The High-Energy Astrophysics of Active Galaxies

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Lecture 1: Particle Acceleration

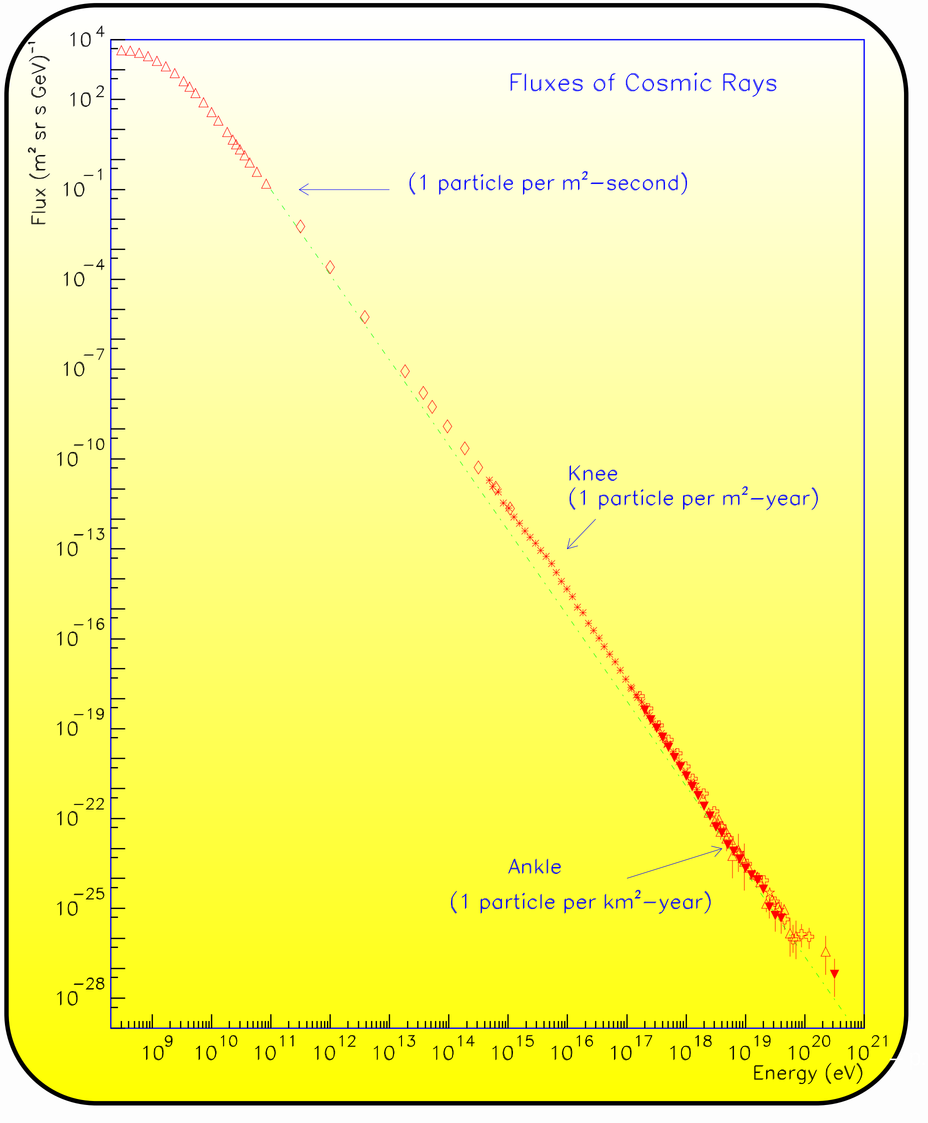
- Stochastic vs non-stochastic mechanisms
- Acceleration at shocks
- The role of current sheets

Energetic particles

- Non-thermal – not in thermodynamic equilibrium with surroundings e.g., cosmic rays, relativistic electrons in supernova remnants, AGN, jets...
- High energy, very low density, e.g., cosmic rays: particle energy 10^{10} eV up to 10^{20} eV (≈ 16 J) number density $10^{-10}\times$ interstellar medium.
- Interactions with background almost exclusively via electromagnetic fields
- $\mathbf{E} = 0$ in highly conducting astrophysical plasma (acceleration problem)

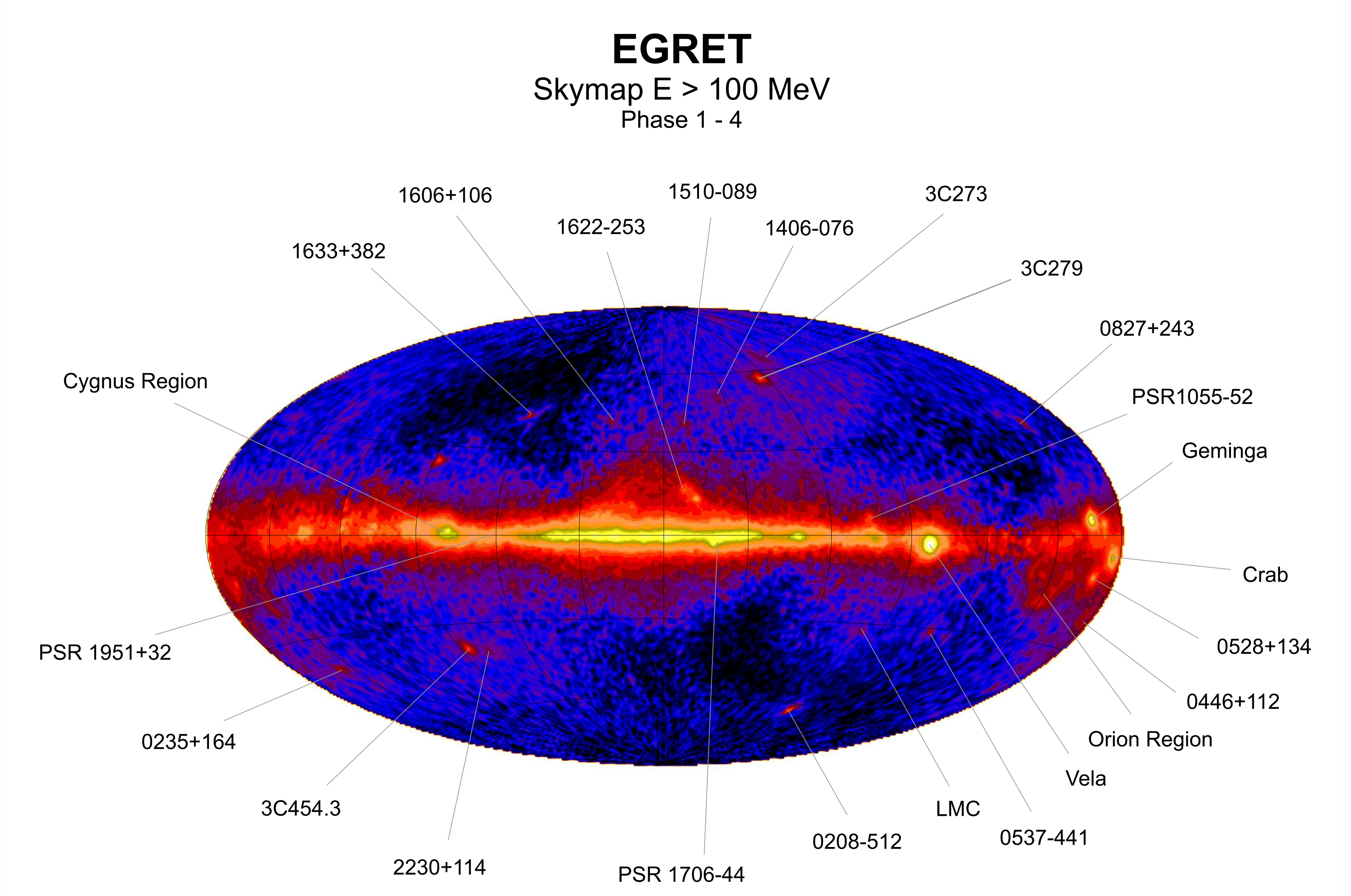
Cosmic Rays

Cosmic Ray Flux at Earth



Magnetic confinement?
Residence time $\sim 10^8$ years,
crossing time $\sim 10^3$ years.

Gamma-rays


EGRET
Skymap E > 100 MeV
Phase 1 - 4
1606+106
1510-089
3C273
1622-253
1406-076
1633+382
3C279
0827+243
Cygnus Region
PSR1055-52
Geminga
Crab
PSR 1951+32
0528+134
0446+112
0235+164
Orion Region
Vela
3C454.3
LMC
0537-441
0208-512
2230+114
PSR 1706-44

Non-stochastic acceleration

Examples

- Pulsar (E-field from rotating B-field)
- Shock drift acceleration (E-field from plasma relative motion)
- Current sheets (E-field from effective resistivity)

Properties

- All particles get the same energy
- Energy limited by losses or finite spatial extent

Stochastic acceleration

General case: acceleration events vs escape

If $\Delta E > 0$ in every event $\Rightarrow E(t)$

Let $P(p)$ be probability that a particle escapes with momentum $> p$ $\quad (p = \sqrt{E^2 - m^2c^4}/c)$

Acceleration $\dot{p} = ap$ $\qquad$ Escape rate b

$$P(p + dp) \;=\; P(p)\,(1 - b\,dt)$$

$$\dot{P} \;=\; -bP \quad \Rightarrow \quad \frac{dP}{dp} = -\frac{bP}{ap}$$

$$P \;\propto\; p^{-b/a}$$

Power-law (scale-free) provided b/a independent of p

Fermi acceleration

- Assume acceleration is by many small events $\left|\Delta p\right| / p \sim \epsilon \ll 1$
- If after averaging, $\left\langle \Delta p \right\rangle / p \gg \left\langle \Delta p^2 \right\rangle / p^2 \to$ 1st order Fermi
- if other way around, 2nd order Fermi
- in general, a *Fokker-Planck equation* results

Homogeneous model

Adding an escape term (number of particles $\mathrm{d}N = n\mathrm{d}p$):

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial p}\Bigg[\underbrace{-\frac{\langle \Delta p \rangle}{\tau} n}_{\text{1st order}} + \underbrace{\frac{1}{2}\frac{\partial}{\partial p}\left(\frac{\langle \Delta p^2 \rangle}{\tau} n\right)}_{\text{2nd order}}\Bigg] - \underbrace{\frac{n}{\tau_{\mathrm{esc}}}}_{\text{escape}}$$

If, without escape, equilibrium is possible, $n = \exp\left(-cp/k_{\mathrm{B}}T\right)$ is a stationary solution, so that

$$\frac{\langle \Delta p \rangle}{\tau} = \left(\frac{c}{k_{\mathrm{B}}T} + \frac{\partial}{\partial p}\right)\frac{\langle \Delta p^2 \rangle}{2\tau}$$

(Compute diffusion, get friction for free!)

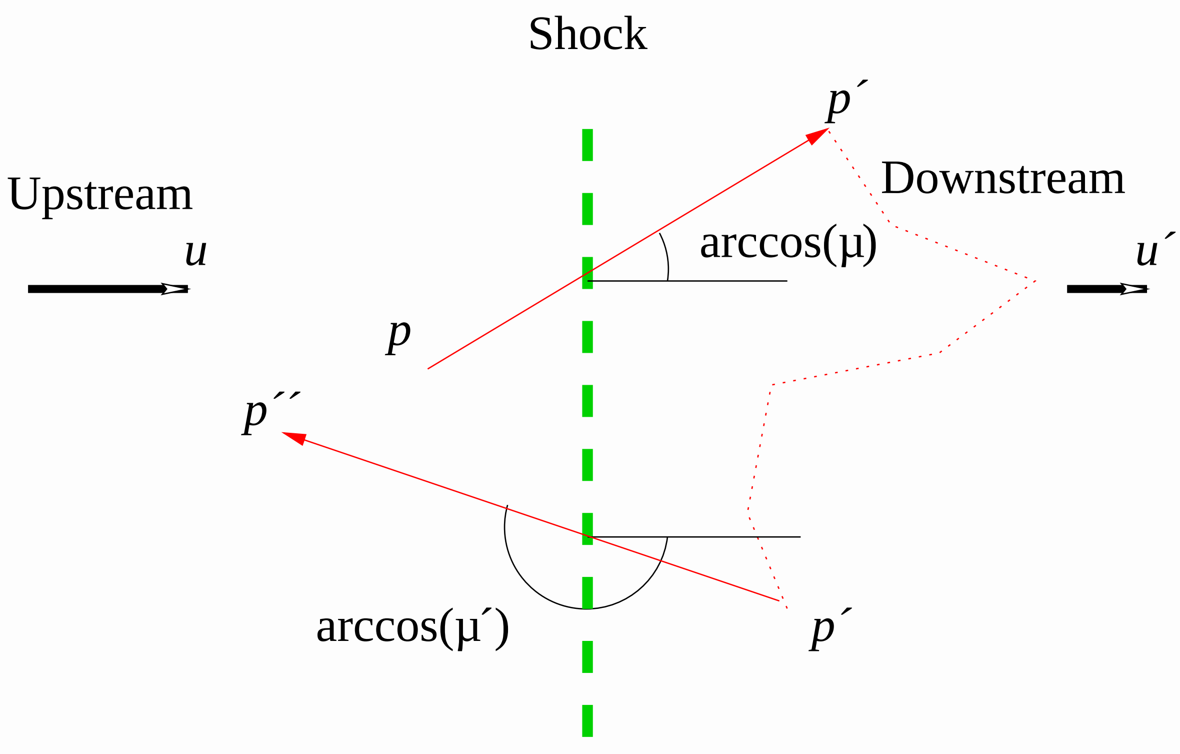
Turbulent acceleration

Scattering by heavy objects (e.g, MHD waves) implies $T \to \infty$ so, for $p \ll k_{\mathrm{B}}T/c$, a *diffusion equation* is found

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial p} D \frac{\partial n}{\partial p} - \frac{n}{\tau_{\mathrm{esc}}}$$

No intrinsic power-law. Injected power-law maintained and accelerated Kardashev 1962. More realistic turbulence description $\to$ include friction term Petrosian & Liu astro-ph/0401585

Acceleration at shock



$$p' = p\left(1 + \frac{\mu \Delta u}{v}\right) \qquad p'' = p'\left(1 - \frac{\mu' \Delta u}{v'}\right) \quad (\Delta u \ll c)$$

to first order in $\Delta u/v$: $\dfrac{\Delta p}{p} = \dfrac{\Delta u}{v}\left(\mu - \mu'\right)$

Diffusive shock acceleration

Average over isotropic distribution (prob. of crossing proportional to relative speed $|\mu v|$):

$$\frac{\langle \Delta p \rangle}{p} = \int_0^{+1} d\mu \int_{-1}^{0} d\mu' |\mu\mu'| \Delta p/p \Big/ \int_0^{+1} d\mu \int_{-1}^{0} d\mu' |\mu\mu'|$$

$$\Rightarrow \langle \Delta p \rangle /p = 4\Delta u/3v$$

Density $n = 2\pi p^2 \int_{-1}^{+1} d\mu f$

$$\mathrm{N^{o.}\ entering/sec} = 2\pi p^2 \int_0^{+1} d\mu(\mu v + u')f = nv/4$$

$$\mathrm{N^{o.}\ leaving/sec} = 2\pi p^2 \int_{-1}^{+1} d\mu(\mu v + u')f = nu'$$

DSA continued

In terms of the phase-space density

$$f(p) \;\propto\; p^{-3-P_{\rm esc}/\langle \Delta p/p\rangle} \equiv p^{-s}\,,$$

$$s = \frac{3u}{\Delta u} = \frac{3r}{r-1}$$

where r is the *compression ratio* of the shock.

A strong shock in a gas with $C_{\rm p}/C_{\rm V} = 5/3$, has $r = 4$, and $s = 4$.

Equations

Upstream

$x < 0$

$u = u_-$

$f \rightarrow 0$ as $x \rightarrow -\infty$

Downstream

$x > 0$

$u = u_+$

f regular at $x \rightarrow +\infty$

Mixed coordinates: t, x in shock frame, p, μ in plasma frame

$$\frac{\partial}{\partial \mu}\left(D_{\mu\mu}\frac{\partial f}{\partial \mu}\right) - (u+\mu)\frac{\partial f}{\partial x} \quad = \quad 0$$

No momentum scale $\Rightarrow$ $f = g(\mu, x)p^{-s}$

Nonrelativistic (DSA) vs. Relativistic

Nonrelativistic (DSA)	Relativistic
pitch-angle diffusion $\Rightarrow$ near-isotropy $\Rightarrow$ spatial diffusion	pitch-angle diffusion, particles in narrow, forward directed cone
solution of PDE in x, p required	solution of PDE in μ, x, p required
small escape probability, small $\langle \Delta p \rangle / p$ per cycle	escape probability ~ 0.5, $\langle \Delta p \rangle / p \sim \Gamma^2$ for first cycle, then ~ 2
power-law of index $s = 3r/(r-1)$, independent of scattering law	Asymptotically, $s = 4.23$, weakly dependent on scattering law

Analytic solution

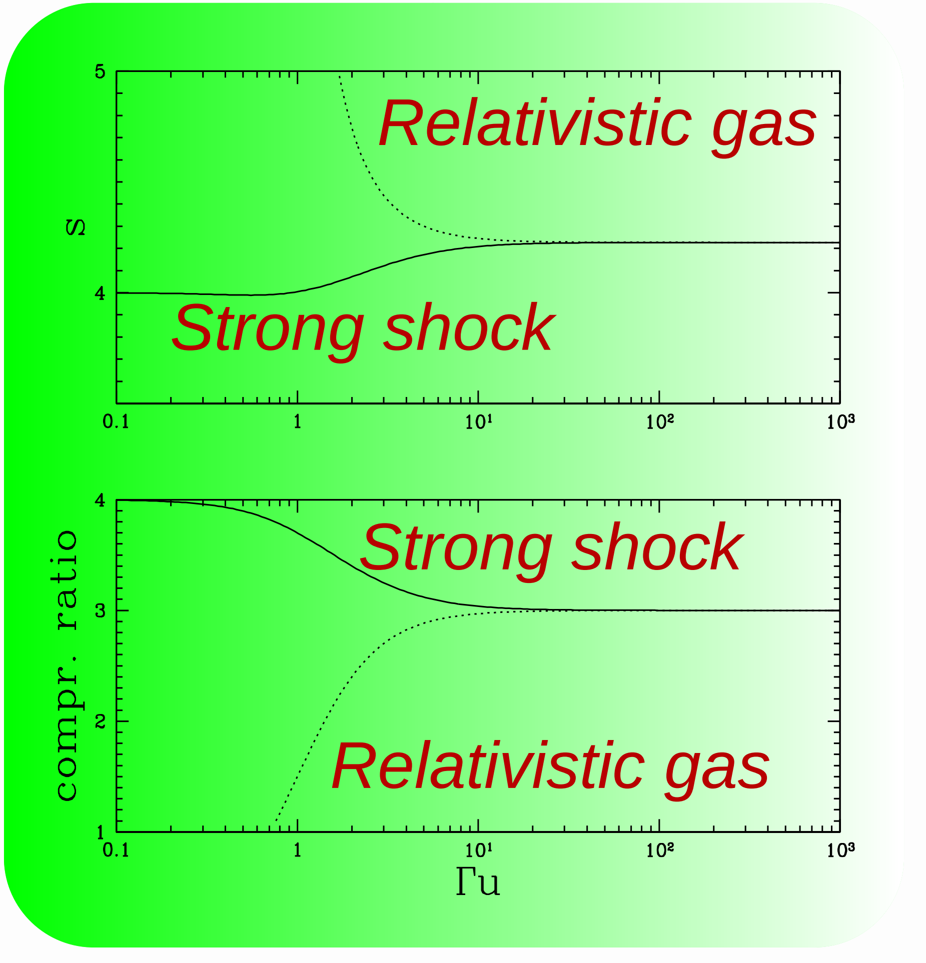
Eigenfunction expansion
$\Rightarrow$ angular dependence:

$$\frac{\exp\left(-\frac{1+\mu_{\rm s}}{1-\mu_{\rm s}u/c}\right)}{\left(1-\mu_{\rm s}u/c\right)^{s}}$$

As $\Gamma \rightarrow \infty$, $s \rightarrow 4.23$

Universal index?

