THE SM VACUUM AND HIGGS INFLATION

Javier Rubio

based on arXiv:1412.3811 F. Bezrukov, J.R., M.Shaposhnikov


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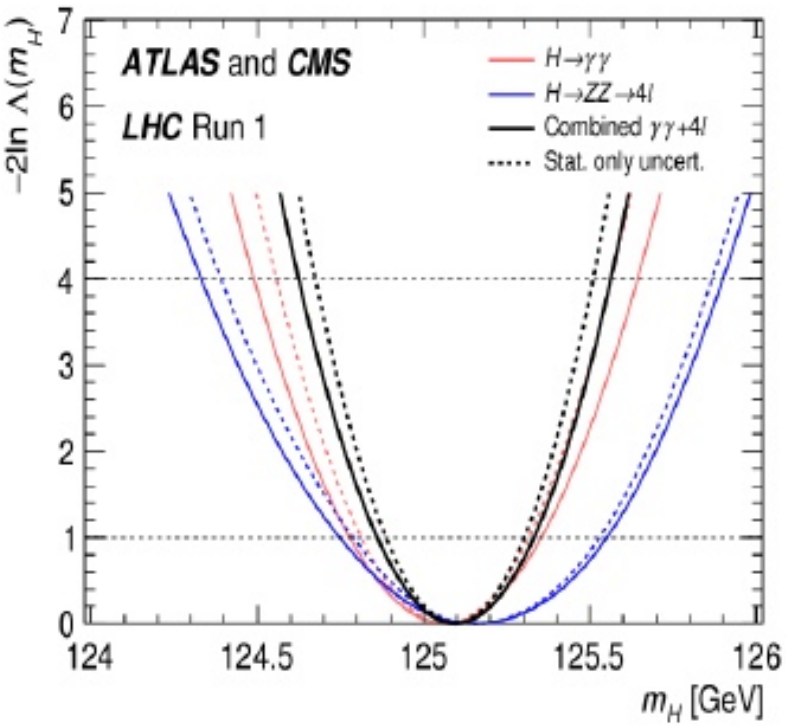
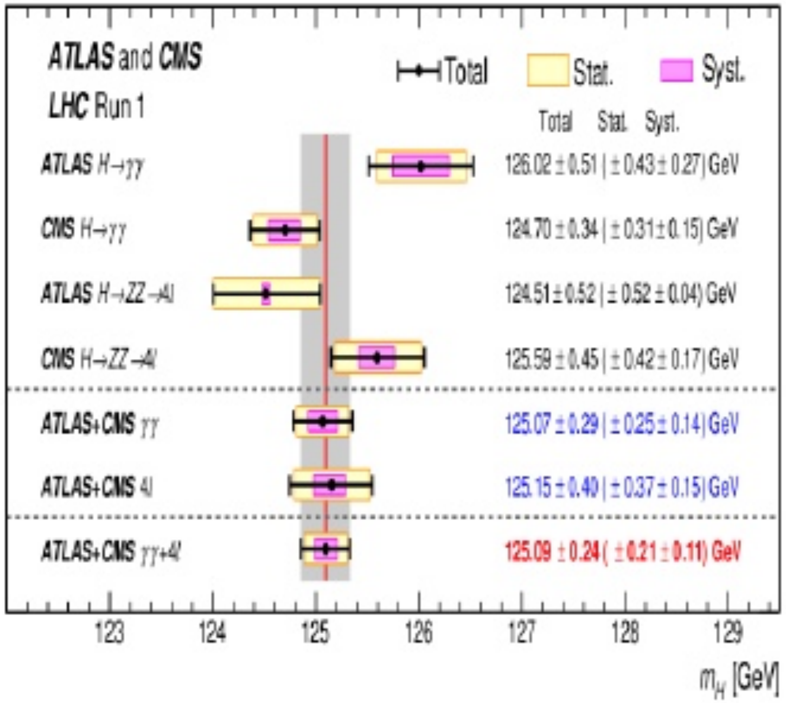
25th International Workshop on Weak Interactions and Neutrinos (WIN2015)

A “fundamental” scalar





ATLAS+CMS, arXiv:1503.07589

$$m_H = 125.09 \pm 0.21(\mathrm{stat})\ \pm 0.11\,(\mathrm{syst})\ \mathrm{GeV}$$

“low” or “high”, depending on your taste...but certainly particular...

Higgs inflation at tree level

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{M_P^2 + \xi_h h^2}{2} R - \frac{1}{2}(\partial h)^2 - \frac{\lambda}{4}(h^2 - v_{EW}^2)^2$$

Scale invariance at $h \gg M_P/\sqrt{\xi_h}$

Moving to the Einstein frame $\tilde{g}_{\mu\nu} = \Omega^2(h) g_{\mu\nu}$

$$\Omega^2 = 1 + \frac{\xi_h h^2}{M_P^2}$$

$$\frac{\tilde{\mathcal{L}}}{\sqrt{-\tilde{g}}} = \frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - U(\phi)$$

All the non-linearities moved to the scalar sector

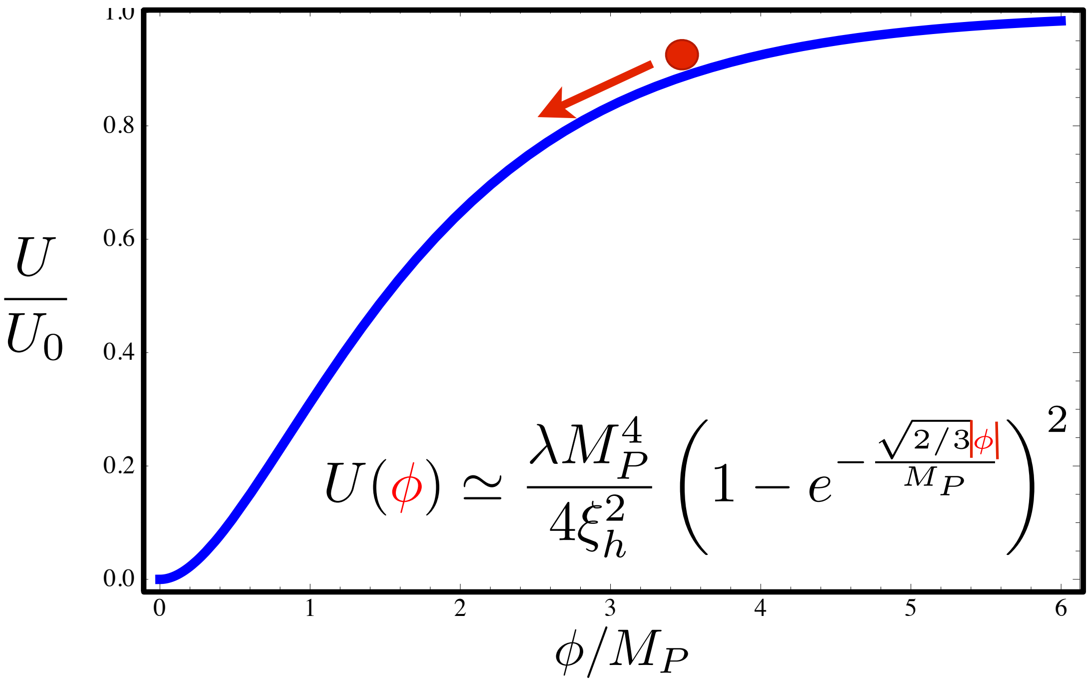
$$U(\phi) = \frac{\lambda}{4}(\phi^2 - v_{EW}^2)^2 \quad \text{for} \quad h < M_P/\xi_h$$

$$U(\phi) = \frac{\lambda M_P^4}{4\xi_h^2}\left(1 - e^{-\frac{\sqrt{2/3}|\phi|}{M_P}}\left(1 + \cancel{\frac{\xi_h v_{EW}^2}{M_P^2}}\right)\right)^2 \quad \text{for} \quad h > M_P/\xi_h$$

F. L. Bezrukov and M. Shaposhnikov Phys. Lett. B659 (2008) 703–706

A sufficiently flat potential

Scale invariance JF --> shift symmetry EF



$$U(\phi) \simeq \frac{\lambda M_P^4}{4\xi_h^2}\left(1 - e^{-\frac{\sqrt{2/3}|\phi|}{M_P}}\right)^2$$

$$\frac{\delta T}{T} \sim 10^{-5}$$

$$\downarrow$$

$$\frac{\lambda}{\xi_h^2} \sim 10^{-11}$$

Only λ/ξ_h^2 is important

The primordial spectra

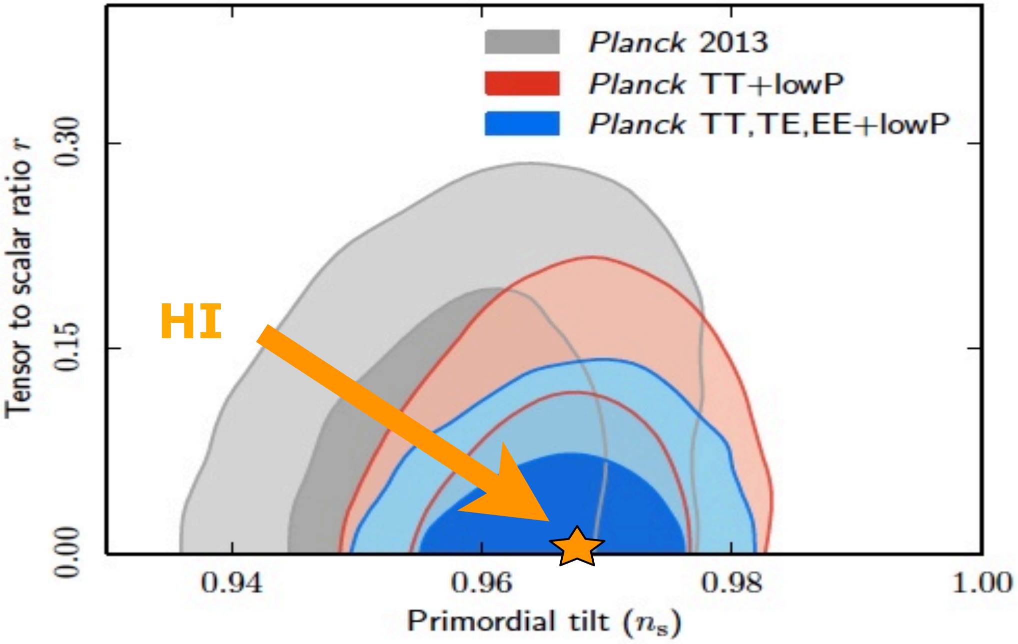
Scalar pert. $$\mathcal{P}_S(k) = \underline{\mathcal{A}_s}\left(\frac{k}{k^*}\right)^{\underline{n_s}-1+\frac{1}{2}\underline{\alpha_s}\ln(k/k^*)+\frac{1}{6}\underline{\beta_s}(\ln(k/k^*))^2+\ldots}$$

