(M_X, M_Y)$

- properties of ε
  - invariant under rephasing of fields
  - independent of phases of f and g
  - basis independent

Decay Asymmetry

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- Decay asymmetry

$$\varepsilon_{Y\to\overline{\Psi}\Psi} = |f|^2 \ \mathrm{Im}\left[I_S\right] \ \mathrm{Im}\left[h_\Psi h_\Sigma^*\right] + |g|^2 \ \mathrm{Im}\left[I_X\right] \ \mathrm{Im}\left[\omega h_\Psi h_\Sigma^*\right]$$

  - cancellation requires delicate adjustment of relative phase $\varphi := \arg(h_\Psi h_\Sigma^*)$
  - for non-degenerate M_S and M_X: $\mathrm{Im}\left[I_S\right] \neq \mathrm{Im}\left[I_X\right]$
    - phase φ unstable under quantum corrections
  - for $\mathrm{Im}\left[I_S\right] = \mathrm{Im}\left[I_X\right]$ & $|f| = |g|$
    - phase φ stable under quantum corrections
    - relations cannot be ensured by outer automorphism of Δ(27)
    - require symmetry larger than Δ(27)

model based on Δ(27) violates CP!

Spontaneous CP Violation with Calculable CP Phase

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

field	X	Y	Z	Ψ	Σ	ϕ
$\Delta(27)$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{1}_8$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_0$
U(1)	$2q_\Psi$	0	$2q_\Psi$	q_Ψ	$-q_\Psi$	0

$$\Delta(27) \subset \mathrm{SG}(54,5): \begin{cases} (X,Z) & : \text{ doublet} \\ (\Psi, \Sigma^C) & : \text{ hexaplet} \\ \phi & : \text{ non–trivial 1–dim. representation} \end{cases}$$

- non–trivial $\langle \phi \rangle$ breaks $\mathrm{SG}(54,5) \to \Delta(27)$
- allowed coupling leads to mass splitting $\mathscr{L}^{\phi}_{\text{toy}} \supset M^2 \left(|X|^2 + |Z|^2\right) + \left[\frac{\mu}{\sqrt{2}} \langle \phi \rangle \left(|X|^2 \underbrace{-}_{\text{CG coefficient of SG(54, 5)}} |Z|^2\right) + \text{h.c.}\right]$
- CP asymmetry with calculable phases

$$\varepsilon_{Y \to \overline{\Psi}\Psi} \propto |g|^2 \, |h_\Psi|^2 \, \mathrm{Im}\left[\underbrace{\omega}_{\text{phase predicted by group theory}}\right] \left(\mathrm{Im}\left[I_X\right] - \mathrm{Im}\left[I_Z\right]\right)$$

Group theoretical origin of CP violation!

M.-C.C., K.T. Mahanthappa (2009)

Summary

Summary

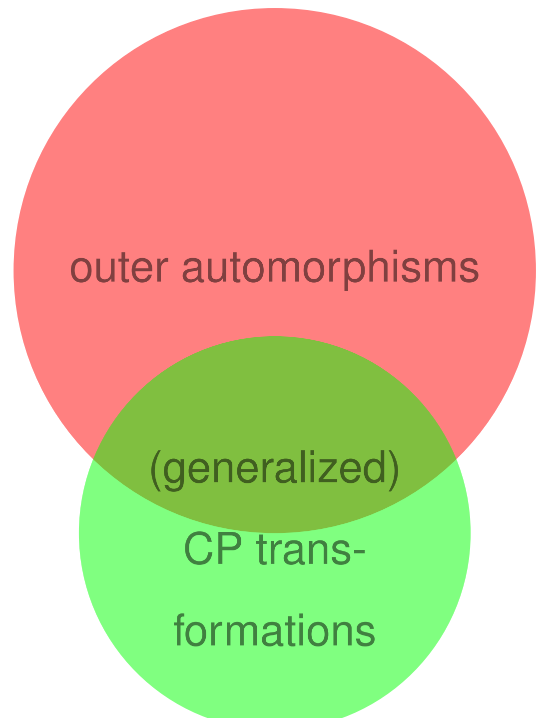
- **NOT all outer automorphisms correspond to physical CP transformations**

