Where does CP violation actually come from?

Mu-Chun Chen, University of California, Irvine
Where can CP violation possibly come from?

Mu-Chun Chen, University of California, Irvine

Based on work in collaboration with Maximillian Fallbacher, K.T. Mahanthappa, Michael Ratz, Andreas Trautner

WIN 2015, Heidelberg, Germany, June 8 - 13, 2015
CP Violation in Nature

- CP violation: required to explain matter-antimatter asymmetry
- So far observed only in flavor sector
  - SM: CKM matrix for the quark sector
    - experimentally established $\delta_{\text{CKM}}$ as major source of CP violation
- Search for new source of CP violation:
  - CP violation in neutrino sector
  - if found $\Rightarrow$ phase in PMNS matrix
- Discrete family symmetries:
  - suggested by large neutrino mixing angles
  - neutrino mixing angles from group theoretical CG coefficients

Discrete (family) symmetries $\Leftrightarrow$ Physical CP violation
Origin of CP Violation

• CP violation ↔ complex mass matrices

\[ \bar{U}_{R,i}(M_u)_{ij}Q_{L,j} + \bar{Q}_{L,j}(M_u^\dagger)_{ji}U_{R,i} \xrightarrow{\text{CP}} \bar{Q}_{L,j}(M_u)_{ij}U_{R,i} + \bar{U}_{R,i}(M_u)^*_{ij}Q_{L,j} \]

• Conventionally, CPV arises in two ways:
  • Explicit CP violation: complex Yukawa coupling constants \( Y \)
  • Spontaneous CP violation: complex scalar VEVs \( <h> \)
A Novel Origin of CP Violation

- Complex CG coefficients in certain discrete groups ⇒ explicit CP violation
  - Real Yukawa couplings, real scalar VEVs
  - CPV in quark and lepton sectors purely from complex CG coefficients
  - No additional parameters needed ⇒ extremely predictive model!

Basic idea

Discrete symmetry $G$

Scalar potential: if $Z_3$ symmetric ⇒ $\langle \Delta_1 \rangle = \langle \Delta_2 \rangle = \langle \Delta_3 \rangle \equiv \langle \Delta \rangle$ real

Complex effective mass matrix: \textbf{phases determined by group theory}

\[
M = \begin{pmatrix}
C_1 & C_3 \\
C_2 & C_4
\end{pmatrix}
\begin{pmatrix}
Y \\ \langle \Delta \rangle
\end{pmatrix}
\begin{pmatrix}
R_1 \\
R_2
\end{pmatrix}
\]

Mu-Chun Chen, UC Irvine
A Novel Origin of CP Violation

- Conventionally:
  - Explicit CP violation: complex Yukawa couplings
  - Spontaneous CP violation: complex Higgs VEVs
- Complex CG coefficients in discrete groups ⇒ explicit CP violation in quark and lepton sectors (e.g. $\delta \neq 0$)

- Conditions for a discrete group to admit real CG’s
  - Existence of a (CP) basis in which all CG coefficients are real
  - $\exists$ automorphism $u$, such that $\lambda_k(R) = \lambda_k(u(R))^*$ for all $R \in G$

---

  - Bickerstaff, Damhus, 1985

---

Mu-Chun Chen, UC Irvine
A Novel Origin of CP Violation


- More generally, for discrete groups that do not have class-inverting, involutory automorphism, CP is generically broken by complex CG coefficients (Type I Group)
- Non-existence of such automorphism $\leftrightarrow$ Physical CP violation

CP Violation from Group Theory!