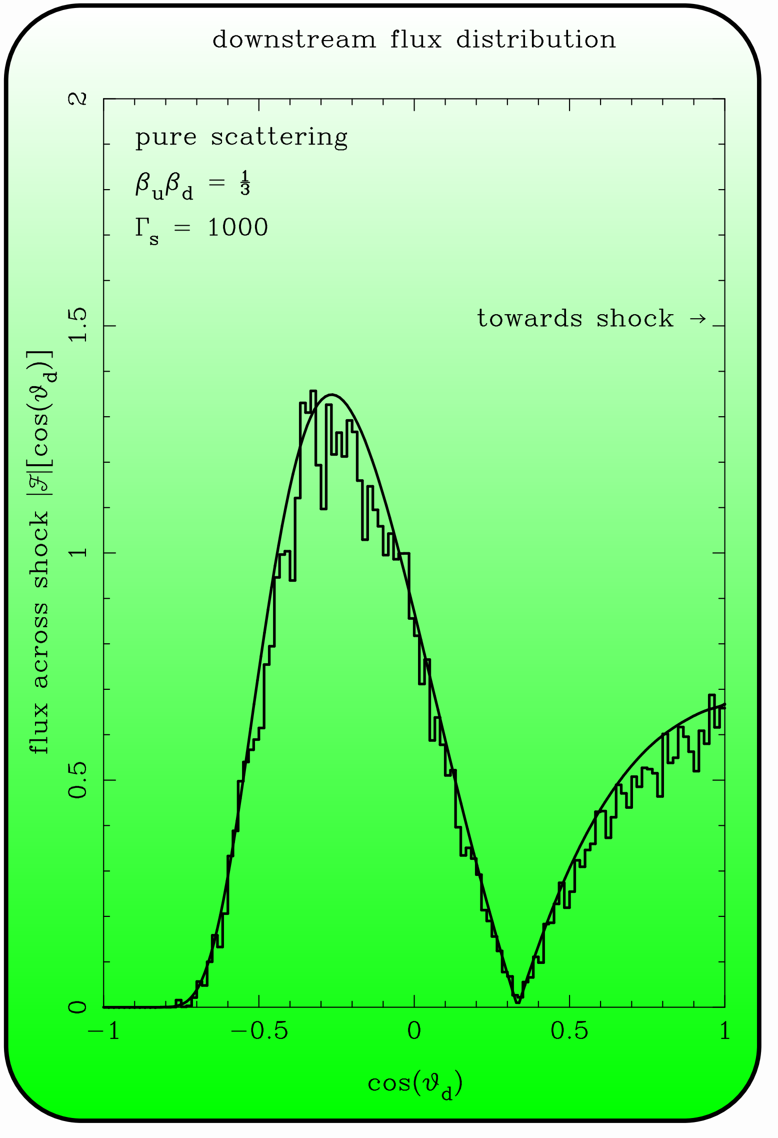
Kirk et al ApJ 542, 235 (2000)



Monte-Carlo

Comparison of MC/analytic angular distributions

Achterberg et al MNRAS 328, 393 (2001)



The box model

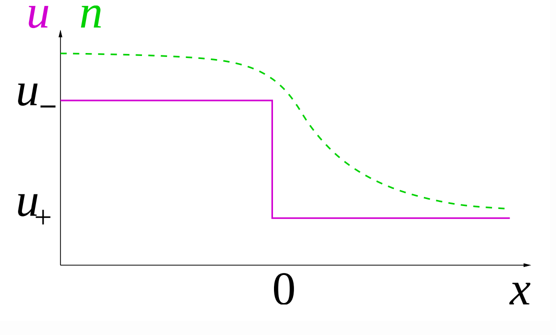
Box model for diffusive shock acceleration – remove spatial dependence:

$$\frac{\partial n}{\partial t} + \frac{\partial \Phi}{\partial x} + \frac{\partial \Psi}{\partial p} = Q_{\rm inj}$$

Space flux: $\Phi = un - (\kappa\partial/\partial x)n$

Momentum flux: $\Psi = [-(p/3)({\rm d}u/{\rm d}x) - \alpha_{\rm synch}p^2]n$

Synchrotron losses: $\alpha_{\rm synch} = (4\sigma_{\rm T}/3m^2c^2)(B^2/8\pi)$



Define box boundaries at which the diffusive flux is negligible and integrate over the box. . .

Drury et al (1999) Kirk (2001)

Two-zone model

In terms of the number of particles in the box $N(p,t)$:

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial p}\left[\frac{p}{t_{\rm acc}} - \alpha_{\rm synch} p^2\right] N + \frac{N}{t_{\rm esc}} \quad = \quad Q_{\rm inj}$$

where $t_{\rm acc} = (s-3)t_{\rm esc}$. Combined with a cooling zone particle density $n_{\rm c}(p,x,t)$

$$\frac{\partial n_{\rm c}}{\partial t} + u_+ \frac{\partial n_{\rm c}}{\partial x} - \frac{\partial}{\partial p}\left(\alpha_{\rm synch} p^2 n_{\rm c}\right) \quad = \quad \frac{N}{t_{\rm esc}}$$

A time-dependent analytic solution is available for (almost) arbitrary energy dependence of $t_{\rm acc}$ and $t_{\rm esc}$

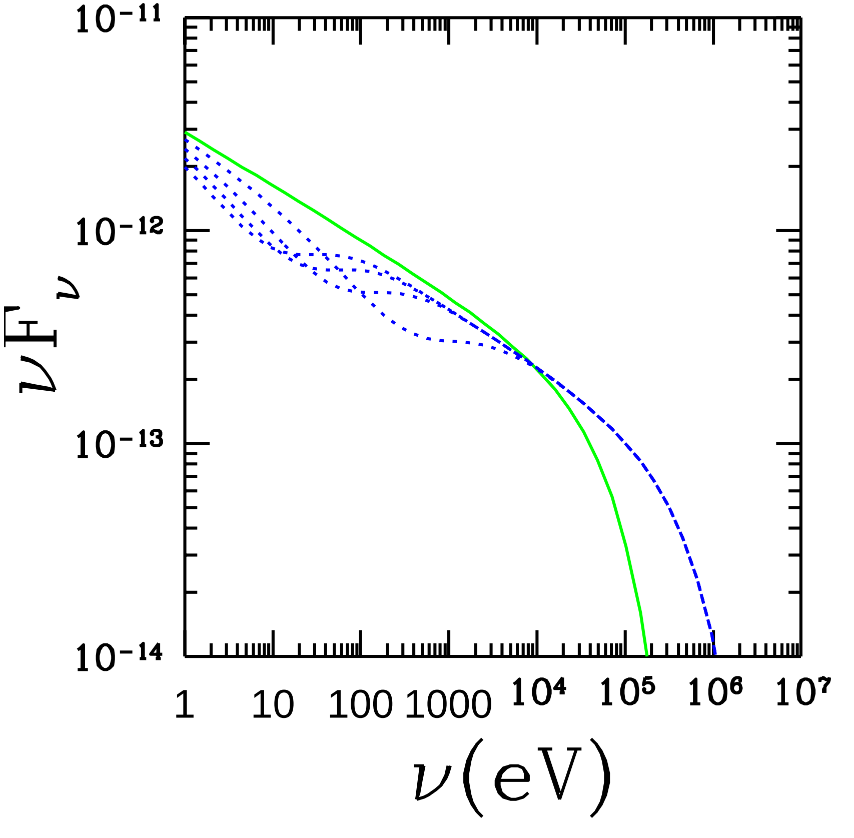
Kardashev (1962), Ball & Kirk (1992), Kirk et al (1998)

Synchrotron spectrum

Easy to model (time-dependent) synchrotron spectra:

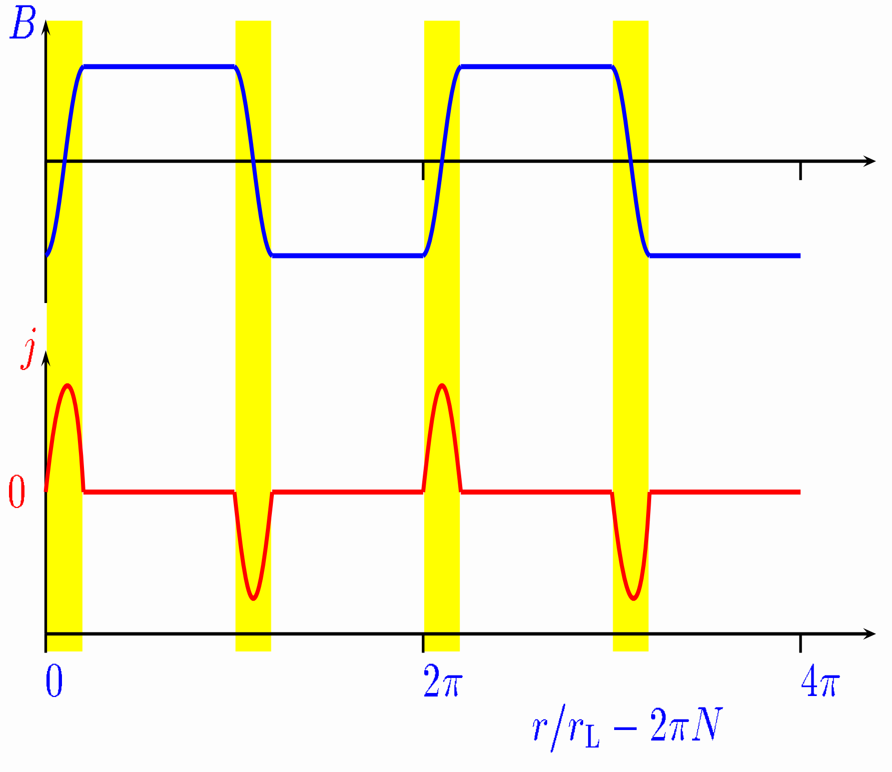
only four parameters, e.g.:

- acceleration timescale $t_{\rm acc}$
- power law index s
- maximum emitted frequency
- flux-level normalisation



Extension to include SSC losses challenging. . .

Current sheets



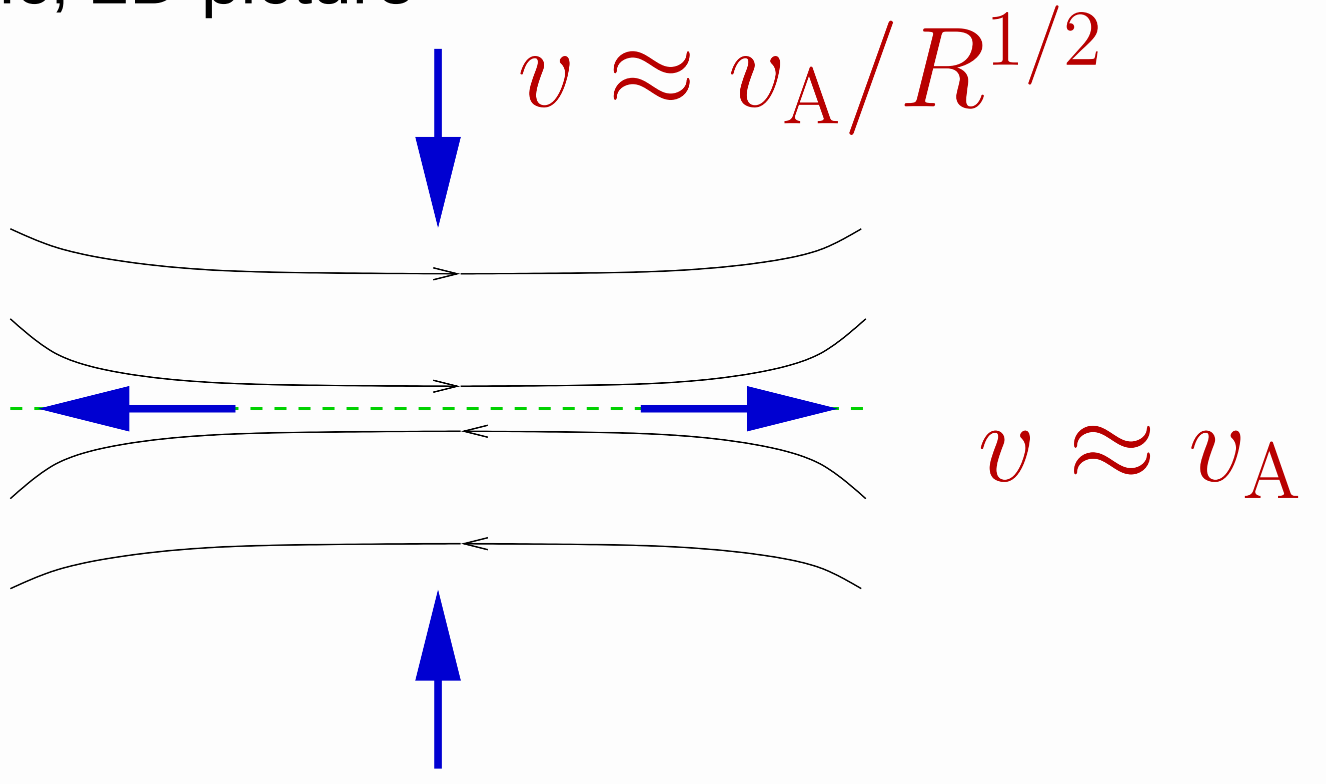
Magnetic pressure balanced by hot plasma in sheet.

Key question:

What controls the dissipation rate?

Sweet-Parker reconnection

Nonrelativistic, 2D picture

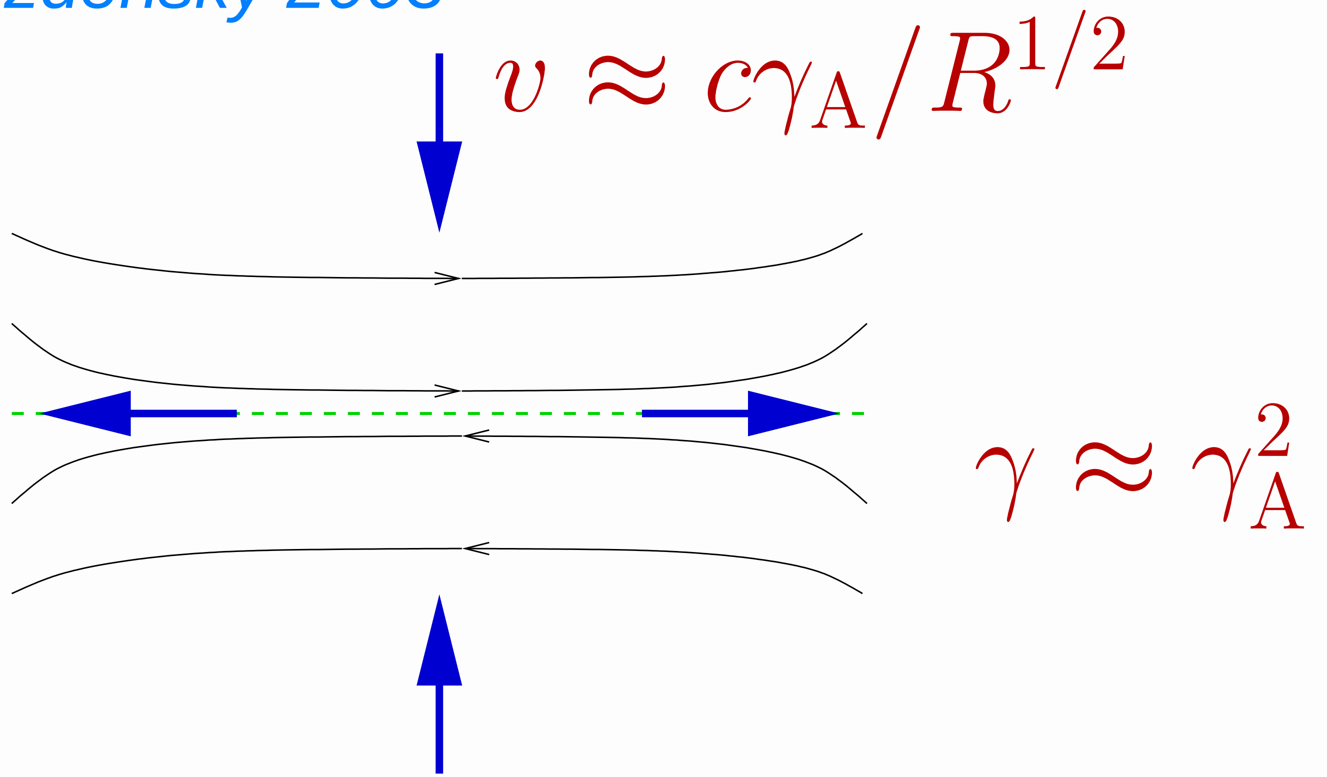


Plasma ejected at approximately Alfvén speed.