- **Condition on automorphism for *physical* CP transformation**

$$\rho_{\boldsymbol{r}_i}\big(u(g)\big) \;=\; U_{\boldsymbol{r}_i}\, \rho_{\boldsymbol{r}_i}(g)^*\, U_{\boldsymbol{r}_i}^\dagger \quad \forall\, g \in G \text{ and } \forall\, i$$

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

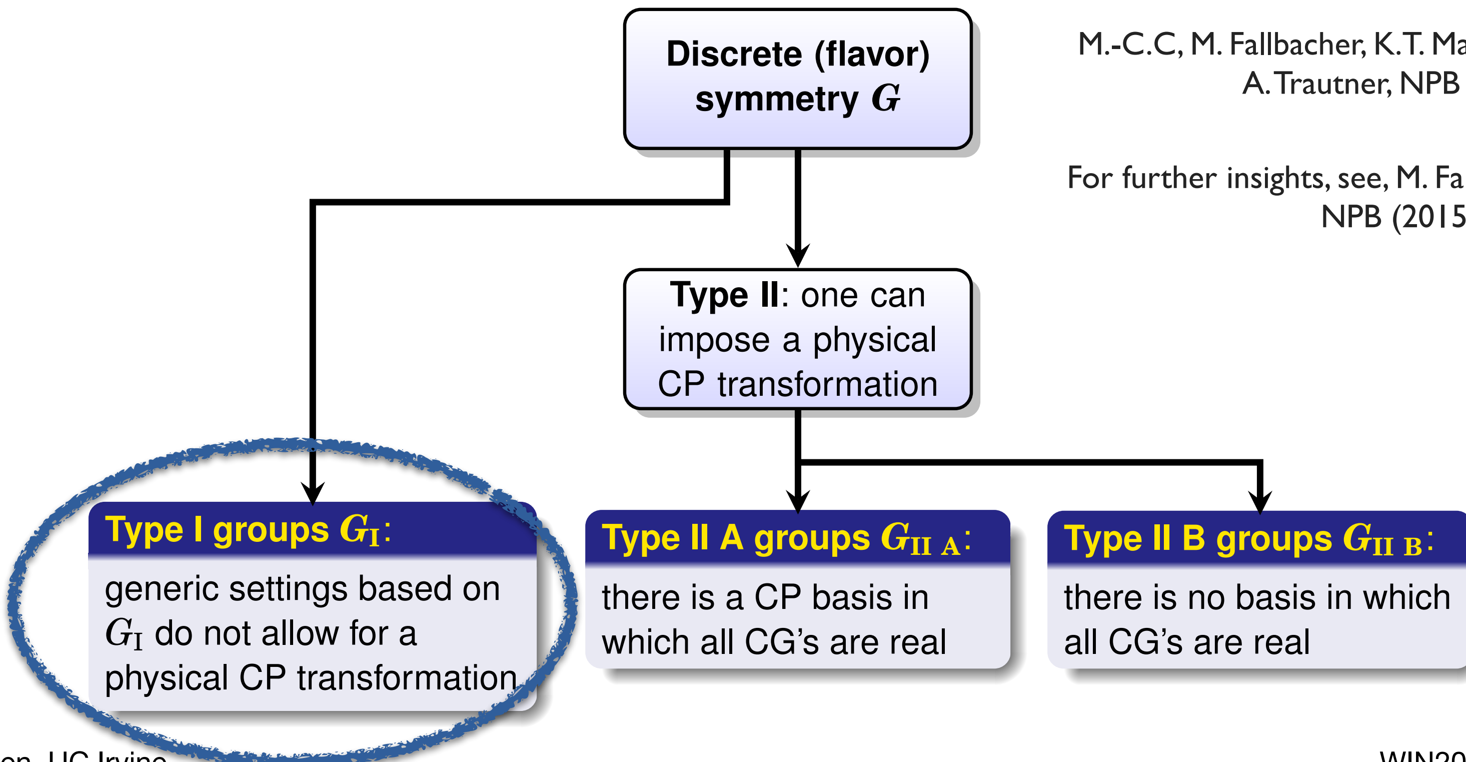
class inverting, involutory automorphisms $\Leftrightarrow$ physical CP transformations