Discrete (flavor) symmetry $G$

Type II: one can impose a physical CP transformation

Type I groups $G_I$: generic settings based on $G_I$ do not allow for a physical CP transformation

Type II A groups $G_{IIA}$: there is a CP basis in which all CG's are real

Type II B groups $G_{IIB}$: there is no basis in which all CG's are real

For further insights, see, M. Fallbacher, A. Trautner, NPB (2015)
Physical CP Transformation

• **NOT all outer automorphisms correspond to physical CP transformations**

• **Condition on automorphism for physical CP transformation**

\[
\rho_{r_i}(u(g)) = U_{r_i} \rho_{r_i}(g)^* U_{r_i}^\dagger \quad \forall \ g \in G \text{ and } \forall \ i
\]

Examples

- **Type I**: all odd order non-Abelian groups

<table>
<thead>
<tr>
<th>group</th>
<th>$\mathbb{Z}_5 \times \mathbb{Z}_4$</th>
<th>$T_7$</th>
<th>$\Delta(27)$</th>
<th>$\mathbb{Z}_9 \times \mathbb{Z}_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG</td>
<td>(20,3)</td>
<td>(21,1)</td>
<td>(27,3)</td>
<td>(27,4)</td>
</tr>
</tbody>
</table>

- **Type IIA**: dihedral and all Abelian groups

<table>
<thead>
<tr>
<th>group</th>
<th>$S_3$</th>
<th>$Q_8$</th>
<th>$A_4$</th>
<th>$\mathbb{Z}_3 \times \mathbb{Z}_8$</th>
<th>$T'$</th>
<th>$S_4$</th>
<th>$A_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG</td>
<td>(6,1)</td>
<td>(8,4)</td>
<td>(12,3)</td>
<td>(24,1)</td>
<td>(24,3)</td>
<td>(24,12)</td>
<td>(60,5)</td>
</tr>
</tbody>
</table>

- **Type IIB**

<table>
<thead>
<tr>
<th>group</th>
<th>$\Sigma(72)$</th>
<th>$((\mathbb{Z}_3 \times \mathbb{Z}_3) \times \mathbb{Z}_4) \times \mathbb{Z}_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG</td>
<td>(72,41)</td>
<td>(144,120)</td>
</tr>
</tbody>
</table>
Example for a type I group:

\[ \Delta(27) \]

- decay asymmetry in a toy model
- prediction of CP violating phase from group theory
Toy Model based on $\Delta(27)$

- **Field content**

<table>
<thead>
<tr>
<th>field</th>
<th>$S$</th>
<th>$X$</th>
<th>$Y$</th>
<th>$\Psi$</th>
<th>$\Sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta(27)$</td>
<td>$1_0$</td>
<td>$1_1$</td>
<td>$1_3$</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$U(1)$</td>
<td>$q_\Psi - q_\Sigma$</td>
<td>$q_\Psi - q_\Sigma$</td>
<td>0</td>
<td>$q_\Psi$</td>
<td>$q_\Sigma$</td>
</tr>
</tbody>
</table>

- **Interactions**

  \[ \mathcal{L}_{\text{toy}} = F^{ij} S \overline{\Psi}_i \Sigma_j + G^{ij} X \overline{\Psi}_i \Sigma_j + H^{ij}_\Psi Y \overline{\Psi}_i \Psi_j + H^{ij}_\Sigma Y \overline{\Sigma}_i \Sigma_j + \text{h.c.} \]

  \[ F = f \mathbf{1}_3 \quad G = g \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \]

  \[ H_{\Psi/\Sigma} = h_{\Psi/\Sigma} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \]

with $\omega := e^{2\pi i/3}$

- "flavor" structures determined by (complex) CG coefficients
- arbitrary coupling constants: $f, g, h_\Psi, h_\Sigma$
Toy Model based on $\Delta(27)$

- Particle decay $Y \rightarrow \bar{\Psi}\Psi$

interference of

with

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)
Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