Tensor pert. $$\mathcal{P}_T(k) = \underline{\mathcal{A}_t}\left(\frac{k}{k^*}\right)^{n_t+\frac{1}{2}\alpha_t\ln(k/k^*)+\ldots}$$

$$n_s \equiv 1 + \frac{d\ln\mathcal{P}_s}{d\ln k} \qquad r(k) \equiv \frac{\mathcal{P}_T(k)}{\mathcal{P}_s(k)} \qquad \alpha_s \equiv \frac{d\,n_s}{d\ln k}. \qquad \beta_s \equiv \frac{d^2 n_s}{d\ln k^2}$$

$$n_s \simeq 1 - \frac{2}{N} \simeq 0.97$$

$$r \simeq \frac{12}{N^2} \simeq 0.0033$$

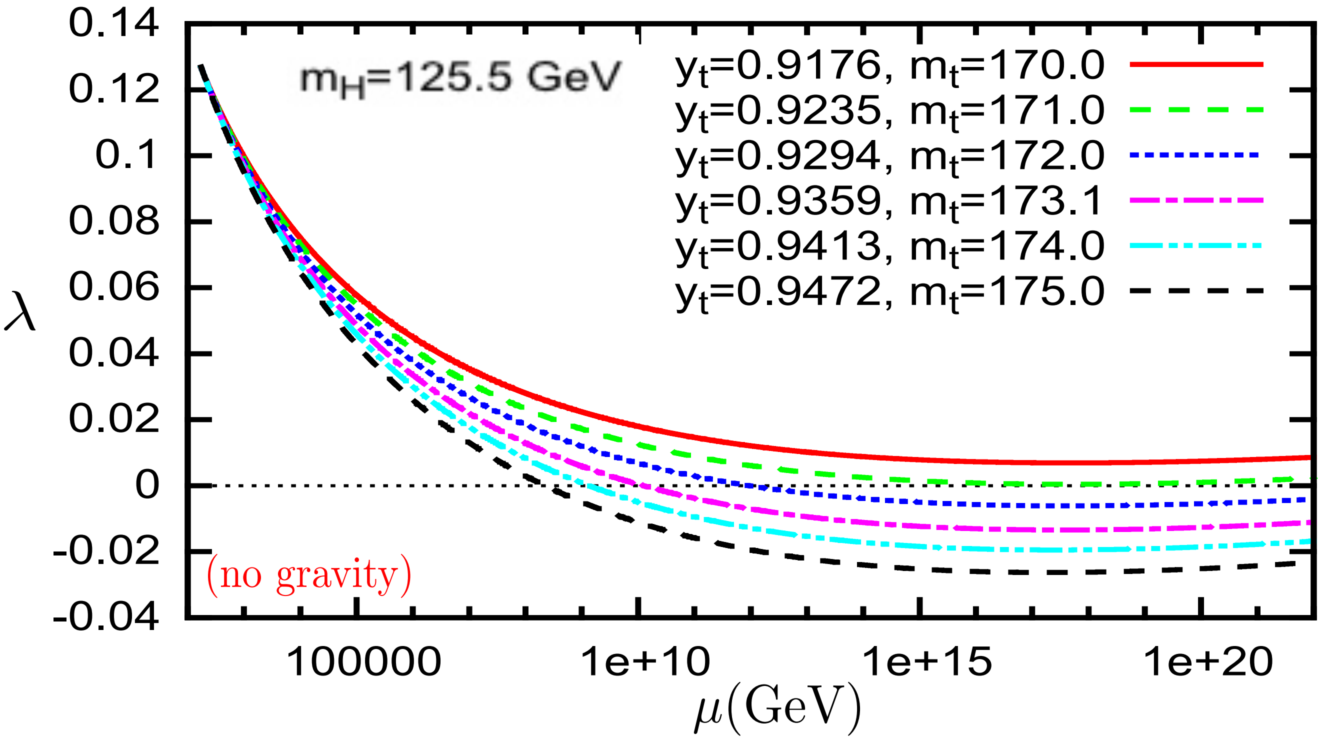


On the edge of stability

- ***SM remains perturbative all the way up till the inflat./Planck scale***
- ***Does the SM vacuum remains stable till those scales?***

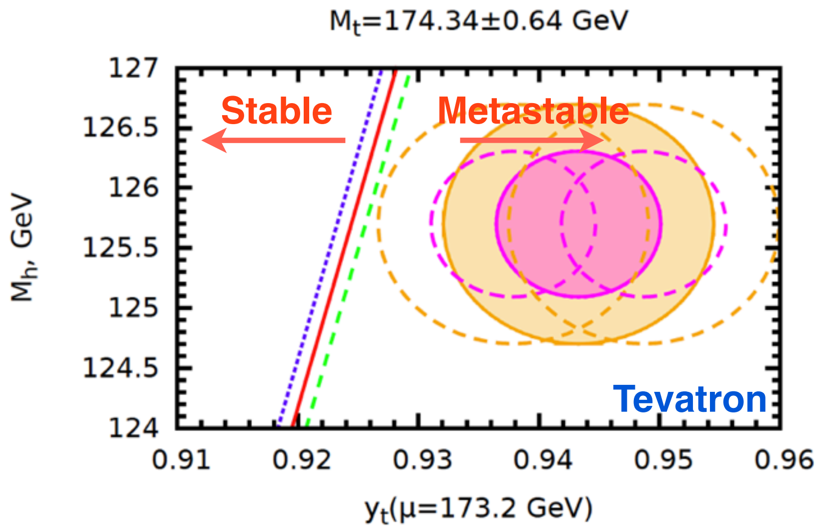
$$\mu\frac{d\lambda(\mu)}{d\log(\mu)} = +\#\lambda^2 + \ldots - \#y_t^4$$

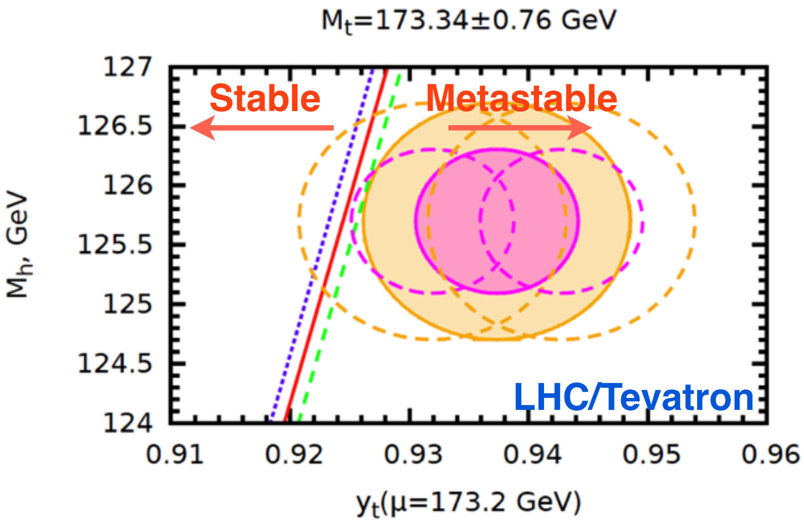
Non trivial interplay between Higgs self-coupling and top quark Yukawa coupling