Dissipation rate controlled by boundary conditions (R)

Resistive relativistic reconnection

Lyutikov & Uzdensky 2003

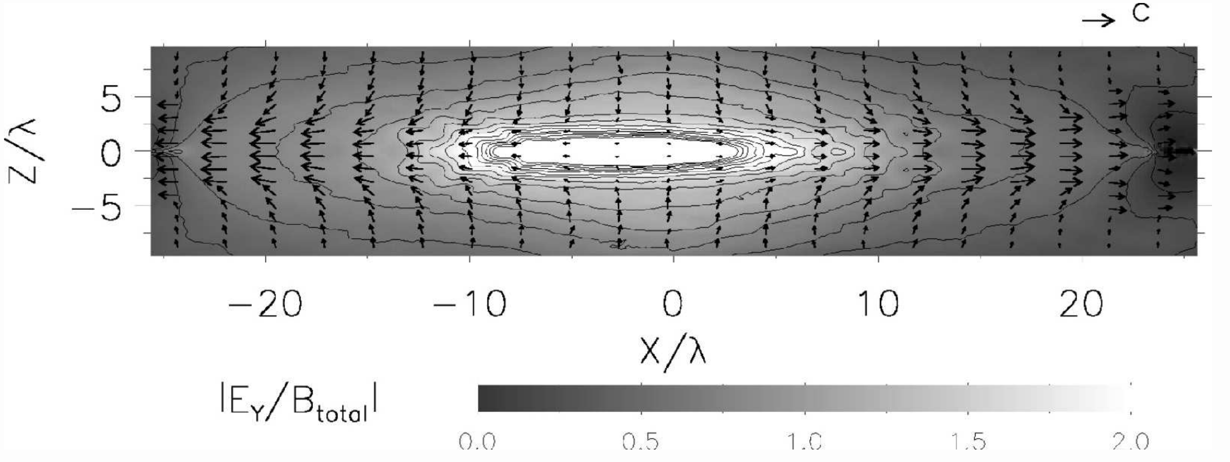


Magnetization parameter $\sigma = B^2/(4\pi w) \approx \gamma_A^2 \gg 1$
Nonrelativistic inflow for $\sigma \ll R$.

Additional regimes with relativistic inflow possible. . .

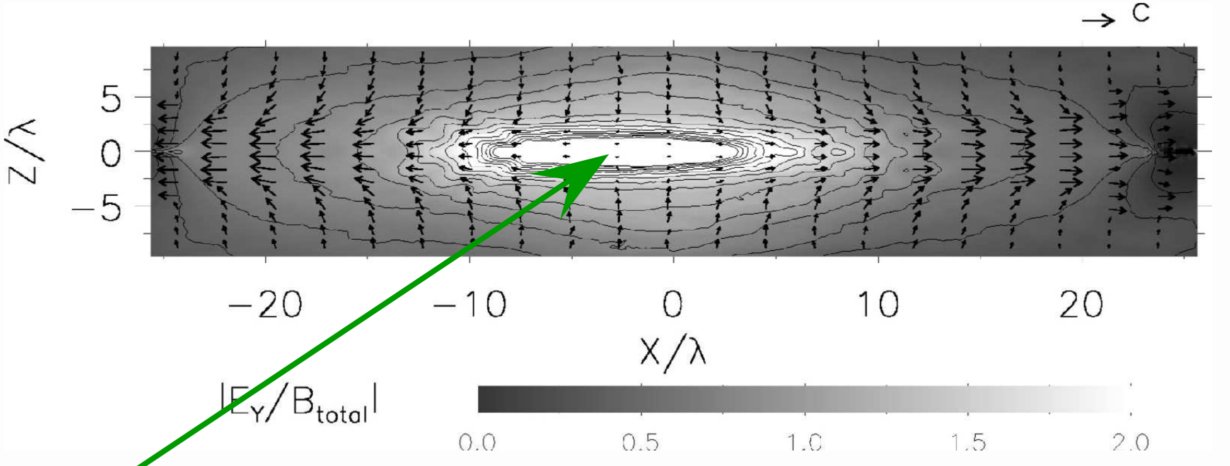
Collisionless relativistic reconnection

Relativistic PIC simulations *Zenitani & Hoshino (2001)*



Collisionless relativistic reconnection

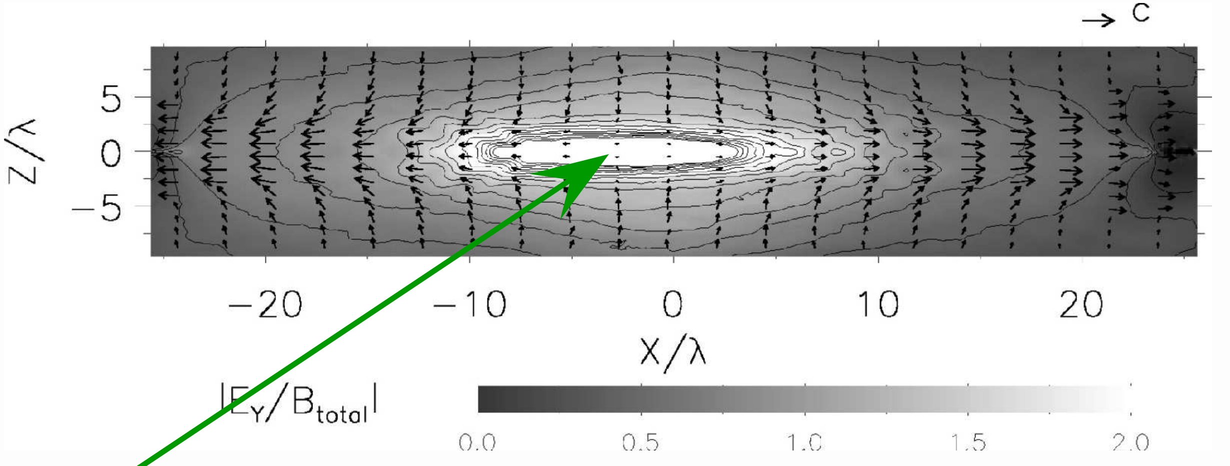
Relativistic PIC simulations *Zenitani & Hoshino (2001)*



Acceleration Region with $E > B$

Collisionless relativistic reconnection

Relativistic PIC simulations *Zenitani & Hoshino (2001)*



Acceleration Region with $E > B$

Box model:

Escape rate $eB_z/\gamma mc \Rightarrow d\ln N/d\ln\gamma = -2B_z/E \approx -1$

Lecture 2: Radiation Mechanisms

- Elements of astrophysical synchrotron theory
- Moving sources
- Inverse Compton scattering
- Synchrotron self-Compton emission
- The *Compton Catastrophe*

General treatment - 1

- General formula for weakly damped waves (Maxwell's eqs. + linear response theory)

$$P(\vec{k}) \;=\; \text{Lim}_{T\to\infty}\frac{4\pi}{TR}\left|\vec{e}\cdot\vec{j}(\omega)\right|^2$$

  E-M wave: $R = 2$, $\vec{e}$ transverse.

- *Gyromagnetic radiation:* $\vec{j}$ from helical motion in $\vec{B}$ (speed $v = \beta c$, Lorentz factor γ, pitch angle θ).
  - cyclotron ($v \ll c$),
  - transrelativistic (!),
  - synchrotron $(\gamma \sin\theta)^3 \gg 1$.

General treatment - 2

Emitted (radiated) power $L_{\rm s.p.}$ is *Lorentz invariant*

$$L_{\rm s.p.} = \frac{2e^2}{3m^2c^3}\left(\frac{{\rm d}p_\mu}{{\rm d}\tau}\frac{{\rm d}p^\mu}{{\rm d}\tau}\right)$$

In a magnetic field

$$\frac{\dot\gamma}{\gamma} = -\frac{2e^2}{3mc^3}\gamma\beta^2\Omega_{\rm L}^2\sin^2\theta$$

$$= -\frac{2\sigma_{\rm T}\gamma\beta^2}{mc}\frac{B^2}{8\pi}\sin^2\theta$$

Beaming etc.

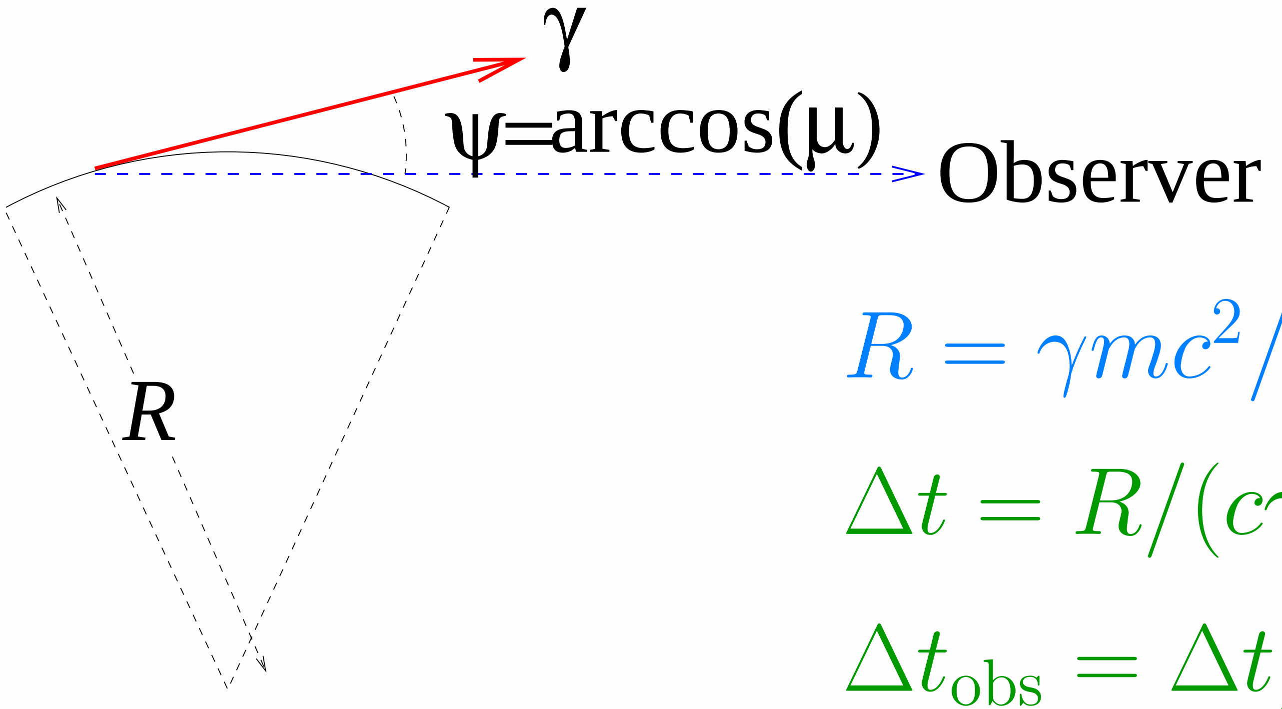
Doppler factor:

$$\mathcal{D} = \frac{1}{\Gamma(1-\beta\mu)}$$
$$\nu = \mathcal{D}\nu'$$

Lorentz scalar: $I_\nu/\nu^3 = I'_{\nu'}/\nu'^3 \Rightarrow I_\nu = \mathcal{D}^3 I'_{\nu'}$

For $\Gamma \gg 1$, $\mathcal{D}$ strongly peaked around $\psi = 0$, width $\sim 1/\Gamma$.

Characteristic frequency:
relativistic particles in magnetic fields



$$R = \gamma m c^2 / e B_\perp$$

$$\Delta t = R/(c\gamma)$$

$$\Delta t_{\rm obs} = \Delta t/\gamma^2$$

Characteristic frequency $= \gamma^2 e B_\perp/(mc)$

Restrictions

Restrictions:

- $B_\perp$ constant over distance $mc^2/eB_\perp$, otherwise jitter radiation
- High harmonic number $s = (\gamma \sin \alpha)^3 \gg 1$, otherwise Very Small Pitch Angle radiation (α: pitch angle)
- Classical regime $B \ll B_{\rm crit}/\Gamma$ otherwise Klein-Nishina-like corrections ($B_{\rm crit} = 4.414 \times 10^{13}$ G)

Approximations

Small parameters: two angles

- Pitch angle: for a smooth distribution of particles $\mathrm{d}N/\mathrm{d}\Omega\mathrm{d}\gamma$ integrate single particle emission over all directions and use

$$\frac{\mathrm{d}L}{\mathrm{d}\Omega\mathrm{d}\nu} = \int \mathrm{d}\gamma \underbrace{\frac{\mathrm{d}L_{\mathrm{s.p.}}}{\mathrm{d}\nu}}_{\text{kernel: function of } \gamma,\ \nu,\ \theta} \frac{\mathrm{d}N}{\mathrm{d}\Omega\mathrm{d}\gamma}$$

  essentially an expansion in $(\mathrm{d}N/\mathrm{d}\mu)\,/\,(\gamma N)$

- Gyrophase: convert sum over harmonics to an integral and replace Bessel functions using *Airy integral approximation* (asymptotic expansion in $1/s$)

Single particle emission - 1

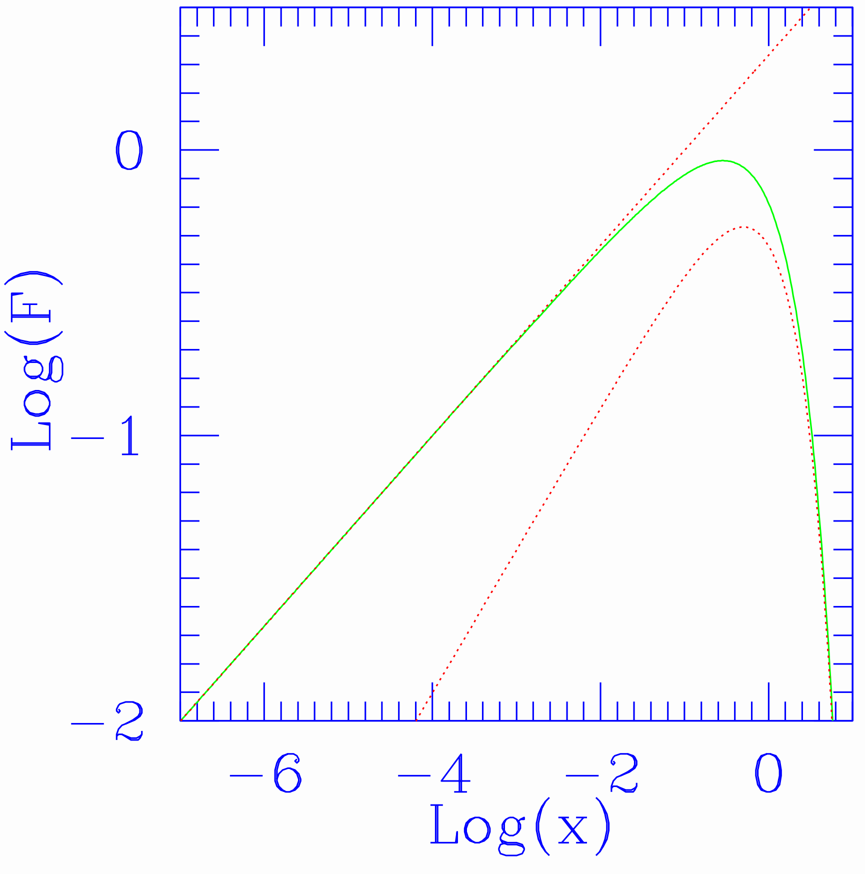
$$\frac{\mathrm{d}L_{\mathrm{s.p.}}}{\mathrm{d}\nu} = \sqrt{3}\,\frac{e^2}{\hbar c}\,\hbar\Omega_{\mathrm{L}}\,\sin\theta\,F\left(\nu/\nu_{\mathrm{c}}\right)$$

$$\nu_{\mathrm{c}}(\gamma,\theta) = \frac{3\Omega_{\mathrm{L}}\sin\theta\gamma^2}{4\pi}$$

$$= \nu_0\gamma^2 \qquad [\nu_0 = 3\Omega_{\mathrm{L}}\sin\theta/(4\pi)]$$

$$F(x) = x\int_x^\infty \mathrm{d}t K_{5/3}(t)$$

Single particle emission - 2



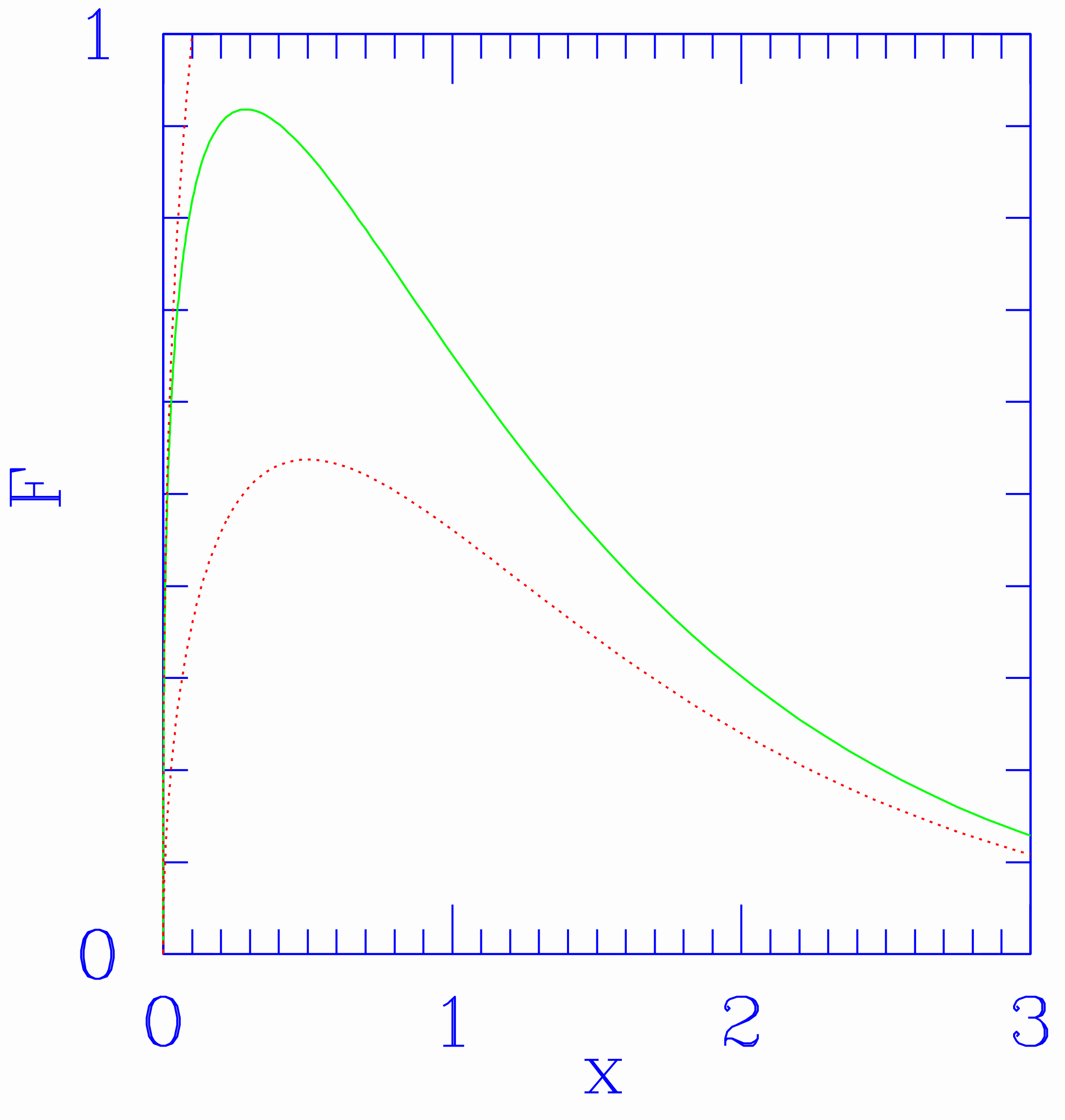
$$F(x) \approx \frac{4\pi}{\sqrt{3}\Gamma(1/3)}\left(\frac{x}{2}\right)^{1/3}$$

$$(x \ll 1)$$

$$\sim \sqrt{\frac{x\pi}{2}}\exp(-x)$$

$$(x \to \infty)$$

Single particle emission - 3



Maximum:

$x = 0.29$

Emissivity power-law distribution

An important special case is

$$\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{r}\mathrm{d}\Omega\mathrm{d}\gamma} = \frac{C}{4\pi}\gamma^{-q} \quad \text{for} \quad \gamma_1 < \gamma < \gamma_2$$

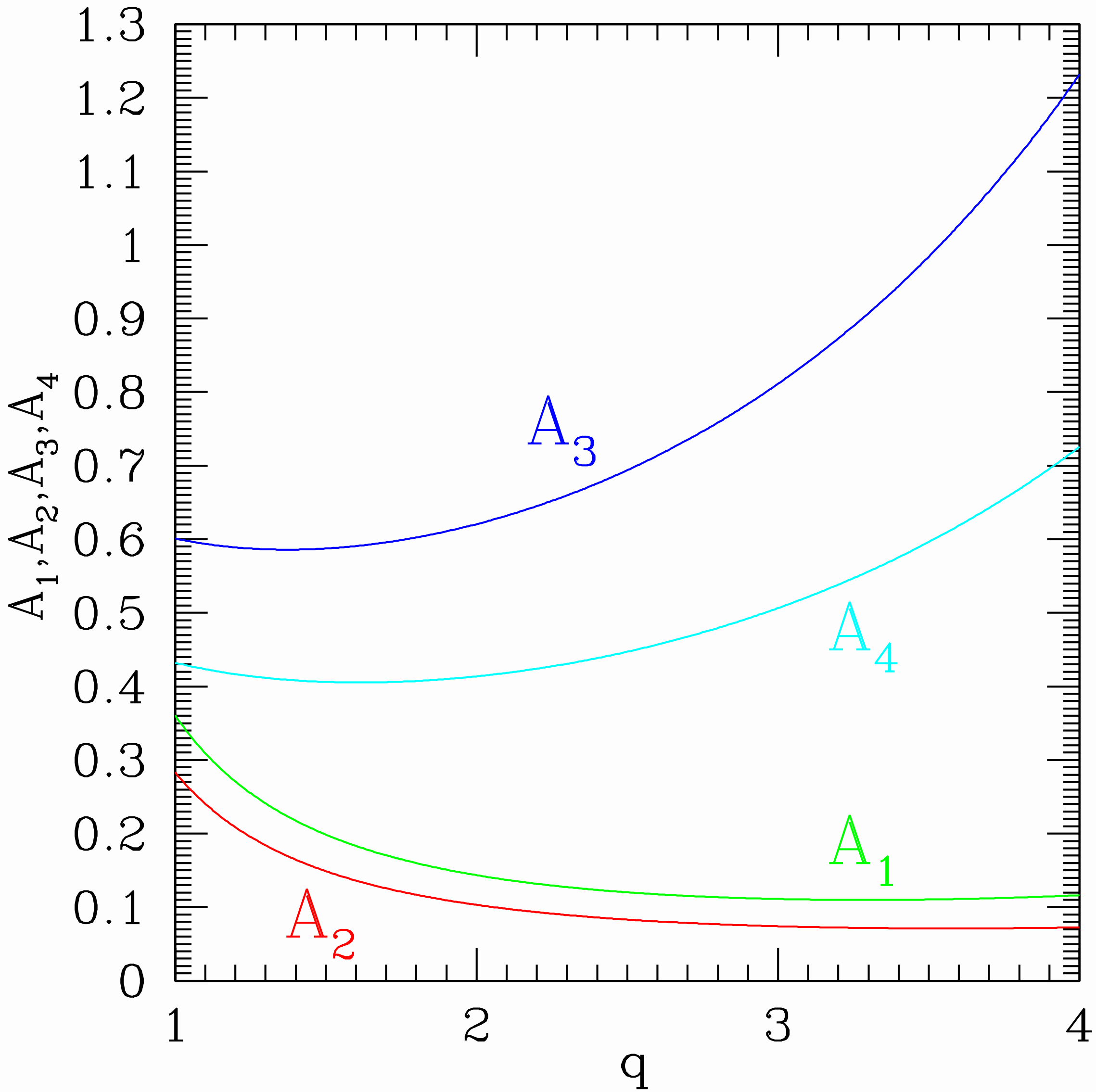
Then (in the optically thin case)

$$P_\nu = \frac{\mathrm{d}L}{\mathrm{d}\Omega\mathrm{d}\nu} = V\,j_\nu = V\int_0^\infty \mathrm{d}\gamma\,\frac{\mathrm{d}L_{\mathrm{s.p.}}}{\mathrm{d}\nu}\,\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{r}\mathrm{d}\Omega\mathrm{d}\gamma}$$

$$= C\,V\,\frac{e^2}{\hbar c}\hbar\Omega_{\mathrm{L}}\sin\theta\left(\frac{\nu}{\nu_0}\right)^{(1-q)/2} A_1(q) \qquad \text{for } q > 1/3$$

provided that $\gamma_1^2 \ll \nu/\nu_0 \ll \gamma_2^2$

The functions $A_{1,2,3,4}$



Angle average:
$j \rightarrow \langle j \rangle$ etc.
in formulae:
$\sin\theta \rightarrow 1$
$\nu_0 \rightarrow \bar{\nu}_0 = 3\Omega_{\mathrm{L}}/(4\pi)$

$A_1(q)$: emission

$A_2(q)$: $\langle$emission$\rangle$

$A_3(q)$: absorption

$A_4(q)$: $\langle$absorption$\rangle$

Synchrotron absorption

Transport equation:

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = j_\nu - \alpha_\nu I_\nu$$

Classical mechanism $\Rightarrow$ Absorption $\approx$ stim. emission.

$$\alpha_\nu = \frac{-1}{2m\nu^2}\int \mathrm{d}\gamma \frac{\mathrm{d}L_{\mathrm{s.p.}}}{\mathrm{d}\nu}\gamma^2 \frac{\mathrm{d}}{\mathrm{d}\gamma}\left(\frac{1}{\gamma^2}\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{r}\mathrm{d}\Omega\mathrm{d}\gamma}\right)$$

(unpolarised radiation). For a power-law distribution

$$\alpha_\nu = \frac{2\pi\alpha_{\mathrm{f}}C}{\sin\theta}\left(\frac{c}{\Omega_{\mathrm{L}}}\right)^2\left(\frac{\hbar\Omega_{\mathrm{L}}}{mc^2}\right)\left(\frac{\nu}{\nu_0}\right)^{-(q+4)/2} A_3(q)$$

Synchrotron spectrum - 1

Optically thin: $I_\nu = L j_\nu \propto \nu^{(1-q)/2}$

Optically thick: $I_\nu = j_\nu/\alpha_\nu \propto \nu^{5/2}$

Transition at $\alpha_\nu R = 1 \Rightarrow \nu_{\rm t} \propto R^{-2/(q+4)} B^{(q+2)/(q+4)}$

Given radiation at ν from electrons of $\gamma = \sqrt{\nu/\nu_0}$
the brightness temperature $k_{\rm B} T_{\rm B}(\nu) = c^2 I_\nu/(2\nu^2)$ is

$$k_{\rm B} T_{\rm B}(\nu) \;\approx\; mc^2\sqrt{\nu/\nu_0} \approx \gamma mc^2$$

in optically thick part of spectrum.

Synchrotron spectrum - 2

Energy density in radiation field near surface:

$$U_{\mathrm{rad}} = \int \frac{\mathrm{d}\Omega \mathrm{d}\nu}{c} I_\nu \approx \frac{2\pi}{c} \int \mathrm{d}\nu I_\nu$$

For $q < 3$, dominant contribution from near the upper cut-off in optically thin region:

$$U_{\mathrm{rad}} \approx \frac{4\pi \nu_{\mathrm{t}}^3 k_{\mathrm{B}} T_{\mathrm{max}}}{c^3} \int_1^{\nu_0 \gamma_2^2 / \nu_{\mathrm{t}}} \mathrm{d}x x^{(1-q)/2}$$

$$\approx 18 \frac{\nu_2 e^2}{mc^3} \left(\frac{k_{\mathrm{B}} T_{\mathrm{max}}}{mc^2} \right)^5 \left(\frac{B^2}{8\pi} \right) \quad \text{for } q = 1$$

Problem: given volume V and $P_\nu = P_1\,(\nu/\nu_1)^{-\alpha}$ for $\nu_1 < \nu < \nu_2$ (V, P_1, $\nu_{1,2}$, α observed quantities), what is the miminum energy requirement in the source?

Variables: C, B; $\quad \nu^\alpha P_\nu = \nu_1^\alpha P_1 = CV(e^2/\hbar c)\hbar\Omega_{\mathrm{L}}\bar{\nu}_0{}^\alpha A_2$

Minimize

$$E = \underbrace{V\frac{B^2}{8\pi}}_{magnetic} + \underbrace{(1+k)}_{dark\ particles}\underbrace{V\,C\int_{\gamma_1}^{\gamma_2}\mathrm{d}\gamma\,\gamma mc^2\gamma^{-2\alpha-1}}_{electrons}$$

$$= V\frac{B^2}{8\pi} + \frac{(1+k)mc^2}{2}V\,C\bar{\nu}_0{}^{(2\alpha-1)/2}\int_{\nu_1}^{\nu_2}\mathrm{d}\nu\,\nu^{-(2\alpha+1)/2}$$

Minimum energy - 2

$$E = V\frac{B^2}{8\pi} + b(1+k)B^{-3/2}$$

with

$$b = P_1 \nu_1^{1/2}\left[\sqrt{\frac{\pi}{3}}\frac{m^{5/2}c^{9/2}}{e^{7/2}}\right]\int_1^{\nu_2/\nu_1} \mathrm{d}x\, x^{-(2\alpha+1)/2}$$

At minimum, $B = [6\pi b(1+k)/V]^{2/7}$ and

$$E_{\min} = \frac{7}{4\,(6\pi)^{3/7}}(1+k)^{4/7}V^{3/7}b^{4/7}$$

Min. Pressure $= E_{\min}/V \propto (P_1/V)^{4/7}$

Moving sources - 1

Remember,

$$\begin{aligned} \nu &= \mathcal{D}\nu' \\ \gamma &= \mathcal{D}\gamma' \end{aligned}$$

Invariance of phase space density:

$$\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{r}\mathrm{d}\Omega\mathrm{d}\gamma} = \mathcal{D}^2\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{r}'\mathrm{d}\Omega'\mathrm{d}\gamma'}$$

and of emitted power:

$$L_{\mathrm{s.p.}} = L'_{\mathrm{s.p.}}$$

Moving sources - 2

Basic formula:

$$P_\nu = \int \mathrm{d}^3\mathbf{r} \int_0^\infty \mathrm{d}\gamma \, \frac{\mathrm{d}L_{\mathrm{s.p.}}}{\mathrm{d}\nu} \, \frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{r}\mathrm{d}\Omega\mathrm{d}\gamma}$$

If limits on $\mathbf{r}$ integration are time-dependent:

$$\int_{\mathrm{source}} \mathrm{d}^3\mathbf{r} = \int \mathrm{d}^3\mathbf{r} \int \mathrm{d}t \, \delta\left(t - \mathbf{r}\cdot\hat{\mathbf{n}}/c\right)$$

$$= \mathcal{D} \int_{\mathrm{source}} \mathrm{d}^3\mathbf{r}'$$

and $V = \mathcal{D}\, V'$ is an *effective* volume. In this case:

$$P_\nu = \mathcal{D}^3 P'_{\nu'}$$

Moving sources - 3

To find the minimum energy magnetic field, need transformation rules for b and V':

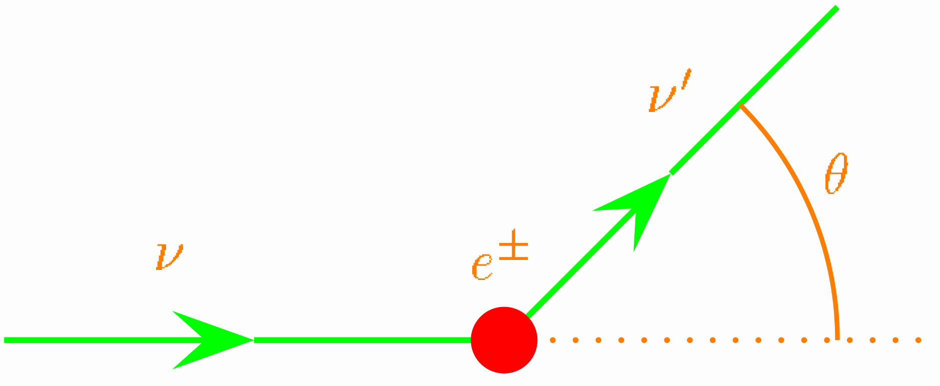
$$\begin{aligned} b &\propto P_1 \nu_1^{1/2} \\ &= \mathcal{D}^{7/2} b' \end{aligned}$$

Estimates of the source volume:

- From angular extent: $V' = (D_{\rm A}\delta\theta)^3 \Rightarrow V = V'$, and $B'_{\rm min} = B_{\rm min}/\mathcal{D}$
- From a variability timescale: $V' = (\mathcal{D}ct_{\rm var})^3 \Rightarrow V = V'/\mathcal{D}^3$ and $B'_{\rm min} = B_{\rm min}/\mathcal{D}^{13/7}$

Compton scattering

Thomson scattering, electron at rest

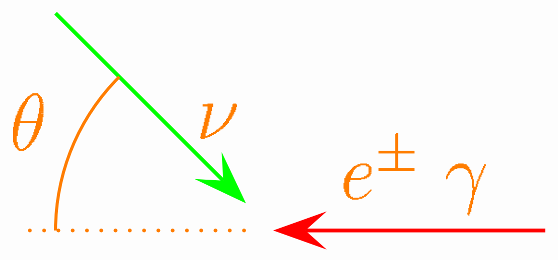


$$
\begin{aligned}
d\sigma/d\Omega &= 3\sigma_{\mathrm{T}}(1+\cos^2\theta)/(16\pi) \\
\nu' &= \nu/\left[1+(h\nu/mc^2)(1-\cos\theta)\right] \\
&\approx \nu, \ \text{for } h\nu \ll mc^2
\end{aligned}
$$

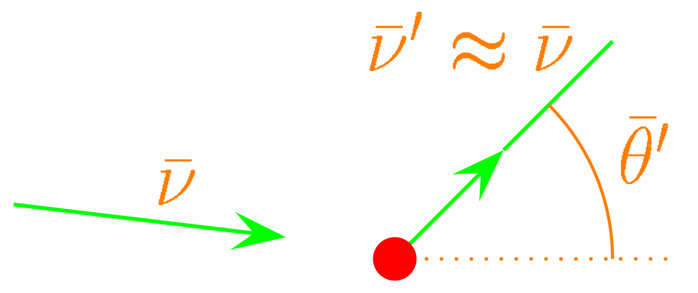
Inverse Compton scattering - 1

Thomson scattering, relativistic electron

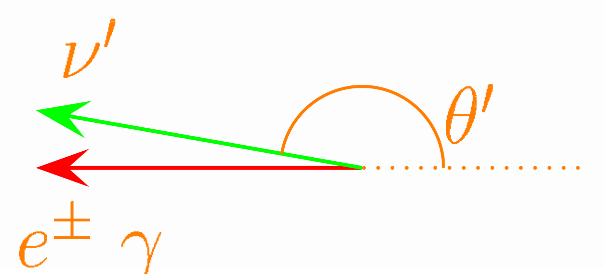
Lab. frame



Rest frame



Lab. frame



$$\bar{\nu} = \nu\gamma\left(1 + v\cos\theta/c\right)$$
$$\bar{\nu}' = \bar{\nu}$$
$$\nu' = \gamma\bar{\nu}'\left(1 + v\cos\bar{\theta}'/c\right)$$

Inverse Compton scattering - 2

Emitted frequency (after angle average):

$$\nu' = 4\gamma^2\nu/3$$

Forward beaming:

$$\begin{aligned} \cos\theta' &= \left(\cos\bar{\theta}' - v/c\right) / \left(1 - v\cos\bar{\theta}'/c\right) \\ &\approx -1 - 1/\left[\gamma^2\left(1 - \cos\bar{\theta}'\right)\right] \end{aligned}$$

Cooling rate (isotropic targets):

$$\frac{-\dot{\gamma}}{\gamma} = \frac{4}{3}\frac{\sigma_{\mathrm{T}}}{mc} \times \text{target energy density} \times \beta^2\gamma$$

Close analogy with synchrotron radiation. . .

