Higgs-Dark Matter Connection and the Scale of New Physics

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The Standard Model and the Higgs

- Discovery of the Higgs @ LHC:

\[ M_H \approx 125 \text{ GeV} \]
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- Standard model:
  - effective theory
  - physical cutoff \( \Lambda \)
  - “New Physics” beyond \( \Lambda \)
The Standard Model and the Higgs

• Discovery of the Higgs @ LHC:
  - ATLAS collaboration (2012)
  - CMS collaboration (2012)

  ⌂ M_H ≈ 125 GeV

• Standard model:
  - effective theory
  - physical cutoff $\Lambda$
  - “New Physics” beyond $\Lambda$

• Range of validity of SM?
  - Gravity effects: $\Lambda \sim M_{Pl} = \sqrt{\hbar c/G} \approx 10^{19}$ GeV
  - Landau pole in $U(1)_{\text{hypercharge}}$: $\Lambda > M_{Pl}$
  - Higgs potential…
• Higgs mass is related to Higgs coupling and vev: 
  \[ m_h = \sqrt{2\lambda_4 \cdot \text{vev}} \]

• Upper bound related to Landau pole
Mechanism for Lower Higgs Mass Bound

- Higgs potential: \( \frac{\mu}{2} H^2 + \frac{\lambda_4}{4} H^4 \)

\[ m_h = \sqrt{2\lambda_4 \cdot vev} \]
\[ m_t = \frac{y}{\sqrt{2}} \cdot vev \]
Mechanism for Lower Higgs Mass Bound

- Higgs potential:
  \[ \frac{\mu}{2} H^2 + \frac{\lambda_4}{4} H^4 \]

- Running Higgs self-coupling:
  \[ \beta \lambda_4 = - \text{top loop} + \text{Higgs loop} + \text{gauge contributions} \]

- Higgs potential:
  \[ m_h = \sqrt{2 \lambda_4} \cdot \text{vev} \]
  \[ m_t = \frac{y}{\sqrt{2}} \cdot \text{vev} \]
Mechanism for Lower Higgs Mass Bound

- **Higgs potential:**
  \[ \frac{\mu}{2} H^2 + \frac{\lambda_4}{4} H^4 \]

- **Running Higgs self-coupling:**
  \[ \beta\lambda_4 = - \] (diagram showing the contribution from the top loop and Higgs loop, with gauge contributions)

- **Choose** \( \lambda = 0 \) at \( \Lambda_{UV} \):
  - minimal value of Higgs mass
  - for smaller \( m_h \):
    - start with unstable potential?!
\[ \beta_{\lambda_4} = \frac{d \lambda_4}{d \log k} = \frac{1}{8\pi^2} \left[ 12\lambda_4^2 + 6\lambda_4 y^2 - 3y^4 - \frac{3}{2}\lambda_4 \left(3g_2^2 + g_1^2\right) + \frac{3}{16} \left(2g_2^4 + (g_2^2 + g_1^2)^2\right) \right] \]

- **Vacuum instability**
  - \( \lambda_4 \) crosses zero in \( \frac{\mu}{2} H^2 + \frac{\lambda_4}{4} H^4 \)
  - Instability of Higgs vacuum
  - ‘Scale of New Physics’ \( \sim 10^{10} \) GeV
  - Strongly depends on top Yukawa:

- **Lower Higgs mass bound**
  \[ m_h(P_{\text{Pl}}) \approx 129 \text{GeV} \]
  \[ m_h^{\text{exp}} \approx 125 \text{GeV} \]
Scenarios at the Scale of New Physics

@ ~ $10^{10}$ GeV several scenarios are possible:

1. *New degrees of freedom* appear that render Higgs potential stable - dark matter?
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@ ~ 10^{10} \text{ GeV} several scenarios are possible:

1. **New degrees of freedom** appear that render Higgs potential stable - dark matter?