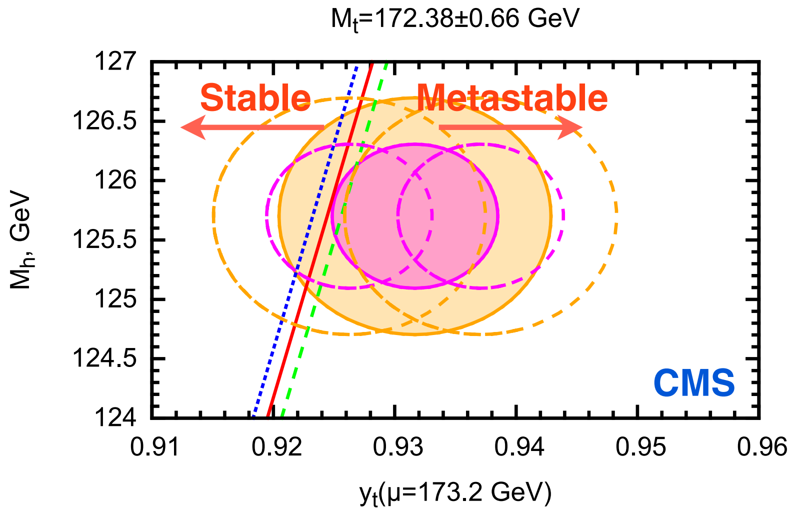


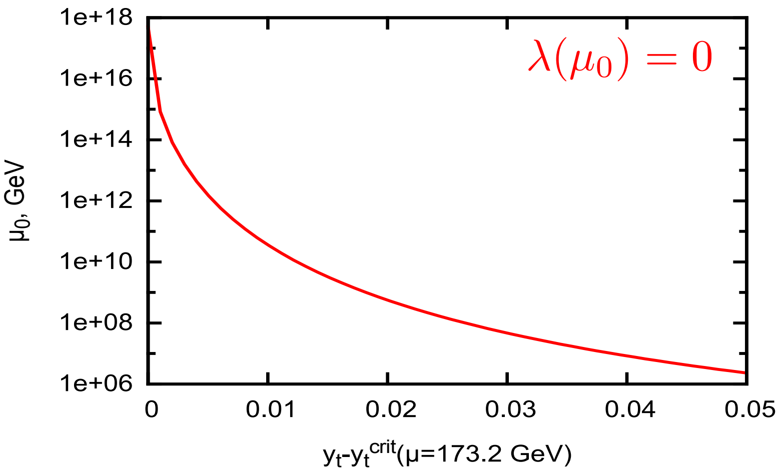
Top quark & vac. instability

$$y_t^{\text{crit}} = 0.9223 + 0.00118\left(\frac{\alpha_s - 0.1184}{0.0007}\right) + 0.00085\left(\frac{M_h - 125.03}{0.3}\right)$$









See F. Bezrukov, M. Shaposhnikov J.Exp.Theor.Phys. 120 (2015) 335-343 and references therein

Imagine that the top and Higgs masses are measured with a precision enough to conclude that our vacuum is not completely stable ...

Should we abandon HI ??

Should/Must new physics appear below or at the scale μ_0 ?

How did we end in the right EW minimum ?

HI is non-renormalizable

Quantum corrections should be introduced by interpreting the theory as an EFT in which a particular set of higher dim. operators are included but.....

Which set of operators?

The most general approach

NO

Add all kind of Planck scale suppressed operators (in the Einstein frame)

This would automatically spoil the flatness of the potential. Indeed, it would generically kill all large-field models of inflation.

The poor man's or self-consistent approach

YES

1. Add only the higher dimensional operators generated by radiative corrections (i.e, those needed to make the theory finite at every order in PT).
2. Require the procedure to maintain the symmetries of the original theory (in particular scale invariance at large field values).

This provides a (partially) controllable link between the low and high energy parameters of the model and selects a particular set of UV completions.

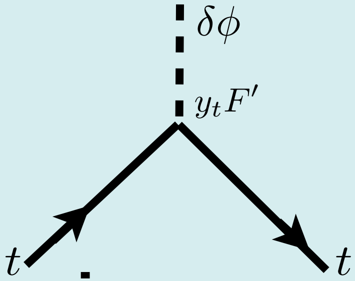
F. Bezrukov, A. Magnin, M. Shaposhnikov, and S. Sibiryakov JHEP 1101 (2011) 016,
See also C.P. Burguess, S.P. Patil, M.Trott JHEP 1406 (2014) 010

Top quark in Higgs background

$$\mathcal{L}_F = \frac{y_f}{\sqrt{2}} h\bar{\psi}\psi \quad \longrightarrow \quad \tilde{\mathcal{L}}_F = \frac{y_f}{\sqrt{2}}\frac{h}{\Omega}\bar{\psi}\psi \equiv \frac{y_f}{\sqrt{2}} F(\phi)\bar{\psi}\psi$$

$$\tilde{\mathcal{L}}_t(\phi + \delta\phi) = \frac{y_t}{\sqrt{2}} F(\phi + \delta\phi)\bar{\psi}_t\psi_t$$

$$= \frac{y_t}{\sqrt{2}} F(\phi)\bar{\psi}_t\psi_t + \frac{y_t}{\sqrt{2}}\frac{dF(\phi)}{d\phi}\delta\phi\bar{\psi}_t\psi_t + \ldots$$



At low energies

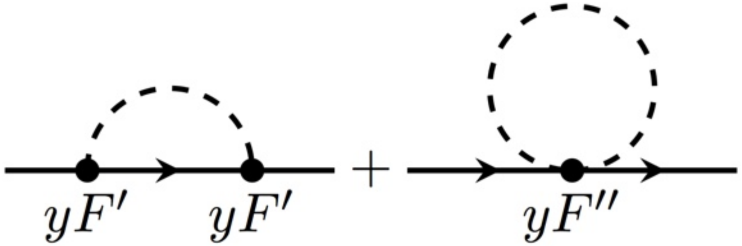
$$F = \phi \quad F'(0) = 1$$

At high energies

$$F = \frac{M_P}{\sqrt{\xi_h}}\left(1 - e^{-\alpha\kappa|\phi|}\right)^{1/2} \quad F'(\infty) = 0$$

Consider the propagation of the top quark

Add counterterms to cancel divergencies

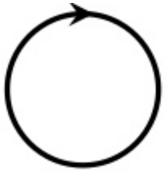


$$\delta\mathcal{L}_{\rm ct} \sim \left(\#\frac{y_t^3}{\bar{\epsilon}} + \delta y_{t1}\right) F'^2 F\bar{\psi}\psi + \left(\#\frac{y_t\lambda}{\bar{\epsilon}} + \delta y_{t2}\right) F''(F^4)''\bar{\psi}\psi;$$

$\phi\psi\psi$ at small ϕ

One-loop effective potential

$$= \frac{1}{2}\,\mathrm{Tr}\ln\left[\Box - \left(\frac{\lambda}{4}(F^4)''\right)^2\right]$$



$$= -\,\mathrm{Tr}\ln\left[i\partial\!\!\!/ + y_t F\right]$$

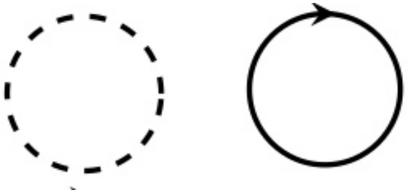
At low energies

$F = \phi \quad F'(0) = 1$

At high energies

$F = const. \quad F'(\phi_0) = 0$

Add counterterms to cancel divergencies



$$\delta\mathcal{L}_{\rm ct} = \left(-\frac{2}{\bar\epsilon}\frac{9\lambda^2}{64\pi^2} + \delta\lambda_1\right)\left(F'^2 + \frac{1}{3}F''F^2\right)^2 F^4 + \left(\frac{2}{\bar\epsilon}\frac{y_t^4}{64\pi^2} - \delta\lambda_2\right)F^4$$

