Exercises to “Standard Model of Particle Physics II”

Winter 2018/19

Prof. Dr. Manfred Lindner and Dr. Werner Rodejohann Sheet 7 November 28, 2018

Problem 14: *Custodial Symmetry and the ρ-Parameter* [**15 Points**]

The 1-loop correction to the ρ-parameter is given by

$$\Delta\rho = \frac{3G_F^2}{8\pi^2 2\sqrt{2}} \left(m_t^2 + m_b^2 - 2\frac{m_t^2 m_b^2}{m_t^2 - m_b^2} \log\frac{m_t^2}{m_b^2} - \frac{11}{9} m_Z^2 \sin^2\theta_W \log\frac{m_h^2}{m_Z^2} \right),$$

where m_t (m_b) is the top (bottom) quark mass, θ_W the Weinberg angle and m_h (m_Z) the mass of the Higgs (Z) boson.

a) Convince yourself that $\Delta\rho = 0$ for $m_t = m_b$ and if the hypercharge gauge coupling is zero.

b) As discussed in the lecture, the Higgs potential is invariant under $\mathrm{SU(2)_L \times SU(2)_R}$. Define

$$L = \begin{pmatrix} t \\ b \end{pmatrix}_L , \quad R = \begin{pmatrix} t \\ b \end{pmatrix}_R \quad \text{and } \Phi = \left(\phi, \tilde{\phi}\right),$$

with ϕ the Higgs doublet, $\tilde{\phi} = i\sigma_2\phi^*$, and σ_2 the second Pauli matrix. Show that the Lagrangian containing the top and bottom Yukawa couplings g_t and g_b is invariant under the $\mathrm{SU(2)_L \times SU(2)_R}$ symmetry only if $g_t = g_b$.

c) Show by analyzing the kinetic term of the Higgs boson that it is invariant under $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$ symmetry only if the hypercharge gauge coupling is zero.

Problem 15: *W-polarisation* [**5 Points**]

For a massive vector boson with four-momentum $k^\mu = (E,\, |k|\vec{n})$ propagation along the direction $\vec{n} = (\sin\theta,\, 0,\, \cos\theta)$, the polarisation vectors corresponding to the helicities $\lambda = 0, \pm 1$ can be written as

$$\epsilon^\mu_{\lambda=0} = m_W^{-1} \left(|k|,\, E\sin\theta,\, 0,\, E\cos\theta\right),$$
$$\epsilon^\mu_{\lambda=\pm1} = \frac{1}{\sqrt{2}} \left(0,\, \mp\cos\theta,\, -i,\, \pm\sin\theta\right).$$

Check that the completeness relation holds, i.e. verify that

$$\sum_\lambda \epsilon^{\mu\,*}_\lambda \epsilon^\nu_\lambda = -g^{\mu\nu} + \frac{k^\mu k^\nu}{m_W^2}.$$

Tutor:

Ingolf Bischer
e-mail: bischer@mpi-hd.mpg.de

Lecture webpage: `https://www.mpi-hd.mpg.de/manitop/StandardModel2/index.html`

Hand-in and discussion of sheet:
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