Exercises to "Standard Model of Particle Physics II"

Winter 2016/17

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Problem 17: *Z-decay and Majorana neutrinos* [**10 Points**]

The Lagrangian for the coupling of a fermion pair f with the Z-boson is

$$\mathscr{L} = \frac{g}{2\cos\theta_W}\bar{f}\gamma^\mu(v_f - a_f\gamma_5)fZ_\mu,$$

For neutrinos we have that $v_\nu = a_\nu = \frac{1}{2}$.

a) Calculate the decay width for $Z \to \bar{\nu}\nu$ in the Standard Model, but keeping a possible neutrino mass in the expression.

b) Neutrinos could be Majorana particles, which obey the relation $\nu^c = \nu$. The superscript c denotes charge conjugation,

$$\nu^c = C(\bar{\nu})^T,$$

with $C = i\gamma_2\gamma_0$ in the Dirac basis.
Show the following properties

$$\begin{aligned} -C = C^T = C^{-1} &= -C^* = C^\dagger, \\ C^{-1}\gamma_\mu C = -\gamma_\mu^T, &\quad C^{-1}\gamma_5 C = \gamma_5^T, \\ \overline{\psi^c} = -\psi^T C^{-1}, &\quad (\psi_L)^c = (\psi^c)_R, \end{aligned}$$

where $(\psi^c)_L = P_L(\psi^c) \equiv \psi_L^c$ and so on.

c) Show that for Majorana neutrinos the vector current $\bar{\nu}\gamma_\mu\nu$ vanishes. What happens with $\bar{\nu}\gamma_5\nu$, $\bar{\nu}\gamma_\mu\gamma_5\nu$ and $\bar{\nu}[\gamma_\mu, \gamma_\nu]\nu$?

d) Using the previous result calculate the decay width $Z \to \nu\nu$ for Majorana neutrinos and compare with a).

Problem 18: *Seesaw type-I* [**10 Points**]

The Higgs mechanism generates Dirac masses for the active neutrinos, when right-handed neutrinos are introduced. The following term appears in the Lagrangian after the Higgs acquires a VEV:

$$\mathscr{L}_{\text{Dirac}} = -\overline{\nu}_L M_D N_R + \text{h.c.},$$

where $\nu_L = (\nu_L^1,\ \nu_L^2,\ \nu_L^3)^T$ is the column vector of the active neutrinos and N_R the corresponding vector for the 3 right-handed neutrinos. The matrix M_D is most generally a complex 3×3 matrix. Furthermore, the right-handed neutrinos, being SM-singlets, can have a Majorana mass term in the Lagrangian:

$$\mathscr{L}_{\text{Majorana}} = -\frac{1}{2}\overline{(N_R)^c}M_R N_R + \text{h.c.},$$

where M_R is a symmetric 3×3 matrix.

a) Prove the identity $\overline{\nu_L}M_D N_R = \overline{(N_R)^c}M_D^T(\nu_L)^c$.

b) Show that it is possible to rewrite the neutrino mass matrix in the flavour basis in the following way

$$\mathscr{L}_{\text{mass}} \equiv \mathscr{L}_{\text{Dirac}} + \mathscr{L}_{\text{Majorana}} = -\frac{1}{2}\overline{\Psi^{\text{c}}} M \Psi + \text{h.c.},$$

with

$$\Psi \equiv \begin{pmatrix} (\nu_L)^{\text{c}} \\ N_R \end{pmatrix} \qquad \text{and} \qquad M \equiv \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}.$$

c) A symmetric complex matrix, such as M, can be diagonalised via a unitary matrix U according to $U^T M U$. Suppose that the entries of M_R are much larger than the ones of M_D ($|M_{Rij}| \gg |M_{Dij}|$). Using the unitary transformation $\Psi = U\chi$, block-diagonalise the mass matrix M keeping only terms of order ρ^2 and less:

$$U^T M U \simeq \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \qquad \text{with} \qquad U = \begin{pmatrix} 1 - \frac{1}{2}\rho\rho^\dagger & \rho \\ -\rho^\dagger & 1 - \frac{1}{2}\rho^\dagger\rho \end{pmatrix}, \tag{1}$$

and U unitary up to $\mathcal{O}(\rho^4)$. M_1 and M_2 are 3×3 symmetric block-matrices and ρ is assumed to be proportional to $M_D M_R^{-1}$.

Determine ρ, M_1, and M_2 from Eq. (1) and verify that the expansion in small ρ is valid. What is the connection between the fields χ_1, χ_2 and the original fields ν_L, N_R? (hint: assume that M_R is invertible.)

Happy Holidays!

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Lecture webpage: `www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in and discussion of sheet:
Wednesday, 14:15, Phil12, R106