2. Stable minimum might appear for large field values
   - True minimum @ large $H (< M_{Pl})$?
   - **Metastability** of Higgs vacuum?
   - Small tunnelling rates to stable minimum?

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![Graph showing running of the gauge couplings and the Higgs potential](image)

**FIG. 1:** Running of the gauge couplings, the top Yukawa and the Higgs self-coupling in the standard model. The Higgs potential arises, invalidating the SM theory. We show that the SM Higgs potential leads to a phenomenologically unacceptable model due to dimensional transmutation. In our frame of reference, we do not plot its behaviour here.

**FIG. 2:** The SM Higgs effective potential as a function of the field value $H$. The effective potential only for field values $H > M_{Pl}$. Indeed, using the perturbative running for a cutoff scale $\mu = 25 M_{Pl}$, and considering the vacuum energy density $\rho_H = V_{\text{eff}}(\mu)$, we can determine the lifetime of the metastable vacuum. The third question is what is the origin of the explicit Higgs mass term? The Higgs potential is given by $V_{\text{eff}}(k = H, H) \approx \frac{1}{4} \lambda_4(H) H^4$.
Scenarios at the Scale of New Physics

@ ~ 10^{10} \text{ GeV} several scenarios are possible:

1. New degrees of freedom appear that render Higgs potential stable - dark matter?

2. Stable minimum might appear for large field values
   - True minimum @ large $H (< M_{\text{Pl}})$?
   - Metastability of Higgs vacuum?
   - Small tunnelling rates to stable minimum?

3. Include higher powers in Higgs field (e.g. $\sim H^6, H^8, \ldots$) to render potential stable
   - Do not appear in perturbatively renormalizable Higgs Lagrangian
   - Appear in effective theories with finite $\Lambda_{\text{UV}}$ when approaching underlying theory
   - New physics appears at higher scales $10^? \text{ GeV} > 10^{10} \text{ GeV}$
   - Link to BSM particle physics models?
Higgs Portal to Dark Matter

• Evidence for DM: *gravitational lensing, galaxy rotation curves, CMB, …*

• Single scalar field serves as **stable DM** candidate (WIMP)

\[
\Gamma_{\text{DM}} = \int d^4x \left( \frac{1}{2} \partial_\mu S \partial^\mu S + \frac{1}{2} m_S^2 S^2 + \frac{\lambda_{11}}{8} S^4 \right)
\]

› with \( \mathbb{Z}_2 \) - symmetry: \( S \rightarrow -S \)

› **Portal coupling to Higgs:** \( \frac{\lambda_{11}}{4} h^2 S^2 \)

• \( S \) can reproduce observed dark matter relic density
  ‣ Condition on scattering cross section
  ‣ Relation between \( m_S \) and \( \lambda_{11} \)
  ‣ For \( m_S > m_h/2 \):

\[
\log_{10} \left( \frac{\lambda_{11}}{2} \right) = -3.63 + 1.04 \log_{10} \left( \frac{m_S}{\text{GeV}} \right)
\]

[Cline et al. (2013)]
Effect of Dark Matter on Higgs Mass

- Running Higgs self-coupling:

\[ \beta_\lambda = - \]

\[ \Lambda_{EW} \quad \Lambda_{UV} \]

\[ \lambda \]

\[ \lambda_{11} > 0: \text{lower Higgs mass} \]
**Effect of Dark Matter on Higgs Mass**

- Running Higgs self-coupling:
  \[ \beta_\lambda = - + \]  

- Contours of fixed cutoff scale:

  **SM:**

  \[ S_{UV} = \int d^4 x \left\{ \bar{\psi} i \gamma^\mu \psi + \frac{1}{2} (\partial_\mu h)^2 + \frac{1}{2} (\partial_\mu S)^2 + i \bar{\psi} h \gamma^\mu \psi + \bar{V}(h, S) \right\} \]
Standard Model as a Low-Energy Effective Theory

