Particle Astrophysics

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Heidelberg, 1st – 6th October 2007

Particle acceleration I

1. Charged particle kinematics
   - Energetic particles
   - Trajectories
2. Non-stochastic acceleration
   - Shock-drift acceleration
3. Stochastic acceleration
   - Magnetic pumping
   - First and second order Fermi mechanisms

Particle acceleration II

4. Acceleration by turbulence
   - Diffusion in energy
   - MHD wave modes
   - Resonance conditions
5. Diffusive shock acceleration
   - Kinematical approach
   - Diffusion-advection equation
   - Nonlinear effects

Radiation mechanisms I

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Radiation mechanisms II

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13. Synchrotron self-Compton emission
14. Photon-photon pair production

Particle acceleration I

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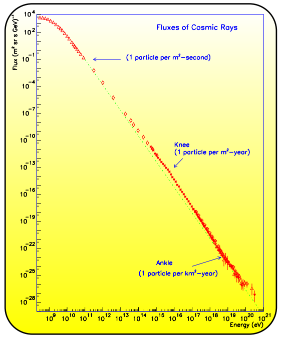
   - First and second order Fermi mechanisms

General properties

- Non-thermal – far from thermodynamic equilibrium with surroundings e.g., cosmic rays, relativistic electrons in supernova remnants, AGN, jets. . .
- High energy, very low density, e.g., cosmic rays: particle energy 10^{10} eV up to 10^{20} eV (≈ 16 J) number density $10^{-10}\times$ interstellar medium.
- Interactions with background almost exclusively via electromagnetic fields
- $\vec{E} = 0$ in highly conducting astrophysical plasma (acceleration problem)

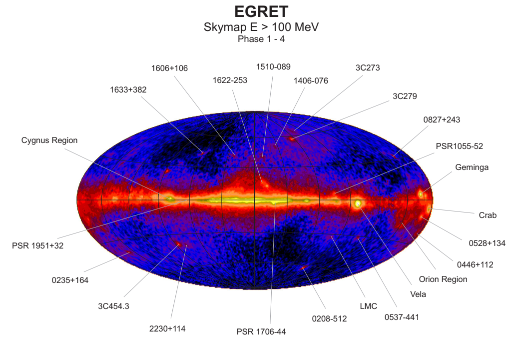
Cosmic Rays

Cosmic Ray Flux at Earth



Magnetic confinement?
Residence time $\sim 10^8$ years,
crossing time $\sim 10^3$ years.

Gamma-rays



Minimalistic electrodynamics

Lorentz force:

$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \frac{1}{c}\vec{v}\wedge\vec{B}\right)$$

Lorentz boosts: three key properties of E-M fields

1. $\vec{E}\cdot\vec{B}$ is a Lorentz scalar
2. So is $|\vec{E}|^2 - |\vec{B}|^2$
3. The components of $\vec{E}$ and $\vec{B}$ parallel to the boost direction remain unchanged.

Kinematical implications

Static, uniform fields

If $\vec{E} = 0$,
Lorentz force$\cdot\vec{p}$ gives

energy is constant

$$\frac{d\,|\vec{p}|^2}{dt} = 0$$

If $\vec{E}\cdot\vec{B} \neq 0$,
Lorentz force$\cdot\vec{B}$ gives

$$\frac{dp_z}{dt} = \text{constant}$$

energy diverges

($\vec{B}$ along $\vec{z}$)

Kinematical implications

Static, uniform, *crossed* fields

If $\vec{E}\cdot\vec{B} = 0$, and $E < B$,
frame exists with $\vec{E} = 0$, moves with $\vec{v} = c\vec{E}\wedge\vec{B}/B^2$ — energy is constant

If $\vec{E}\cdot\vec{B} = 0$ and $E > B$,
frame exists with $\vec{B} = 0$ — energy diverges

If $\vec{E}\cdot\vec{B} = 0$ and $E = B$,
vacuum wave — energy also diverges

Acceleration in static fields

Integration of particle orbits in prescribed $\vec{E}$ and $\vec{B}$ fields. e.g.,

- $\vec{E}\cdot\vec{B} \neq 0$ Pulsar (E-field from rotating B-field)
- $\vec{E}\cdot\vec{B} = 0$ Current sheets ($\vec{B}=0$ on neutral line)

General properties

- Given initial conditions, final energy determined
- Energy limited by losses or finite spatial extent

Grad-B drift

Nonuniform, unidirectional (along $\vec{z}$) B field, with $\vec{E}=0$:

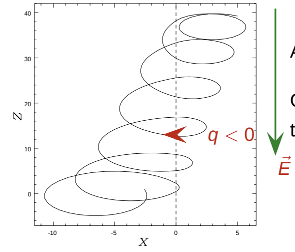
$$R = \frac{c\beta}{\omega\sin\alpha}$$

R: Radius of curvature
$\beta = v/c$: (constant)
$\omega = eB/mc$: gyro frequency
$\alpha = \cos(p_z/p)$: pitch angle (constant, since p_z is constant)

R larger where B weaker

Grad-B drift: perp. to $\vec{B}$ and to ∇B

Perpendicular shock: $\vec{E} = -E\vec{z} = \text{constant}$; $\vec{B} = B_{\pm}\vec{y}$;
$B_{+} = rB_{-} > E > 0$; $v_{\pm} = E\vec{x}/B_{\pm}$, $\vec{E}\wedge\vec{B}$ "drift"



Approx. conservation of ${p_\perp}^2/B$

Oblique shock:
transmission *and* reflection

Let $\vec{B} = B(t)\vec{z}$ (e.g., very long wavelength (compressional) magnetosonic wave)

$$\frac{\dot{p}_\perp}{p_\perp} = \frac{\dot{B}}{2B} \qquad \dot{p}_z = 0$$

The distribution function satisfies Liouville's eq:

$$\frac{\partial f}{\partial t} + \dot{p}_\perp\frac{\partial f}{\partial p_\perp} + \dot{p}_z\frac{\partial f}{\partial p_z} = 0$$

so that

$$\frac{\partial f}{\partial t} + \frac{\dot{B}}{2B}p_\perp\frac{\partial f}{\partial p_\perp} = 0$$

The pressure is

$$P = \frac{1}{3}\int\frac{d^3p}{E}p^2 f$$

If $\langle p_z^2\rangle \ll \langle {p_\perp}^2\rangle$, and assuming relativistic relativistic particles

$$\Rightarrow PB^{-3/2} = \text{constant}$$

Adiabat for 4 degrees of freedom
($B \propto$ density)

In spherical polars:

$$\frac{\dot{p}}{p} = \frac{(1-\mu^2)}{2}\frac{\dot{B}}{B} \qquad \frac{\dot{\mu}}{\mu} = -\frac{(1-\mu^2)}{2}\frac{\dot{B}}{B}$$

So that Liouville's eq. is

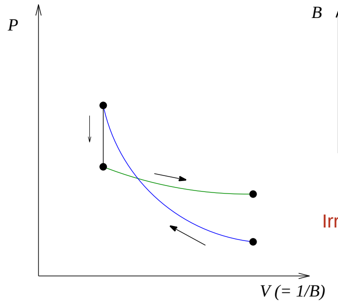
$$\frac{\partial f}{\partial t} + \frac{\dot{B}}{2B}(1-\mu^2)\left(p\frac{\partial f}{\partial p} - \mu\frac{\partial f}{\partial \mu}\right) = 0$$