• Decay asymmetry

\[ \varepsilon_{Y \to \overline{\Psi}\Psi} = \frac{\Gamma(Y \to \overline{\Psi}\Psi) - \Gamma(Y^* \to \overline{\Psi}\Psi)}{\Gamma(Y \to \overline{\Psi}\Psi) + \Gamma(Y^* \to \overline{\Psi}\Psi)} \]

\[ \propto \text{Im} [I_S] \text{Im} \left[ \text{tr} \left( F^\dagger H_{\Psi} F H_{\Sigma}^\dagger \right) \right] + \text{Im} [I_X] \text{Im} \left[ \text{tr} \left( G^\dagger H_{\Psi} G H_{\Sigma}^\dagger \right) \right] \]

\[ = |f|^2 \text{Im} [I_S] \text{Im} [h_{\Psi} h_{\Sigma}^*] + |g|^2 \text{Im} [I_X] \text{Im} [\omega h_{\Psi} h_{\Sigma}^*] . \]

one-loop integral \( I_S = I(M_S, M_Y) \)

one-loop integral \( I_X = I(M_X, M_Y) \)

• properties of \( \varepsilon \)
  • invariant under rephasing of fields
  • independent of phases of \( f \) and \( g \)
  • basis independent
Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Decay asymmetry
  \[ \varepsilon_{Y \rightarrow \overline{\Psi}} = |f|^2 \text{Im} [I_S] \text{Im} [h_{\Psi} h_{\Sigma}^*] + |g|^2 \text{Im} [I_X] \text{Im} [\omega h_{\Psi} h_{\Sigma}^*] \]

- Cancellation requires delicate adjustment of relative phase \( \varphi := \arg(h_{\Psi} h_{\Sigma}^*) \)

- For non-degenerate \( M_S \) and \( M_X \):
  \[ \text{Im} [I_S] \neq \text{Im} [I_X] \]
  - Phase \( \varphi \) unstable under quantum corrections

- For \( \text{Im} [I_S] = \text{Im} [I_X] \) and \( |f| = |g| \)
  - Phase \( \varphi \) stable under quantum corrections
  - Relations cannot be ensured by outer automorphism of \( \Delta(27) \)
  - Require symmetry larger than \( \Delta(27) \)

Model based on \( \Delta(27) \) violates CP!
Spontaneous CP Violation with Calculable CP Phase

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

<table>
<thead>
<tr>
<th>field</th>
<th>$X$</th>
<th>$Y$</th>
<th>$Z$</th>
<th>$\Psi$</th>
<th>$\Sigma$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta(27)$</td>
<td>$1_1$</td>
<td>$1_3$</td>
<td>$1_8$</td>
<td>$3$</td>
<td>$3$</td>
<td>$1_0$</td>
</tr>
<tr>
<td>$U(1)$</td>
<td>$2q_\Psi$</td>
<td>$0$</td>
<td>$2q_\Psi$</td>
<td>$q_\Psi$</td>
<td>$-q_\Psi$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

$\Delta(27) \subset \text{SG}(54, 5)$:

\[
\begin{align*}
(X, Z) & : \text{doublet} \\
(\Psi, \Sigma^C) & : \text{hexaplet} \\
\phi & : \text{non–trivial 1–dim. representation}
\end{align*}
\]

- non–trivial $\langle \phi \rangle$ breaks $\text{SG}(54, 5) \rightarrow \Delta(27)$

- allowed coupling leads to mass splitting

\[
\mathcal{L}^\phi_{\text{toy}} \supset M^2 (|X|^2 + |Z|^2) + \left[ \frac{\mu}{\sqrt{2}} \langle \phi \rangle (|X|^2 - |Z|^2) + \text{h.c.} \right]
\]

- CP asymmetry with calculable phases

\[
\varepsilon_{Y \rightarrow \bar{Y} \Psi} \propto |g|^2 |h_\Psi|^2 \text{Im} \left[ \omega \right] (\text{Im} [I_X] - \text{Im} [I_Z])
\]

Group theoretical origin of CP violation!

