

Exercises to “Standard Model of Particle Physics II”

Winter 2018/19

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Sheet 8

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Problem 16: *Running couplings* [10 Points]

- a) In QED the coupling $\alpha = e^2/4\pi$ is a running coupling. For low energies one obtains $\alpha(\mu = m_e) = 1/137$. The running of the QED coupling with only one lepton is given by

$$\alpha_{\text{QED}}(\mu) = \frac{\alpha(m_e)}{1 - \frac{2\alpha(m_e)}{3\pi} \ln\left(\frac{\mu}{m_e}\right)}. \quad (1)$$

At which scale does $\alpha_{\text{QED}}(\mu)$ become infinite? (If further charged leptons and quarks are considered, the coupling increases even faster.)

- b) In QCD, with $\alpha_s = g_3^2/4\pi$, the running coupling

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + \frac{(33-2f)\alpha_s(\mu_0)}{6\pi} \ln\left(\frac{\mu}{\mu_0}\right)} \quad (2)$$

is obtained, where f denotes the respective number of quark flavours with mass $2m_q \leq \mu$ in the considered energy range.

The experimental boundary conditions are $\alpha_s(\mu_0 = m_Z = 91 \text{ GeV}) = 0.12$, $m_t = 175 \text{ GeV}$, $m_b = 4.8 \text{ GeV}$, $m_c = 1.4 \text{ GeV}$ and $m_u \simeq m_d \simeq m_s \simeq 0$.

- i) Determine the pole $\mu = \Lambda_{\text{QCD}}$ of eq. (2).
 - ii) For which μ does the coupling $\alpha_s(\mu)$ become very small (asymptotic freedom), and where does perturbation theory break down?
 - iii) Determine the value of $\alpha_s(\mu)$ in the different energy ranges ($2m_q \leq \mu \leq 2m_t$ etc.) at the thresholds.
 - iv) $\alpha_s(\mu)$ should be continuous. However, from the threshold values calculated in the last part we saw that this is not yet true. To make $\alpha_s(\mu)$ continuous, promote μ_0 to a function $\mu_0(f)$ with $\mu_0(f = 5) = 91 \text{ GeV}$. Find a relation between $\mu_0(f)$ and $\mu_0(f - 1)$ assuming that $\alpha_s(\mu_0(f)) = 0.12$ for all f . With this relation determine the values of $\mu_0(4)$ and $\mu_0(6)$.
- c) Draw $\alpha_s^{-1}(\mu)$ and $\alpha_{\text{QED}}^{-1}(\mu)$ as functions of $\ln(\mu)$. What could the intersection of the curves indicate?

Problem 17: Integrating out the W -boson [10 Points]

Heavy particles decouple from a given theory when the relevant energy scale of a process is well below their mass. An important example are the W^\pm bosons of the SM, which we will consider in this exercise. The relevant part of the SM Lagrangian reads

$$\mathcal{L}_W = -\frac{1}{2}W^-_{\mu\nu}W^{+\mu\nu} + m_W^2 W^+_\mu W^{-\mu} + \frac{g}{\sqrt{2}} W^+_\mu J^{-\mu} + \frac{g}{\sqrt{2}} W^-_\mu J^{+\mu},$$

with the electroweak charged currents acting as source terms,

$$J^+_\mu = \bar{e}_L \gamma_\mu \nu_L, \quad J^-_\mu = \bar{\nu}_L \gamma_\mu e_L.$$

Derive the classical equation of motion for the W^\pm bosons and expand to lowest order in $\frac{p}{m_W}$, where p is the momentum of the W , i.e. $\partial_\mu W^\pm_\nu = -ip_\mu W^\pm_\nu$. Solve the equation to lowest order in $\frac{p}{m_W}$ ignoring non-linear interactions among the W 's. Show that this solution can be used to obtain the Lagrangian of Fermi's effective theory of weak interactions,

$$\mathcal{L}_{\text{Fermi}} = \frac{G_F}{\sqrt{2}} J^+_\mu J^{-\mu}$$

and determine G_F in terms of the W mass, m_W , and the weak gauge coupling, g .

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Lecture webpage: <https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html>

Hand-in and discussion of sheet:

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