- **Potential at UV scale:** all operators compatible with symmetries

  \[ V_{UV} = \frac{\lambda_4(\Lambda)}{4} H^4 + \frac{\lambda_6(\Lambda)}{8\Lambda^2} H^6 + \ldots \]

- **Towards IR:** irrelevant operators follow canonical scaling

  \[ \lambda_6 \] becomes tiny very fast!
  - Nevertheless: impact on mass bounds
  - Or: impact on maximal UV extension

References:
- Fodor et al. (2008)
- Branchina & Messina (2013)
- Gies et al. (2013)
**Standard Model as a Low-Energy Effective Theory**

- **Potential at UV scale**: all operators compatible with symmetries

\[
V_{UV} = \frac{\lambda_4(\Lambda)}{4} H^4 + \frac{\lambda_6(\Lambda)}{8 \Lambda^2} H^6 + \ldots
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- **Towards IR**: irrelevant operators follow canonical scaling

  \[\lambda_6 \rightarrow 0\] becomes tiny very fast!

  - Nevertheless: impact on mass bounds
  - Or: impact on maximal UV extension

- **Mechanism to go below lower mass bound:**

  1. Choose Higgs-self-coupling < 0 at UV scale
  2. Choose \(\Phi^6\)-coupling > 0 \(\rightarrow\) potential is stable

\[\text{Obtain smaller Higgs-self-coupling in the IR} \rightarrow \text{Higgs mass lower than lower bound!}\]
Functional RG

- Use functional RG method as an appropriate tool to obtain $\beta$ functions:
  - allows to include all quantum fluctuations in presence of higher-dim operators
  - flowing action $\Gamma_k$ with RG scale $k$ interpolates between
    - microscopic action ($k \to \Lambda$): $\Gamma_k[\Phi] \to S[\Phi]$
    - full effective action ($k \to 0$): $\Gamma_k[\Phi] \to \Gamma[\Phi]$

- FRG flow equation:
  \[
  \partial_t \Gamma_k[\Phi] = \frac{1}{2} \text{STr} \left\{ \left[ \Gamma_k^{(2)} [\Phi] + R_k \right]^{-1} (\partial_t R_k) \right\}
  \]

$\beta$ functions for model couplings…

…(reproduce 1-loop $\beta$ functions from PT, include threshold effects, higher-dim operators,…)
\[ S_\Lambda = \int d^4x \left[ \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (\partial_{\mu} \varphi)^2 + V_{\text{eff}}(\Lambda) + i \sum_{j=1}^{n_f} \overline{\psi}_j i\mathcal{D} \psi_j + i \frac{y}{\sqrt{2}} \sum_{j=1}^{n_y} \varphi \overline{\psi}_j \psi_j \right] \]

- **Potential at UV scale:** completely stable with unique minimum at \( H=0 \)

\[ V_{\text{UV}} = \frac{\lambda_4(\Lambda)}{4} H^4 + \frac{\lambda_6(\Lambda)}{8\Lambda^2} H^6 + \ldots \]
\[ S_A = \int d^4x \left[ \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (\partial_\mu \varphi)^2 + V_{\text{eff}}(\Lambda) + i \sum_{j=1}^{n_f} \bar{\psi}_j \slashed{D} \psi_j + i y \sqrt{2} \sum_{j=1}^{n_y} \varphi \bar{\psi}_j \psi_j \right] \]

**Potential at UV scale:** completely stable with unique minimum at \( H=0 \)

\[ V_{\text{UV}} = \frac{\lambda_4(\Lambda)}{4} H^4 + \frac{\lambda_6(\Lambda)}{8\Lambda^2} H^6 + \ldots \]

