Exercises to “Standard Model of Particle Physics II”

Winter 2016/17

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Problem 15: *Discrete symmetries* [**5 Points**]

A CP transformation takes a left-handed neutrino and transforms it into a right-handed anti-neutrino. Consider the probability $P(\nu_\alpha \to \nu_\beta,\, t)$ of a neutrino of flavour α to oscillate into a flavour β after time t of propagation.

a) How do the discreet symmetries CP, T, and CPT change the initial and final states of the process given above, i.e. calculate

$$(\mathrm{CP})^{-1} P(\nu_\alpha \to \nu_\beta,\, t)(\mathrm{CP}),$$
$$(\mathrm{T})^{-1} P(\nu_\alpha \to \nu_\beta,\, t)(\mathrm{T}),$$
$$(\mathrm{CPT})^{-1} P(\nu_\alpha \to \nu_\beta,\, t)(\mathrm{CPT}).$$

b) The CPT theorem states that every QFT described by a Lorentz invariant, local Lagrangian is invariant under CPT transformations. How could the above result be used to test this theorem in Nature? (Hint: Consider the case $\alpha = \beta$, i.e. the survival probability of (anti-) neutrinos.)

Problem 16: *Neutrino Oscillations and CP violation* [**15 Points**]

Neutrinos are produced and detected in weak interactions, but the neutrino mass matrix is diagonal in a different basis. The unitary matrix U rotates the flavour basis into the mass eigenbasis. Consider a neutrino of flavour α produced at time $t = 0$,

$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = U^*_{\alpha j} |\nu_j\rangle, \tag{1}$$

as a superposition of mass eigenstates $|\nu_j\rangle$.

a) Using the following approximations, derive the most general expression for neutrino oscillations in vacuum,

$$P_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta,\, t) = \delta_{\alpha\beta} - 4\sum_{j>i} \mathrm{Re}\left(\mathcal{I}^{ij}_{\alpha\beta}\right) \sin^2\left(\frac{\Delta_{ij}}{2}\right) + 2\sum_{j>i} \mathrm{Im}\left(\mathcal{I}^{ij}_{\alpha\beta}\right) \sin\left(\Delta_{ij}\right), \tag{2}$$

where $\mathcal{I}^{ij}_{\alpha\beta} = U^*_{\alpha i} U_{\beta i} U^*_{\beta j} U_{\alpha j}$ and $\Delta_{ij} = \frac{m_i^2 - m_j^2}{2E}$. To arrive at this expressions, you have to make the following assumptions:

i) Neutrinos are highly relativistic, such that the distance travelled is $L \simeq t$ $(c = 1)$ and $E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i \left(1 + \frac{m_i^2}{2p_i^2}\right)$.

ii) The central momentum of the neutrino beam is given by $p_i \simeq p \approx E$ for all i.

b) Derive the expression for the CP asymmetry $\mathcal{A}_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta, t) - P(\overline{\nu}_\beta \to \overline{\nu}_\alpha, t)$. (Hint: How does the state (1) change under a CP transformation? Use this fact and Eq. (2)!)

c) Consider now the special case of two neutrino flavours, say e and μ, hence

$$U = \begin{pmatrix} \cos\theta & e^{-i\delta}\sin\theta \\ -e^{i\delta}\sin\theta & \cos\theta \end{pmatrix}.$$

Calculate the CP asymmetry for $e - \mu$ oscillations. What are the conditions for CP violation to occur?

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Lecture webpage: `www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in and discussion of sheet:
Wednesday, 14:15, Phil12, R106