Exercises to "Standard Model of Particle Physics II"

Winter 2015/16

Prof. Dr. Manfred Lindner, Dr. Werner Rodejohann — Sheet 9 — 16.12.15

Exercise 19: Custodial Symmetry and the ρ-Parameter [**15 Points**]

The 1-loop correction to the ρ-parameter is given by

$$\Delta\rho = \frac{3G_F^2}{8\pi^2 2\sqrt{2}}\left(m_t^2 + m_b^2 - 2\frac{m_t^2 m_b^2}{m_t^2 - m_b^2}\log\frac{m_t^2}{m_b^2} - \frac{11}{9}m_Z^2\sin^2\theta_W\log\frac{m_h^2}{m_Z^2}\right),$$

where m_t (m_b) is the top (bottom) quark mass, θ_W the Weinberg angle and m_h (m_Z) the mass of the Higgs (Z) boson.

a) Convince yourself that $\Delta\rho = 0$ for $m_t = m_b$ and if the hypercharge gauge coupling is zero.

b) As discussed in the lecture, the Higgs potential is invariant under $SU(2)_L \times SU(2)_R$. Defining

$$L = \begin{pmatrix} t \\ b \end{pmatrix}_L, \quad R = \begin{pmatrix} t \\ b \end{pmatrix}_R \quad \text{and } \Phi = \left(\phi, \tilde{\phi}\right),$$

with ϕ the Higgs doublet, show that the Lagrangian containing the top and bottom Yukawa couplings g_t and g_b is invariant under the $SU(2)_L \times SU(2)_R$ symmetry only if $g_t = g_b$.

c) Show by analyzing the kinetic term of the Higgs boson (covariant derivative!) that it is invariant under $SU(2)_L \times SU(2)_R$ symmetry only if the hypercharge gauge coupling is zero.

d) The ρ-parameter at tree level is given by

$$\rho = \frac{\sum v_i^2\left(4T_i(T_i+1) - Y_i^2\right)}{2\sum v_i^2 Y_i^2},$$

where T_i, Y_i and v_i are the isospin, hypercharge and vev of the involved Higgs multiplets. Convince yourself that the Standard Model Higgs doublet gives $\rho = 1$. Adding a Higgs triplet for the type II seesaw mechanism as in Exercise 16, how is the ρ-parameter modified? After a doublet, what is the next highest multiplet compatible with $\rho = 1$ and electrically neutral vevs? What are the electric charges of the members of that multiplet?

Exercise 20: Real Scalar Singlet [**5 Points**]

Adding a real scalar singlet S to the Standard Model implies, with a suitable Z_2 symmetry, the following scalar potential:

$$V = m_h^2(H^\dagger H) + m_s^2 S^2 + \lambda_h (H^\dagger H)^2 + \lambda_s S^4 + \lambda_{hs}(H^\dagger H)S^2 \,.$$

Here H is the Standard Model Higgs. In "unitary gauge" we can write $H = (0, h/\sqrt{2})^T$ and $S = s/\sqrt{2}$. Demanding that the potential has a minimum at $\langle h \rangle = v$ and $\langle S \rangle = u$, obtain v^2 and u^2 in terms of the parameters m_h, m_s, λ_s, λ_h and λ_{hs}. Show that the condition for $\langle h \rangle = v$ and $\langle S \rangle = u$ to be local minimum is $4\lambda_h\lambda_s - \lambda_{hs}^2 > 0$. Obtain the physical mass eigenstates and give the mixing angle that diagonalizes the mass matrix.

Enjoy your Holidays !!!

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Lecture homepage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index_WS15.html`

Hand-in and discussion of sheet:
Tuesday 12.01.16 4.15 pm, INF 501 / CIP R. 103