Klein-Nishina regime

For $h\bar{\nu} \sim mc^2$ in electron rest frame ($\gamma h\nu \sim mc^2$) *recoil* important.

$\Rightarrow$ Maximum photon energy $\approx \gamma mc^2$

$\Rightarrow$ Cross-section reduced

$\Rightarrow$ Pair-production threshold exceeded

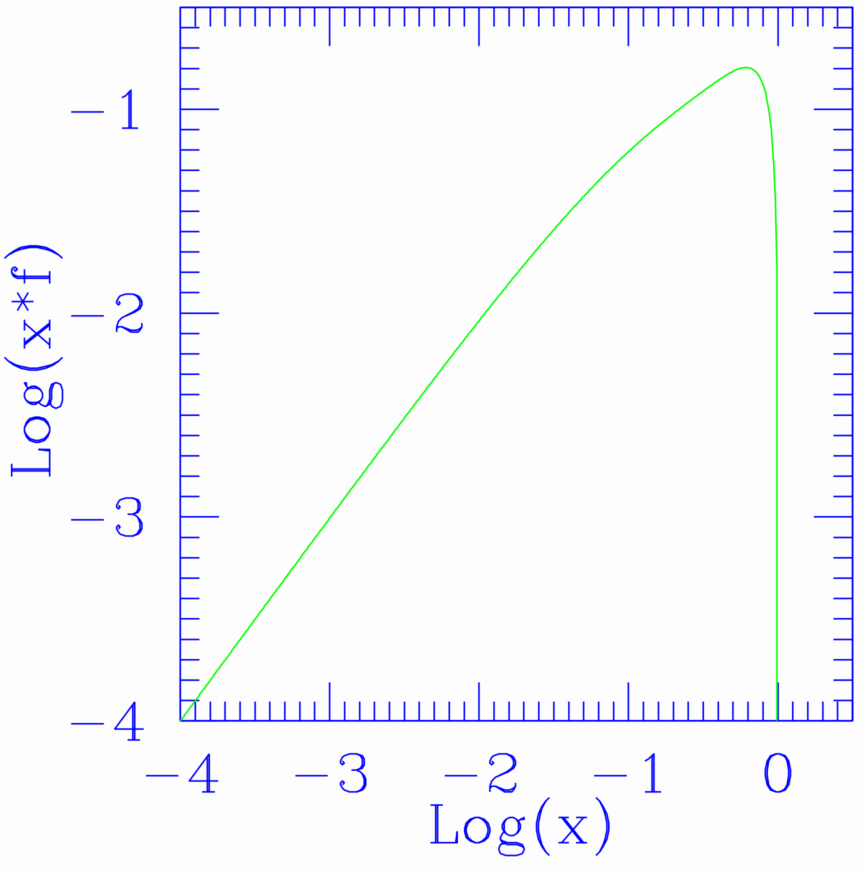
(For $\gamma + \gamma \to e^+ + e^-$ require $\nu'\nu > m^2c^4/h^2$.)

Single particle emissivity - 1

For *relativistic electrons* ($\gamma \gg 1$) in inverse Compton scattering ($\nu > \nu_\mathrm{s}$) off *isotropic* target photons (spectral energy density $\mathrm{d}U_\mathrm{s}/\mathrm{d}\nu_\mathrm{s}$) in the *Thomson limit*:

$$
\begin{aligned}
\frac{\mathrm{d}L_\mathrm{s.p.}}{\mathrm{d}\nu} &= 3\sigma_\mathrm{T} c \int \mathrm{d}\nu_\mathrm{s} \frac{\mathrm{d}U_\mathrm{s}}{\mathrm{d}\nu_\mathrm{s}} \frac{\nu}{\nu_\mathrm{IC}} f\left(\nu/\nu_\mathrm{IC}\right) \\
\nu_\mathrm{IC}(\gamma) &= 4\gamma^2 \nu_\mathrm{s} \\
f(x) &= \begin{cases} 2x\ln x + x + 1 - 2x^2 & \text{for } x < 1 \\ 0 & \text{for } x > 1 \end{cases}
\end{aligned}
$$

Single particle emissivity - 2



Exact kinematics
⇒ $f(x) = 0$,
for $x < 1/(4\gamma^2)$.

Emissivity power-law distribution

In the source frame, $\int \mathrm{d}{r'}^3 = V'$ and

$$\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{r}'\mathrm{d}\Omega'\mathrm{d}\gamma'} = \frac{C}{4\pi}{\gamma'}^{-q} \quad \text{for} \quad \gamma_1 < \gamma' < \gamma_2$$

Angular distribution of targets?

- Cosmic background radiation: isotropic in galaxy frame
- Self-produced synchrotron photons: isotropic in source frame

IC off the CMB

In the galaxy frame $\int \mathrm{d}r^3 \to \mathcal{D}V'$ and

$$\left(\frac{\mathrm{d}N}{\mathrm{d}^3 r\,\mathrm{d}\Omega\,\mathrm{d}\gamma}\right) = \frac{\mathcal{D}^2 C}{4\pi}\left(\frac{\gamma}{\mathcal{D}}\right)^{-q}$$

$$P_\nu = \mathcal{D}V' \int_0^\infty \mathrm{d}\gamma\, \frac{\mathrm{d}L_{\mathrm{s.p.}}}{\mathrm{d}\nu}\, \frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{r}\,\mathrm{d}\Omega\,\mathrm{d}\gamma}$$

$$= \frac{\mathcal{D}^{3+q} C V' \sigma_{\mathrm{T}} c U_{\mathrm{CMB}}}{\nu_{\mathrm{CMB}}}\left(\frac{\nu}{\nu_{\mathrm{CMB}}}\right)^{(1-q)/2} A_{\mathrm{IC}}(\alpha)$$

Provided $4\nu_{\mathrm{CMB}}\gamma_1^2 \ll \nu \ll 4\nu_{\mathrm{CMB}}\gamma_2^2$

$[h\nu_{\mathrm{CMB}} = 3k_{\mathrm{B}}T_{\mathrm{CMB}},\, U_{\mathrm{CMB}} = aT_{\mathrm{CMB}}^4]$

Synchrotron self-Compton (SSC)

Compute in the source frame, then use $P_\nu = \mathcal{D}^3 P'_{\nu'}$
Geometry important for target density:

$$\frac{\mathrm{d}U_\mathrm{s}}{\mathrm{d}\nu_\mathrm{s}} = \frac{P_{\nu_\mathrm{s}}}{\langle \text{Area} \rangle\, c}$$

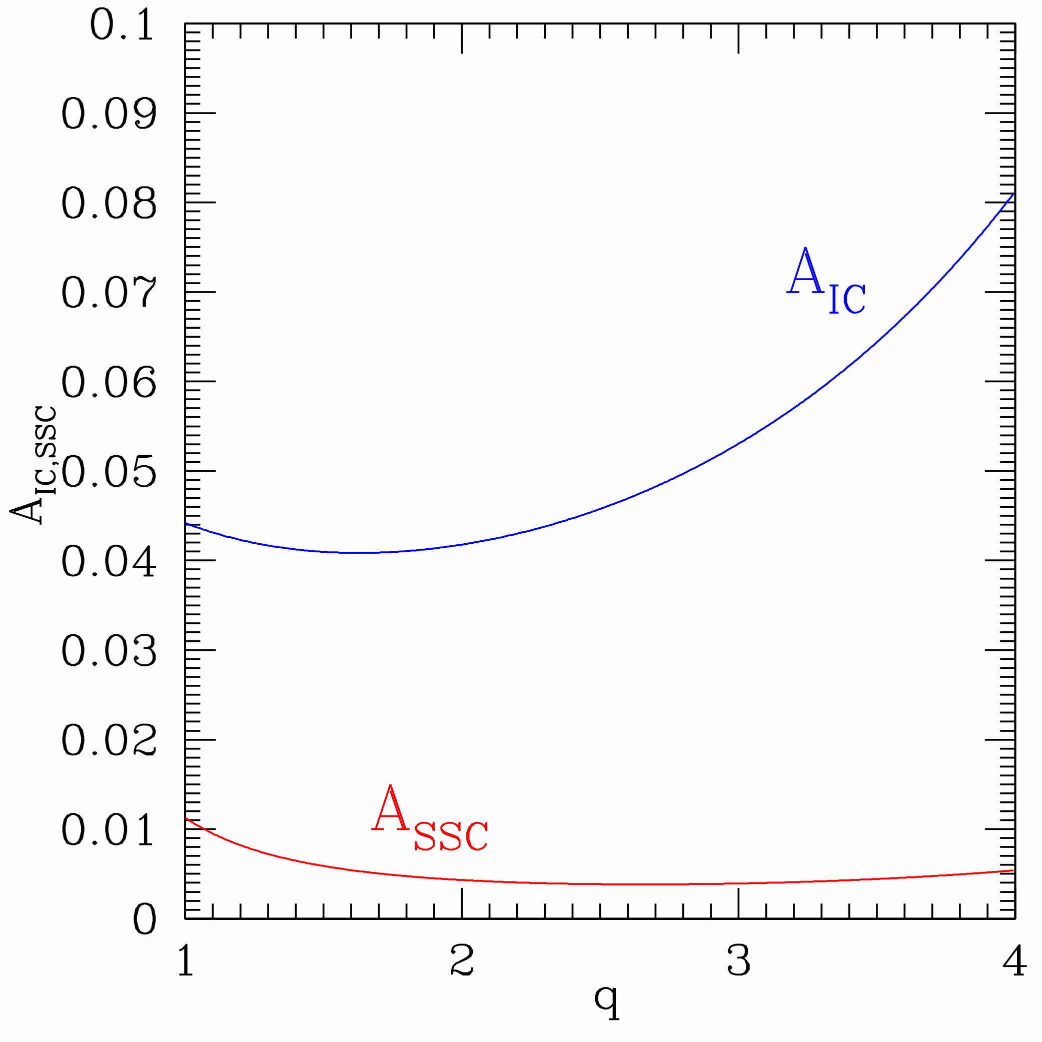
Simplest solution: set $\langle \text{Area} \rangle = V^{2/3}$

$$P_\nu = \mathcal{D}^{(5+q)/2} C^2 \sigma_\mathrm{T} V^{4/3} \frac{e^2}{\hbar c} \hbar\Omega_\mathrm{L} \left(\frac{\nu}{\nu_0}\right)^{(1-q)/2} \ln \Sigma_\mathrm{G}\, A_\mathrm{SSC}(q)$$

where Σ_G is Gould's *Compton logarithm*

[A&A 76, 306 (1979)]

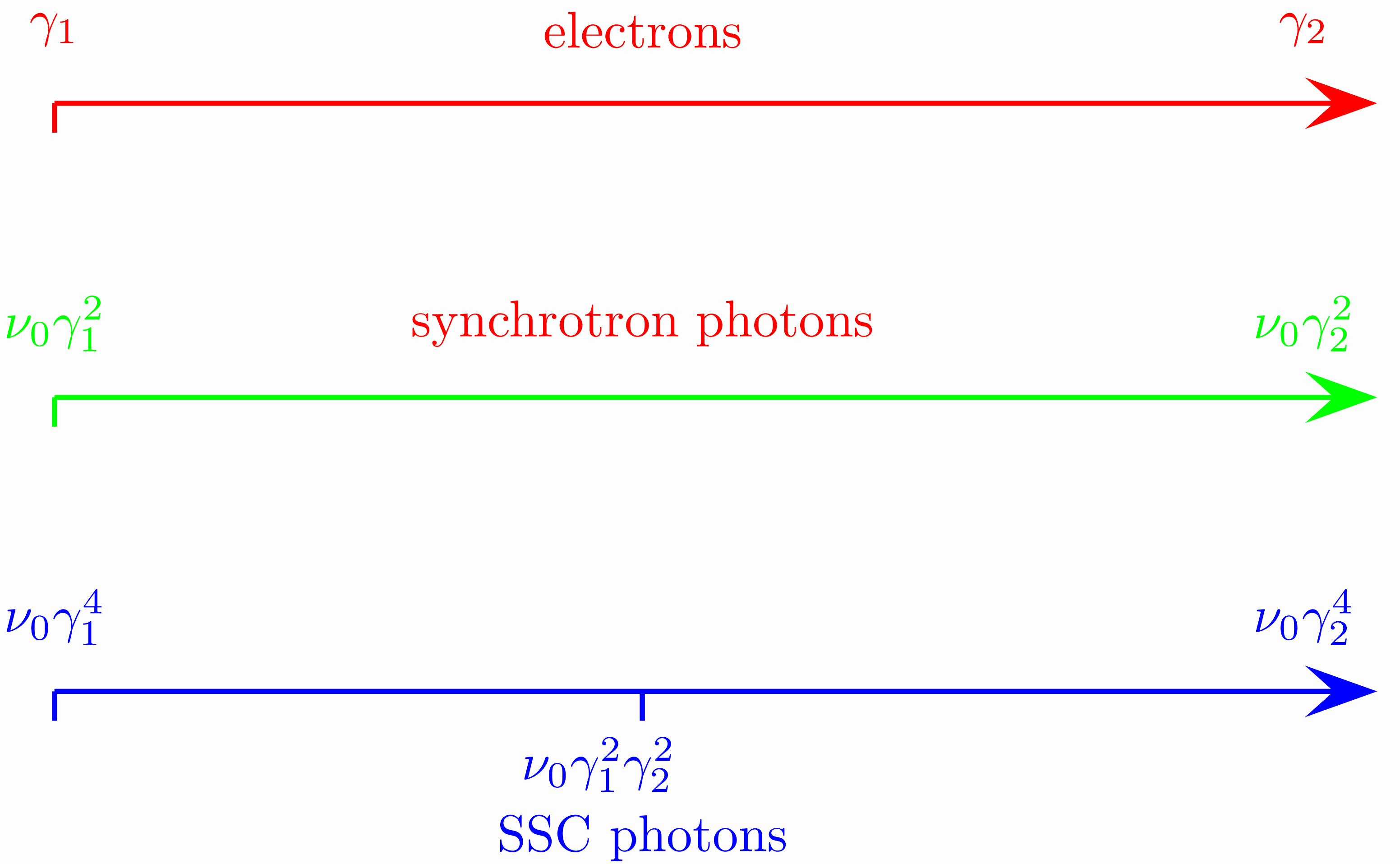
The functions $A_{IC,SSC}$



The functions $A_{\rm IC}(q)$ and $A_{\rm SSC}$ for inverse Compton scattering by a power-law distribution of electrons off either a black body distribution or self-produced synchrotron target photons

