

Exercises to “Standard Model of Particle Physics II”

Winter 2014/15

Dr. Werner Rodejohann

Sheet 8

07.01.15

Exercise 18: Neutrino oscillations in matter [15 Points]

The MSW effect describes oscillations of neutrinos in matter. Consider the case of two flavour oscillations in a realistic (non-constant) mass distribution.

The time evolution of the eigenfunctions of the Hamiltonian in matter is described by the following expression:

$$i \frac{d}{dt} \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix} = \begin{pmatrix} E_A(t) & -i\dot{\theta}(t) \\ i\dot{\theta}(t) & E_B(t) \end{pmatrix} \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix}$$

The eigenstates in matter are $|\nu\rangle = (|\nu_A\rangle, |\nu_B\rangle)^T$ and to any given time they are connected to the flavor states via: $|\tilde{\nu}\rangle = \tilde{U}(t)|\nu\rangle$ with

$$|\tilde{\nu}\rangle = \begin{pmatrix} |\tilde{\nu}_e\rangle \\ |\tilde{\nu}_\mu\rangle \end{pmatrix} \quad \tilde{U} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

and the angle

$$\tan(2\theta) = \frac{\frac{\Delta m^2}{2E} \sin(2\theta_0)}{\frac{\Delta m^2}{2E} \cos(2\theta_0) - \sqrt{2} G_F N_e(t)}, \quad (1)$$

where θ_0 is constant in time, and $N_e(t)$ denotes the electron number density in matter.

The effective Hamiltonian is not diagonal, since matter eigenstates and thus the mixing angle θ are not constant. If, however, $\dot{\theta}$ is small, i.e. the change in θ occurs slowly, we obtain the adiabatic approximation, where we can neglect $\dot{\theta}$. Suppose that at the time $t = t_i$ an electron neutrino is produced,

$$|\tilde{\nu}_\alpha(t_i)\rangle \equiv |\tilde{\nu}_e\rangle = \cos\theta_i |\nu_A(t_i)\rangle + \sin\theta_i |\nu_B(t_i)\rangle.$$

- What is the expression for $|\tilde{\nu}_\alpha(t_f)\rangle$ in terms of $|\nu_A(t_i)\rangle$, $|\nu_B(t_i)\rangle$ at some later time t_f in the adiabatic approximation?
- Show that in the adiabatic approximation the result for the transition probability is given by

$$P(\nu_e \rightarrow \nu_\mu) = \frac{1}{2} - \frac{1}{2} \cos(2\theta_i) \cos(2\theta_f) - \frac{1}{2} \sin(2\theta_i) \sin(2\theta_f) \cos\Phi_{AB}$$

with

$$\Phi_{IJ} \equiv \int_{t_i}^{t_f} [E_I(t) - E_J(t)] dt,$$

where $I, J \in \{A, B\}$.

- Use Eq. (1) to find an expression for $\sin^2(2\theta)$. The formula for $\sin^2(2\theta)$ describes a Breit-Wigner distribution. We define the resonance width at half height, Δr , as the width at which the curve of the distribution is larger than a half [$\sin^2(2\theta) > 1/2$]. Find an approximate expression for Δr .
- The adiabatic parameter at the MSW resonance is

$$\gamma_r \equiv \frac{\sin^2(2\theta_0) \Delta m^2}{\cos(2\theta_0) 2E} L \quad .$$

The scale $L = |\dot{N}_e/N_e|_{\text{res}}^{-1}$ is the characteristic length scale where the electron density varies in the resonance region. Use the expression for the oscillation length $l_{\text{res}} = 2\pi/|E_A - E_B|_{\text{res}} = (4\pi E/\Delta m^2)/\sin(2\theta_0)$ and your expressions for Δr to express γ_r in another way, and discuss the adiabatic condition $\gamma_r > 3$.

Exercise 19: Neutrino masses [5 Points]

The best-fit values for the three-flavor oscillation parameters (without error) are given in Table 1.

parameter	normal ordering	inverted ordering
$\sin^2 \theta_{12}$	0.304	0.304
$\theta_{12} [^\circ]$	33.48	33.48
$\sin^2 \theta_{23}$	0.452	0.579
$\theta_{23} [^\circ]$	42.3	49.5
$\sin^2 \theta_{13}$	0.0218	0.0219
$\theta_{13} [^\circ]$	8.50	8.51
$\delta_{\text{CP}} [^\circ]$	306	254
$\Delta m_{21}^2 [10^{-5} \text{ eV}^2]$	7.50	7.50
$\Delta m_{3l}^2 [10^{-3} \text{ eV}^2]$	+2.457	-2.449

Table 1: Three-flavor oscillation parameters (best-fit values without error). In the last line $\Delta m_{3l}^2 \equiv \Delta m_{31}^2 > 0$ for normal ordering and $m_{3l}^2 \equiv \Delta m_{32}^2 < 0$ for inverted ordering. Values taken from arXiv: 1409.5439.

Use the values from Table 1 to draw the following functions in double logarithmic plots as functions of the lightest neutrino mass from 10^{-4} eV to 2 eV :

- the neutrino masses m_1 , m_2 and m_3 for normal ordering,
- the neutrino masses m_1 , m_2 and m_3 for inverted ordering,
- the electron neutrino mass m_β (as measured from the β -decay spectrum endpoint energy e.g. by KATRIN) for both orderings.

Tutor:

Pascal Humbert, email: pascal.humbert@mpi-hd.mpg.de

Tutorials homepage: <http://www.mpi-hd.mpg.de/manitop/StandardModel2/exercise.html>

Hand-in and discussion of sheet:

during tutorial on Thursday, 15.01.15, 9.15 am, kHs, Philosophenweg 12