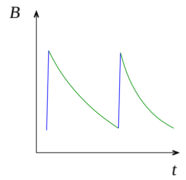
Introducing strong isotropization:

$$\Rightarrow PB^{-4/3} = \text{constant}$$

Adiabat for 6 degrees of freedom

Adiabats for no scattering and isotropization





Irreversibility $\Rightarrow$ acceleration

details: Saas Fee lectures, http://www.mpi-hd.mpg.de/~kirk/publications/saasfee.pdf

Fokker-Planck approach

Stochastic interactions with $\Delta p/p \ll 1$.
Evolution of an isotropic distribution on timescale long compared to that between individual interactions:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left[-Fp^2 f + \frac{\partial}{\partial p}\left(Dp^2 f\right)\right]$$

dynamical friction: $F = \frac{\langle \Delta p\rangle}{\Delta t}$ diffusion: $D = \frac{\langle(\Delta p)^2\rangle}{2\Delta t}$

Computation of F and D:
turbulent waves *two-body collisions*

Fermi acceleration

Two-body collisions:

$$\vec{p} + \vec{P} = \vec{p}' + \vec{P}'$$

$$\sqrt{p^2+m^2} + \sqrt{P^2+M^2} = \sqrt{p'^2+m^2} + \sqrt{P'^2+M^2}$$

For non-relativistic targets, and $|p'-p| \ll p$

$$\frac{p'-p}{p} \approx \frac{\sqrt{p^2+m^2}}{p^2 M}\left[\vec{P}\cdot\Delta\vec{p} + \text{second-order terms}\right]$$

$$\sim \frac{V}{v} \ll 1$$

Head-on collisions ($\vec{P}\cdot\Delta\vec{p} > 0$) $\Rightarrow$ energy gain
Tail-on collisions ($\vec{P}\cdot\Delta\vec{p} < 0$) $\Rightarrow$ energy loss

Fokker-Planck coefficients:

$$\begin{pmatrix} \frac{\langle p'-p\rangle}{p\Delta t} \\ \frac{\langle (p'-p)^2\rangle}{p^2\Delta t}\end{pmatrix} = \int \mathrm{d}^3 P f_{\mathrm{T}}(\vec{P}) \int \mathrm{d}\Omega' \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega'} v_{\mathrm{rel}} \begin{pmatrix} \frac{V}{v}\vec{P}\cdot\Delta\vec{p} \\ \left(\frac{V}{v}\right)^2\left(\vec{P}\cdot\Delta\vec{p}\right)^2\end{pmatrix}$$

($\vec{P} = \vec{P}/P$, $\Delta\vec{p} = \Delta\vec{p}/p$.)
If head-on and tail-on collisions equally probable (to lowest order in $|p'-p|/p$) then $F \sim (p'-p)^2/p^2 \sim D$
$\Rightarrow$ *second order Fermi process*
Under special conditions (e.g., anisotropic f_{T}) $F \sim |p'-p|/p$
$\Rightarrow$ *first order Fermi process*

Particle acceleration II

4 Acceleration by turbulence
- Diffusion in energy
- MHD wave modes
- Resonance conditions

5 Diffusive shock acceleration
- Kinematical approach
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Fermi I and II

Fokker-Planck coefficients:

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Diffusion in energy

Maxwellian targets, density n_{T}, temperature $T = MV_{\mathrm{th}}^2$ ($k_{\mathrm{B}} = c = 1$):

$$\frac{\left\langle (\Delta p)^2 \right\rangle}{p^2 \Delta t} = 2n_{\mathrm{T}} \frac{\sigma V_{\mathrm{th}}^2}{v} \left(1 + \frac{1}{2} \langle \cos\theta \rangle \right)$$ θ = scattering angle

Requiring the equilibrium solution $f \propto \mathrm{e}^{-\sqrt{p^2+m^2}/T}$ to be stationary gives (specialising to relativistic particles, $v = 1$):

$$F = -n_{\mathrm{T}} \bar{\sigma} p^2 / M + n_{\mathrm{T}} V_{\mathrm{th}}^2 \frac{1}{p^2} \frac{\partial}{\partial p} \left(\bar{\sigma} p^4 \right)$$ $\bar{\sigma} = \sigma(1 + \langle\cos\theta\rangle/2)$

second derivative of D cancels!
Kinematical effect, true for all distributions, not just equilibrium

Diffusion in energy

recoil   Doppler

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{1}{p^2} \frac{\partial}{\partial p} \left[n_{\mathrm{T}} \bar{\sigma} p^4 \left(\frac{1}{M} f + V_{\mathrm{th}}^2 \frac{\partial f}{\partial p} \right) \right]$$

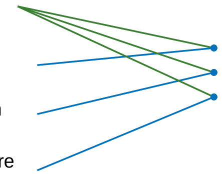
For heavy scatterers (T, $M \to \infty$, V_{th} finite)

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 D \frac{\partial f}{\partial p} \right)$$

Diffusion equation
Applies also to wave turbulence ($\hbar k \gg \hbar\omega / v$, $v \gg v_{\mathrm{phase}}$)
For $D \propto p^2$, ($\bar{\sigma}$ constant) always acceleration
Tends to stationary spectrum $f \propto p^{-3}$

At low frequency, physics dictated by:

- plasma inertia
- plasma pressure
- magnetic tension
- magnetic pressure



sound wave
Alfvén wave
fast wave

Čerenkov and cyclotron resonances

Effective interaction only when

$$\omega - k_{\parallel} v_{\parallel} + n\Omega = 0$$

$n = \pm 1$ cyclotron resonance $n = 0$ Čerenkov res.

Alfvén wave: $\omega = k_{\parallel} v_{\mathrm{A}}$

$\Rightarrow$ cyclotron resonance ($v_{\mathrm{A}} \ll v$)

Fast wave: $\omega = k v_{\mathrm{A}}$ (cold plasma)

$\Rightarrow$ Čerenkov resonance

Scattering

Alfvén wave: static in frame moving along $\vec{B}$ at speed $\pm v_{\mathrm{A}}$.

- For unidirectional waves, $\Rightarrow$ pitch-angle diffusion
- With both forward and backward waves, diffusion also in p.