Summary

- **For discrete groups that do not have class-inverting, involutory automorphism, CP is generically broken by complex CG coefficients (Type I Group)**
- **Non-existence of such automorphism ⇔ physical CP violation**

CP Violation from Group Theory!



M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

For further insights, see, M. Fallbacher, A. Trautner, NPB (2015)

Backup Slides

CP Transformation

- Canonical CP transformation

$$\phi(x) \xrightarrow{\mathcal{CP}} \eta_{\mathcal{CP}}\ \phi^*(\mathcal{P}x)$$

freedom of re–phasing fields

- Generalized CP transformation

Ecker, Grimus, Konetschny (1981); Ecker, Grimus, Neufeld (1987); Grimus, Rebelo (1995)

$$\Phi(x) \xrightarrow{\widetilde{\mathcal{CP}}} U_{\mathrm{CP}}\, \Phi^*(\mathcal{P}\ x)$$

unitary matrix

Generalized CP Transformation

Ecker, Grimus, Konetschny (1981); Ecker, Grimus, Neufeld (1987)

- setting w/ discrete symmetry G **G and CP transformations do not commute**
- generalized CP transformation Feruglio, Hagedorn, Ziegler (2013); Holthausen, Lindner, Schmidt (2013)
- invariant contraction/coupling in A_4 or T'

$$\left[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}\right]_{\mathbf{1}_0} \propto \phi \left(x_1 y_1 + \omega^2 x_2 y_2 + \omega x_3 y_3\right)$$

$\omega = \mathrm{e}^{2\pi\,\mathrm{i}/3}$

- canonical CP transformation maps A_4/T' invariant contraction to something non–invariant

➥ need generalized CP transformation $\widetilde{\mathcal{CP}}$: $\phi \xmapsto{\widetilde{\mathcal{CP}}} \phi^*$ as usual but

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \xmapsto{\widetilde{\mathcal{CP}}} \begin{pmatrix} x_1^* \\ x_3^* \\ x_2^* \end{pmatrix} \quad \& \quad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \xmapsto{\widetilde{\mathcal{CP}}} \begin{pmatrix} y_1^* \\ y_3^* \\ y_2^* \end{pmatrix}$$

The Bickerstaff-Damhus automorphism (BDA)

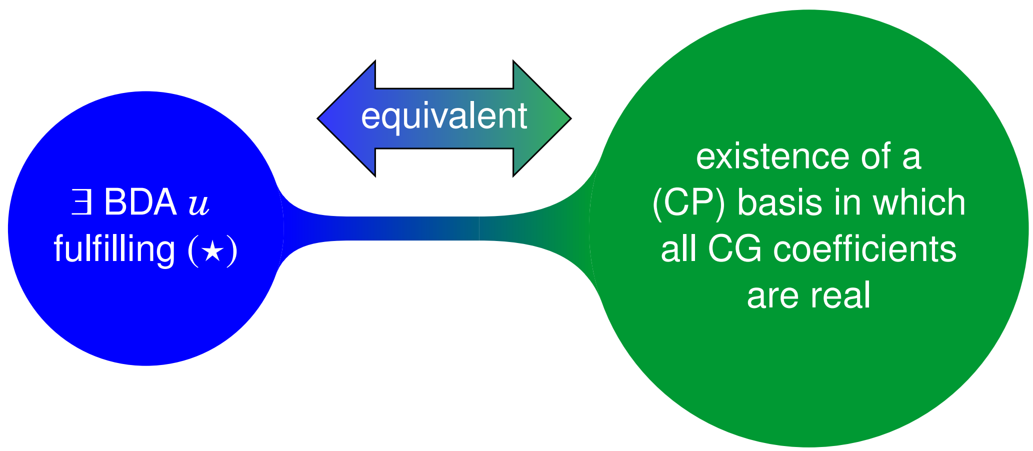
- Bickerstaff-Damhus automorphism (BDA) $\boldsymbol{u}$