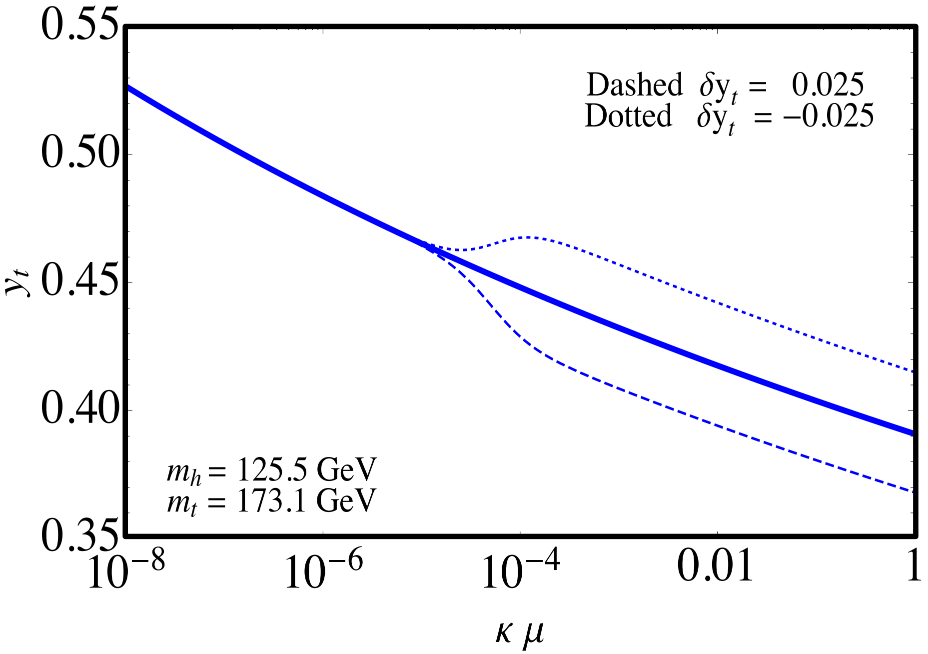
$\downarrow$ new

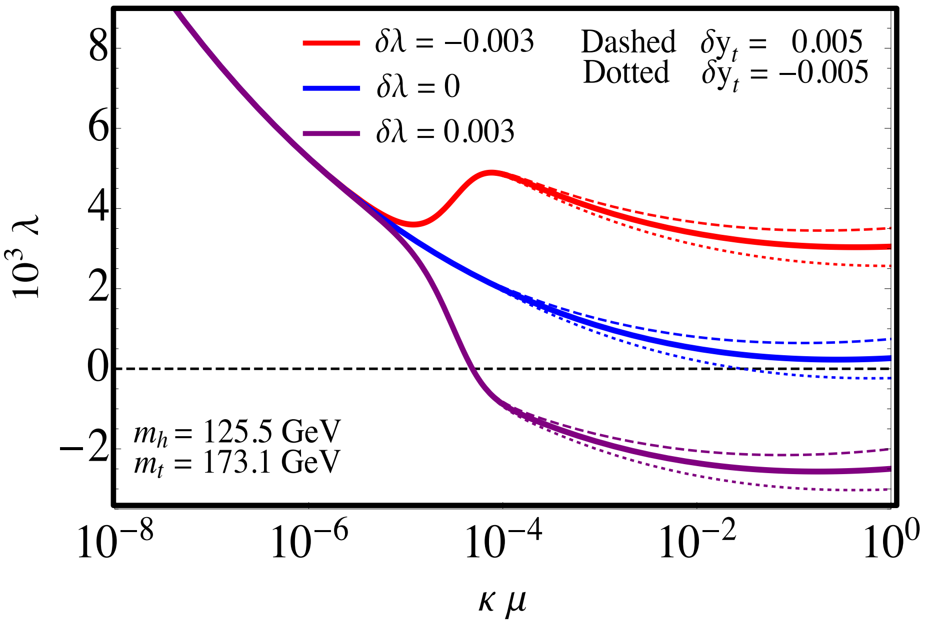
$\downarrow$ $\sim$ tree level

Eff. behaviour of coupling constants

Neglecting the running of $\delta\lambda_1$ and $\delta_{y_{t1}}$
between $\mu \sim M_P/\xi_h$ and $M_P/\sqrt{\xi_h}$

$$y_t(\mu) \longrightarrow y_t(\mu) + \delta y_t \left[F'^2 - 1\right] + \dots \qquad \lambda(\mu) \longrightarrow \lambda(\mu) + \delta\lambda \left[\left(F'^2 + \frac{1}{3}F''F\right)^2 - 1\right]$$





The shape of the asymptotics is maintained

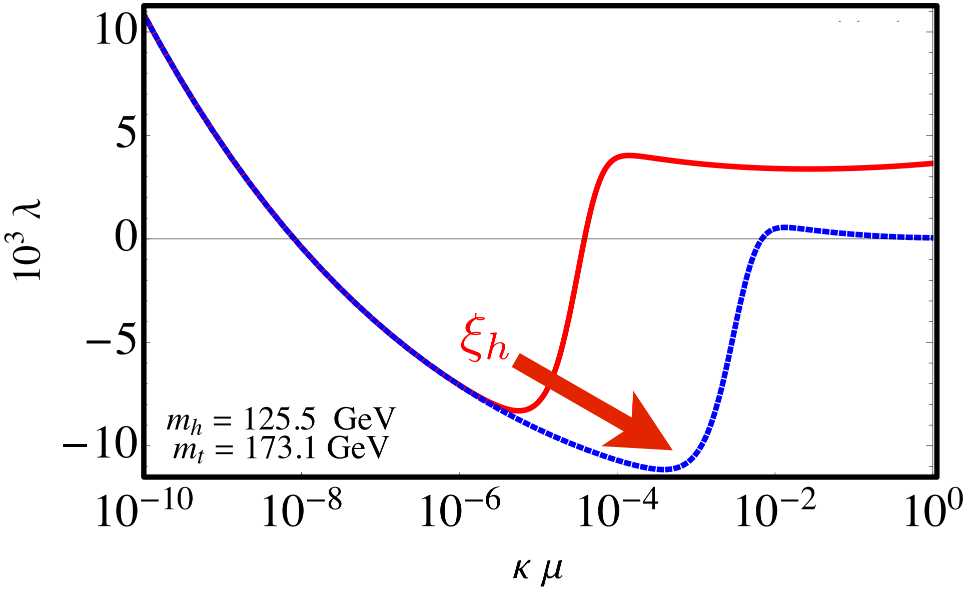
Restoring Higgs inflation

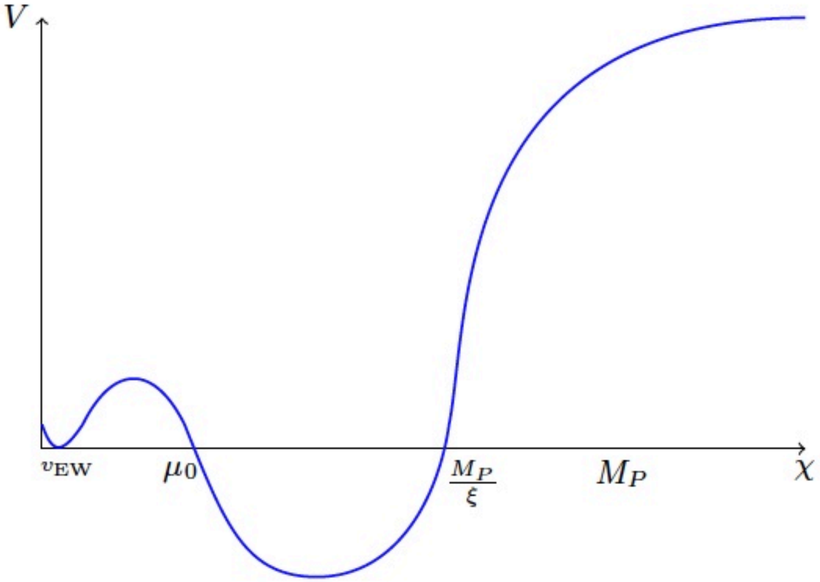
$$\delta\lambda \ll \lambda(M_P/\xi)$$
$$\delta y_t \ll y_t(M_P/\xi)$$

- **Higgs inflation requires absolute stability of the SM vacuum**

$$\xcancel{\delta\lambda \ll \lambda(M_P/\xi)}$$
$$\xcancel{\delta y_t \ll y_t(M_P/\xi)}$$

- **Higgs inflation can be possible even in the case of a metastable vacuum.**

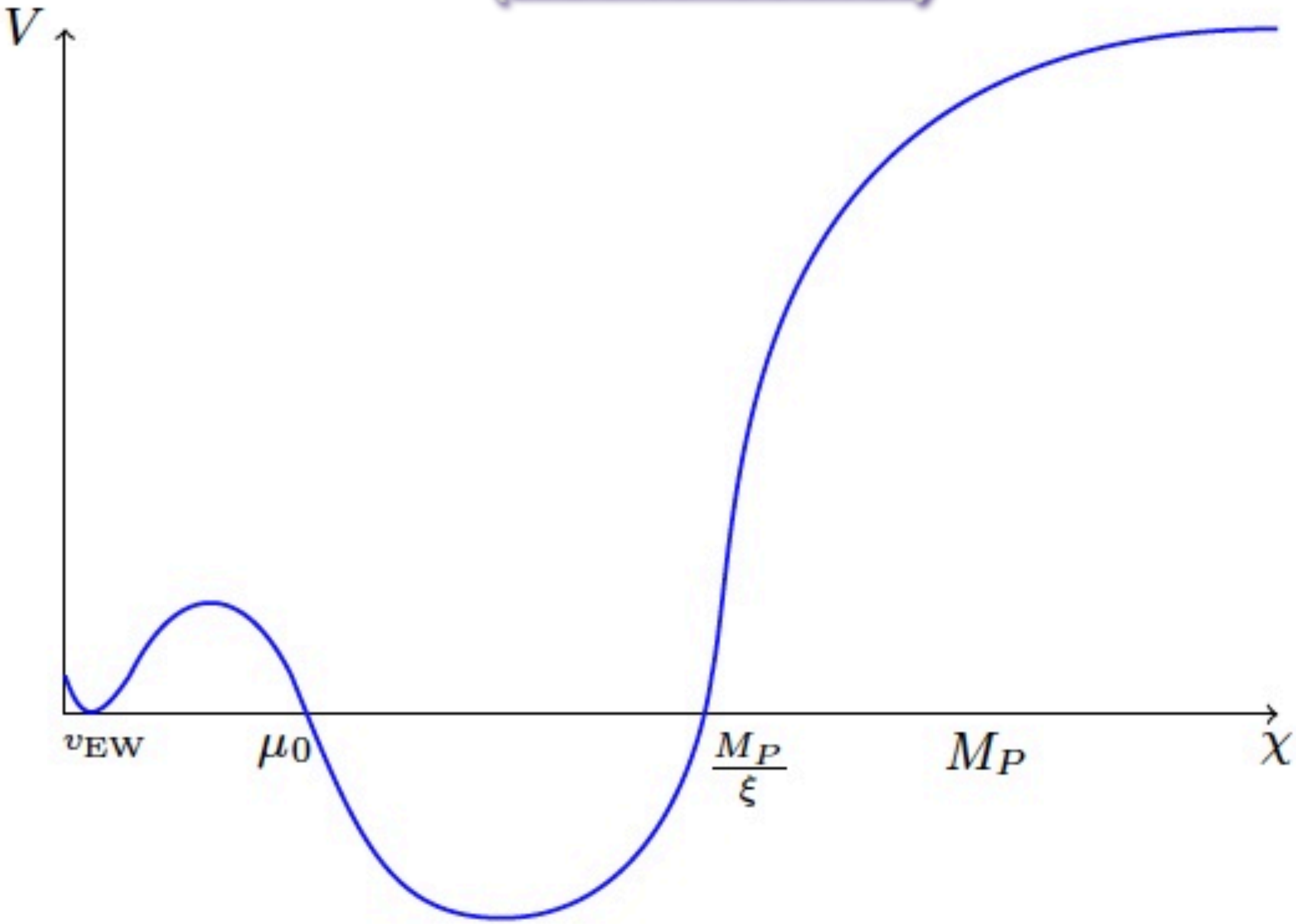




But how to avoid finishing in the wrong vacuum ???