- Potential completely stable during entire RG flow
- Extend UV cutoff by orders of magnitude (~2)
Gauged Higgs-Top Model - Higher-dimensional operators

\[ S_\Lambda = \int d^4x \left[ \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (\partial_\mu \varphi)^2 + V_{\text{eff}}(\Lambda) + i \sum_{j=1}^{n_f} \bar{\psi}_j \gamma_\mu \psi_j + i \frac{y}{\sqrt{2}} \sum_{j=1}^{n_y} \varphi \bar{\psi}_j \psi_j \right] \]

- Potential at UV scale: completely stable with unique minimum at \( H=0 \)

\[ V_{\text{UV}} = \frac{\lambda_4(\Lambda)}{4} H^4 + \frac{\lambda_6(\Lambda)}{8\Lambda^2} H^6 + \ldots \]

- Potential completely stable during entire RG flow
- Extend UV cutoff by orders of magnitude (\( \sim 2 \))
- Potential develops 2nd Minimum during RG flow
  - Min @ \( H=0 \) only metastable
  - Small \( \lambda_6 \) sufficient to stabilize UV potential
  - Further studies required…
While retaining the full stability of the electroweak vacuum. This can be seen from the green region I in Fig. 6.

With this choice, the limit therefore essentially reduces to the usual longevity limit for the meta-stable region as the point of expansion will have to shed further light on the global renormalization group flow of the electroweak minimum reliably. Future studies based on a different UV boundary conditions as indicated in the plot.

The dependence of the cuto\(\Lambda\) at the electroweak scale can be increased by approximately two orders of magnitude for observed Higgs mass \(m_h\). Negative values imply that the Higgs mass resulting from finite ultraviolet values of the cuto\(\Lambda\) is less effective for large cuto\(\Lambda\).

Our numerical study based on a more advanced approximation as described in App. C shows that already with the cuto\(\Lambda\) fixed at the TeV scale the cuto\(\Lambda\) dependence of \(m_h\) flattens toward higher cuto\(\Lambda\). This corresponds to shifting the curve \(m_h(\Lambda)\) at a fixed \(\Lambda\) with a linear dependence on \(\Lambda\) and a constant mass bound \(\approx 129\text{ GeV}\) for observed Higgs mass \(m_h\).

 shifts at level of 1-5% seem viable

\[ m_h(M_{Pl}) \approx 129\text{ GeV} \]
\[ m_h^{\exp} \approx 125\text{ GeV} \]
Moderately small $\lambda_6(\Lambda)$ extend UV cutoff by 2 orders of magnitude at full stability (green)

- Pseudo-stable region (blue) - allows for more orders of magnitude
  - extend FRG study
  - possibility of meta-stable effective potential at $k \approx 0$
Higgs Mass (Bounds) with Light Dark-Matter Scalar

- toy model: \[ S_{UV} = \int d^4x \{ \bar{\psi}i\partial\psi + \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(\partial_\mu S)^2 + i\bar{\psi}h\psi + \hat{V}(h, S) \} \]

- Fix vev = 246 GeV and \( m_{top} = 173 \text{ GeV} \)
- Choose \( \lambda_4 = -0.05 \) and \( \lambda_6 = 0.5 \)

\[ \Rightarrow \text{Higgs masses with dark matter and new couplings:} \]

\[ \text{toy model!} \]
**Conclusion**

- measured Higgs mass very close to lower bound $m_h(\Lambda = M_{Pl})$
- perturbative analysis: Higgs potential loses stability around $10^{10}$ GeV
- this statement can be relaxed:
  - higher-dimensional operators at UV scale $\Lambda$
  - non-perturbative treatment allows for more general values of higher-dim couplings
- Higgs masses below lower bound are possible
- with completely stable potential, we can extend UV cutoff by 2 orders of magnitude
- DM fluctuations allow for smaller Higgs masses at fixed $\Lambda$ ($\sim$ a few GeV @ $\Lambda = M_{Pl}$)

**Question:** What type of physics can predict higher-dim operators of suitable size?
- we have investigated simple SM extension with heavy scalars
- required parameter choices in simple model are at border to non-perturbative