Bickerstaff, Damhus (1985)

$$\rho_{\boldsymbol{r}_i}(u(g)) \;=\; U_{\boldsymbol{r}_i}\;\rho_{\boldsymbol{r}_i}(g)^*\,U_{\boldsymbol{r}_i}^{\dagger} \quad \forall\, g \in G \text{ and } \forall\, i \qquad (\,\star\,)$$

unitary & symmetric

- BDA vs. Clebsch-Gordan (CG) coefficients



Twisted Frobenius-Schur Indicator

- How can one tell whether or not a given automorphism is a BDA?
- Frobenius-Schur indicator:

$$\mathrm{FS}(\boldsymbol{r}_i) := \frac{1}{|G|} \sum_{g \in G} \chi_{\boldsymbol{r}_i}(g^2) = \frac{1}{|G|} \sum_{g \in G} \mathrm{tr}\left[\rho_{\boldsymbol{r}_i}(g)^2\right]$$

$$\mathrm{FS}(\boldsymbol{r}_i) = \begin{cases} +1, & \text{if } \boldsymbol{r}_i \text{ is a real representation,} \\ 0, & \text{if } \boldsymbol{r}_i \text{ is a complex representation,} \\ -1, & \text{if } \boldsymbol{r}_i \text{ is a pseudo–real representation.} \end{cases}$$