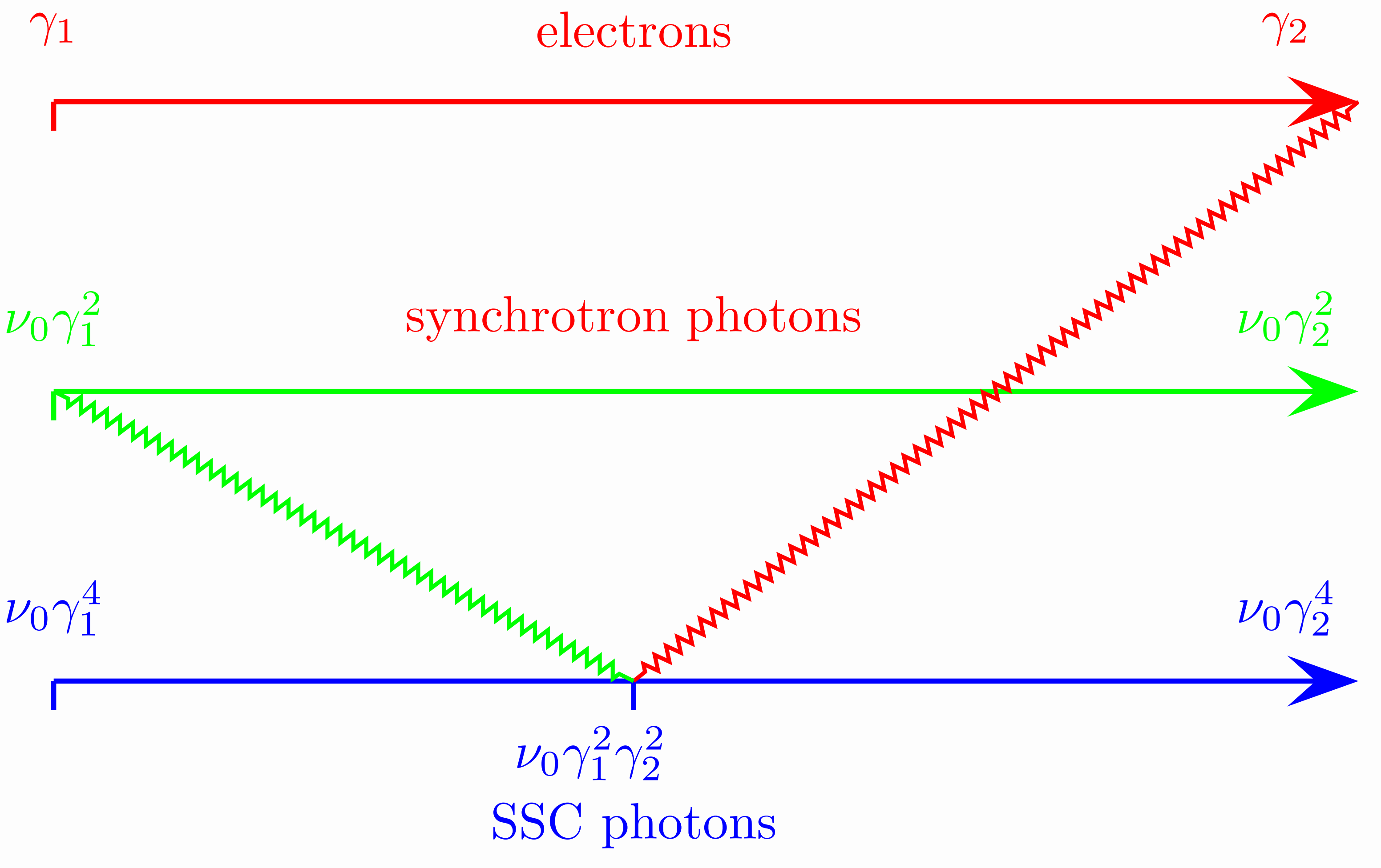
Synchrotron self-Compton - 2

Frequency dependence of the Compton logarithm



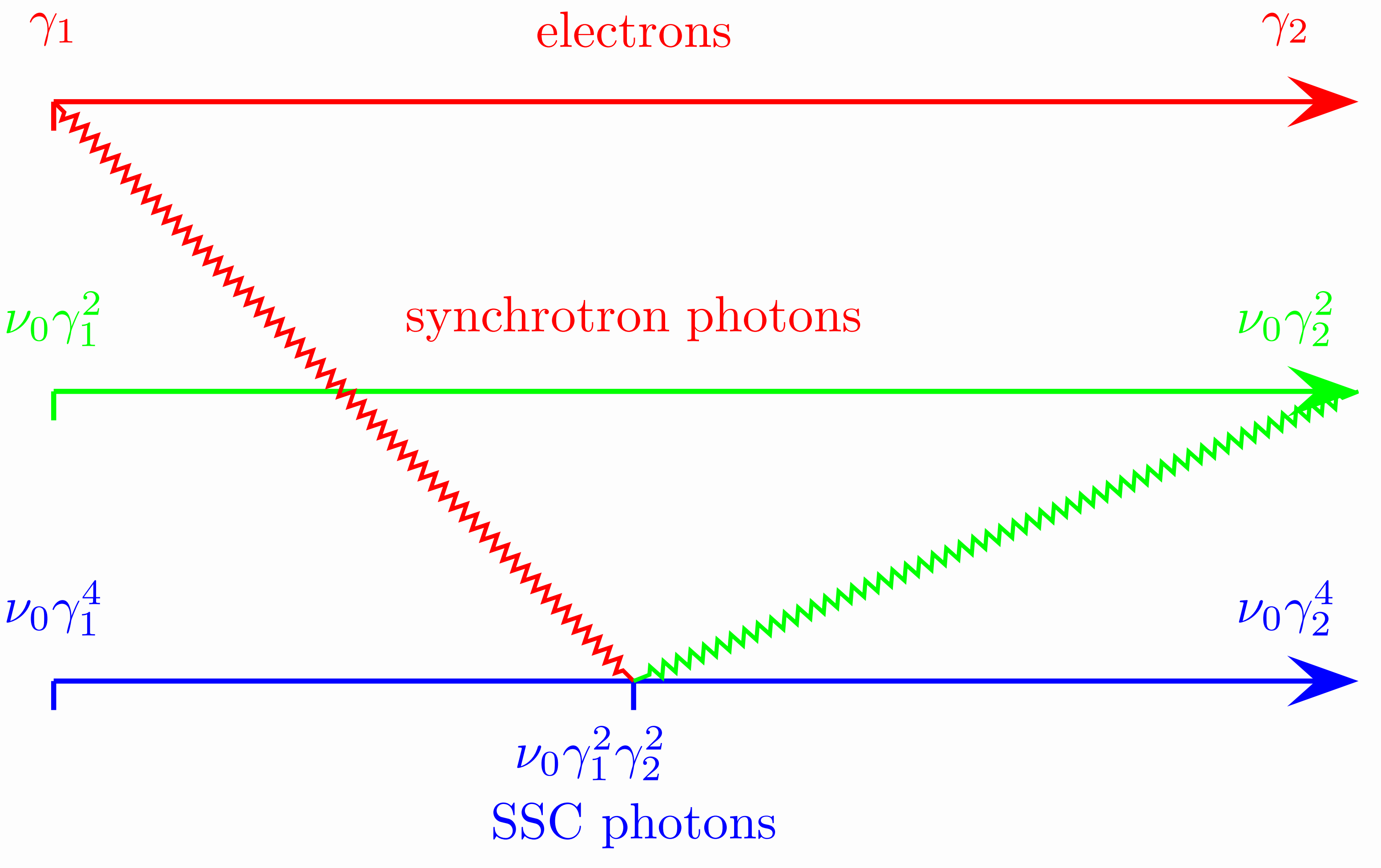
Synchrotron self-Compton - 2

Frequency dependence of the Compton logarithm



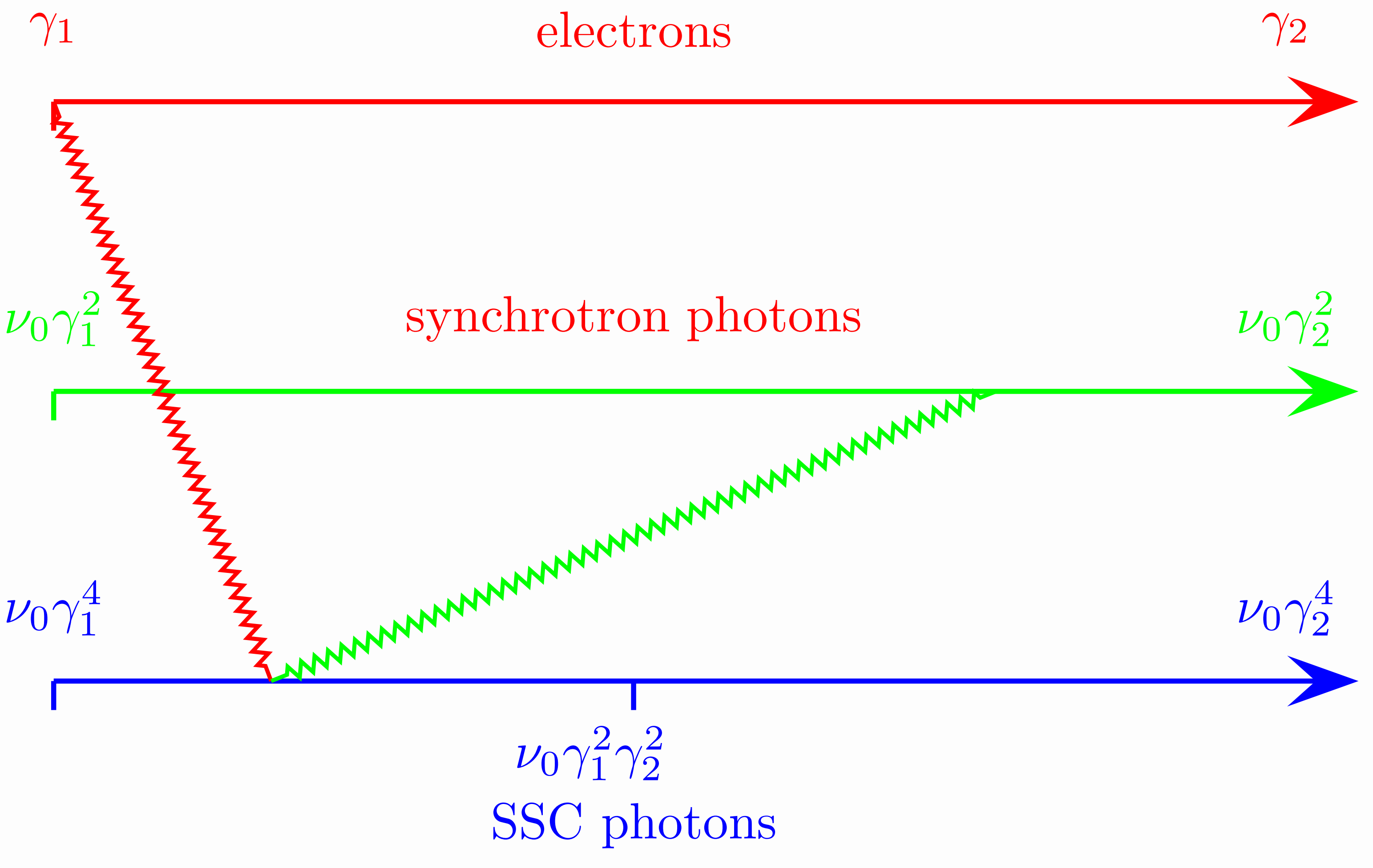
Synchrotron self-Compton - 2

Frequency dependence of the Compton logarithm



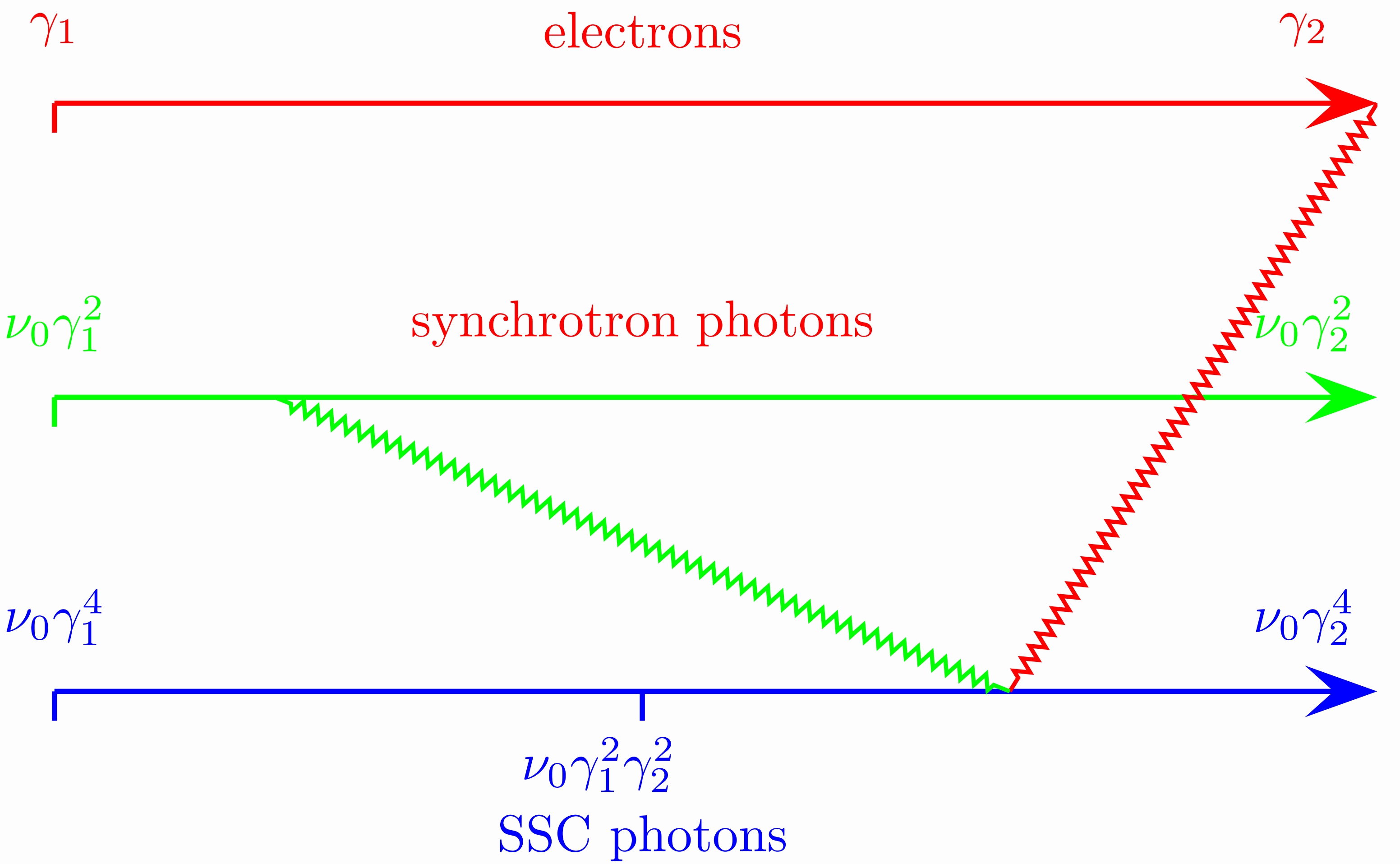
Synchrotron self-Compton - 2

Frequency dependence of the Compton logarithm

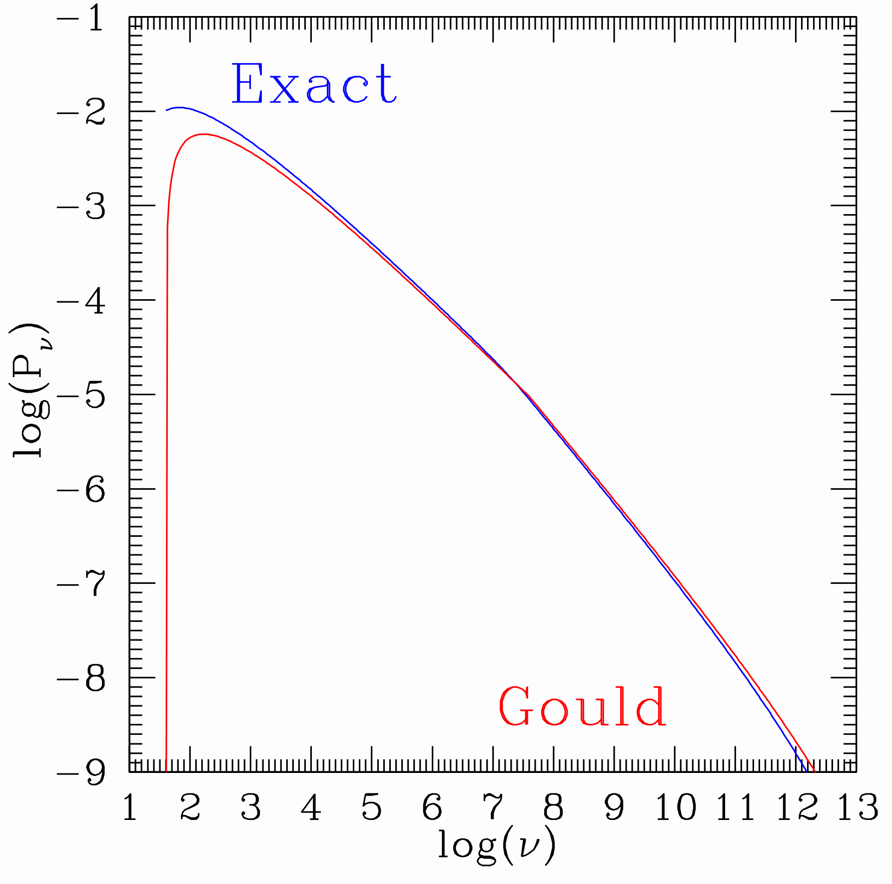


Synchrotron self-Compton - 2

Frequency dependence of the Compton logarithm



Compton logarithm



SSC spectrum for dynamic range $\gamma_2/\gamma_1 = 10^5$

$\alpha = (q-1)/2 = 0.7$

Blue (“exact”): numerical evaluation

Red: Gould’s approx.

Spectral break at $\nu = \gamma_1^2\gamma_2^2\nu_0 = 10^7$.

Compton Catastrophe - 1

$$\frac{L_{\rm C}}{L_{\rm s}} = \frac{L/\left(R^2 c\right)}{U_B} \quad \text{and} \quad L = L_{\rm C} + L_{\rm s}$$

$$\Rightarrow L = \frac{L_{\rm s}}{(1-\zeta)}$$

where $\zeta = \left(L_{\rm s}/R^2 c\right)/U_B$. *Catastrophe* when $\zeta \geq 1$

for $q = 1$: $\zeta = 18\frac{\nu_2 e^2}{mc^3}\left(\frac{k_{\rm B}T_{\rm max}}{mc^2}\right)^5 = 2.3\left(\frac{\nu_2}{100\,\text{GHz}}\right)\left(\frac{T_{\rm max}}{10^{12}\text{K}}\right)^5$

Kellermann & Pauliny-Toth, ApJ 155, L71 (1969)

Compton Catastrophe - 2

Escape routes:

1. Softer synchrotron spectrum?
   Weak dependence $T_{\rm max} \propto \nu_2^{1/5}$
2. Re-absorption of IC photons? Would need $I \propto \nu^2$ for $h\nu < \gamma mc^2$
3. Klein-Nishina effects? No. of generations $\approx \ln(mc^2/h\nu_2)$ (~ 25 for $\nu_2 \sim 1$ GHz)
4. Relativistic boosting? Unresolved source: $T_{\rm max} \propto \mathcal{D}^{-3}$, resolved source $T_{\rm max} \propto \mathcal{D}^{-1}$.