Fast wave:

- For perpendicular propagation, reversible change in $p_{\perp}$
- For oblique propagation — reflection off moving magnetic compressions $\Rightarrow$ diffusion in p_z

Summary

Model properties:

- Based on diffusion equation
- Spatial homogeneity assumed
- Isotropisation by Alfvén waves
- Energisation by fast wave or Alfvén waves or their higher frequency manifestations (cyclotron, whistler, etc.) dependent on particle species and energy range
- Details in, for example:
  Achterberg, A& A 1981;
  Miller, Larosa & Moore, ApJ 1996;
  Petrosian & Liu, ApJ, 2004

Scale-free spectra

General case: acceleration events vs escape
If $\Delta E > 0$ in every event $\Rightarrow E(t)$
Let $P(p)$ be probability that a particle escapes with momentum $> p$ $(p = \sqrt{E^2 - m^2c^4}/c)$

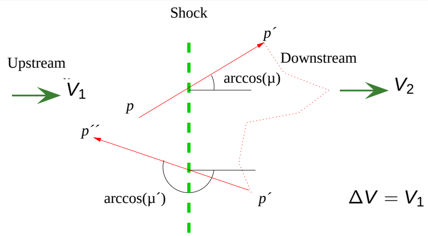
Acceleration $\dot{p} = ap$ Escape rate b

$$P(p + dp) = P(p)\,(1 - b\,dt)$$
$$\dot{P} = -bP \Rightarrow \frac{dP}{dp} = -\frac{bP}{ap}$$

$$P \propto p^{-b/a}$$

Power-law (scale-free) provided b/a independent of p

Acceleration at shock



$\Delta V = V_1 - V_2$

$$p' = p\left(1 + \frac{\mu\Delta V}{v}\right) \qquad p'' = p'\left(1 - \frac{\mu'\Delta V}{v'}\right) \quad (\Delta V \ll c)$$

to first order in $\Delta V/v$: $\frac{\Delta p}{p} = \frac{\Delta V}{v}(\mu - \mu')$

Average over isotropic distribution (prob. of crossing proportional to relative speed $|\mu v|$):

$$\frac{\langle \Delta p \rangle}{p} = \int_0^{+1} d\mu \int_{-1}^{0} d\mu' |\mu\mu'| \Delta p/p \Big/ \int_0^{+1} d\mu \int_{-1}^{0} d\mu' |\mu\mu'|$$

$$\Rightarrow \langle \Delta p \rangle / p = 4\Delta V/3v$$

Density $n = 2\pi p^2 \int_{-1}^{+1} d\mu f$

$$\text{N}^{\text{o.}}\ \text{entering/sec} = 2\pi p^2 \int_0^{+1} d\mu(\mu v + V_2) f = nv/4$$
$$\text{N}^{\text{o.}}\ \text{leaving/sec} = 2\pi p^2 \int_{-1}^{+1} d\mu(\mu v + V_2) f = nV_2$$

$$\Rightarrow \text{Escape Prob.} = 4V_2/v$$

In terms of the phase-space density

$$f(p) \propto p^{-3-P_{\rm esc}/\langle \Delta p/p \rangle} \equiv p^{-s},$$

$$s = \frac{3u}{\Delta u} = \frac{3r}{r-1}$$

where $r = V_1/V_2$ is the *compression ratio* of the shock.

A strong shock in a gas with $C_{\rm p}/C_{\rm V} = 5/3$, has $r = 4$, and $s = 4$.

Diffusion approximation $f(p, \mu) = f^{(0)}(p) + f^{(1)}(p, \mu)$

$$\frac{\partial}{\partial x}\left(\kappa \frac{\partial f^{(0)}}{\partial x}\right) - V\frac{\partial f^{(0)}}{\partial x} = 0$$

(constant $V = V_{1,2}$, *unmodified* shock). General solution:

$$f^{(0)}_{1,2} = A_{1,2}(p) + C_{1,2}(p)\exp\left(\int_0^x {\rm d}x' V_{1,2}/\kappa_{1,2}\right)$$

Exponential decay upstream and growth downstream
$\Rightarrow C_2 = 0$

No diffusive flux downstream $f^{(1)}_2 = 0$

Parallel shock (no change of $\vec{p}$ in shock frame)

$$p_2 \approx p_1(1 + \Delta V \mu_1/v_1)$$

At shock front, Liouville's theorem gives

$$f_1(p_1, \mu_1) = f_2(p_2, \mu_2)$$
$$= f^{(0)}_2(p_2)$$
$$\approx f^{(0)}_2(p_1) + \mu_1 \frac{\Delta V}{v_1} p_1 \left.\frac{\partial f^{(0)}_2}{\partial p}\right|_{p_1}$$

Integrate over μ_1: $\Rightarrow A_1 + C_1 = A_2$

No density jump across shock

Fick's law just upstream:

$$-\kappa_1 \frac{\partial f_1^{(0)}}{\partial x} = \frac{v_1}{2}\int_{-1}^{+1} \mathrm{d}\mu_1 \mu_1 f_1$$

so that

$$-V_1 C_1 \approx \frac{\Delta V}{3} p_1 \left.\frac{\partial f_2^{(0)}}{\partial p}\right|_{p_1, x=0}$$

and the matching condition is

$$\frac{\Delta V}{3V_1} p \frac{\mathrm{d}f_2^{(0)}}{\mathrm{d}p} + f_2^{(0)} = A_1$$

General solution

$$f_2^{(0)}(p) = ap^{-s} + s\int_0^p \frac{\mathrm{d}p'}{p}\left(\frac{p'}{p}\right)^s A_1(p')$$

where

$$s = \frac{3V_1}{\Delta V} = \frac{3r}{r-1}$$

- a denotes particles *injected* at the shock
- $A_1(p)$ denotes incoming upstream particles, for $A_1 = A_0 p^{-q}$, $f_2^{(0)} = \frac{s}{s-q} A_0 p^{-q}$

Shock modification

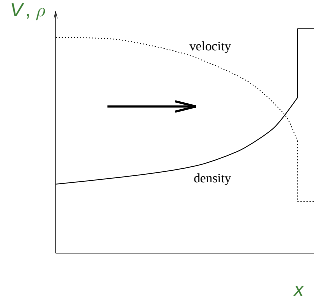
Energy density
$U = \int \mathrm{d}^3 p \sqrt{p^2 + m^2} f(p)$

For $f \propto p^{-s}$, U diverges at large p if $s \leq 4$, (i.e., $r > 4$)

Strong shock:

$$r = \frac{\hat{\gamma}+1}{\hat{\gamma}-1}$$

divergence for $\hat{\gamma} \leq 5/3$



Shock modification

Two feedback effects important for the spectrum in nonlinear case:

- Relativistic particles have $\hat{\gamma} = 4/3$, and so soften the equation of state
  - overall compression ratio increases
  - high energy particles (with long mean free path) get a harder spectrum
- Pressure gradient decelerates and heats incoming plasma
  - Mach number (strength) of sub-shock reduced
  - low energy particles (with short mean free path) get a softer spectrum

⇒ concave spectrum

Solutions:

- Steady state requires escape of high energy particles
  Malkov & Drury, *Rep. Prog. Phys.* 64, 429 (2001)
- Time dependent equations (applied to SNR) avoid this
  Berezhko & Völk, *Astroparticle Phys.* 14, 201 (2000)

Problems:

- *Injection* arbitrary
- Possible feedback effect of magnetic amplification not understood
- Transport properties in driven turbulence unknown

Radiation mechanisms I

Reaction rates

Three ingredients for models:

- Energy loss rates — needed to compute the distribution of radiating particles (along with an acceleration model and non-radiative loss-rates)
- Emissivities — to predict intrinsic spectra of "optically thin" sources and/or compare with observation
- Absorption coefficients — account for radiation transfer effects

In particle astrophysics, usually two-body interactions/decays. Starting point is usually a cross section:

$$\text{rate} = \text{cross-section} \times \text{flux}$$

Important processes for relativistic hadrons:

- CR interactions with 'cold' gas, e.g., $p + p \to p + p + \pi^0$
- Photo disintegration e.g., $A + \gamma \to (A-1)^* + n \to (A-1) + \gamma + n$
- Photo pion production e.g., $p + \gamma \to n + \pi^+$
- Bethe-Heitler pair production, $p + \gamma \to p + e^+ + e^-$

Two kinds of targets:

- Slow (cold gas) — rate given in rest frame of targets
- Fast (photons) — rate given in rest frame of relativistic particle

and a special case

- Photon-photon interactions — rate given in C.M.S.