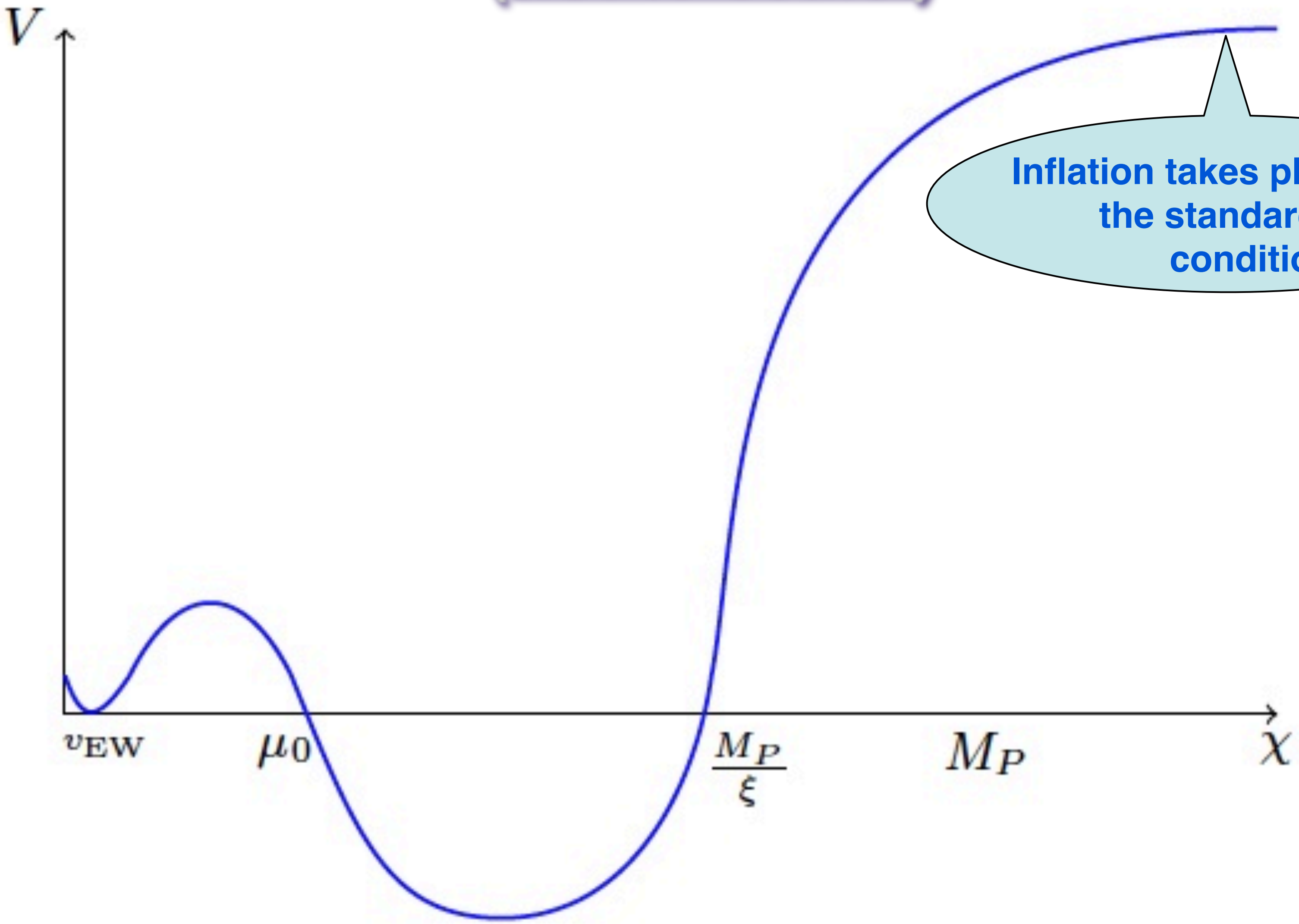
Sketch of effective potential

(not to scale!)



Sketch of effective potential

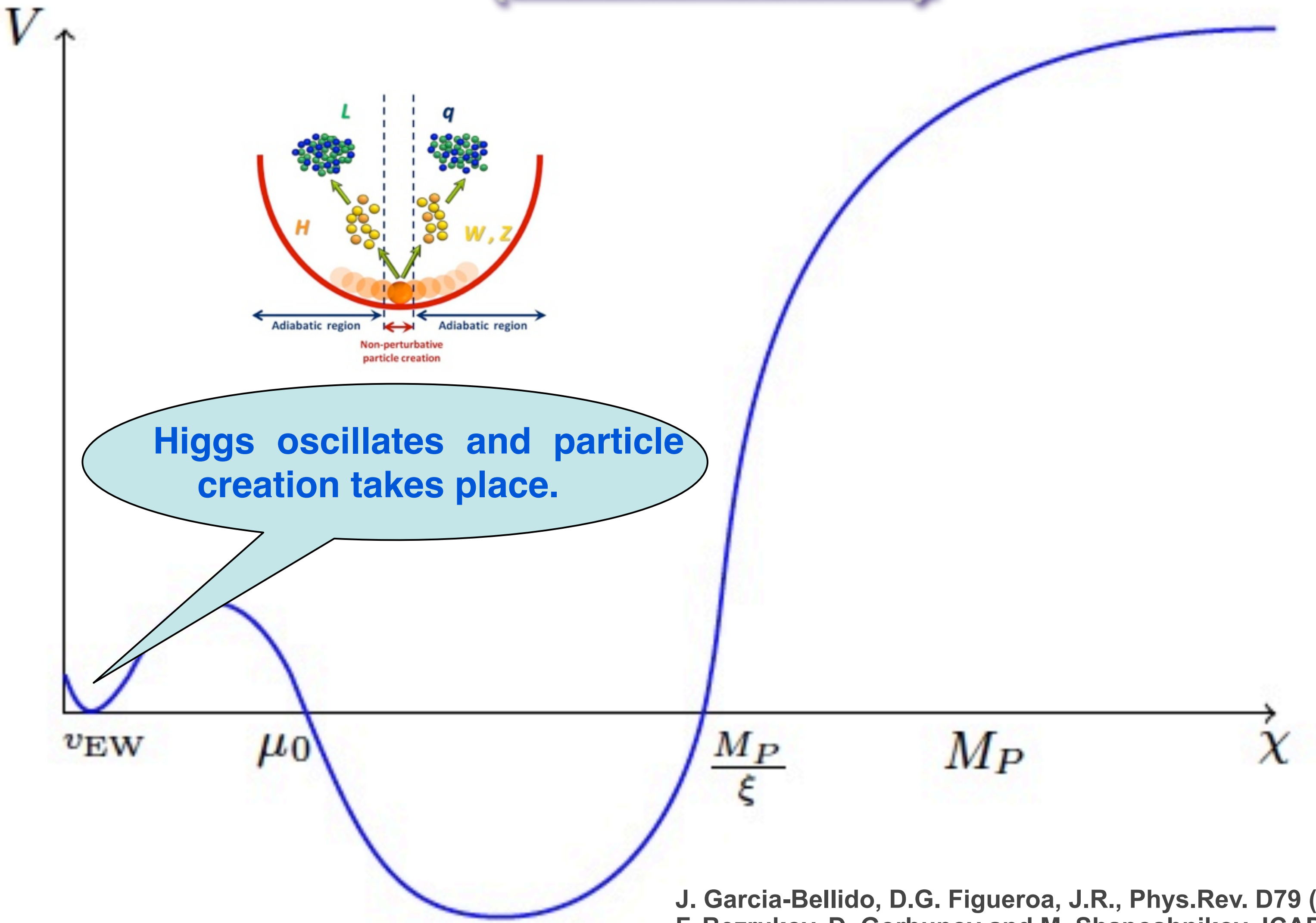
(not to scale!)



Inflation takes place with the standard initial conditions

Sketch of effective potential

(not to scale!)