Lecture 3: Spectral Modelling

- Intergalactic absorption
- Breaks/cut-offs
- One and two-zone models for gamma-ray blazars

Intergalactic absorption

Optical depth of an isotropic photon field:

$$\tau \quad = \quad 2\pi \int \mathrm{d}r\, R \int_0^\infty \mathrm{d}\varepsilon_\mathrm{s} n(\varepsilon_\mathrm{s}) \int_0^\pi \mathrm{d}\psi \sin\psi (1+\cos\psi)\sigma_{\gamma\gamma}(\varepsilon')$$

$\varepsilon_\mathrm{s} = h\nu_\mathrm{s}/mc^2$, $n(\varepsilon_\mathrm{s}) = (mc^2/h)\mathrm{d}N/\mathrm{d}\Omega\mathrm{d}\nu$

Three integrations:

Intergalactic absorption

Optical depth of an isotropic photon field:

$$\tau = 2\pi \int \mathrm{d}r \, R \int_0^\infty \mathrm{d}\varepsilon_\mathrm{s} n(\varepsilon_\mathrm{s}) \int_0^\pi \mathrm{d}\psi \sin\psi (1 + \cos\psi) \sigma_{\gamma\gamma}(\varepsilon')$$

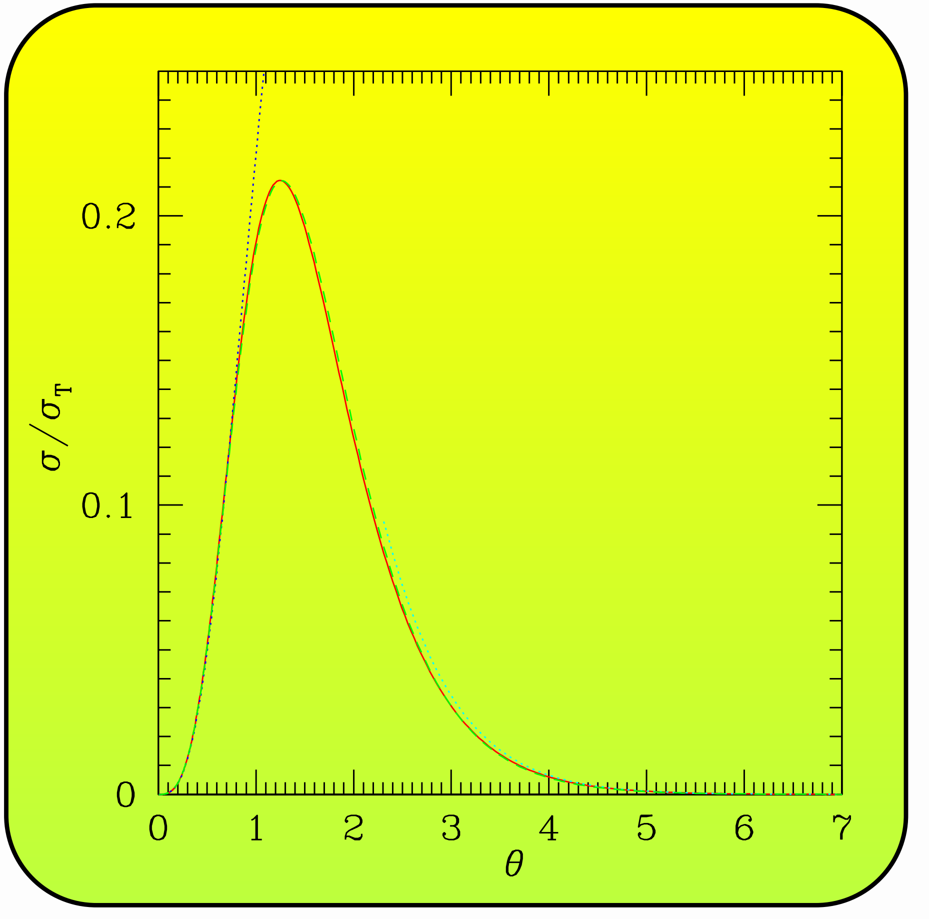
$\varepsilon_\mathrm{s} = h\nu_\mathrm{s}/mc^2$, $n(\varepsilon_\mathrm{s}) = (mc^2/h)\mathrm{d}N/\mathrm{d}\Omega\mathrm{d}\nu$

Three integrations:

1. angle

Angle-averaged γ-γ cross-section

Pair production for isotropic targets
$\cosh\theta$ = Lorentz factor of produced $e^{\pm}$
(in COM frame)



Intergalactic absorption

Optical depth of an isotropic photon field:

$$\tau \;=\; 2\pi \int \mathrm{d}r\, R \int_0^{\infty} \mathrm{d}\varepsilon_{\mathrm{s}} n(\varepsilon_{\mathrm{s}}) \int_0^{\pi} \mathrm{d}\psi \sin\psi (1+\cos\psi)\sigma_{\gamma\gamma}(\varepsilon')$$

$\varepsilon_{\mathrm{s}} = h\nu_{\mathrm{s}}/mc^2$, $n(\varepsilon_{\mathrm{s}}) = (mc^2/h)\mathrm{d}N/\mathrm{d}\Omega\mathrm{d}\nu$

Three integrations:

1. angle

Intergalactic absorption

Optical depth of an isotropic photon field:

$$\tau \;=\; 2\pi \int \mathrm{d}r\, R \int_0^{\infty} \mathrm{d}\varepsilon_{\rm s} n(\varepsilon_{\rm s}) \int_0^{\pi} \mathrm{d}\psi \sin\psi (1+\cos\psi)\sigma_{\gamma\gamma}(\varepsilon')$$

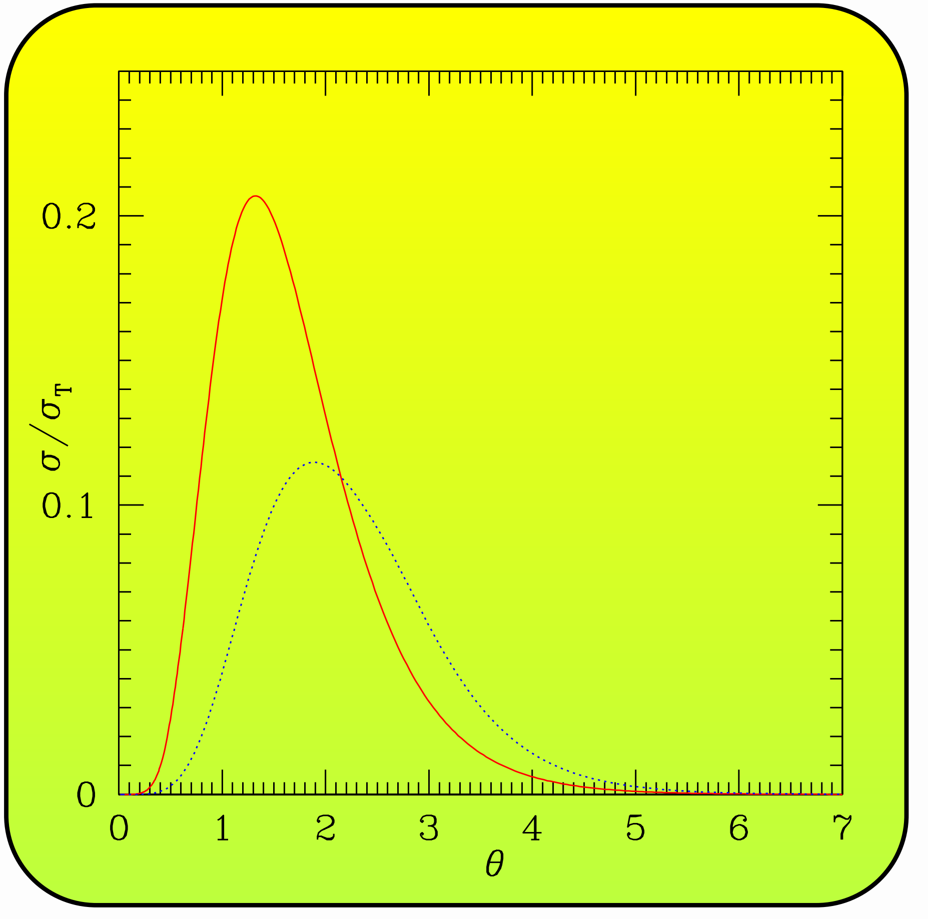
$\varepsilon_{\rm s} = h\nu_{\rm s}/mc^2$, $n(\varepsilon_{\rm s}) = (mc^2/h)\mathrm{d}N/\mathrm{d}\Omega\mathrm{d}\nu$

Three integrations:

1. angle
2. redshift

Redshift-averaged γ-γ cross-section

Pair production off non-evolving radiation field "Consensus" cosmology.
$z = 0.1$ (red) and
$z = 5$ (blue)
$\cosh\theta$ = Lorentz factor of produced $e^{\pm}$ (in COM frame)



Intergalactic absorption

Optical depth of an isotropic photon field:

$$\tau \;=\; 2\pi \int \mathrm{d}r\, R \int_0^\infty \mathrm{d}\varepsilon_\mathrm{s} n(\varepsilon_\mathrm{s}) \int_0^\pi \mathrm{d}\psi \sin\psi (1+\cos\psi)\sigma_{\gamma\gamma}(\varepsilon')$$

$\varepsilon_\mathrm{s} = h\nu_\mathrm{s}/mc^2$, $n(\varepsilon_\mathrm{s}) = (mc^2/h)\mathrm{d}N/\mathrm{d}\Omega\mathrm{d}\nu$

Three integrations:

1. angle
2. redshift

Intergalactic absorption

Optical depth of an isotropic photon field:

$$\tau = 2\pi \int \mathrm{d}r\, R \int_0^\infty \mathrm{d}\varepsilon_\mathrm{s} n(\varepsilon_\mathrm{s}) \int_0^\pi \mathrm{d}\psi \sin\psi (1+\cos\psi)\sigma_{\gamma\gamma}(\varepsilon')$$

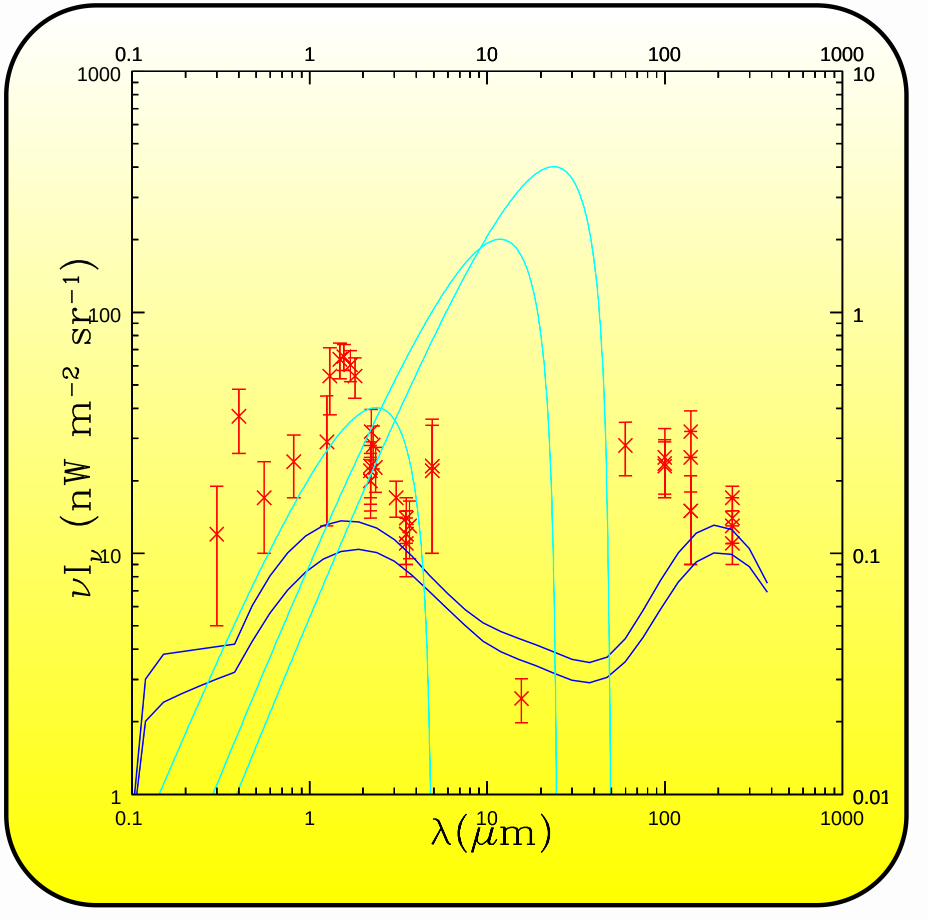
$\varepsilon_\mathrm{s} = h\nu_\mathrm{s}/mc^2$, $n(\varepsilon_\mathrm{s}) = (mc^2/h)\mathrm{d}N/\mathrm{d}\Omega\mathrm{d}\nu$

Three integrations:

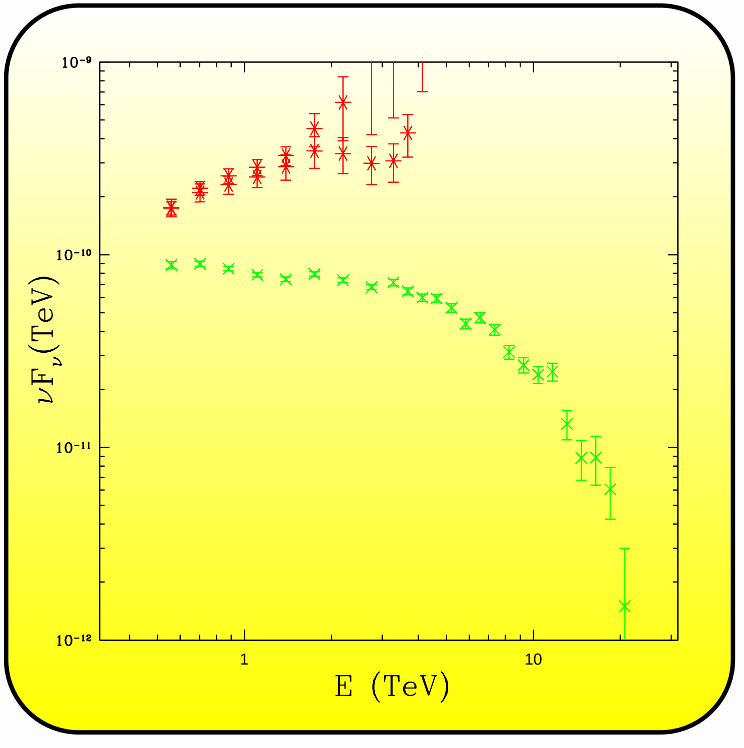
1. angle
2. redshift
3. target spectrum

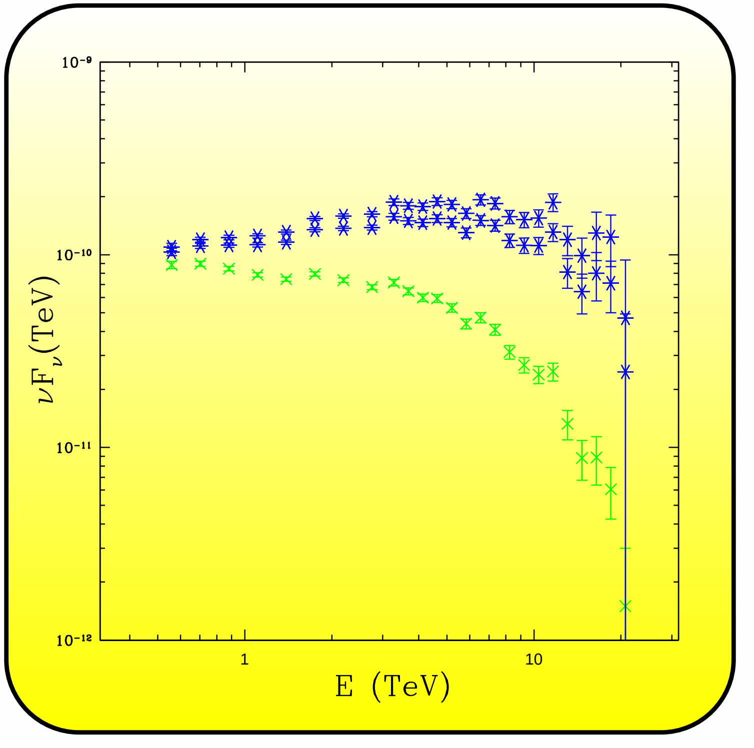
Target spectrum

Diffuse infra-red
background measurements
and models
together with
averaged γ-γ cross-section
for $z = 0.031$ (Mkn 501)
at
$h\nu = 1,\ 5,\ 10$ TeV



Mkn 501





De-absorbed spectrum of Mkn 501 in 1997:
Interpolated DIBR (left)
Malkan & Stecker models (right)

Cooling - 1

Power emitted by particles in range $d\gamma$:

$$dP = (-\dot{\gamma} mc^2) \frac{\mathrm{d}N}{\mathrm{d}\gamma} d\gamma$$

$$\Rightarrow \frac{dP}{d\gamma} \propto \underbrace{\gamma^2}_{\dot{\gamma}} \underbrace{\gamma^{-q}}_{\mathrm{d}N/\mathrm{d}\gamma}$$

($\mathrm{d}N/\mathrm{d}\gamma = CV\gamma^{-q}$). Monochromatic approx. $\nu \propto \gamma^2$:

$$\frac{dP}{d\nu} \propto \nu^{(1-q)/2} \equiv \nu^{-\alpha}$$

Assumes $\mathrm{d}N/\mathrm{d}\gamma$ not changed by the energy lost in radiation — applies when $t \ll t_{\mathrm{cool}}$.

Cooling - 2

If cooling important, specify not $n(\gamma) \equiv \mathrm{d}N/\mathrm{d}\gamma$, but $Q(\gamma)$ (rate of injection). Continuity eq. in phase space:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial \gamma}\left(\dot{\gamma} n\right) \;=\; Q(\gamma)$$

Stationary solution depends on particles injected at *higher* energy:

$$n(\gamma) \;=\; \frac{1}{-\dot{\gamma}}\int_{\gamma}^{\infty} d\gamma' Q(\gamma')$$

$\Rightarrow$ *steepened* or *cooled* spectrum: $Q \propto \gamma^{-q}$, $n \propto \gamma^{-q-1}$

$$\Rightarrow \; dP/d\nu \;\propto\; \nu^{-q/2}$$

Spectral breaks

Power-law spectrum $\Longleftrightarrow$ no preferred energy scale
E.g. for γ such that

$$t_{\rm acc} \ll t_{\rm cool} \ll t_{\rm esc}$$

or

$$t_{\rm acc} \ll t_{\rm esc} \ll t_{\rm cool}$$

Breaks/cut-offs occur at energies/frequencies where, for example

$$t_{\rm acc} \approx t_{\rm cool}$$

or

$$t_{\rm cool} \approx t_{\rm esc}$$

Implies *different dependence of timescales on* γ

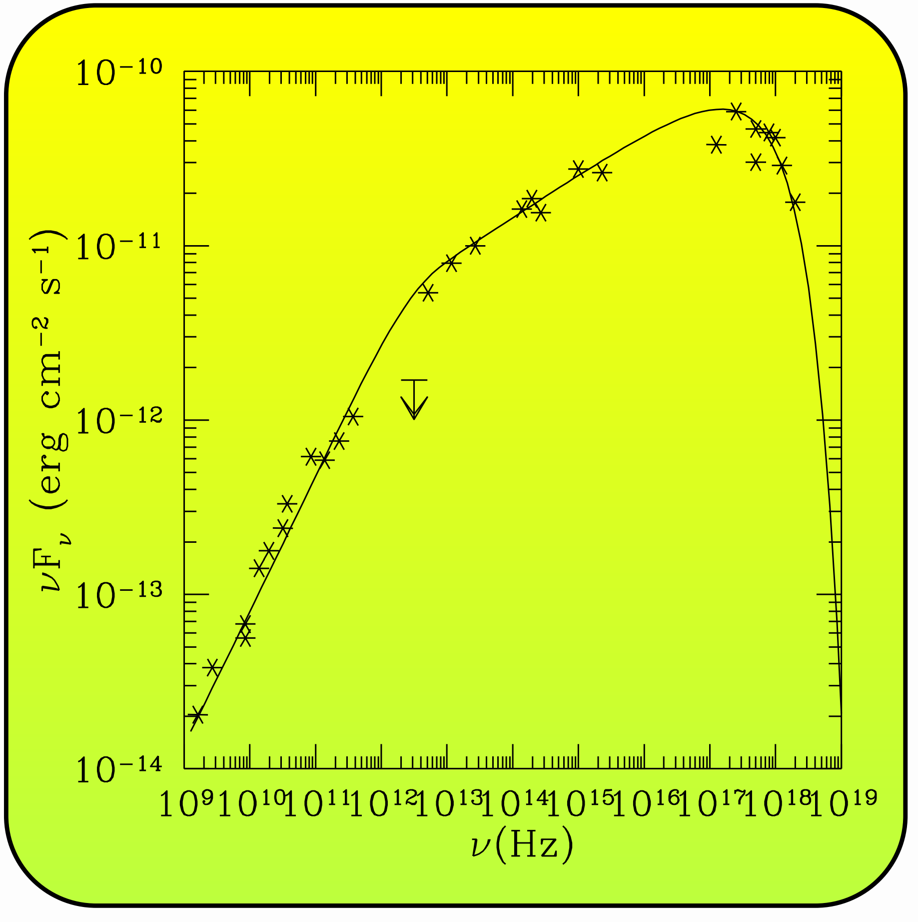
One-zone models - 1

- Kinetic equations for photons and electrons (and/or protons)
- Prescribed particle injection/escape
- Photon escape on crossing time
- Numerical sols. including all processes linear and quadratic in particle/photon densities

Problems: $t_{\rm cool} \ll t_{\rm cross}$ for X-ray-emitting electrons

Acceleration model?