Lorentz invariants

Need to transform distribution functions:

$$dN = \underbrace{f}_{\text{phase-space density}} \times \underbrace{d^3p\, d^3x/h^3}_{\text{phase-space element}}$$

Both f and $d^3p d^3x$ are Lorentz scalars.
f is equivalent to an occupation number.
Bosons in thermodynamic equilibrium $f = \left(e^{h\nu/k_B T + \mu} - 1\right)^{-1}$
For photons ($\mu = 0$, two polarisations), the Planck distribution follows:

$$B(\nu, T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/k_B T} - 1}$$

Lorentz invariants

The specific intensity of radiation is

$$I_\nu = h\nu c \frac{dN_\gamma}{d\nu d\Omega d^3x} = \frac{h\nu^3}{c^2} f$$

so I_ν/ν^3, or, equivalently

$$\frac{1}{\nu^2}\frac{dN}{d\nu d\Omega d^3x} = \frac{n_\epsilon}{\epsilon^2} \quad \text{in the notation of Blumenthal and Gould}$$

is a Lorentz scalar.

In vacuum I_ν/ν^3 is also constant along rays.

Lorentz invariants

Using spherical polars in momentum space $(h\nu/c, \theta, \phi)$ a boost along the axis gives

$$\nu = \nu'\Gamma(1 - \beta\cos\theta')$$
$$\nu' = \nu\Gamma(1 + \beta\cos\theta)$$

Doppler factor: $\mathcal{D} = 1/\left[\Gamma\left(1 - \beta\cos\theta'\right)\right] = \Gamma\left(1 + \beta\cos\theta\right)$

$$\cos\theta = \left(\cos\theta' - \beta\right)/\left(1 - \beta\cos\theta'\right)$$
$$\cos\theta' = \left(\cos\theta + \beta\right)/\left(1 + \beta\cos\theta\right)$$
$$\phi = \phi'$$

Differentiating gives another useful Lorentz scalar:

$$\nu d\nu d(\cos\theta) d\phi = \nu d\nu d\Omega \quad (\text{special case of } d^3p/E)$$

CR-nucleon collisions

The invariant *inclusive* cross-sections are written

$$\sigma_n(\sqrt{s}, p_\parallel^*, p_\mathrm{T}) = E_1 \times E_2 \times \ldots E_n \frac{\mathrm{d}\sigma}{\mathrm{d}^3 p_1 \mathrm{d}^3 p_2 \ldots \mathrm{d}^3 p_n}$$

- In astrophysics, need the total cross-section σ_0 and the inclusive cross-sections for emission of a "messenger"
- *Ab initio* computation is not feasible — accelerator data or, when out of range, a scaling hypothesis

$$\sigma_n(\sqrt{s}, p_\parallel^*, p_\mathrm{T}) = \sigma_n(x^*, p_\mathrm{T})$$

with, for example, $x^* = 2p_\parallel^*/\sqrt{s}$ (see Gaisser, Chap. 2)

CR-nucleon collisions

- Total inelastic p–p cross-section almost constant at 30 mb ($= 3 \times 10^{-26}\,\mathrm{cm}^2$) from 2 GeV to 2 TeV, ($\Delta$ resonance)
- Inelasticity $K \approx 0.5$
- Gamma-ray yield mainly from decay of π^0
- Neutrino yield from $\pi^\pm$ decay (ν_μ) and subsequent *in flight* decays of the muons ($\nu_{\mu,e}$).
- Fitting formulae available for different spectra of incoming CR's (Drury et al 1994, Gaisser, Chap 7)
- E.g.,

$$\dot{N}_{E_\gamma > 1\,\mathrm{TeV}} = qE_\mathrm{CR} M_\mathrm{target}/\mu$$

$q \approx 10^{-17}\,\mathrm{s\,cm\,g^{-2}}$, E_CR: energy density of CR, M_target: total mass of gas in source region, $\mu =$: mean molecular weight.

Photo-disintegration

Photon on nucleus:

- From threshold 10 MeV to 30 MeV *Giant Dipole Resonance* dominates. (collective oscillation of protons and neutrons)
- Large cross-section $\sigma \approx A \times 10^{-27}\,\mathrm{cm}^2$
- After emission of nucleon, daughter nucleus de-excites radiatively ⇒ MeV photon
- Total cross-section for absorption is $\sigma(\epsilon')$, (ϵ' is photon energy in rest frame of nucleus). For a relativistic nucleus, reaction rate in the lab. frame is

$$R = \int_0^\infty \mathrm{d}\epsilon \frac{n(\epsilon)}{2\gamma^2\epsilon^2} \int_0^{2\gamma\epsilon} \mathrm{d}\epsilon' \epsilon' \sigma(\epsilon')$$

So one needs target photons of energy greater than $10\,\mathrm{MeV}/(2\gamma)$

Photo-disintegration

Radiative deexcitation of daughter nucleus:

- For CMB targets $\epsilon = 3k_\mathrm{B}T_\mathrm{CMB}$, $\gamma > 10^{10}$
- For radiation field of a massive star $T = 2 \times 10^4\,\mathrm{K}$, $\gamma > 10^6$.
- In rest frame of nucleus, $\epsilon_\gamma \approx 1\,\mathrm{MeV}$
- Emitted photon, $\gamma \times 1\,\mathrm{MeV}$ CMB: $= 10^{16}$ eV, massive star: $= 1$ TeV.
- Effectively photon scattering with an inelasticity of 90 % and an energy boost of a factor γ^2 (similar to inverse Compton scattering).
- Neutrino yield from decay of neutron ejected from nucleus — relatively low yield.

Photo-pion and pair production

Pion production

- Large cross-section $\sigma \sim 10^{-26}\,\mathrm{cm}^2$.
- Significant inelasticity - at threshold $\sim (m_\mathrm{p} - m_\pi)/m_\mathrm{p}$
- Threshold energy relatively high $\epsilon > m_\pi(1 + 2m_\pi/m_\mathrm{p}) = 145\,\mathrm{MeV}$
- Roughly equal amounts of energy into photons via $\pi^0 \to 2\gamma$ and neutrinos via π^+ decay chain.

Pair production

- Small effective cross-section $\sigma \sim 10^{-28}\,\mathrm{cm}^2$.
- Low inelasticity: at threshold produce e^+ and e^- each with $\gamma = \gamma_\mathrm{proton}$
- Threshold energy low $\epsilon > 2m_\mathrm{e}(1 + m_\mathrm{e}/m_\mathrm{p}) = 1\,\mathrm{MeV}$
- All energy into photons via synchrotron/inverse Compton

Radiation mechanisms II

Important processes for relativistic leptons/photons

- synchrotron radiation
- inverse Compton scattering
- Bremsstrahlung
- Photon-photon pair production $\gamma + \gamma \rightarrow e^+ + e^-$

Synchrotron theory Moving sources Inverse Compton scattering Synchrotron self-Compton emission Photon-photon pair produc

General treatment - 1

- General formula for weakly damped waves (Maxwell's eqs. + linear response theory)

$$P(\vec{k}) = \mathrm{Lim}_{T\rightarrow\infty} \frac{4\pi}{TR} \left| \vec{e}\cdot\vec{j}(\omega) \right|^2$$

  E-M wave: $R = 2$, $\vec{e}$ transverse.