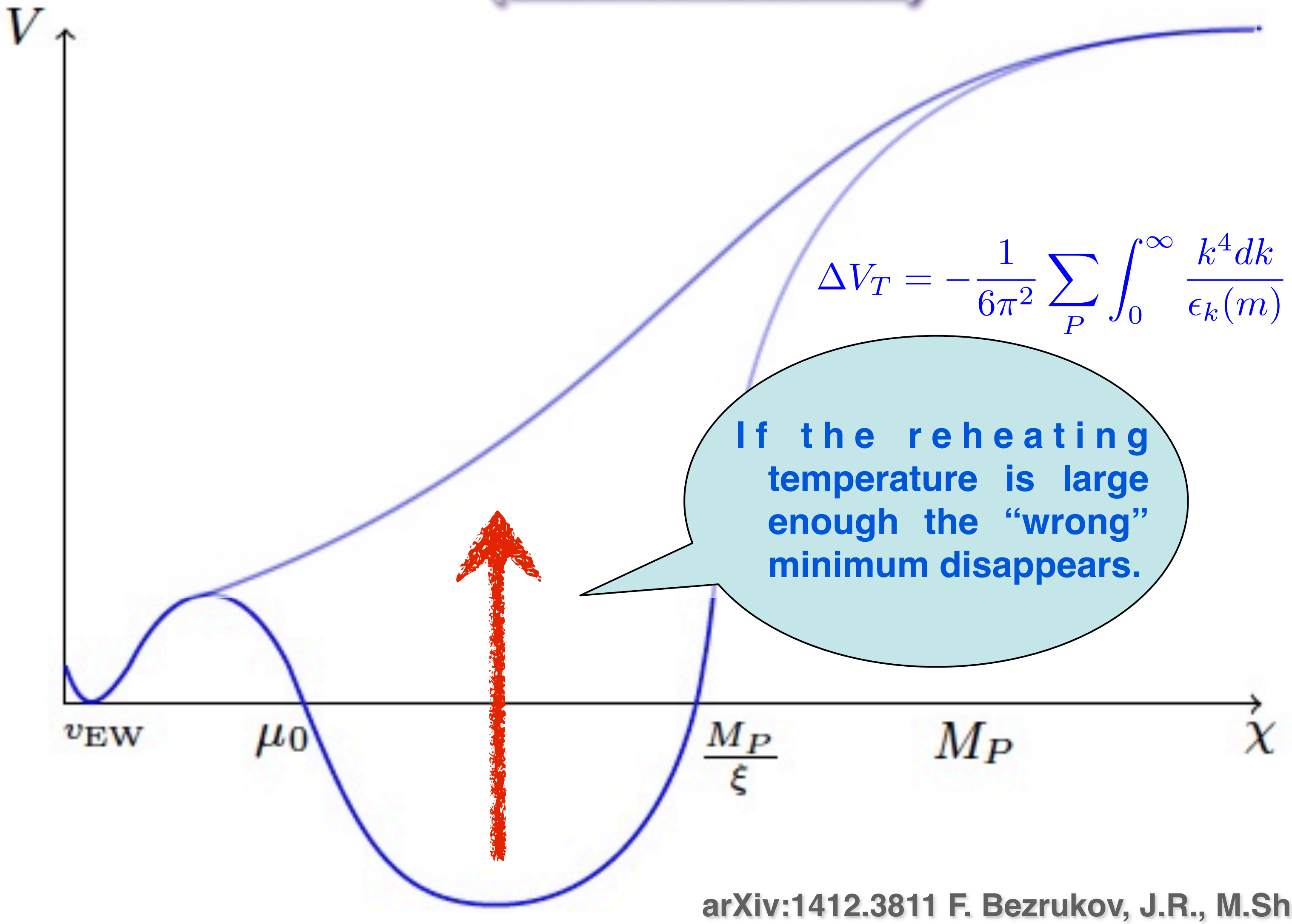


Higgs oscillates and particle creation takes place.

J. Garcia-Bellido, D.G. Figueroa, J.R., Phys.Rev. D79 (2009) 063531
F. Bezrukov, D. Gorbunov and M. Shaposhnikov JCAP 0906 (2009) 029
J. Repond, J. R, M. Shaposhnkov, in preparation

Sketch of effective potential

(not to scale!)



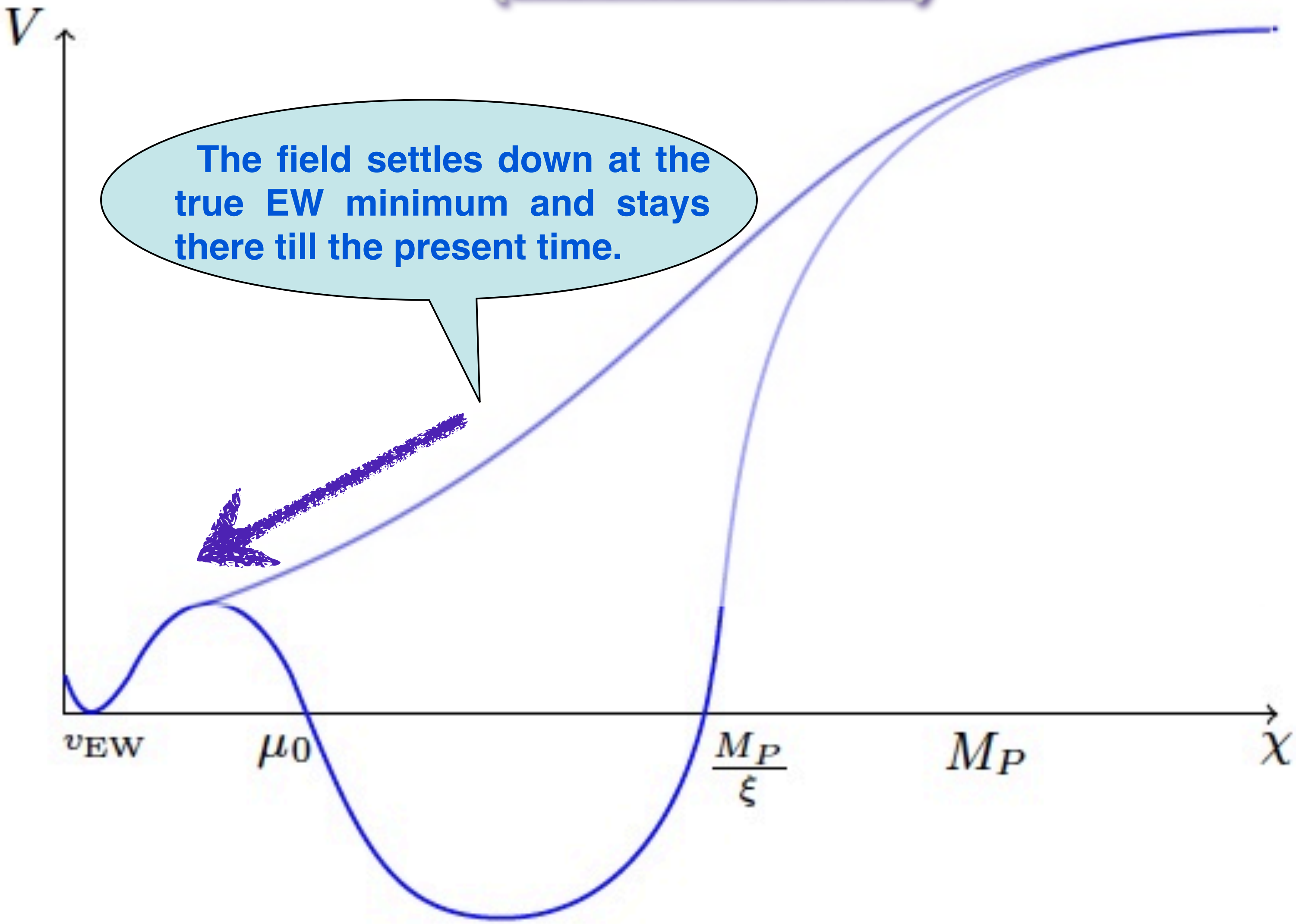
$$\Delta V_T = -\frac{1}{6\pi^2}\sum_P \int_0^\infty \frac{k^4 dk}{\epsilon_k(m)} \frac{1}{e^{\epsilon_k(m)T} \mp 1}$$

If the reheating temperature is large enough the “wrong” minimum disappears.

arXiv:1412.3811 F. Bezrukov, J.R., M.Shaposhnikov

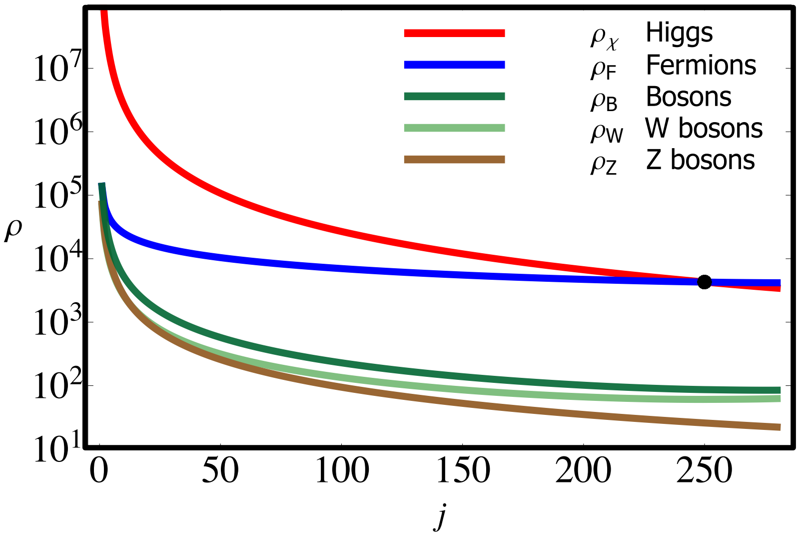
Sketch of effective potential

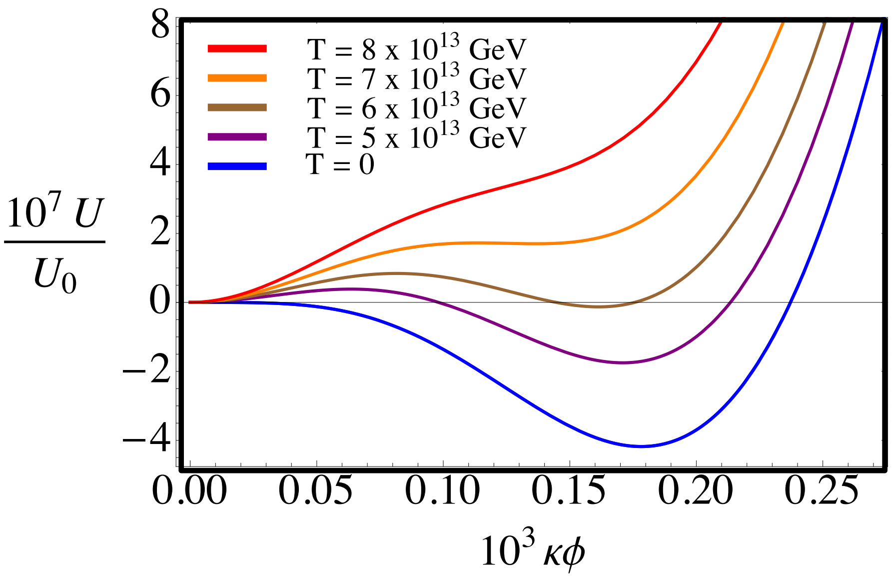
(<u>not to scale!</u>)



The field settles down at the true EW minimum and stays there till the present time.