- Twisted Frobenius-Schur indicator

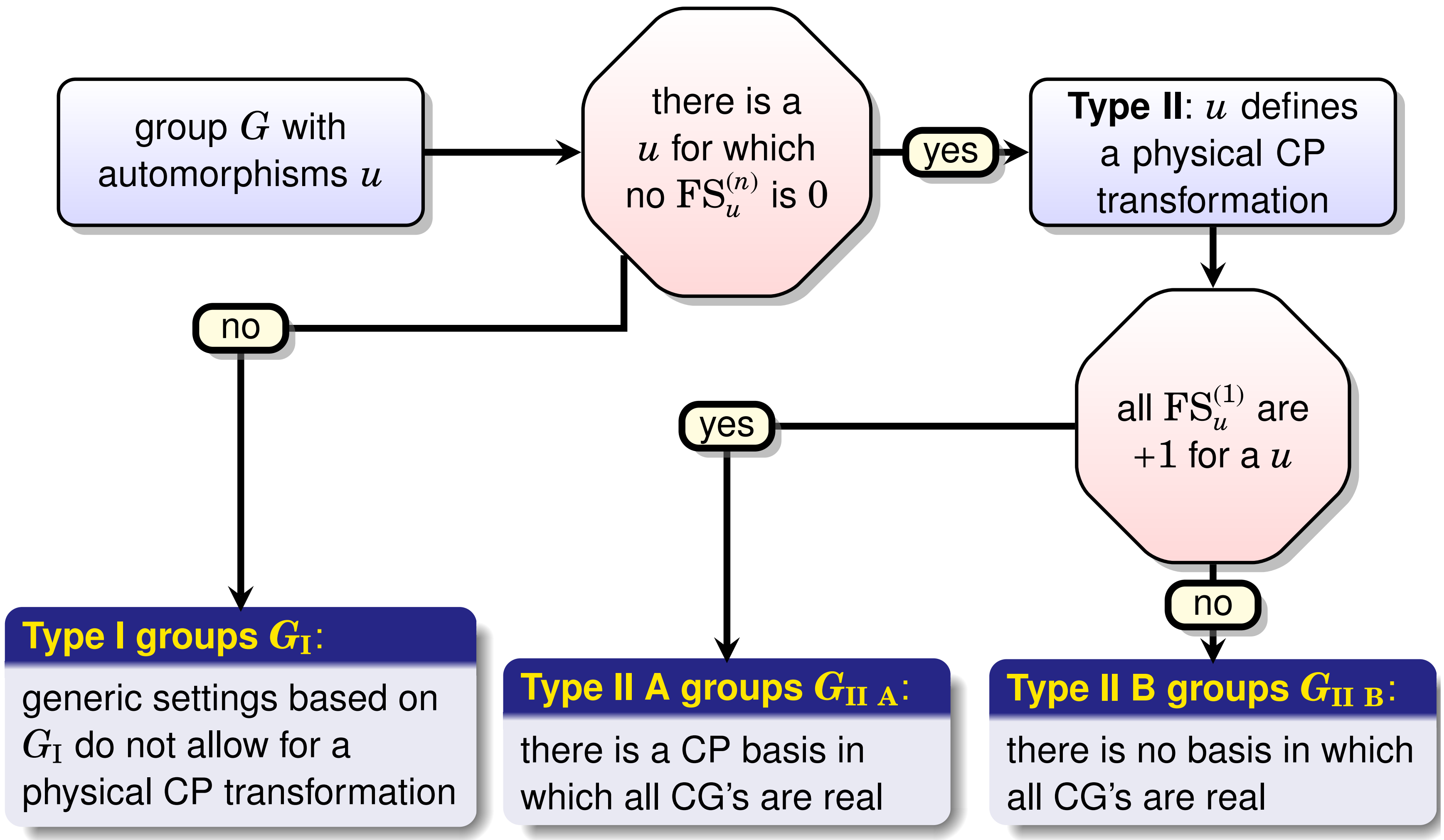
Bickerstaff, Damhus (1985); Kawanaka, Matsuyama (1990)

$$\mathrm{FS}_u(\boldsymbol{r}_i) = \frac{1}{|G|} \sum_{g \in G} \left[\rho_{\boldsymbol{r}_i}(g)\right]_{\alpha\beta} \left[\rho_{\boldsymbol{r}_i}(u(g))\right]_{\beta\alpha}$$

$$\mathrm{FS}_u(\boldsymbol{r}_i) = \begin{cases} +1 \quad \forall\, i, & \text{if } u \text{ is a BDA,} \\ +1 \text{ or } -1 \quad \forall\, i, & \text{if } u \text{ is class–inverting and involutory,} \\ \text{different from } \pm 1, & \text{otherwise.} \end{cases}$$

Three Types of Finite Groups

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)



CP Conservation vs Symmetry Enhancement

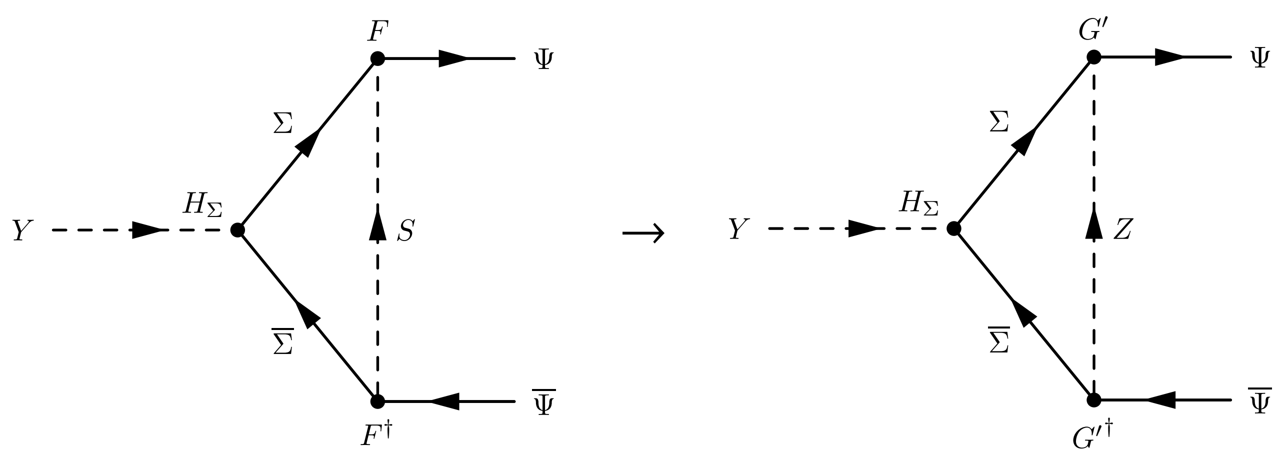
M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