One-zone models - 2



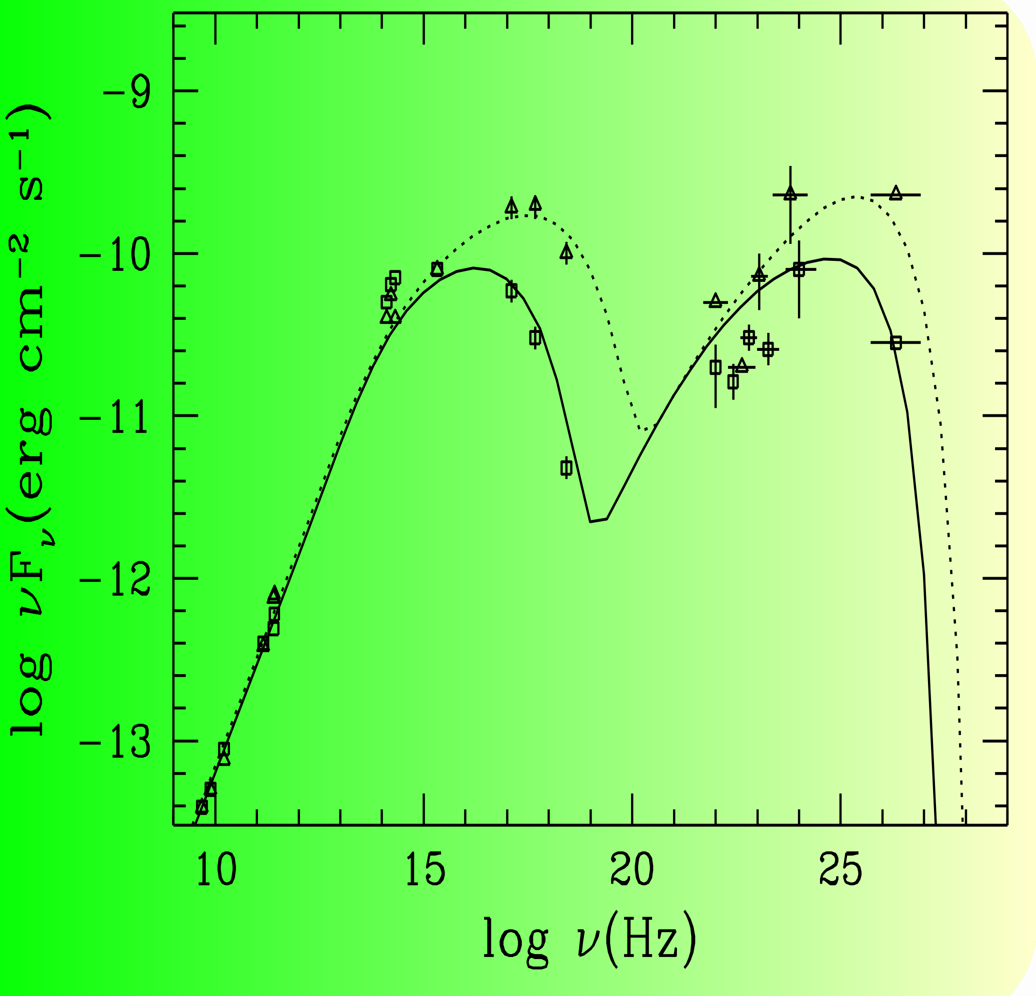
Spectrum of Mkn 501 fit to synchrotron emission of a *single* homogeneous source.

Mastichiadis & Kirk A&A 320, 19 (1997)

Homogeneous SSC model - 1

Observables

1. IC peak ν_{27}
2. Synch. peak ν_{18}
3. Ratio IC/Synch η
4. Luminosity L_{46}
5. Variability t_3



Homogeneous SSC model - 2

Parameters

1. $\gamma_{\rm max}$
2. B
3. $U_{\rm rad}$
4. Source size R
5. Doppler factor δ

Peak frequencies:

$$\gamma_{\rm max} = 3 \times 10^6 \nu_{27} \delta^{-1}$$

$$B = 5 \times 10^{-3} \nu_{18} \nu_{27}^{-2} \delta$$

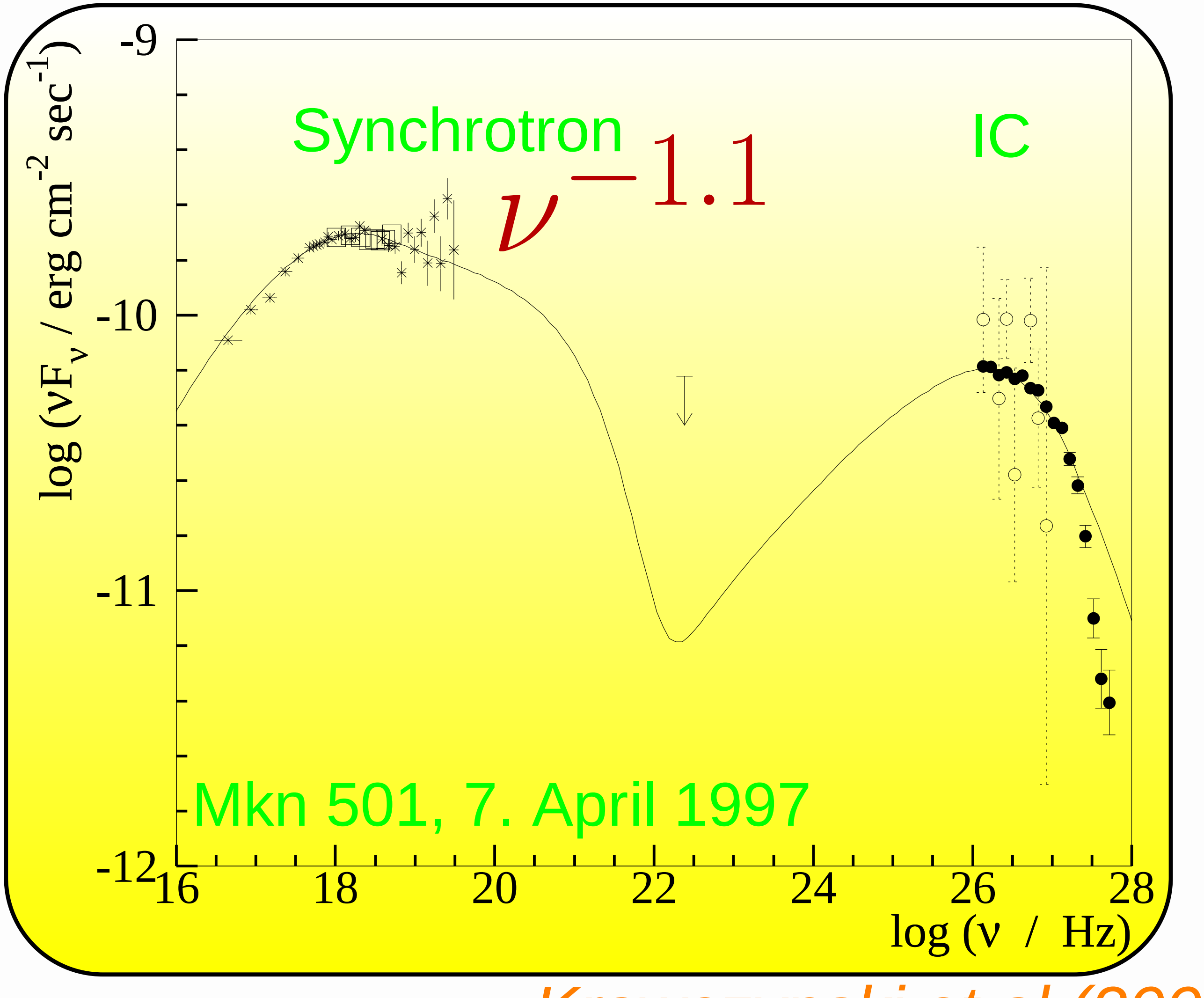
Fluxes:

$$U_{\rm rad} = \eta B^2 / 8\pi$$

$$U_{\rm rad} = \frac{4\pi}{c} \left(\frac{\nu I_\nu}{\delta^4} \right) = 3.3 \times 10^{35} \frac{L_{46}}{\delta^4 R^2}$$

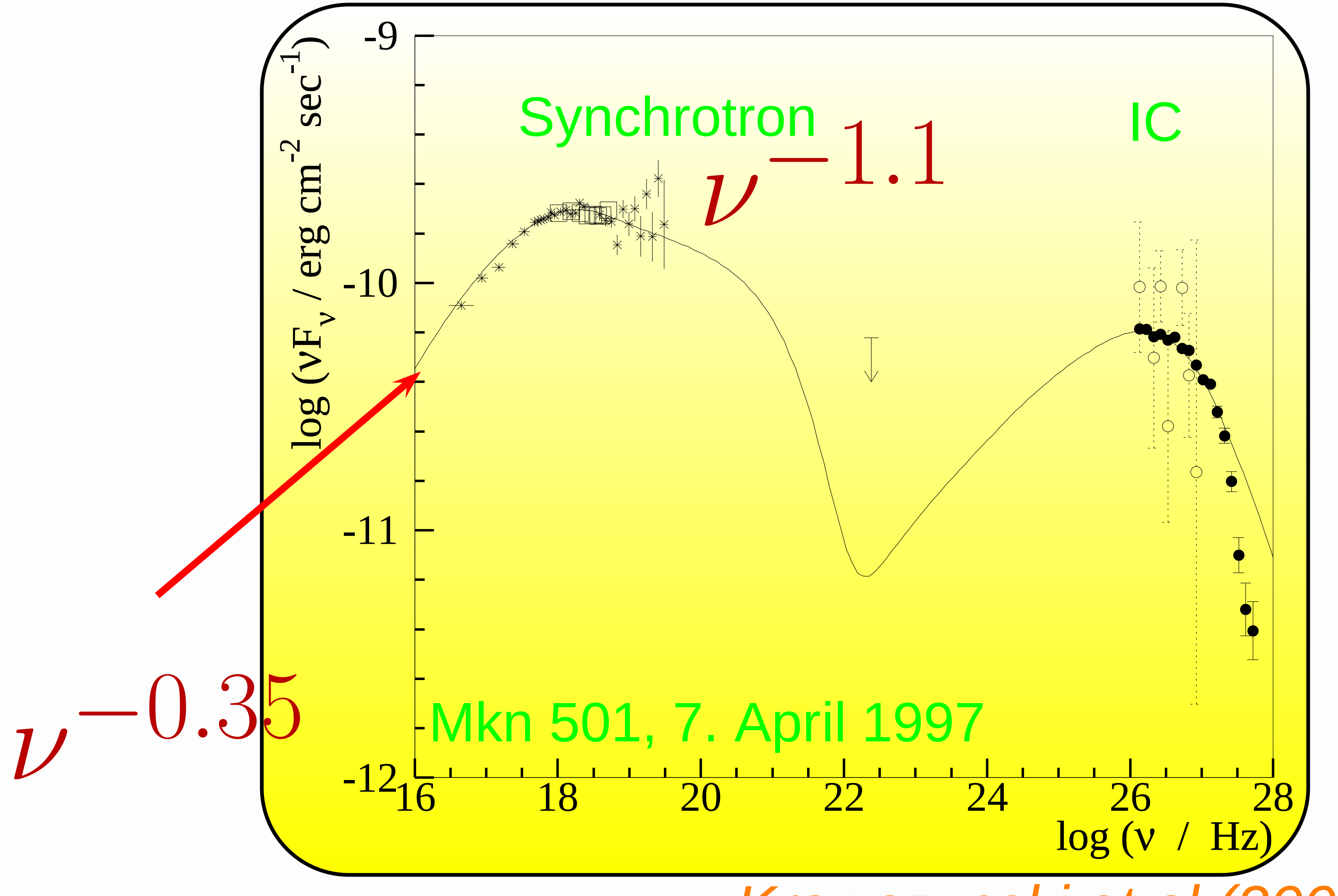
$$t_{\rm var} \approx R/(c\delta) \Rightarrow \delta \approx 66\, t_3^{-1/4} \eta^{-1/8} L_{46}^{1/8} \nu_{18}^{-1/4} \nu_{27}^{1/2}$$

Homogeneous SSC model - 3



Krawczynski et al (2000)

Homogeneous SSC model - 3



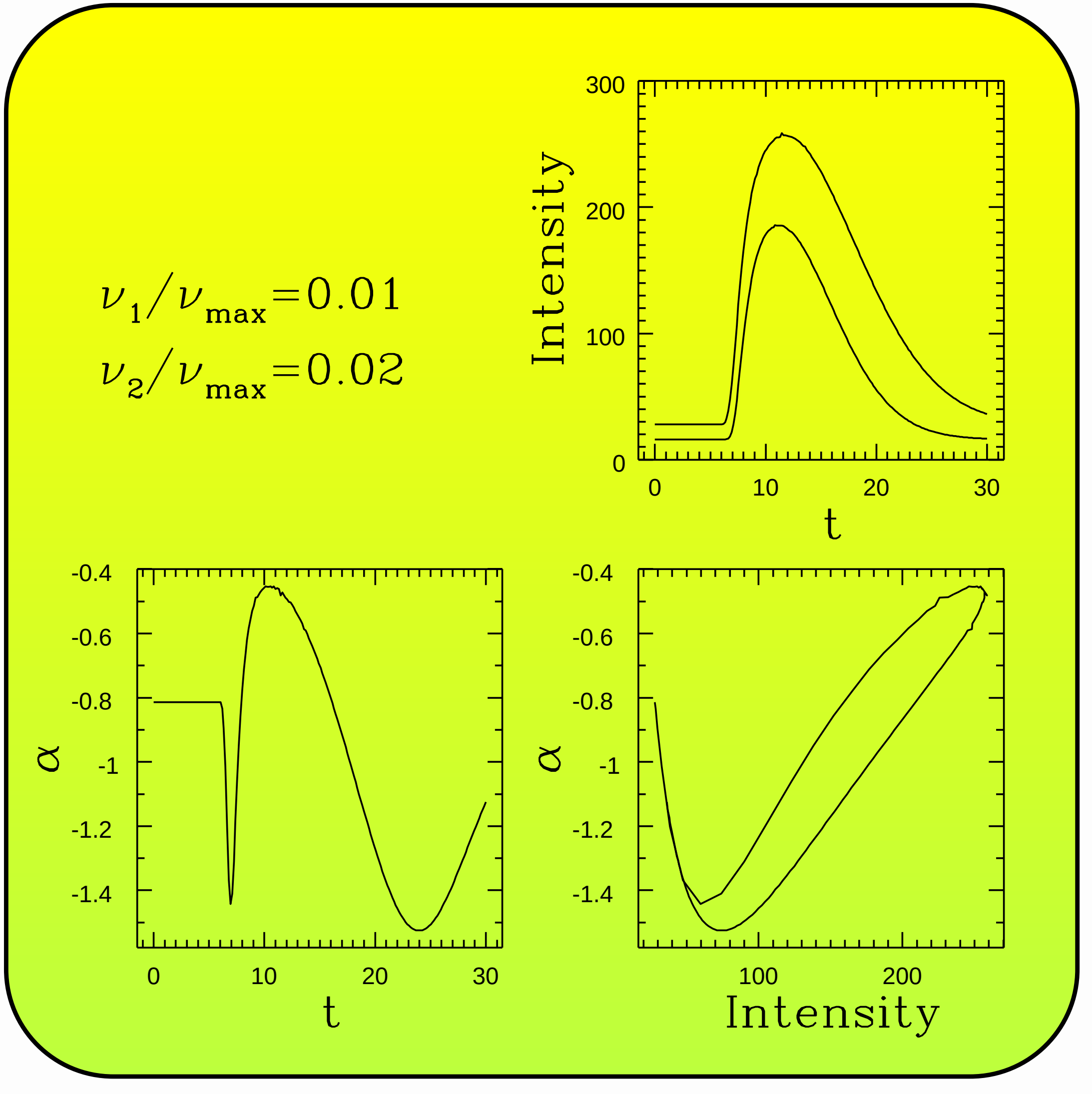
Krawczynski et al (2000)

Two-zone models - 1

- Box model: kinetic equations for electrons in each zone
- Numerical sols. include linear processes - Compton cooling off external photons and synchrotron radiation

Problem: inclusion of self-produced photons sensitive to geometry

Two-zone models - 2



Asymmetric flare:

$t_{\rm cool} \gg t_{\rm acc} \gg t_{\rm flare}$

Kirk, Rieger, Mastichiadis

A&A 333, 452 (1998)

& Proceedings of Turku Blazar Conference (1998)