Symmetry restoration





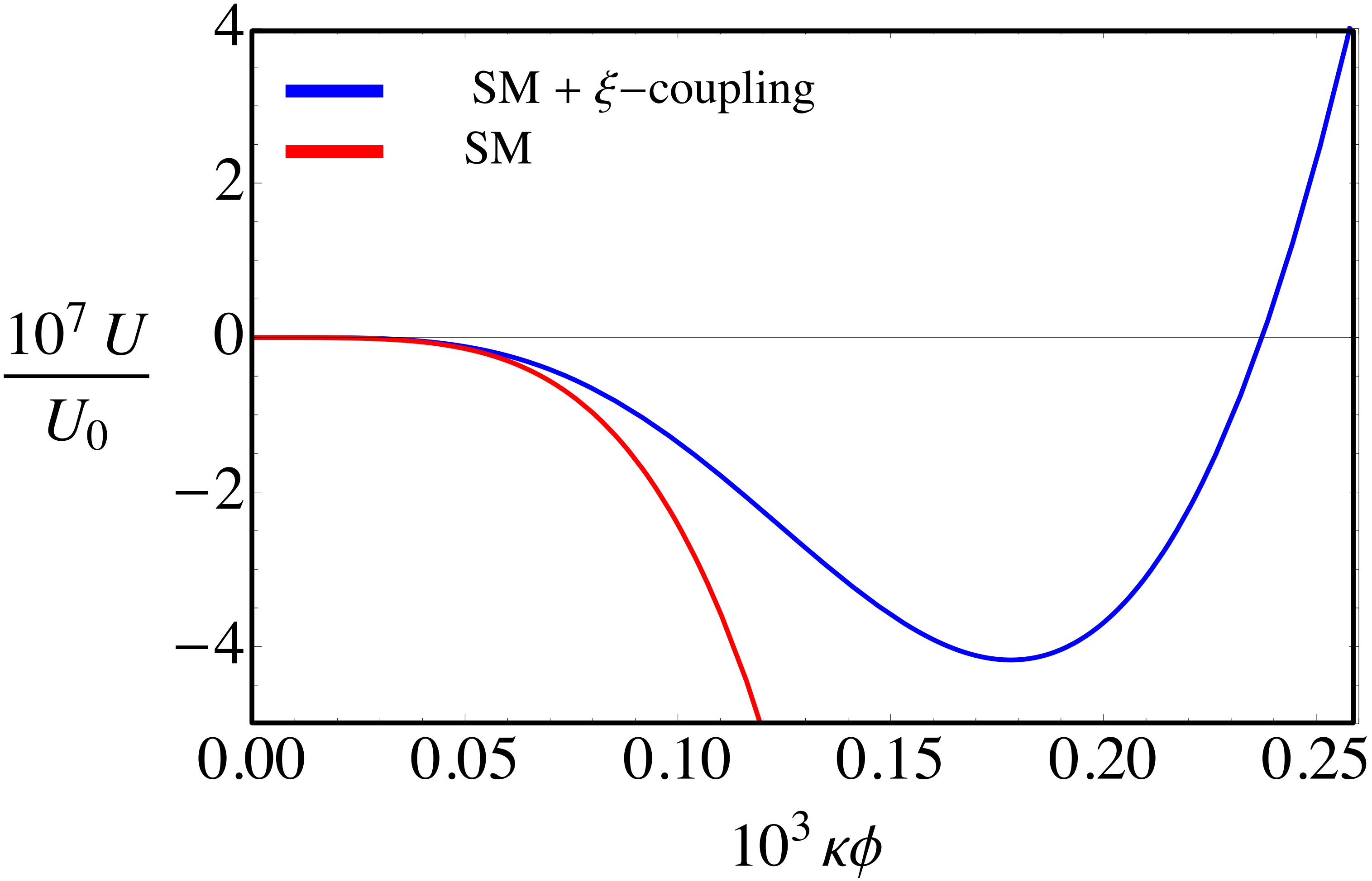
$$T_R \simeq 1.8 \times 10^{14}\,\mathrm{GeV} \qquad T_+ \simeq 7 \times 10^{13}\,\mathrm{GeV}$$

$$T_R > T_+$$

Higgs inflation can be possible

even if our vacuum is not completely stable

What about lifetime?



Z. Lalak,M. Lewicki, P. Olszewki JHEP 1405 (2014) 119 ,
V. Branchina, E. Messina Phys.Rev.Lett. 111 (2013) 241801 etc...

For SM computation see : J. R. Espinosa and M. Quiros, Phys. Lett. B 353 (1995) 257
J. R. Espinosa, G. F. Giudice and A. Riotto, JCAP 0805 (2008) 002

CONCLUSIONS

- ✓ **The Higgs field can inflate the Universe**
- ✓ **HI provides universal predictions if the UV completion respects SI**

$$n_s \simeq 1 - \frac{2}{N} \simeq 0.97 \qquad r \simeq \frac{12}{N^2} \simeq 0.0033$$

- ✓ **The relation of these predictions to LE observables contains an irreducible theoretical uncertainty.**

UV completion?

- ✓ **Higgs inflation can be possible *even if our vacuum is metastable***
- ✓ **The HI scenario is just a particular realization of a general idea. Vacuum instability is not necessarily a problem if:**
  - **The potential is modified below the scale of inflation**
  - **The reheating process is efficient enough as to make the wrong minimum disappear temporally.**

BACKUP
SLIDES

CONSISTENCY IS

The new finite parts should be promoted to new independent coupling constants with their own RG equations ...

$$\frac{d\lambda}{d\log\mu} = \beta_\lambda(\lambda, \lambda_1, \ldots) \qquad \frac{d\lambda_1}{d\log\mu} = \beta_{\lambda_1}(\lambda, \lambda_1, \ldots)$$

$$\frac{dy_t}{d\log\mu} = \beta_{y_t}(y_t, y_{t1}, \ldots) \qquad \frac{dy_{t1}}{d\log\mu} = \beta_{y_{t1}}(y_t, y_{t1}, \ldots)$$

$$\vdots \qquad\qquad \vdots$$

$$\vdots \qquad\qquad \vdots$$

but since we are dealing with a non-renormalizable theory, the set of RG equations is not closed...

Truncation

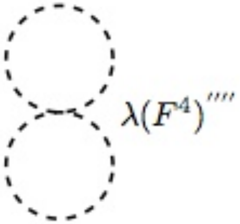
The one-loop diagrams associated to the new counterterms are given by

$$\sim\ \delta\lambda[(F'^2+\frac{1}{3}F''F)^2F^4]''\lambda(F^4)''$$

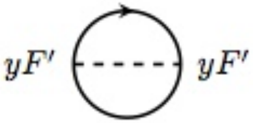
$$\sim\ \delta yF'^2Fy^3F^3$$

The two-loop contributions generated by the original Lagrangian

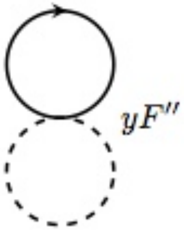
$$\sim\ (\lambda(F^4)''')^2\lambda(F^4)''\,,$$



$$\sim\ \lambda(F^4)''''(\lambda(F^4)'')^2$$



$$\sim\ (yF')^2(yF)^4\,,$$



$$\sim\ yF''(yF)^3\lambda(F^4)''\,,$$

3. In order to have a good perturbative expansion, the finite parts must be of the same order (in power counting) than the loops producing them.

$$\delta\lambda\sim\mathcal{O}(\lambda^2,y^4)\qquad\delta y\sim\mathcal{O}(y^3,y\lambda)\qquad\lambda\sim\mathcal{O}(y^2)$$

Example

$$\phi = \bar{\phi} + \delta\phi \qquad g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

1. Compute the quadratic lagrangian (Jordan F.)

$$\mathcal{L}^{(2)} = -\frac{M_P^2 + \xi\bar{\phi}^2}{8}\left(h^{\mu\nu}\Box h_{\mu\nu} + 2\partial_\nu h^{\mu\nu}\partial^\rho h_{\mu\rho} - 2\partial_\nu h^{\mu\nu}\partial_\mu h - h\Box h\right) \\ +\frac{1}{2}(\partial_\mu\delta\phi)^2 + \xi\bar{\phi}\left(\Box h - \partial_\lambda\partial_\rho h^{\lambda\rho}\right)\delta\phi\ ,$$