- *Gyromagnetic radiation:* $\vec{j}$ from helical motion in $\vec{B}$ (speed $v = \beta c$, Lorentz factor γ, pitch angle θ).
  - cyclotron ($v \ll c$),
  - trans-relativistic (!),
  - synchrotron $(\gamma \sin\theta)^3 \gg 1$.

Synchrotron theory Moving sources Inverse Compton scattering Synchrotron self-Compton emission Photon-photon pair produc

General treatment - 2

Emitted (radiated) power $L_{\mathrm{s.p.}}$ is *Lorentz invariant*

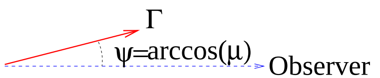
$$L_{\mathrm{s.p.}} = \frac{2e^2}{3m^2c^3}\left(\frac{\mathrm{d}p_\mu}{\mathrm{d}\tau}\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau}\right)$$

In a magnetic field

$$\frac{\dot{\gamma}}{\gamma} = -\frac{2e^2}{3mc^3}\gamma\beta^2\Omega_L^2\sin^2\theta = -\frac{2\sigma_T\gamma\beta^2}{mc}\frac{B^2}{8\pi}\sin^2\theta$$

Synchrotron theory Moving sources Inverse Compton scattering Synchrotron self-Compton emission Photon-photon pair produc

Beaming etc.



Doppler factor:

$$\mathcal{D} = \frac{1}{\Gamma(1-\beta\mu)}$$
$$\nu = \mathcal{D}\nu'$$

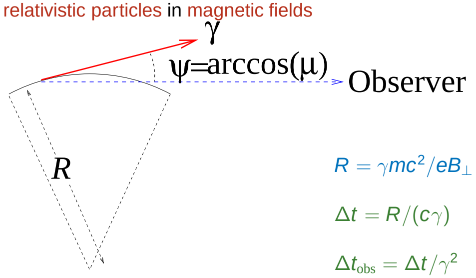
Lorentz scalar: $I_\nu/\nu^3 = I'_{\nu'}/\nu'^3 \Rightarrow I_\nu = \mathcal{D}^3 I'_{\nu'}$

For $\Gamma \gg 1$, $\mathcal{D}$ strongly peaked around $\psi = 0$, width $\sim 1/\Gamma$.

Synchrotron theory Moving sources Inverse Compton scattering Synchrotron self-Compton emission Photon-photon pair produc

Synchrotron radiation

Characteristic frequency:
relativistic particles in magnetic fields



$R = \gamma mc^2/eB_\perp$

$\Delta t = R/(c\gamma)$

$\Delta t_{\mathrm{obs}} = \Delta t/\gamma^2$

Characteristic frequency $= \gamma^2 eB_\perp/(mc)$

Synchrotron theory Moving sources Inverse Compton scattering Synchrotron self-Compton emission Photon-photon pair produc

Restrictions

Restrictions:

- $B_\perp$ constant over distance $mc^2/eB_\perp$, otherwise jitter radiation
- High harmonic number $s = (\gamma\sin\alpha)^3 \gg 1$, otherwise Very Small Pitch Angle radiation (α: pitch angle)
- Classical regime $B \ll B_{\mathrm{crit}}/\Gamma$ otherwise Klein-Nishina-like corrections ($B_{\mathrm{crit}} = 4.414 \times 10^{13}$ G)

Approximations

Small parameters: two angles

- Pitch angle: for a smooth distribution of particles $\mathrm{d}N/\mathrm{d}\Omega\mathrm{d}\gamma$ integrate single particle emission over all directions and use

$$\frac{\mathrm{d}L}{\mathrm{d}\Omega\mathrm{d}\nu} = \int \mathrm{d}\gamma \underbrace{\frac{\mathrm{d}L_{\mathrm{s.p.}}}{\mathrm{d}\nu}}_{\text{kernel: function of } \gamma,\, \nu,\, \theta} \frac{\mathrm{d}N}{\mathrm{d}\Omega\mathrm{d}\gamma}$$

  essentially an expansion in $(\mathrm{d}N/\mathrm{d}\mu)/(\gamma N)$
- Gyrophase: convert sum over harmonics to an integral and replace Bessel functions using *Airy integral approximation* (asymptotic expansion in $1/s$)

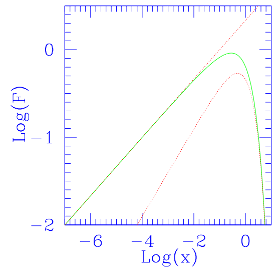
Single particle emission - 1

$$\frac{\mathrm{d}L_{\mathrm{s.p.}}}{\mathrm{d}\nu} = \sqrt{3}\frac{e^2}{\hbar c}\hbar\Omega_{\mathrm{L}} \sin\theta\, F(\nu/\nu_{\mathrm{c}})$$

$$\nu_{\mathrm{c}}(\gamma,\theta) = \frac{3\Omega_{\mathrm{L}}\sin\theta\gamma^2}{4\pi} = \nu_0\gamma^2 \quad [\nu_0 = 3\Omega_{\mathrm{L}}\sin\theta/(4\pi)]$$

$$F(x) = x\int_x^\infty \mathrm{d}t K_{5/3}(t)$$

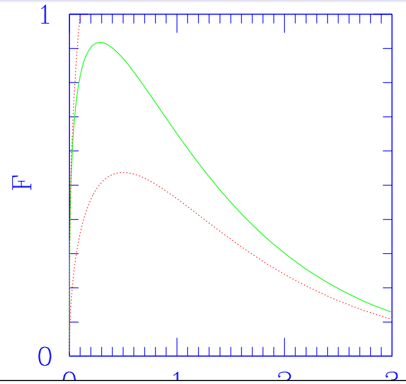
Single particle emission - 2



$$F(x) \approx \frac{4\pi}{\sqrt{3}\Gamma(1/3)}\left(\frac{x}{2}\right)^{1/3} \quad (x \ll 1)$$

$$\sim \sqrt{\frac{x\pi}{2}}\exp(-x) \quad (x\to\infty)$$

Single particle emission - 3



Maximum:
$x = 0.29$

Emissivity power-law distribution

An important special case is

$$\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{r}\mathrm{d}\Omega\mathrm{d}\gamma} = \frac{C}{4\pi}\gamma^{-q} \quad \text{for} \quad \gamma_1 < \gamma < \gamma_2$$

