Exercises to "Standard Model of Particle Physics II"

Winter 2014/15

Dr. Werner Rodejohann | Sheet 9 | 14.01.15

Exercise 20: Fermion mass matrix diagonalization [**5 Points**]

a) A hermitian matrix $M = M^\dagger$ can always be diagonalized by a unitary transformation

$$UMU^\dagger = D \equiv \mathrm{diag}(m_1,\ m_2, \ldots,\ m_n),$$

where the eigenvalues m_i can be negative as well as positive.

Show that one can choose an appropriate biunitary transformation to diagonalize M so that all diagonal elements are non-negative.

b) Show that for any complex $n \times n$ matrix A one can find two unitary transformation matrices U and V such that $UAV^\dagger$ is diagonal with non-negative elements.

c) Now we know that the mass matrices for the SM fermions can be diagonalized by biunitary transformations with non-negative mass eigenvalues. The diagonalization in the charged lepton sector leads to a mixing for the charged leptons

$$e_\mathrm{L} = U^\dagger e'_\mathrm{L} \qquad\qquad e_\mathrm{R} = V^\dagger e'_\mathrm{R},$$

where the primed (unprimed) fields denote the fields in the flavour (mass) basis.

Show that in the case of massless neutrinos this mixing for the charged leptons would have no physical effect.

Exercise 21: Charge conjugation and Majorana fields [**5 Points**]

a) Show for two spinor fields ψ and χ that

$$\overline{\psi}^\mathrm{c}_\mathrm{L} \chi^\mathrm{c}_\mathrm{R} = \overline{\chi}_\mathrm{L} \psi_\mathrm{R}.$$

Note that $\psi^\mathrm{c}_\mathrm{L} = (\psi^\mathrm{c})_\mathrm{L}$ etc.

b) The Lagrange density in the Majorana basis is given by

$$\mathscr{L}_\mathrm{mass} = -\frac{1}{2} \begin{pmatrix} \overline{\psi}_\mathrm{L} & \overline{\chi}_\mathrm{L} \end{pmatrix} \begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix} \begin{pmatrix} \psi^\mathrm{c}_\mathrm{R} \\ \chi^\mathrm{c}_\mathrm{R} \end{pmatrix} + \mathrm{h.c.}$$

Show that this corresponds to a spinor field with Dirac mass term

$$\mathscr{L}_\mathrm{Dirac} = -m\overline{\Psi}\Psi.$$

Exercise 22: Z-decay [**10 Points**]

The Lagrange density for the coupling of a fermion pair f with the Z-boson is

$$\mathscr{L} = \frac{g}{2\cos\theta_\mathrm{w}} \overline{f}\gamma^\mu(v_f - a_f\gamma^5) f Z_\mu$$

For neutrinos we have $v_f = a_f = 1/2$.

a) Calculate the decay rate for the process $Z \to \overline{\nu}\nu$ in the case of massive ($m \neq 0$) Dirac neutrinos.

b) Neutrinos could be Majorana particles, which obey the relation $\nu^\mathrm{c} = \nu$. Show that the vector current $\overline{\nu}\gamma^\mu\nu$ vanishes in the case of Majorana neutrinos.

c) Calculate the decay rate for the process $Z \to \overline{\nu}\nu$ in the case of massive Majorana neutrinos. What is the ratio between the rate from part a) and the one computed here?

Remark: For your solution use the following definitions:
The charge conjugate of a spinor ψ is defined

$$\psi^\mathrm{c} = C(\overline{\psi})^\mathrm{T},$$

with the charge conjugation matrix $C = \mathrm{i}\gamma^2\gamma^0$.
The charge conjugation matrix has the following properties:

$$C^\dagger = C^{-1} \qquad C^\mathrm{T} = -C \qquad C^{-1}\gamma^\mu C = -(\gamma^\mu)^\mathrm{T}.$$

You can work with the Dirac matrices in the chiral representation:

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^5 \equiv \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix},$$

Bonus: Why does the labelling *chiral* representation fit here?

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Tutorials homepage: `http://www.mpi-hd.mpg.de/manitop/StandardModel2/exercise.html`

Hand-in and discussion of sheet:
during tutorial on Thursday, 22.01.15, 9.15 am, kHs, Philosophenweg 12