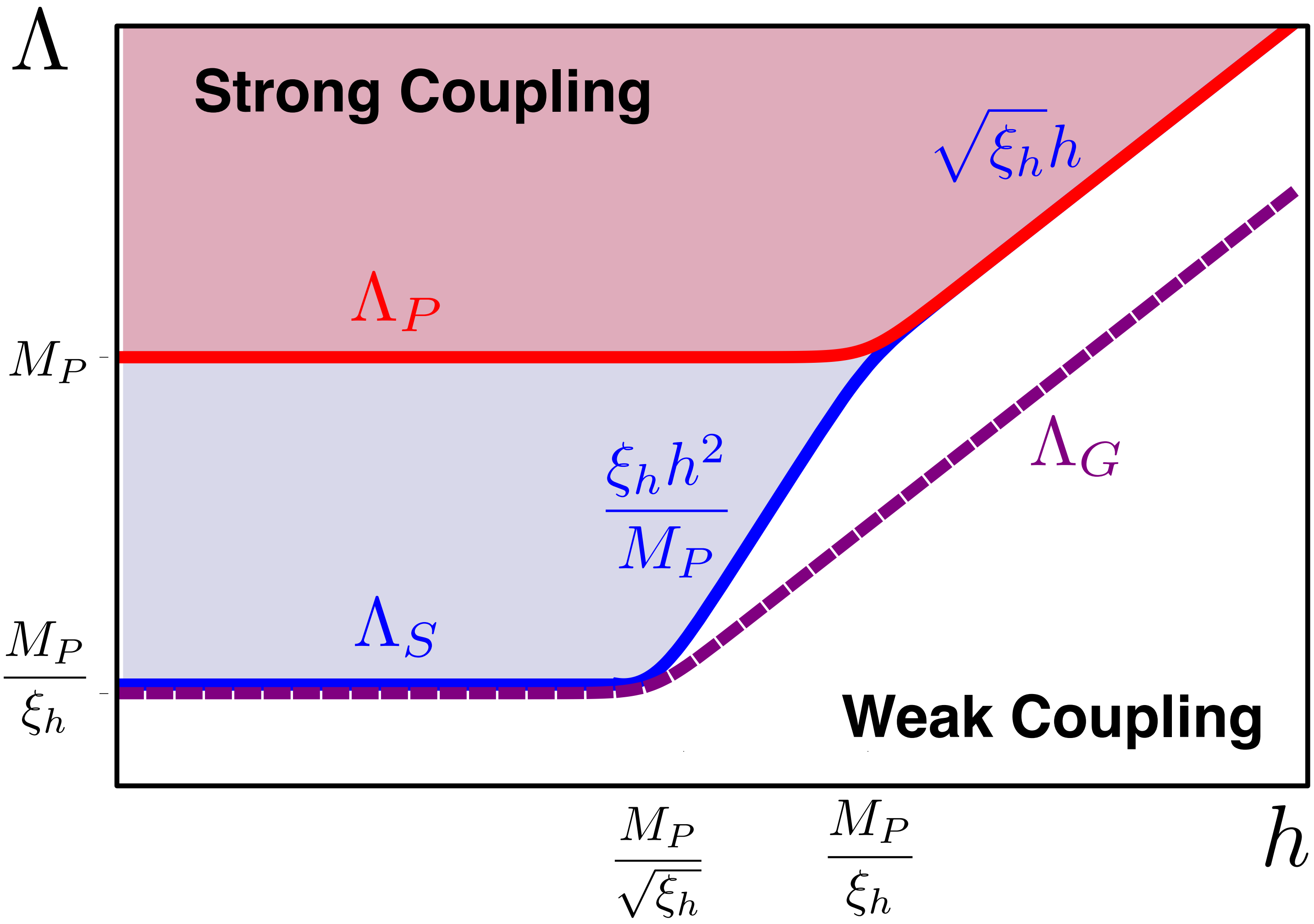
2. Get rid of the mixings in the quadratic action

$$\delta\phi = \sqrt{\frac{M_P^2 + \xi\bar{\phi}^2}{M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2}}\,\delta\hat{\phi}\ ,$$

$$h_{\mu\nu} = \frac{1}{\sqrt{M_P^2 + \xi\bar{\phi}^2}}\,\hat{h}_{\mu\nu} - \frac{2\xi\bar{\phi}}{\sqrt{(M_P^2 + \xi\bar{\phi}^2)(M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2)}}\,\bar{g}_{\mu\nu}\,\delta\hat{\phi}$$

3. Read out the cutoff from higher order operat.

$$\frac{\xi\sqrt{M_P^2 + \xi\bar{\phi}^2}}{M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2}(\delta\hat{\phi})^2\Box\hat{h}$$



A consistent EFT : **Cutoffs are parametrically larger than all the energy scales involved in the history of the Universe**

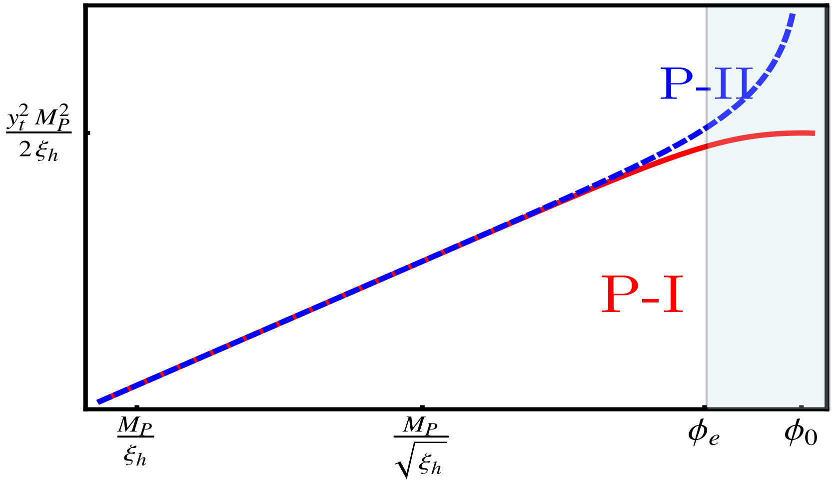
Respect scale invariance -> Dimensional regularization

The choice of μ

In renormalizable theories, it is arbitrary and field-independent

We modify this prescription in our non-renormalizable theory

$$\lambda = \mu^{2\epsilon}\left(\lambda_R + \sum_n \frac{a_n}{\epsilon_n}\right)$$



Jordan frame def.

P-I	$M_P^2 + \xi_h h^2$
P-II	M_P^2

Einstein frame equiv.

P-I	M_P^2
P-II	$M_P^4/(M_P^2 + \xi_h h^2)$

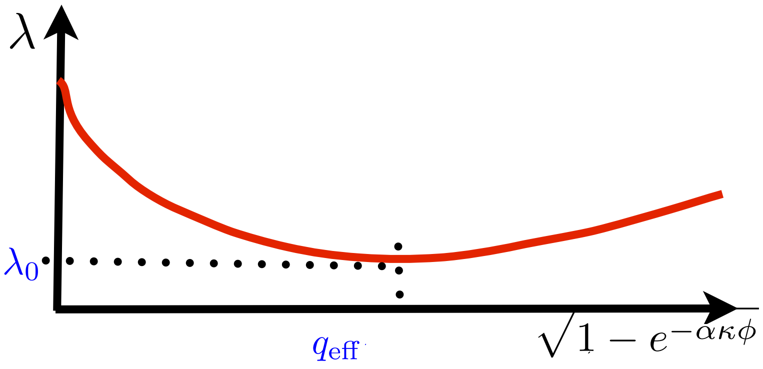
Lets look at the asymptotics

$$\lambda(\mu(\phi)) = \lambda_0 + b\log^2\left(\frac{\sqrt{1-e^{-\alpha\kappa\phi}}}{q_{\rm eff}}\right)$$

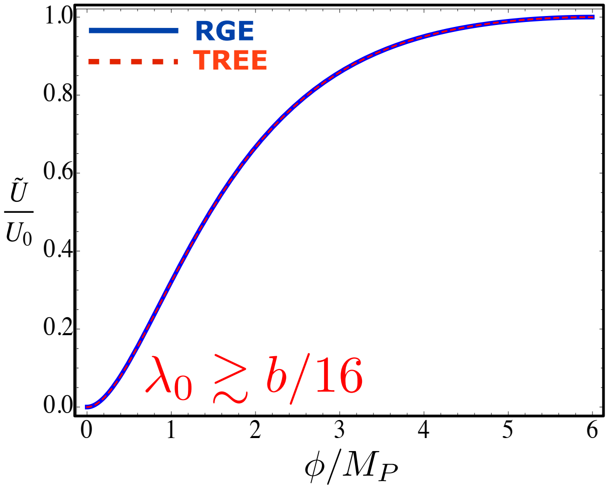
$$b \simeq 2.3\times 10^{-5}$$

$$\lambda_0 = \lambda_0(m_h^*, m_t^*) \ll 1$$

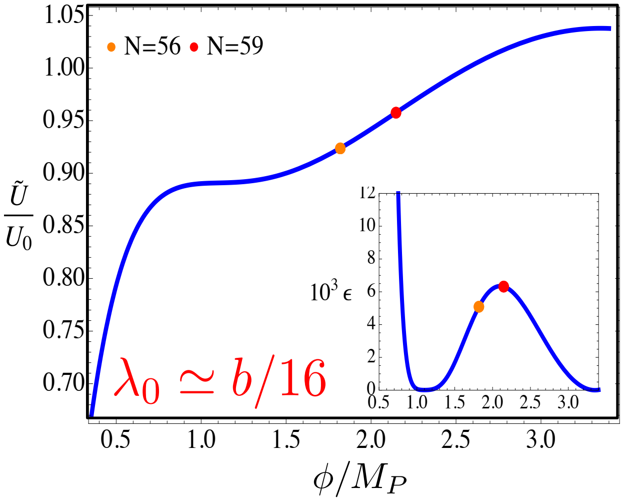
$$q_{\rm eff} = q_{\rm eff}(m_h^*, m_t^*) \sim \mathcal{O}(1)$$



UNIVERSALITY



CRITICALITY



NO INFLATION