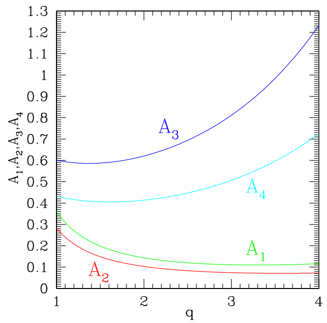
Then (in the optically thin case)

$$P_\nu = \frac{\mathrm{d}L}{\mathrm{d}\Omega\mathrm{d}\nu} = V j_\nu = V\int_0^\infty \mathrm{d}\gamma \frac{\mathrm{d}L_{\mathrm{s.p.}}}{\mathrm{d}\nu}\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{r}\mathrm{d}\Omega\mathrm{d}\gamma}$$

$$= C\,V\frac{e^2}{\hbar c}\hbar\Omega_{\mathrm{L}}\sin\theta\left(\frac{\nu}{\nu_0}\right)^{(1-q)/2} A_1(q) \quad \text{for } q > 1/3$$

provided that $\gamma_1^2 \ll \nu/\nu_0 \ll \gamma_2^2$

The functions $A_{1,2,3,4}$



Angle average:
$j \to \langle j \rangle$ etc.
in formulae:
$\sin\theta \to 1$
$\nu_0 \to \bar{\nu}_0 = 3\Omega_{\mathrm{L}}/(4\pi)$
$A_1(q)$: emission
$A_2(q)$: $\langle$emission$\rangle$
$A_3(q)$: absorption
$A_4(q)$: $\langle$absorption$\rangle$

Synchrotron absorption

Transport equation:

$$\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu$$

Classical mechanism ⇒ Absorption ≈ stim. emission.

$$\alpha_\nu = \frac{-1}{2m\nu^2}\int d\gamma \frac{dL_{\rm s.p.}}{d\nu}\gamma^2 \frac{d}{d\gamma}\left(\frac{1}{\gamma^2}\frac{dN}{d^3\mathbf{r}d\Omega d\gamma}\right)$$

(unpolarised radiation). For a power-law distribution

$$\alpha_\nu = \frac{2\pi\alpha_{\rm f}C}{\sin\theta}\left(\frac{c}{\Omega_{\rm L}}\right)^2\left(\frac{\hbar\Omega_{\rm L}}{mc^2}\right)\left(\frac{\nu}{\nu_0}\right)^{-(q+4)/2} A_3(q)$$

Synchrotron spectrum - 1

Optically thin: $I_\nu = Lj_\nu \propto \nu^{(1-q)/2}$
Optically thick: $I_\nu = j_\nu/\alpha_\nu \propto \nu^{5/2}$
Transition at $\alpha_\nu R = 1 \Rightarrow$
$\nu_{\rm t} \propto R^{-2/(q+4)} B^{(q+2)/(q+4)}$

Given radiation at ν from electrons of $\gamma = \sqrt{\nu/\nu_0}$ the brightness temperature $k_{\rm B}T_{\rm B}(\nu) = c^2 I_\nu/(2\nu^2)$ is

$$k_{\rm B}T_{\rm B}(\nu) \approx mc^2\sqrt{\nu/\nu_0} \approx \gamma mc^2$$

in optically thick part of spectrum.

Synchrotron spectrum - 2

Energy density in radiation field near surface:

$$U_{\rm rad} = \int \frac{d\Omega d\nu}{c} I_\nu \approx \frac{2\pi}{c}\int d\nu I_\nu$$

For $q < 3$, dominant contribution from near the upper cut-off in optically thin region:

$$U_{\rm rad} \approx \frac{4\pi\nu_{\rm t}^3 k_{\rm B}T_{\rm max}}{c^3}\int_1^{\nu_0\gamma_2^2/\nu_{\rm t}} dx\, x^{(1-q)/2}$$

$$\approx 18\frac{\nu_2 e^2}{mc^3}\left(\frac{k_{\rm B}T_{\rm max}}{mc^2}\right)^5\left(\frac{B^2}{8\pi}\right) \quad \text{for } q = 1$$

Minimum energy - 1

Problem: given volume V and $P_\nu = P_1(\nu/\nu_1)^{-\alpha}$ for $\nu_1 < \nu < \nu_2$ (V, P_1, $\nu_{1,2}$, α observed quantities), what is the minimum energy requirement in the source?
Variables: C, B; $\nu^\alpha P_\nu = \nu_1^\alpha P_1 = CV(e^2/\hbar c)\hbar\Omega_{\rm L}\bar{\nu}_0{}^\alpha A_2$
Minimize

$$E = \underbrace{V\frac{B^2}{8\pi}}_{magnetic} + \underbrace{(1+k)}_{dark\ particles}\underbrace{V\,C\int_{\gamma_1}^{\gamma_2} d\gamma\,\gamma mc^2\gamma^{-2\alpha-1}}_{electrons}$$

$$= V\frac{B^2}{8\pi} + \frac{(1+k)mc^2}{2}V\,C\bar{\nu}_0{}^{(2\alpha-1)/2}\int_{\nu_1}^{\nu_2} d\nu\,\nu^{-(2\alpha+1)/2}$$

Minimum energy - 2

$$E = V\frac{B^2}{8\pi} + b(1+k)B^{-3/2}$$

with

$$b = P_1\nu_1^{1/2}\left[\sqrt{\frac{\pi}{3}}\frac{m^{5/2}c^{9/2}}{e^{7/2}}\right]\int_1^{\nu_2/\nu_1} dx\, x^{-(2\alpha+1)/2}$$

At minimum, $B = [6\pi b(1+k)/V]^{2/7}$ and

$$E_{\rm min} = \frac{7}{4\,(6\pi)^{3/7}}(1+k)^{4/7}V^{3/7}b^{4/7}$$

Min. Pressure $= E_{\rm min}/V \propto (P_1/V)^{4/7}$

Moving sources - 1

Remember,

$$\nu = \mathcal{D}\nu'$$
$$\gamma = \mathcal{D}\gamma'$$

Invariance of phase space density:

$$\frac{dN}{d^3\mathbf{r}d\Omega d\gamma} = \mathcal{D}^2\frac{dN}{d^3\mathbf{r}'d\Omega' d\gamma'}$$

and of emitted power:

$$L_{\rm s.p.} = L'_{\rm s.p.}$$

Moving sources - 2

Basic formula:

$$\frac{\mathrm{d}L}{\mathrm{d}\nu\mathrm{d}\Omega} \equiv P_\nu = \int \mathrm{d}^3\mathbf{r} \int_0^\infty \mathrm{d}\gamma \frac{\mathrm{d}L_{\mathrm{s.p.}}}{\mathrm{d}\nu} \frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{r}\mathrm{d}\Omega\mathrm{d}\gamma}$$

If limits on $\mathbf{r}$ integration are time-dependent:

$$\int_{\mathrm{source}} \mathrm{d}^3\mathbf{r} = \int \mathrm{d}^3\mathbf{r} \int \mathrm{d}t\, \delta(t - \mathbf{r}\cdot\mathbf{n}/c) = \mathcal{D}\int_{\mathrm{source}} \mathrm{d}^3\mathbf{r}'$$

and $V = \mathcal{D}\, V'$ is an *effective* volume. In this case:

$$P_\nu = \mathcal{D}^3 P'_{\nu'}$$

Moving sources - 3

To find the minimum energy magnetic field, need transformation rules for b and V':

$$b \propto P_1\nu_1^{1/2} = \mathcal{D}^{7/2} b'$$

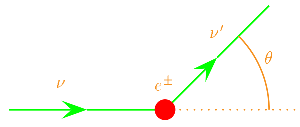
Estimates of the source volume:

- From angular extent: $V' = (D_{\mathrm{A}}\delta\theta)^3 \Rightarrow$ no dependence on Doppler factor $B'_{\mathrm{min}} = B_{\mathrm{min}}/\mathcal{D}$
- From a variability timescale: $V' = (\mathcal{D}ct_{\mathrm{var}})^3 \Rightarrow$ apparent volume $= V'/\mathcal{D}^3$ and $B'_{\mathrm{min}} = B_{\mathrm{min}}/\mathcal{D}^{13/7}$

N.B. apparent, not effective volume

Compton scattering

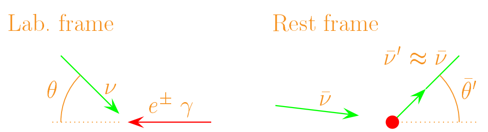
Thomson scattering, electron at rest

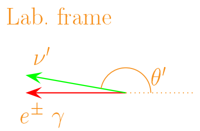


$$d\sigma/d\Omega = 3\sigma_{\mathrm{T}}(1+\cos^2\theta)/(16\pi)$$
$$\nu' = \nu/\left[1+(h\nu/mc^2)(1-\cos\theta)\right] \approx \nu,\ \text{for } h\nu \ll mc^2$$

Inverse Compton scattering - 1

Thomson scattering, relativistic electron





$\bar\nu = \nu\gamma\,(1 + v\cos\theta/c)$
$\bar\nu' = \bar\nu$
$\nu' = \gamma\bar\nu'\,(1 + v\cos\bar\theta'/c)$

Inverse Compton scattering - 2

Emitted frequency (after angle average):

$$\nu' = 4\gamma^2\nu/3$$

Forward beaming:

$$\cos\theta' = (\cos\bar\theta' - v/c)\,/\,(1 - v\cos\bar\theta'/c) \approx -1 - 1/\left[\gamma^2\,(1-\cos\bar\theta')\right]$$

Cooling rate (isotropic targets):

$$\frac{-\dot\gamma}{\gamma} = \frac{4}{3}\frac{\sigma_{\mathrm{T}}}{mc} \times \text{target energy density} \times \beta^2\gamma$$

Close analogy with synchrotron radiation...

Klein-Nishina regime I

Full KN cross-section: (' indicates electron rest frame)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega'_{\mathrm{out}}\mathrm{d}\epsilon'_{\mathrm{out}}} = \sigma_{\mathrm{T}} g\left(\epsilon'_{\mathrm{in}}, \epsilon'_{\mathrm{out}}, \cos\theta'_{\mathrm{sc}}\right)\delta\left[\epsilon_{\mathrm{out}} - f\left(\epsilon'_{\mathrm{in}}, \cos\theta'_{\mathrm{sc}}\right)\right]$$
$$g(x,y,z) = \frac{3}{16\pi}\left(\frac{y}{x}\right)^2\left(\frac{y}{x}+\frac{x}{y}-1+z^2\right)$$
$$f(x,z) = x/[1+x(1-z)]$$

For $\hbar\nu/m_ec^2 = \epsilon' \sim 1$ (i.e., $\gamma h\nu \sim mc^2$) *recoil* important.

- $\Rightarrow$ Cross-section reduced
- $\Rightarrow$ Maximum photon energy $\approx \gamma mc^2$
- $\Rightarrow$ Pair-production threshold exceeded

Klein-Nishina regime II

- In extreme KN regime, photon takes most of the electron energy
- IC spectrum softens from $\nu^{(1-q)/2}$ to ν^{-q} (transition region large!)
- Isotropic targets: Jones, Phys.Rev. 167, 1159 (1968), Blumenthal & Gould Rev. Mod. Phys. 42, 237 (1970)
- Anisotropic targets: Ho & Epstein ApJ 343, 277 (1989), Kirk et al Astroparticle Phys. 10, 31 (1999), Dubus et al arXiv:astro-ph0710.0968
- Head-on ("δ-function") approximation Ball & Kirk Astroparticle Physics (2000)

Single particle emissivity - 1

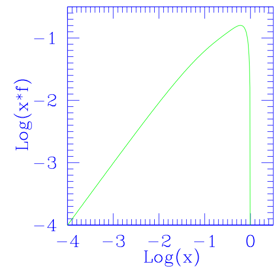
For *relativistic electrons* ($\gamma \gg 1$) in *inverse* Compton scattering ($\nu > \nu_s$) off *isotropic* target photons (spectral energy density $dU_s/d\nu_s$) in the *Thomson limit*:

$$\frac{dL_{\rm s.p.}}{d\nu} = 3\sigma_T c \int d\nu_s \frac{dU_s}{d\nu_s}\frac{\nu}{\nu_{IC}} f(\nu/\nu_{IC})$$

$$\nu_{IC}(\gamma) = 4\gamma^2\nu_s$$

$$f(x) = \begin{cases} 2x\ln x + x + 1 - 2x^2 & \text{for } x < 1 \\ 0 & \text{for } x > 1 \end{cases}$$

Single particle emissivity - 2



Exact kinematics
$\Rightarrow f(x) = 0$,
for $x < 1/(4\gamma^2)$.

Emissivity power-law distribution

In the source frame, $\int dr'^3 = V'$ and

$$\frac{dN}{d^3\mathbf{r}'d\Omega'd\gamma'} = \frac{C}{4\pi}\gamma'^{-q} \quad \text{for} \quad \gamma_1 < \gamma' < \gamma_2$$

Angular distribution of targets?

- Cosmic background radiation: isotropic in galaxy frame
- Self-produced synchrotron photons: isotropic in source frame

IC off the CMB

In the galaxy frame $\int dr^3 \to \mathcal{D}V'$ and

$$\left(\frac{dN}{d^3r\,d\Omega d\gamma}\right) = \frac{\mathcal{D}^2 C}{4\pi}\left(\frac{\gamma}{\mathcal{D}}\right)^{-q}$$

$$P_\nu = \mathcal{D}V'\int_0^\infty d\gamma \frac{dL_{\rm s.p.}}{d\nu}\frac{dN}{d^3\mathbf{r}d\Omega d\gamma}$$

$$= \frac{\mathcal{D}^{3+q}CV'\sigma_T c U_{CMB}}{\nu_{CMB}}\left(\frac{\nu}{\nu_{CMB}}\right)^{(1-q)/2} A_{IC}(\alpha)$$

Provided $4\nu_{CMB}\gamma_1^2 \ll \nu \ll 4\nu_{CMB}\gamma_2^2$
[$h\nu_{CMB} = 3k_B T_{CMB}$, $U_{CMB} = aT_{CMB}^4$]

Synchrotron self-Compton (SSC)

Compute in the source frame, then use $P_\nu = \mathcal{D}^3 P'_{\nu'}$
Geometry important for target density:

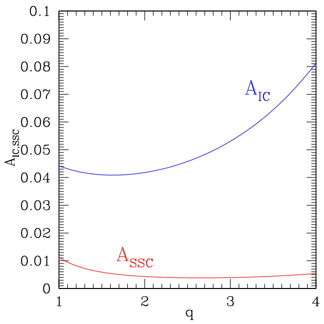
$$\frac{dU_s}{d\nu_s} = \frac{P_{\nu_s}}{\langle \text{Area}\rangle c}$$