M.-C.C., K.T. Mahanthappa (2009)
Summary
Summary

• NOT all outer automorphisms correspond to physical CP transformations

• Condition on automorphism for physical CP transformation

\[ \rho_{r_i}(u(g)) = U_{r_i} \rho_{r_i}(g)^* U_{r_i}^\dagger \quad \forall \ g \in G \text{ and } \forall \ i \]


Mu-Chun Chen, UC Irvine
Summary

- For discrete groups that do not have class-inverting, involutory automorphism, CP is generically broken by complex CG coefficients (Type I Group)
- Non-existence of such automorphism $\Leftrightarrow$ physical CP violation

**CP Violation from Group Theory!**

- **Type I groups $G_I$:**
  - generic settings based on $G_I$ do not allow for a physical CP transformation

- **Type II: one can impose a physical CP transformation**

- **Type II A groups $G_{II A}$:**
  - there is a CP basis in which all CG's are real

- **Type II B groups $G_{II B}$:**
  - there is no basis in which all CG's are real


For further insights, see, M. Fallbacher, A. Trautner, NPB (2015)
Backup Slides
CP Transformation

- Canonical CP transformation

\[ \phi(x) \xrightarrow{CP} \eta_{CP} \phi^*(Px) \]

freedom of re-phasing fields

- Generalized CP transformation

\[ \Phi(x) \xrightarrow{\tilde{CP}} U_{CP} \Phi^*(P x) \]

unitary matrix

Generalized CP Transformation

- setting w/ discrete symmetry $G$
- generalized CP transformation
- invariant contraction/coupling in $A_4$ or $T'$

$$
\left[ \phi_{12} \otimes (x_3 \otimes y_3) \right]_{10} \propto \phi \left( x_1 y_1 + \omega^2 x_2 y_2 + \omega x_3 y_3 \right)
$$

- canonical CP transformation maps $A_4/T'$ invariant contraction to something non–invariant

- need generalized CP transformation $\tilde{CP}$: $\phi \overset{\tilde{CP}}{\rightarrow} \phi^*$ as usual but

\[
\begin{pmatrix}
x_1 \\
x_2 \\
x_3
\end{pmatrix}
\overset{\tilde{CP}}{\leftrightarrow}
\begin{pmatrix}
x_1^* \\
x_2^* \\
x_3^*
\end{pmatrix}
\quad \& \quad 
\begin{pmatrix}
y_1 \\
y_2 \\
y_3
\end{pmatrix}
\overset{\tilde{CP}}{\leftrightarrow}
\begin{pmatrix}
y_1^* \\
y_2^* \\
y_3^*
\end{pmatrix}
\]
The Bickerstaff-Damhus automorphism (BDA)

- Bickerstaff-Damhus automorphism (BDA) \( u \)

\[
\rho_{r_i}(u(g)) = U_{r_i} \rho_{r_i}(g)^* U_{r_i}^\dagger \quad \forall \, g \in G \text{ and } \forall \, i \quad (\star)
\]

- BDA vs. Clebsch-Gordan (CG) coefficients

existence of a (CP) basis in which all CG coefficients are real

\exists \text{ BDA } u \text{ fulfilling } (\star)

Mu-Chun Chen, UC Irvine
Twisted Frobenius-Schur Indicator

- How can one tell whether or not a given automorphism is a BDA?
- Frobenius-Schur indicator:
  \[ \text{FS}(r_i) := \frac{1}{|G|} \sum_{g \in G} \chi_{r_i}(g^2) = \frac{1}{|G|} \sum_{g \in G} \text{tr} \left[ \rho_{r_i}(g)^2 \right] \]
  \[ \text{FS}(r_i) = \begin{cases} 
  +1, & \text{if } r_i \text{ is a real representation,} \\
  0, & \text{if } r_i \text{ is a complex representation,} \\
  -1, & \text{if } r_i \text{ is a pseudo-real representation.} 
\end{cases} \]