☞ replace $S \sim \mathbf{1}_0$ by $Z \sim \mathbf{1}_8$ $\curvearrowright$ interaction

$$\mathscr{L}^{Z}_{\mathrm{toy}} = g' \left[Z_{\mathbf{1}_8} \otimes \left(\overline{\Psi} \Sigma \right)_{\mathbf{1}_4} \right]_{\mathbf{1}_0} + \text{h.c.} = (G')^{ij} \, Z \, \overline{\Psi}_i \Sigma_j + \text{h.c.}$$

$$G' = g' \begin{pmatrix} 0 & 0 & \omega^2 \\ 1 & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix}$$

and leads to new interference diagram



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☞ replace $S \sim \mathbf{1}_0$ by $Z \sim \mathbf{1}_8 \curvearrowright$ interaction

$$\mathscr{L}_{\text{toy}}^{Z} = g' \left[Z_{\mathbf{1}_8} \otimes \left(\overline{\Psi}\Sigma\right)_{\mathbf{1}_4} \right]_{\mathbf{1}_0} + \text{h.c.} = (G')^{ij}\; Z\, \overline{\Psi}_i \Sigma_j + \text{h.c.}$$

➥ different contribution to decay asymmetry: $\varepsilon^{S}_{Y \to \overline{\Psi}\Psi} \;\to\; \varepsilon^{Z}_{Y \to \overline{\Psi}\Psi}$

☞ total CP asymmetry of the Y decay vanishes if $\begin{cases} \text{(i)} & M_Z = M_X \\ \text{(ii)} & |g| = |g'| \\ \text{(iii)} & \varphi = 0 \end{cases}$

☞ relations (i)—(iii) can be due to an outer automorphism

$$X \overset{u_3}{\longleftrightarrow} Z\,, \quad Y \overset{u_3}{\longrightarrow} Y\,, \quad \Psi \overset{u_3}{\longrightarrow} U_{u_3}\, \Sigma^C \quad \& \quad \Sigma \overset{u_3}{\longrightarrow} U_{u_3}\, \Psi^C$$

requires $q_\Sigma = -q_\Psi$

$$U_{u_3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

. . . BUT this enlarges $\Delta(27) \to \mathrm{SG}(54,5) \simeq \Delta(27) \rtimes \mathbb{Z}_2^{u_3}$

$\mathrm{SG}(54,5)$: group name from GAP library

Summary

Three examples:

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

☞ Type I group: $\Delta(27)$

- generic settings based on $\Delta(27)$ violate CP!
- spontaneous breaking of type II A group $\mathrm{SG}(54,5) \to \Delta(27)$
  $\curvearrowright$ prediction of CP violating phase from group theory!

☞ Type II A group: T'

- CP basis exists but has certain shortcomings
- advantageous to work in a different basis & impose generalized CP transformation
- CP constrains phases of coupling coefficients

☞ Type II B group: $\Sigma(72)$

- absence of CP basis but generalized CP transformation ensures physical CP conservation
- CP forbids couplings