Simplest solution: set $\langle \text{Area}\rangle = V^{2/3}$

$$P_\nu = \mathcal{D}^{(5+q)/2}C^2\sigma_T V^{4/3}\frac{e^2}{\hbar c}\hbar\Omega_L\left(\frac{\nu}{\nu_0}\right)^{(1-q)/2}\ln\Sigma_G\, A_{SSC}(q)$$

where Σ_G is Gould's *Compton logarithm*
[A&A 76, 306 (1979)]

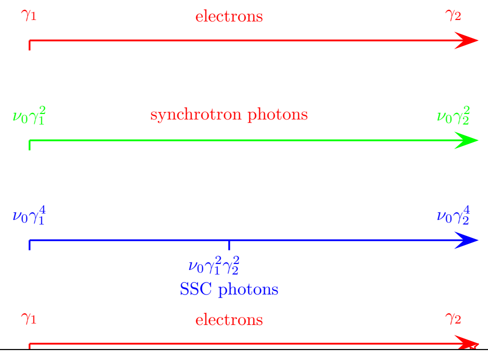
The functions $A_{IC,SSC}$



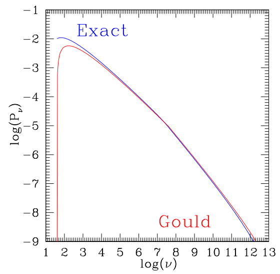
The functions $A_{\rm IC}(q)$ and $A_{\rm SSC}$ for inverse Compton scattering by a power-law distribution of electrons off either a black body distribution or self-produced synchrotron target photons

Synchrotron self-Compton - 2

Frequency dependence of the Compton logarithm



Compton logarithm



SSC spectrum for dynamic range $\gamma_2/\gamma_1 = 10^5$
$\alpha = (q-1)/2 = 0.7$
Blue ("exact"): numerical evaluation
Red: Gould's approx.
Spectral break at $\nu = \gamma_1^2\gamma_2^2\nu_0 = 10^7$.

$\gamma + \gamma \rightarrow e^+ + e^-$

Look in CMS frame — photon energy $\bar\epsilon/m_e c^2$.

$$\sigma_{\gamma\gamma} = \frac{3\sigma_T}{16}\left(1-\zeta^2\right)\left[\left(3-\zeta^4\right)\ln\left(\frac{1+\zeta}{1-\zeta}\right) - 2\zeta\left(2-\zeta^2\right)\right]$$

$\zeta = \sqrt{\bar\epsilon^2 - 1}/\bar\epsilon$.
Transform to lab frame:

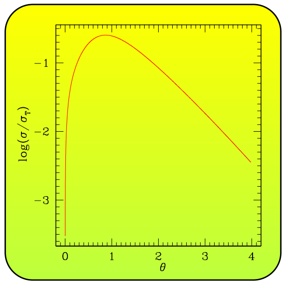
$$\bar\epsilon = \sqrt{(1-\cos\psi)\,\epsilon_1\epsilon_2/2}$$

ψ is angle between photons.
Threshold: $\bar\epsilon \geq 1$, for head-on, $\epsilon_1 > 1/\epsilon_2$.

γ-γ cross-section

$$\cosh\theta = \bar\epsilon/m_e c^2$$

(Lorentz factor of produced $e^\pm$ in CMS frame)



Intergalactic absorption

Optical depth of an isotropic photon field:

$$\tau = 2\pi \int {\rm d}r\, R \int_0^\infty {\rm d}\varepsilon_s n(\varepsilon_s) \int_0^\pi {\rm d}\psi \sin\psi(1+\cos\psi)\sigma_{\gamma\gamma}(\varepsilon')$$

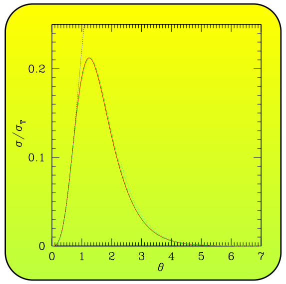
$\varepsilon_s = h\nu_s/mc^2$, $n(\varepsilon_s) = (mc^2/h){\rm d}N/{\rm d}\Omega{\rm d}\nu$

Three integrations:

1. angle angle
2. redshift
3. target spectrum

Angle-averaged γ-γ cross-section

Pair production for isotropic targets
$\cosh\theta =$ Lorentz factor of produced $e^{\pm}$ (in COM frame)



Intergalactic absorption

Optical depth of an isotropic photon field:

$$\tau = 2\pi \int \mathrm{d}r\, R \int_0^\infty \mathrm{d}\varepsilon_s n(\varepsilon_s) \int_0^\pi \mathrm{d}\psi \sin\psi (1+\cos\psi)\sigma_{\gamma\gamma}(\varepsilon')$$

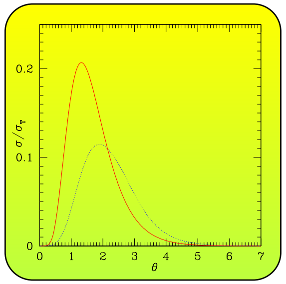
$\varepsilon_s = h\nu_s/mc^2$, $n(\varepsilon_s) = (mc^2/h)\mathrm{d}N/\mathrm{d}\Omega\mathrm{d}\nu$

Three integrations:

1. angle
2. redshift redshift
3. target spectrum

Redshift-averaged γ-γ cross-section

Pair production off non-evolving radiation field
"Consensus" cosmology
$z = 0.1$ $z = 5$
$\cosh\theta =$ Lorentz factor of produced $e^{\pm}$ (in COM frame)



Intergalactic absorption

Optical depth of an isotropic photon field:

$$\tau = 2\pi \int \mathrm{d}r\, R \int_0^\infty \mathrm{d}\varepsilon_s n(\varepsilon_s) \int_0^\pi \mathrm{d}\psi \sin\psi (1+\cos\psi)\sigma_{\gamma\gamma}(\varepsilon')$$

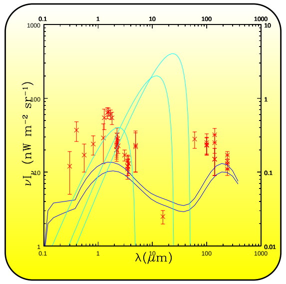
$\varepsilon_s = h\nu_s/mc^2$, $n(\varepsilon_s) = (mc^2/h)\mathrm{d}N/\mathrm{d}\Omega\mathrm{d}\nu$

Three integrations:

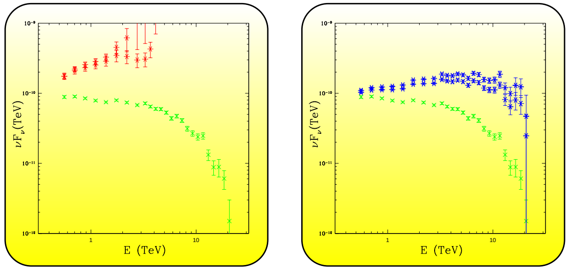
1. angle
2. redshift
3. target spectrum target spectrum

Target spectrum

Diffuse infra-red background measurements and models
together with
averaged γ-γ cross-section for
$z = 0.031$ (Mkn 501)
at
$h\nu = 1, 5, 10$ TeV



Mkn 501



De-absorbed spectrum of Mkn 501 in 1997:
Interpolated DIBR (left)
Malkan & Stecker models (right)