- Twisted Frobenius-Schur indicator
  \[ \text{FS}_u(r_i) = \frac{1}{|G|} \sum_{g \in G} \left[ \rho_{r_i}(g) \right]_{\alpha\beta} \left[ \rho_{r_i}(u(g)) \right]_{\beta\alpha} \]
  \[ \text{FS}_u(r_i) = \begin{cases} 
  +1 & \forall i, \text{ if } u \text{ is a BDA,} \\
  +1 \text{ or } -1 & \forall i, \text{ if } u \text{ is class-inverting and involutory,} \\
  \text{different from } \pm 1, & \text{otherwise.} 
\end{cases} \]
Three Types of Finite Groups

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- **Type I groups** $G_I$: generic settings based on $G_I$ do not allow for a physical CP transformation
- **Type II A groups** $G_{II A}$: there is a CP basis in which all CG’s are real
- **Type II B groups** $G_{II B}$: there is no basis in which all CG’s are real

- Group $G$ with automorphisms $u$
  - there is a $u$ for which no $FS_u^{(n)}$ is 0
    - yes: Type II: $u$ defines a physical CP transformation
      - all $FS_u^{(1)}$ are +1 for a $u$
        - yes
          - Type II A groups $G_{II A}$: there is a CP basis in which all CG’s are real
          - no
            - Type II B groups $G_{II B}$: there is no basis in which all CG’s are real
    - no
replace $S \sim 1_0$ by $Z \sim 1_8 \sim$ interaction

\[ \mathcal{L}_{toy} = g' \left[ Z_{1_8} \otimes (\overline{\Psi} \Sigma)_{1_4} \right]_{1_0} + \text{h.c.} = (G')^{ij} Z \overline{\Psi} \Sigma_j + \text{h.c.} \]

\( G' = g' \begin{pmatrix} 0 & 0 & \omega^2 \\ 1 & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix} \)

and leads to new interference diagram
**CP Conservation vs Symmetry Enhancement**

Mu-Chun Chen, UC Irvine

---

.replace $S \sim 1_0$ by $Z \sim 1_8 \sim$ interaction

$\mathcal{L}_{\text{toy}}^Z = g' \left[ Z_{18} \otimes (\Psi \Sigma)^1_{14} \right]_{10} + \text{h.c.} = (G')^{ij} Z \bar{\Psi}_i \Sigma_j + \text{h.c.}$

Different contribution to decay asymmetry: $\varepsilon^{S}_{Y \rightarrow \bar{\Psi} \Psi} \rightarrow \varepsilon^{Z}_{Y \rightarrow \bar{\Psi} \Psi}$

- total CP asymmetry of the $Y$ decay vanishes if $\begin{cases} (i) & M_Z = M_X \\ (ii) & |g| = |g'| \\ (iii) & \varphi = 0 \end{cases}$

- relations (i)–(iii) can be due to an outer automorphism

$X \leftrightarrow u_3 Z, \quad Y \xrightarrow{u_3} Y, \quad \Psi \xrightarrow{u_3} U_{u_3} \Sigma^C \quad & \quad \Sigma \xrightarrow{u_3} U_{u_3} \Psi^C$

requires $q_\Sigma = -q_\Psi$

... BUT this enlarges $\Delta(27) \rightarrow \text{SG}(54, 5) \simeq \Delta(27) \times \mathbb{Z}_2^{u_3}$

$U_{u_3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$

SG(54, 5): group name from GAP library
Summary

Three examples:

を持っているもの

Type I group: \( \Delta(27) \)

- generic settings based on \( \Delta(27) \) violate CP!
- spontaneous breaking of type II A group \( \mathbb{S}G(54, 5) \rightarrow \Delta(27) \)
  \( \sim \) prediction of CP violating phase from group theory!

Type II A group: \( T' \)

- CP basis exists but has certain shortcomings
- advantageous to work in a different basis & impose generalized CP transformation
- CP constrains phases of coupling coefficients

Type II B group: \( \Sigma(72) \)

- absence of CP basis but generalized CP transformation ensures physical CP conservation
- CP forbids couplings